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Faculty of Engineering and Natural Sciences

CS301 – Algorithms

Homework 1

Due: March 6, 2024 @ 23.55
(upload to SUCourse)

PLEASE NOTE:

- Provide only the requested information and nothing more. Unreadable, unintelligible, and irrelevant answers will not be considered.
- Submit only a PDF file. (-20 pts penalty for any other format)
- Not every question of this homework will be graded. We will announce the question(s) that will be graded after the submission.
- You can collaborate with your `TA/INSTRUCTOR ONLY` and discuss the solutions of the problems. However, you have to write down the solutions on your own.
- Plagiarism will not be tolerated.

Late Submission Policy:

- Your homework grade will be decided by multiplying what you normally get from your answers by a "submission time factor (STF)".
- If you submit on time (i.e. before the deadline), your STF is 1. So, you don't lose anything.
- If you submit late, you will lose 0.01 of your STF for every 5 mins of delay.
- We will not accept any homework later than 500 mins after the deadline.
- SUCourse's timestamp will be used for STF computation.
- If you submit multiple times, the last submission time will be used.

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8.öğr

Question 1

The recurrence relation of a recursive divide and conquer algorithm is given. Explain this recurrence, verbally, in terms of the size of each sub-problem, the cost of dividing the problem, and combining solutions.

$$T(n) = 3T(\frac{n}{4}) + 2n + n^3$$

Answer:

The problem gets divided into three subproblems that are $1/4$ th of the original problem.

$D(n) = 2n$ is the cost of dividing the problem

$C(n) = n^3$ is the cost of combining the solutions

Note: the actual functionalities of the terms can't be derived from the equation alone. Hence, the $C(n) = 2n$, $D(n) = n^3$ may also be correct.

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1. ödev

Question 2

Find an asymptotically tight lower bound for the following recurrence by using the substitution method.

$$T(n) = T(\tfrac{n}{3}) + T(\tfrac{2n}{3}) + \Theta(n)$$

Answer:

I claim that $T(n) = \Omega(n.\log.n)$

$\Rightarrow T(n) > c.n.\log.n, \ \exists c > 0, \ \forall n \geq n_0$

$\Rightarrow T(n) > c.n.\log.n$ assuming that $T(m) \geq c.m.\log m$ for $\forall m < n$, if $m = \frac{n}{3}$, $T(\frac{n}{3}) > c.\frac{n}{3}\log\frac{n}{3}$

$T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + \Theta(n)$

$\Rightarrow T(n) \geq c.\frac{n}{3}\log\frac{n}{3} + c.\frac{2n}{3}\log\frac{2n}{3} + \Theta(n)$

$\Rightarrow T(n) \geq c.\frac{n}{3}(\log\frac{n}{3} + 2\log\frac{2n}{3}) + \Theta(n)$

$\Rightarrow T(n) \geq c.\frac{n}{3}(\log n - \log 3 + \log 4n^2 - \log 9) + \Theta(n)$

$\Rightarrow T(n) \geq c.\frac{n}{3}(3.\log n - 3\log 3 + 2) + \Theta(n)$

$\Rightarrow T(n) \geq c.n.\log n \underbrace{- c.n.\log 3 + \frac{2cn}{3} + \Theta(n)}_{\text{Something Positive}}$

Therefore; $T(n) \geq cn\log n$

$\therefore T(n) = \Omega(n\log n)$

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Question 3

For the following recurrences, either solve it by using the master method or show that it cannot be solved with the master method.

(a) $T(n) = T(\frac{n}{2}) + \Theta(1)$

Answer: $a = 1$, $b = 2$, $f(n) = \Theta(1)$ which is asymptotically positive; $a \geq 1$, $b > 1$ therefore master theorem can be applied.
$\log_b a = \log_2 1$, $f(n) \neq O(n^{\log_2 1 - \varepsilon})$ when $\varepsilon > 0$
Therefore case I does not hold.
$f(n) = \Theta(n^{\log_2 1}) = \Theta(1)$ therefore case II holds.
Hence, $T(n) = \Theta(n^{\log_2 1} . \log n) = \Theta(\log n)$

(b) $T(n) = 3T(\frac{n}{4}) + n \lg n$

Answer: $a = 3$, $b = 4$ $f(n) = n \log_2 n$ which is asymptotically positive, therefore, Master Theorem can be applied.
$\log_b a = \log_4 3$, $f(n) \neq O(n^{\log_4 3 - \varepsilon})$ for $\varepsilon > 0$
Therefore case I does not hold!
$f(n) \neq \Theta(n^{\log_4 3})$ since $n^{\log_4 3}$ cannot be an upper bound for $f(n)$ thus case II does not hold.
$f(n) = \Omega(n^{\log_4 3 + \varepsilon})$ for some $\varepsilon > 0$. $3 . f(n/4) = 3 . \frac{n}{4} . \log \frac{n}{4}$ $< c . n . \log n$ which holds for $c = \frac{3}{4}$, Hence $T(n) = \Theta(n \log_2 n)$