Partial derivatives of the Huxley-model

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Contents

1 gammadot

1.1 the original differential equation

$$\dot{\gamma} = \begin{cases} \frac{stim-\gamma}{\tau_{act}}, & \text{if } stim \geq \gamma \\ \frac{stim-\gamma}{\tau_{deact}} & \text{otherwise} \end{cases} \tag{1}$$

1.2 pd with respect to gamma

$$\frac{\partial \dot{\gamma}}{\partial \gamma} = \begin{cases} \frac{-1}{\tau_{act}}, & \text{if } stim \geq \gamma \\ \frac{-1}{\tau_{deact}} & \text{otherwise} \end{cases} \tag{2}$$

1.3 pd with respect to n

$$\frac{\partial \dot{\gamma}}{\partial n} = 0 \tag{3}$$

1.4 pd with respect to lce

$$\frac{\partial \dot{\gamma}}{\partial lce} = 0 \tag{4}$$

2 ndot

2.1 the original differential equation

$$\dot{n} = fisomrel(lce) * q(\gamma) * f(x) - (f(x) + g(x)) * n \tag{5}$$

2.2 pd with respect to gamma

2.2.1 derivative of q with respect to gamma

$$q = \frac{(1 + k^N) * \gamma^N}{\gamma^N + k^N} \tag{6}$$

$$\frac{\mathrm{d}q}{\mathrm{d}\gamma} = \frac{(\gamma^N + k^N) * (1 + k^N) * N * \gamma^{N-1} - (1 + k^N) * \gamma^N * N * \gamma^{N-1}}{(\gamma^N + k^N)^2} \tag{7}$$

2.2.2 partial derivative of ndot with respect to gamma

$$\frac{\partial \dot{n}}{\partial \gamma} = fisomrel(lce) * f(x) * \frac{\mathrm{d}q}{\mathrm{d}\gamma} \tag{8}$$

2.3 pd with respect to n

$$\frac{\partial \dot{n}}{\partial n} = -(f(x) + g(x)) \tag{9}$$

2.4 pd with respect to lce

2.4.1 derivative of x with respect to lce

$$x = x0 + scalefac * \frac{lce - lce0}{lce_{opt}} \tag{10}$$

$$\frac{\mathrm{d}x}{\mathrm{d}lce} = \frac{scalefac}{lce_{opt}} \tag{11}$$

2.4.2 derivative of fisomrel with respect to lce

$$fisomrel = c_{norm} * (\frac{lce}{lce_{opt}} - 1)^4 + 1 \tag{12}$$

Chain rule:

$$\alpha = \frac{lce}{lce_{opt}} \tag{13}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}lce} = \frac{1}{lce_{opt}} \tag{14}$$

$$\beta = \alpha - 1 \tag{15}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}lce} = \frac{\mathrm{d}\beta}{\mathrm{d}\alpha} * \frac{\mathrm{d}\alpha}{\mathrm{d}lce} = 1 * \frac{1}{lce_{opt}} = \frac{1}{lce_{opt}} \tag{16}$$

$$\sigma = \beta^4 \tag{17}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}lce} = \frac{\mathrm{d}\sigma}{\mathrm{d}\beta} * \frac{\mathrm{d}\beta}{\mathrm{d}lce} = 4 * \beta^3 * \frac{1}{lce_{opt}} = 4 * (\frac{lce}{lce_{opt}} - 1)^3 * \frac{1}{lce_{opt}} \tag{18}$$

$$\chi = c_{norm} * \sigma + 1 \tag{19}$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}lce} = \frac{\mathrm{d}\chi}{\mathrm{d}\sigma} * \frac{\mathrm{d}\sigma}{\mathrm{d}lce} = c_{norm} * 4 * (\frac{lce}{lce_{opt}} - 1)^3 * \frac{1}{lce_{opt}} \tag{20}$$

$$\frac{\mathrm{d}fisomrel}{\mathrm{d}lce} = \frac{\mathrm{d}\chi}{\mathrm{d}lce} = c_{norm} * 4 * (\frac{lce}{lce_{opt}} - 1)^3 * \frac{1}{lce_{opt}} \tag{21}$$

