

Rajk College for Advanced Studies

Resilience Evaluation of Budapest Public Transport Network
Using Network Analysis and Physarum Polycephalum Slime
Mould Modelling

Made by:
Dániel Kofrán

Máté Mizesák

Network Science course

Lecturers: Balázs Lengyel, Gergő Tóth

Abstract

This research aims to analyse the current state of Budapest's public transport network and explore avenues to enhance its efficiency and resilience. By employing network analysis methods and *Physarum polycephalum* slime mould modelling, the study investigates the system's resilience and proposes strategies for its improvement. Targeted attacks on routes are executed to assess the system's ability to withstand disruptions, revealing that while it shows strong resilience against route-focused attacks, it can rapidly collapse if critical stops and important squares are targeted. Simulations indicate that constructing a fifth metro line on the Buda-side could enhance the network's resilience and overall quality. The research incorporates a comprehensive network created through modifications to a publicly available dataset, combined with a manually created dataset on average capacity. The network is analysed using well-known methods in network analysis. The study also introduces a novel approach to model the spread of *Physarum polycephalum* slime mould. The paper's structure includes discussions on network analysis tools, research execution process, and presentation of key findings and conclusions. The research highlights the significance of a resilient public transport system for well-being, development, and disaster management. Further research in *Physarum polycephalum* modelling and resilience, as well as comparative studies across European cities, is recommended for enhancing understanding and improving public transport networks.

Table of contents

1. Introduction	1
2. Network analysis.....	1
2.1. Fundamentals of network analysis	1
2.2. Spatial networks	2
2.3. Most widely applied network analysis tools	3
2.3.1 Centrality	3
2.3.2 Community detection	4
2.3.3 Network resilience.....	4
2.4. Network analysis and public transport systems.....	4
2.4.1. Most common applications of network analysis on public transport systems.....	5
2.4.2. Importance of resilience in public transportation	5
2.5. Budapest's public transport and network analysis	6
2.5.1. Brief history of Budapest's public transport.....	6
2.5.2. Literature on the Budapest's public transport network.....	7
2.6. <i>Physarum polycephalum</i> slime mould-based approach of network development.....	7
3. Network analysis of Budapest's public transport network	8
3.1. Fundaments of Budapest's public transport network.....	8
3.1.1 Descriptives of Budapest's public transport network.....	8
3.1.2. Clustering measures of Budapest's public transport network	9
3.1.3. Network resilience of Budapest's public transport network, node removal.....	9
3.1.4. Network resilience of Budapest's public transport network, edge removal	11
3.2. Resilient network expansion of Budapest's public transport	12
3.2.1 Introducing the <i>Physarum polycephalum</i> imitation model.....	12
3.2.2. Setting up the first stage of the <i>Physarum polycephalum</i> imitation model	13
3.2.3. Simulating the first stage	14
3.2.4. The second stage of the <i>Physarum polycephalum</i> imitation model	16
4. Discussion and research restraints.....	17
5. Conclusion.....	18
6. Citations	20
Appendix	22

1. Introduction

In today's world, the creation of a resilient and stable public transport system is of utmost importance for the overall well-being and development of a city. The goal of this research is to analyse the current state of Budapest's public transport network and explore strategies to enhance its efficiency. To achieve this, the study examines the system's resilience and utilises *Physarum polycephalum* slime mould modelling to propose potential improvements.

The research involves combining publicly available data with a manually curated dataset that captures the average capacity of buses, trams, metros, and trolleybuses to construct a comprehensive graph of the Budapest public transport network. Various well-established network analysis methods are then employed to gain insights into the network's characteristics and behaviour. Additionally, a unique modelling approach utilising *Physarum polycephalum* slime mould is employed to further understand potential enhancements for the network.

The study begins by discussing the significance of resilient and stable public transport systems in today's world. It then delves into the methodology, describing how the research data is collected, combined, and analysed. The findings of the analysis provide valuable insights and recommendations for optimising Budapest's public transport network, with the aim of improving its efficiency and resilience.

2. Network analysis

In the following section we introduce the most important and relevant basics that we used in the analysis. First, we introduce the fundamentals of network analysis, expanded by the literature of spatial networks closely followed by the most commonly used measures in the field. We then turn our attention to the literature of network analysis and public transportation, with a special focus on literature concerning Budapest.

2.1. Fundamentals of network analysis

Networks are basic structures used to represent relationships and interactions between entities. They have found widespread applications in various fields, including social sciences, computer science, biology, transport and communications systems. Network science is an interdisciplinary field of research into the properties, structures and dynamics of networks. A network consists of a set of nodes (or vertices) connected to an edge (or link). (Barabási, 2016)

Nodes represent individual entities, while edges represent relationships and connections between entities. Networks can be classified on the basis of various features, including connectivity, size, and edge direction. Networks have certain properties that shape their structure and behaviour. Network analysis gives valuable insight into the structure, dynamics and functions of complex systems. It enables the study of information dissemination, social influence, disease spread, transportation efficiency, and interconnected system resilience. (Barabási, 2016)

According to Barabási (2016), networks provide a powerful framework for understanding and analysing relationships and interactions in different fields. It can discover hidden patterns, identify important entities, and understand the functions and behaviours of complex systems by studying networks. Using network analysis techniques, we can better understand the complexity of social networks, biological systems, technological infrastructures, and many other interconnected phenomena.

2.2. Spatial networks

In addition to the growing importance of network analysis on a broader scale, there is a larger area with untapped potential: spatial networks. Along with many natural networks, there is a plethora of man-made networks that are inherently spatial and thus could be analysed using network analytic tools. Marc Barthélemy (2006, pp. 1) defines spatial networks as “networks for which the nodes are located in a space equipped with a metric,” usually referring to complex networks that are embedded in a physical space and whose connections are dependent on some spatial proximity.

