

Problem 2. (40 points)

Grade:.....

- (a) Explain (step by step) how the Bellman-Ford algorithm can be used to solve the following difference constraints:

$$\begin{aligned}x_1 - x_2 &\leq 7 \\x_2 - x_3 &\leq -3 \\x_1 - x_3 &\leq 4.\end{aligned}$$

- (b) The longest path in a graph can be computed by negating the costs of all edges in the graph and then running the Floyd-Warshall algorithm.

True or False?

Explain: If it is True then give a proof. If it is False, then give a counterexample and explain what the algorithm returns.

- (c) Suppose that there are 3 students, $\{A, B, C\}$, and 4 graduation projects, $\{p, q, r, y\}$. Each student specifies a set of projects they would like to work on, and each supervisor of the project specifies a set of students whom they would like to work with:

$$\begin{array}{ll}A : \{p, q, r\} & p : \{A, B, C\} \\B : \{r, y\} & q : \{C\} \\C : \{q, r\} & r : \{C, D\} \\ & y : \{A, B\}\end{array}$$

Explain (step by step) how the Ford-Fulkerson algorithm can be used to assign projects to the maximum number of students, such that the following two conditions hold: there is no student matched to two different projects, and there is no project assigned to two different students.