2.4.3 derivative of f(x) with respect to lce

TODO

2.4.4 derivative of g(x) with respect to lce

TODO

2.4.5 partial derivative of ndot with respect to lce

Chain rule:

$$\mu = fisomrel * q \tag{22}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}lce} = q * \frac{\mathrm{d}fisomrel}{\mathrm{d}lce} \tag{23}$$

$$\nu = \mu * f(x) \tag{24}$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}lce} = \frac{\mathrm{d}\mu}{\mathrm{d}lce} * f(x) + \mu * \frac{\mathrm{d}f}{\mathrm{d}lce} = q * \frac{\mathrm{d}fisomrel}{\mathrm{d}lce} * f(x) + fisomrel * q * \frac{\mathrm{d}f}{\mathrm{d}lce} \tag{25}$$

$$\epsilon = f(x) + g(x) \tag{26}$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}lce} = \frac{\mathrm{d}f}{\mathrm{d}lce} + \frac{\mathrm{d}g}{\mathrm{d}lce} \tag{27}$$

$$\zeta = \epsilon * n \tag{28}$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}lce} = \frac{\mathrm{d}\zeta}{\mathrm{d}\epsilon} * \frac{\mathrm{d}\epsilon}{\mathrm{d}lce} = n * (\frac{\mathrm{d}f}{\mathrm{d}lce} + \frac{\mathrm{d}g}{\mathrm{d}lce}) \tag{29}$$

$$\frac{\partial \dot{n}}{\partial lce} = \frac{\mathrm{d}\nu}{\mathrm{d}lce} - \frac{\mathrm{d}\zeta}{\mathrm{d}lce} = q * \frac{\mathrm{d}fisomrel}{\mathrm{d}lce} * f(x) + fisomrel * q * \frac{\mathrm{d}f}{\mathrm{d}lce} - n * (\frac{\mathrm{d}f}{\mathrm{d}lce} + \frac{\mathrm{d}g}{\mathrm{d}lce}) \tag{30}$$

3 lcedot

3.1 the original differential equation

$$\dot{lce} = \frac{kse(lse) * \dot{lmtc} - I_{\dot{n}x}}{I_n * \frac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse)} \tag{31}$$

with

$$I_{\dot{n}x} = kf * \int_{-\infty}^{\infty} \dot{n} * x \, dx \tag{32}$$

$$I_n = kf * \int_{-\infty}^{\infty} n \, dx \tag{33}$$

To apply the chain rule:

$$\alpha = kse(lse) * \dot{lmtc} \tag{34}$$

$$\beta = I_{\dot{n}x} \tag{35}$$

$$\epsilon = \alpha - \beta \tag{36}$$

$$\zeta = I_n \tag{37}$$

$$\eta = \zeta * \frac{s}{2 * h * lce_{opt}} \tag{38}$$

$$\theta = kpe(lce) + kse(lse) \tag{39}$$

$$\lambda = \eta + \theta \tag{40}$$

$$\mu = \frac{\epsilon}{\lambda} \tag{41}$$

3.2 pd with respect to gamma

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\gamma} = 0 \tag{42}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}\gamma} = kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial \gamma} * x \, dx \tag{43}$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}\gamma} = \frac{\mathrm{d}\alpha}{\mathrm{d}\gamma} - \frac{\mathrm{d}\beta}{\mathrm{d}\gamma} = 0 - kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial \gamma} * x \, dx = -kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial \gamma} * x \, dx \tag{44}$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}\gamma} = 0 \tag{45}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}\gamma} = 0 \tag{46}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\gamma} = 0 \tag{47}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\gamma} = 0 \tag{48}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}\gamma} = \frac{(\frac{\mathrm{d}\epsilon}{\mathrm{d}\gamma})}{\lambda} = \frac{-kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial \gamma} * x \, dx}{I_n * \frac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse)} \tag{49}$$

$$\frac{\partial \dot{lce}}{\partial \gamma} = \frac{\mathrm{d}\mu}{\mathrm{d}\gamma} = \frac{-kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial \gamma} * x \, dx}{I_n * \frac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse)} \tag{50}$$

