Math of Finance 6740 Spring 2016

Instructor: Chjan Lim, Professor Mathematical Sciences

Worksheet 1 for Presentation this Friday 02/19

1. Let S_0 be the t = 0 price of a risky asset and $S^d = S^u < S_0\,(1+r)$ where r > 0 is the riskless interest rate in a 2 by 2 single period market in payoff format for one riskless and one risky asset. Construct explicitly an arbitrage for this market.

2. Consider the 3 by 3 incomplete market M = $\begin{bmatrix} 1 & 1+r & 1+s \\ 1 & 1+r & 1+s \\ 1 & 1+r & 1+s \end{bmatrix}$ where 0 < r < s < 1, and the t = 0 prices of the three assets are 1, 1, 1. Does this market have an arbitrage. Construct one explicitly if there is one.

3. Give an explicit example of a 2 by 2 market (M, S_0) with unifom risk neutral probability (that is equal for up and down market states).

4. For the incomplete 3 by 3 market in Q2, find a risk neutral probability vector q = $\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$ or show there is none.

5. For the generic 2 by 2 market (M, S_0) that is arbitrage free (AF), calculate the prices $C_0(V)$ of the claims $V = \binom{1}{0}$ and $V = \binom{0}{1}$ and then price the generic claim $V = \binom{a}{b}$ where a and b are any real numbers.

6. Calculate the expected values $E_q\,[V]$ of the claims $V = \binom{1}{0}$ and $V = \binom{0}{1}$ under the risk neutral probability q = $\binom{q_1}{q_2}$ for the AF generic 2 by 2 market in Q5.