Sabancı University
Faculty of Engineering and Natural Sciences

CS301 – Algorithms

Homework 4

Due: April 2, 2024 @ 23.55
(upload to SUCourse)

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[illegible]

PLEASE NOTE:

- Provide only the requested information and nothing more. Unreadable, unintelligible, and irrelevant answers will not be considered.
- Submit only a PDF file. (-20 pts penalty for any other format)
- Not every question of this homework will be graded. We will announce the question(s) that will be graded after the submission.
- You can collaborate with your `TA/INSTRUCTOR ONLY` and discuss the solutions of the problems. However, you have to write down the solutions on your own.
- Plagiarism will not be tolerated.

Late Submission Policy:

- Your homework grade will be decided by multiplying what you normally get from your answers by a "submission time factor (STF)".
- If you submit on time (i.e. before the deadline), your STF is 1. So, you don't lose anything.
- If you submit late, you will lose 0.01 of your STF for every 5 mins of delay.
- We will not accept any homework later than 500 mins after the deadline.
- SUCourse's timestamp will be used for STF computation.
- If you submit multiple times, the last submission time will be used.

Question 1

A palindrome is a sequence of characters that reads the same backward as forward. In other words, if you reverse the sequence, you get the same sequence. For example, "a", "bb", "aba", "ababa", "aabbaa" are palindromes. We also have real words which are palindromes, like "noon", "level", "rotator", etc.

We also have considered the concept of a subsequence in our lectures. Given a word A, B is a subsequence of A, if B is obtained from A by deleting some symbols in A. For example, the following are some of the subsequences of the sequence "*abbdcacdb*": "*da*", "*bcd*", "*abba*", "*abcd*", "*bcacb*", "*bbccdb*", etc.

The Longest Palindromic Subsequence (LPS) problem is finding the longest subsequences of a string that is also a palindrome. For example, for the sequence "*abbdcacdb*", the longest palindromic subsequence is "*bdcacdb*", which has length 7. There is no subsequence of "*abbdcacdb*" of length 8 which is a palindrome.

One can find the length of LPS of a given sequence by using dynamic programming. As common in dynamic programming, the solution is based on a recurrence.

Given a sequence $A = a_1 a_2 \dots a_n$, let $A[i, j]$ denote the sequence $a_i a_{i+1} \dots a_j$. Hence it is part of the sequence that starts with a_i and ends with a_j (including these symbols). For example, if $A = abcdef$, $A[2, 4] = bcd$, $A[1, 5] = abcde$, $A[3, 4] = cd$, etc.

For a sequence $A = a_1 a_2 \dots a_n$, let us use the function $L[i, j]$ to denote the length of the longest palindromic subsequence in $A[i, j]$.

(a) If we have a sequence $A = a_1 a_2 \dots a_n$, for which values of i and j, $L[i, j]$ would refer to the length of the longest palindromic subsequence in A?

L[1,N] refers to the length of the longest subsequence

(b) Write the recurrence for $L[i, j]$.

$$L[i,j] = \begin{cases} 0 & \text{if } i > j \\ 1 \dots\dots\dots\dots & \text{if } i = j \\ L[i+1, j-1] + 2 \dots & \text{if } i < j \text{ and } a_i = a_j \\ \max(L[i+1, j], L[i, j-1]) & \text{if } i < j \text{ and } a_i \neq a_j \end{cases}$$

(c) What would be the worst case time complexity of the algorithm in Θ notation? Why?