3.3 pd with respect to n

$$\frac{\mathrm{d}\alpha}{\mathrm{d}n} = 0 \tag{51}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}n} = kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial n} * x \, dx \tag{52}$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}n} = \frac{\mathrm{d}\alpha}{\mathrm{d}n} - \frac{\mathrm{d}\beta}{\mathrm{d}n} = 0 - kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial n} * x \, dx = -kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial n} * x \, dx \tag{53}$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}n} = kf * \int_{-\infty}^{\infty} 1\, dx (??) \tag{54}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}n} = \frac{\mathrm{d}\eta}{\mathrm{d}\zeta} * \frac{\mathrm{d}\zeta}{\mathrm{d}n} = \frac{s}{2 * h * lce_{opt}} * kf * \int_{-\infty}^{\infty} 1\, dx \tag{55}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}n} = 0 \tag{56}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}n} = \frac{\mathrm{d}\eta}{\mathrm{d}n} + \frac{\mathrm{d}\theta}{\mathrm{d}n} = \frac{s}{2 * h * lce_{opt}} * kf * \int_{-\infty}^{\infty} 1\, dx + 0 = \frac{s}{2 * h * lce_{opt}} * kf * \int_{-\infty}^{\infty} 1\, dx \tag{57}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}n} = \frac{\lambda * \frac{\mathrm{d}\epsilon}{\mathrm{d}n} - \epsilon * \frac{\mathrm{d}\lambda}{\mathrm{d}n}}{\lambda^2} = \frac{\begin{array}{r}(I_n * \frac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse)) * -kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial n} * x\, dx \\ - (kse(lse) * l\dot{m}tc - I_{\dot{n}x}) * (\frac{s}{2*h*lce_{opt}} * kf * \int_{-\infty}^{\infty} 1\, dx)\end{array}}{(I_n * \frac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse))^2} \tag{58}$$

$$\frac{\partial l\dot{c}e}{\partial n} = \frac{\mathrm{d}\mu}{\mathrm{d}n} = \frac{\begin{array}{r}(I_n * \frac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse)) * -kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial n} * x\, dx \\ - (kse(lse) * l\dot{m}tc - I_{\dot{n}x}) * (\frac{s}{2*h*lce_{opt}} * kf * \int_{-\infty}^{\infty} 1\, dx)\end{array}}{(I_n * \frac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse))^2} \tag{59}$$

3.4 pd with respect to lce

3.4.1 derivative of kpe with respect to lce

$$kpe = 2 * c_{ce} * (lce - lce0) \tag{60}$$

$$\frac{\mathrm{d}kpe}{\mathrm{d}lce} = 2 * c_{ce} \tag{61}$$

3.4.2 partial derivative of lcedot with respect to lce

$$\frac{\mathrm{d}\alpha}{\mathrm{d}lce} = 0 \tag{62}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}lce} = kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial lce} * x + \frac{\mathrm{d}x}{\mathrm{d}lce} * \dot{n}\, dx \tag{63}$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}lce} = \frac{\mathrm{d}\alpha}{\mathrm{d}lce} - \frac{\mathrm{d}\beta}{\mathrm{d}lce} = 0 - kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial lce} * x + \frac{\mathrm{d}x}{\mathrm{d}lce} * \dot{n}\, dx = -kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial lce} * x + \frac{\mathrm{d}x}{\mathrm{d}lce} * \dot{n}\, dx \tag{64}$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}lce} = 0 \tag{65}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}lce} = \frac{\mathrm{d}\eta}{\mathrm{d}\zeta} * \frac{\mathrm{d}\zeta}{\mathrm{d}lce} = \frac{s}{2 * h * lce_{opt}} * 0 = 0 \tag{66}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}lce} = \frac{\mathrm{d}kpe}{\mathrm{d}lce} = 2 * c_{ce} \tag{67}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}lce} = \frac{\mathrm{d}\eta}{\mathrm{d}lce} + \frac{\mathrm{d}\theta}{\mathrm{d}lce} = 0 + 2 * c_{ce} = 2 * c_{ce} \tag{68}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}lce} = \frac{\lambda * \frac{\mathrm{d}\epsilon}{\mathrm{d}lce} - \epsilon * \frac{\mathrm{d}\lambda}{\mathrm{d}lce}}{\lambda^2} = \frac{\begin{gathered}(I_n * \tfrac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse)) * \\ (-kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial lce} * x + \frac{\mathrm{d}x}{\mathrm{d}lce} * \dot{n}\, dx) - (kse(lse) * l\dot{mtc} - I_{\dot{n}x}) * 2 * c_{ce}\end{gathered}}{(I_n * \tfrac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse))^2} \tag{69}$$

$$\frac{\partial l\dot{ce}}{\partial lce} = \frac{\mathrm{d}\mu}{\mathrm{d}lce} = \frac{\begin{gathered}(I_n * \tfrac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse)) * \\ (-kf * \int_{-\infty}^{\infty} \frac{\partial \dot{n}}{\partial lce} * x + \frac{\mathrm{d}x}{\mathrm{d}lce} * \dot{n}\, dx) - (kse(lse) * l\dot{mtc} - I_{\dot{n}x}) * 2 * c_{ce}\end{gathered}}{(I_n * \tfrac{s}{2*h*lce_{opt}} + kpe(lce) + kse(lse))^2} \tag{70}$$