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For Q1 – 3, Stock price S(t) is given by an exponential expression involving μ the mean rate of return per unit time and σ (variance per unit time), namely

$\boldsymbol{S(t) = S(n\Delta t) = S(0)exp(\mu t + \sigma\sqrt{t})}$. **This lognormal model will be discretized using $\boldsymbol{t = n(\Delta t)}$ in Q 1 – 3.**

1. **Calculate (a) mean and variance of X(n), where X(n) equals the number of Heads minus the number of Tails in n coin tosses.**

2. **Calculate mean and variance of (2X(n) – n) /sqrt(n)**

3. **Use the Central Limit thm to show that (2X(n) – n) /sqrt(n) converges as Δt tends to zero, to a Normal distribution with mean = sqrt(t) (μ + σ^2/2 – r) / σ and variance equal to 1.**

Hint: what are the limits of the mean and var in Q2 as the time period Δt goes to 0

3 b) Verify that S(t) is lognormally distributed with mean = log S(0) + (r – σ^2/2) t and var = (σ^2) t.

For Q4 and more, S(n) is NOT stock price but given by definition below.

4. **Consider the symmetric random walk:**

S(n) = S(n-1) + X(n) with S(0) = 0 and X(n) = -1 or + 1 with equal probability.

Show that the stochastic process {S(n)} is a Martingale

5. **Calculate E[S(n)] and var (S(n))**

6. **Harder problems on First Exit Times: The first time t at which the random walker above hits the level S(t) = 10 say is a random variable, so we can ask for its mean and variance. We will briefly outline without some of the proofs the methods for calculating such Optional Stopping times, this and next week.**