CS301, Spring 2020, Final Exam
May 28, 2020 **Name:......................**

Problem 1. (30 points) **Grade:.....**

Consider the following two graph algorithms. Each algorithm takes two input: a connected graph $G = (V, E)$, and a weight function w that maps each edge in E to a unique positive integer. Each algorithm returns a subset T of E.

Algorithm A	**Algorithm B**

$S \leftarrow$ Sort the edges in E in a list,
by decreasing order of their weights.
$T = E$
for $i = 1$ to $\lvert S \rvert$ **do**
if $T \setminus \{S[i]\}$ is a connected graph **then**
$T = T \setminus \{S[i]\}$
return T
	$S \leftarrow$ Put the edges in E in a list,
	in an arbitrary order.
	$T = \emptyset$
	for $i = 1$ to $\lvert S \rvert$ **do**
	if $T \cup \{S[i]\}$ has no cycle **then**
	$T = T \cup \{S[i]\}$
	return T

For each algorithm, check whether it finds a minimum spanning tree (MST) in a given graph.

(a) Does Algorithm A find an MST (i.e., is T an MST in G)? **Yes** or **No**

If Algorithm A is an MST algorithm:

1) give an example and show the value of T at each step of the algorithm, and
2) analyze the asymptotic time complexity of Algorithm A (do not forget to analyze the running time of checking whether $T \setminus \{S[i]\}$ is connected).

If Algorithm A is not an MST algorithm: give a counterexample and show the value of T at each step.

(b) Does Algorithm B find an MST (i.e., is T an MST in G)? **Yes** or **No**

If Algorithm B is an MST algorithm:

1) give an example and show the value of T at each step of the algorithm, and
2) analyze the asymptotic time complexity of Algorithm A (do not forget to analyze the running time of checking whether $T \cup \{S[i]\}$ has a cycle).

If Algorithm B is not an MST algorithm: give a counterexample and show the value of T at each step.