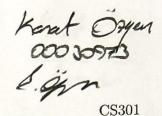
Homework 1



# Sabancı University Faculty of Engineering and Natural Sciences

CS301 - Algorithms

### Homework 1

Due: March 6, 2024 @ 23.55 (upload to SUCourse)

#### PLEASE NOTE:

- Provide only the requested information and nothing more. Unreadable, unintelligible, and irrelevant answers will not be considered.
- Submit only a PDF file. (-20 pts penalty for any other format)
- Not every question of this homework will be graded. We will announce the question(s) that will be graded after the submission.
- You can collaborate with your TA/INSTRUCTOR ONLY and discuss the solutions of the problems. However, you have to write down the solutions on your own.
- Plagiarism will not be tolerated.

#### Late Submission Policy:

- Your homework grade will be decided by multiplying what you normally get from your answers by a "submission time factor (STF)".
- If you submit on time (i.e. before the deadline), your STF is 1. So, you don't lose anything.
- If you submit late, you will lose 0.01 of your STF for every 5 mins of delay.
- We will not accept any homework later than 500 mins after the deadline.
- SUCourse's timestamp will be used for STF computation.
- If you submit multiple times, the last submission time will be used.





CS301

#### Question 1

The recurrence relation of a recursive divide and conquer algorithm is given. Explain this recurrence, verbally, in terms of the size of each sub-problem, the cost of dividing the problem, and combining solutions.

$$T(n) = 3T(\frac{n}{4}) + 2n + n^3$$

Answer:

The problem gets diwded into three Supposem.

Plut on 1/4 M of the engined problem.

D(n) = 2n is the cost of dividing the Problem.

C(n) = n³ is the cost of combining the Solutions.

Note: the coluct functionalities of the terms contained the cleaned from the composition done. Here, the C(n) = 2n, D(n) = n³ may also be correct.



CS301

#### Question 2

Find an asymptotically tight lower bound for the following recurrence by using the substitution method.

$$T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + \Theta(n)$$

Answer: I claim that T(N) = D (n.lay.N) => T(N)> c.N.lay.N, =c>0, Un>No =) T(n) > c.N.ley.N assumy that T(m) > c.n.logn for bonco, if M== 3, T(3) > c. = lough T(n) = T(3) + T(2) + O(n) =) T(n) >, C. & ley 1 + C. 2 n ley 2 + O(n) => T(N), C. ~ (lay = + 2 lay 2 ) + O(N) =) T(n) 7 C. ~ ( ley N - lay 3 + lay 42 - ley 9)+ O(n) =) T(1) >C. 2 (3. lay N - 3 lay 3 +24) + th) =) T(2) 7, C. N. Ley N - C.N. Lay 3 + 2CN + O(N) Souther Positive Therefore; T.(n) > cologn C. T(r)= a (rlan)



CS301

## Question 3

For the following recurrences, either solve it by using the master method or show that it cannot be solved with the master method.

(a) 
$$T(n) = T(\frac{n}{2}) + \Theta(1)$$

Answer:  $\alpha = 1$ , b = 2,  $f(n) = \Theta(L)$  which is esymptotically Positive; a > 1, b > 1 therefore next theorem can be applied. leaf  $a = \log_2 1$ ,  $f(n) \neq O(n\log_2 1 - \epsilon)$  who  $\epsilon > 0$  therefore case T does not held.  $f(n) = \Theta(n\log_2 1) = \Theta(1)$  therefore case T helds.

There  $T(n) = \Theta(n\log_2 1 + \log_2 1) = O(\log_2 1) = O(\log_2 1)$ 

# (b) $T(n) = 3T(\frac{n}{4}) + n \lg n$

Answer:  $\alpha = 3$ , b = 4 f(x) = n by x which is esymptotically posting, therefore, News Therem can be applied.

loy  $b = a = \log_4 3$ ,  $f(x) \neq O(x \log_4 3 - 2)$  for e > 0Therefore and  $E = \log_4 3$  since  $E = \log_4 3$  and be supported for  $E = \log_4 3$  since  $E = \log_4 3$  and  $E = \log_4 3$  beind for  $E = \log_4 3$  for some E > 0.  $E = \log_4 3$  by  $E = \log_4 3$