$\Theta(n^2)$, no best cases, each subproblems must be solved

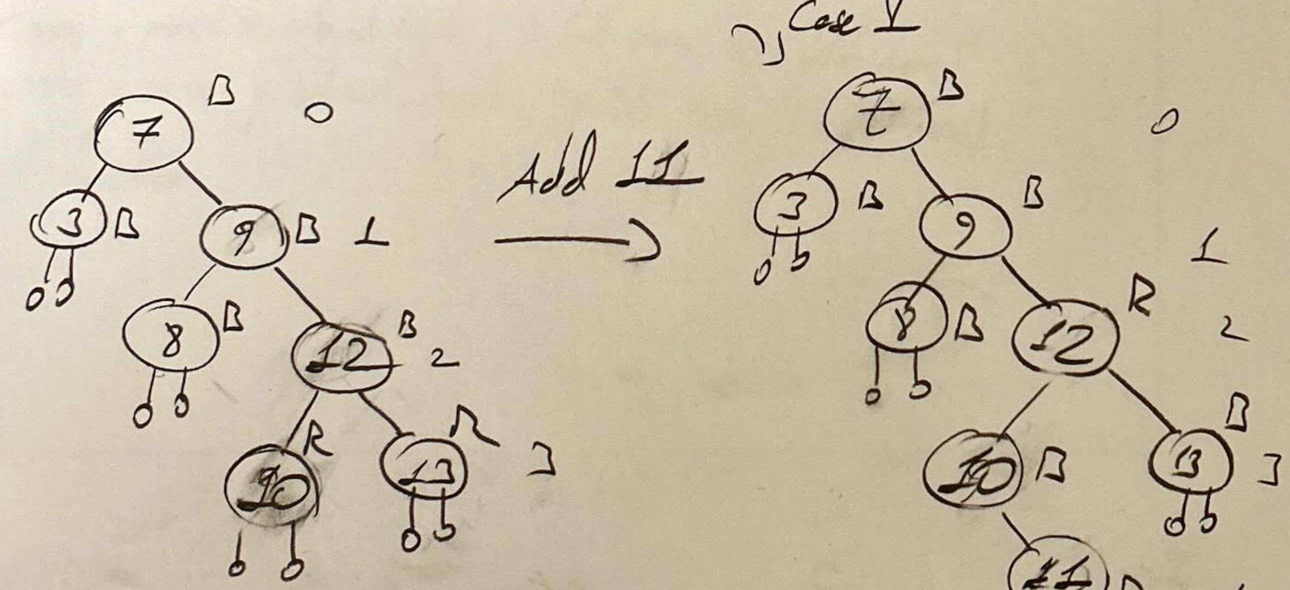
(d) Considering a memoization-based approach, complete the following pseudocode for the dynamic programming solution to the LPS problem. Assume the existence of a 2D array 'dp' of size $n \times n$ initialized to -1, where n is the length of sequence A, and a function 'LPS(i, j)' that computes the length of the longest palindromic subsequence in $A[i, j]$.

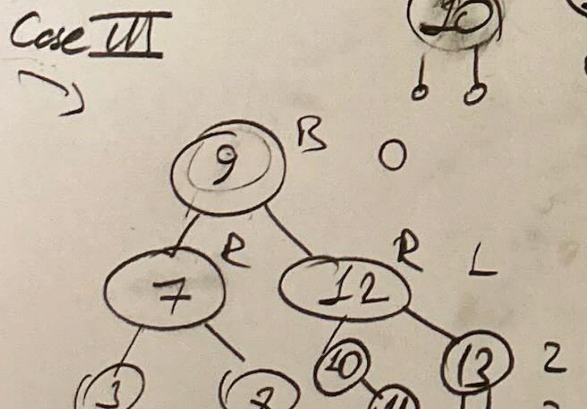
```
function LPS(i, j):
    if dp[i][j] != -1:
        return dp[i][j]
    if i > j:
        dp[i][j] = 0
    elif i == j:
        dp[i][j] = 1
    elif A[i] == A[j]:
        dp[i][j] = LPS(i+1, j-1) +2
    else:
        dp[i][j] = max(LPS(i+1, j), LPS(i, j-1))
    return dp[i][j]
```

Question 2

To compute the depth of a given node in an RBT in constant time, consider augmenting the depths of nodes as additional attributes in the nodes of the RBT. Please explain by an example why this augmentation increases the asymptotic time complexity of inserting a node into an RBT in the worst case.

when a node is inserted to the tree, other nodes should be updated too. In an unbalanced tree, the amount of updates to be done has a complexity that can approach $O(n)$, rather than $O(\log n)$.





A lot of height changes. The whole tree has heights changed, $O(n)$ worst case.

Question 3

Write pseudocode for LEFT-ROTATE that operates on nodes in an interval tree and updates the max attributes in $O(1)$ time.

```
function leftRotate (tree, x)
 y = x.right
 x.right = y.left
 if y.left != NULL
   tree.root = y
 else if x == x.parent.left
   x.parent.left = y
 else
   x.parent.right = y
 y.left = x
 x.parent = y
 x.max = max( x.interval.high , x.left.max , x.right.max)
 y.max = max (y.interval.high , y.left.max, y.right.max)
end function
```