They are usually classified as metric networks and topological networks. Metric networks are based on continuous spatial embedding, which means that it accounts for the distances between the nodes, as well as their spatial location. Topological networks lack this characteristic, their focus is on relative connectivity between nodes, irrespective of the actual distances (Barthélemy, 2006). Budapest's public transport system is fundamentally a metric spatial network, with all the advantages and limitations that this entails. In our paper the main implication of this is observable in the presence of spatial constraints as they lead to increased clustering and shorter average path lengths, indicating a higher degree of local interconnectedness (Barthélemy, 2006).

In the next chapter we introduce three major network analysis tools which are suitable for measuring important aspects of spatial networks: centrality, community detection and network resilience.

2.3. Most widely applied network analysis tools

Our analysis builds on some of the most widely applied network analysis tools. In the next section we introduce the four most common centrality measures (degree, interdependence, closeness and eigenvector centrality), then we lay the foundations of both community detection and network resilience.

2.3.1 Centrality

Centrality is a fundamental concept of network science that quantifies the importance and influence of network nodes. It provides useful insights into the roles and positions of nodes and helps to identify key players, influential individuals, or key elements of a network. Several central measures have been developed to capture various aspects of node importance. A commonly used central measure is degree centrality. The degree of centrality simply counts the number of direct connections to the network node. High-centralised nodes are considered central hubs because they have many connections and can quickly disseminate information and resources. Degree centrality has been widely studied and applied in various fields such as social networks, reference networks, and biological networks. (Newman, 2010)

Newman (2010) says that another important centralization measurement is interdependence centralization, which determines the nodes acting as bridges or intermediaries between other nodes in the network. It measures how many nodes are placed on the shortest paths between pairs of other nodes. High-segment central nodes have important control over information flow because they serve as important connections between various parts of the network. Betweenness centrality is widely used in transport networks, social networks, and organisational networks.

Closeness centrality is a measure of the speed with which a node can reach other nodes on the network. It quantifies the average distance between nodes and all other nodes on the network. (Newman, 2010)

Eigenvector centrality is another widely used centrality measurement that considers the centrality of neighbours of nodes. It considers both the quantity and quality of the connection of a node. A node with a high centrality of a given vector is connected to a dominant node and often has a high influence itself. (Bonacich, 1987)

2.3.2 Community detection

Community, also known as clustering or modules according to Newman (2006) is an essential concept in network science. Community detection involves identifying a dense network of nodes within the network and a few nodes outside the network. Understanding network community structures provides an overview of complex system organisational, functional, and dynamic dynamics. Various methods and algorithms for community detection in networks have been developed. A common approach is modularity optimization. Modularity is a measure of the quality of network partitioning into communities. It compares the actual number of edges within the community with the expected number of such edges in the random network. Maximising the modularity value indicates the optimal separation with the most important community structure.

2.3.3 Network resilience

According to Barabási and his co-writers (2016) the resilience in network contexts refers to the ability of a network to maintain its functionality and structural integrity in the event of disturbances, failures, or attacks. It involves understanding how the network recovers, adapts and continues to provide essential services, even in the event of a disruption. Resilience to networks is a key aspect of the study of the ability to cope with network problems, failures, and attacks while maintaining its functionality and structural integrity. It involves understanding how networks recover from disturbances, adapt to changes in conditions and continue to provide essential services. Network resilience is widely studied in various fields such as transportation systems, communication networks, and social networks. (Barabási et al., 2016).

The researchers developed resilience metrics, modelling techniques, and optimization strategies to improve network resilience and reduce the impact of damage. It is important to analyse the robustness and vulnerability of the network in order to identify crucial elements and design strategies to improve its resilience. The study of the resilience of the network can help to develop effective disaster management plans, optimise resource allocation and improve the overall performance of network systems. (Barabási, 2016)

2.4. Network analysis and public transport systems

In the next section we dive into the intersection of network analysis and public transport systems. We introduce the most relevant parts of the literature on different applications, then explain the importance of resilience in this field.

2.4.1. Most common applications of network analysis on public transport systems

Network analysis application to public transportation systems has become an important area of research, with several studies that highlight its various applications. A key application is the design of transit networks, in which graph theory and network science techniques are used to optimise the configuration and efficiency of public transport networks. These studies aim to identify the most effective routes, connections and service frequencies to improve accessibility, reduce travel time and improve the overall performance of the transit system. By analysing network connectivity, central measurement, and network flow dynamics, researchers gain insight into the principles and functional characteristics of public transportation networks. (Derrible – Kennedy, 2011)

According to Luo and his co-writers another important application of network analysis in public transport systems is the assessment of accessibility. Researchers have combined network science with accessibility analysis to evaluate the level of service provided by transit networks. Using travel times, route options and connectivity, these studies assess the ease of reaching different destinations in the network. Comparison assessments are often carried out to identify differences in accessibility between different regions or modes of transport. Such analyses help policymakers and transportation planners identify areas that need improvement, support decision-making processes, and promote equitable access to public transport services.

In addition, Derrible (2012) says that network analysis techniques have been used to examine the structure and topological characteristics of public transport networks. By analysing network metrics such as degree distribution, clustering coefficients and path lengths, researchers gain an idea of network robustness, efficiency, and vulnerability. Understanding network structures helps identify key stations, key connections, and potential areas of network expansion or improvement. These insights help to improve the resilience and efficiency of public transport systems. (Derrible, 2012)

2.4.2. Importance of resilience in public transportation

In the event of natural disasters and other unexpected disruptions, the resilience of the transport system is crucial to ensuring the stability and reliability of the transport network. To date, several studies have been carried out to investigate the vulnerability of public transport in specific cities. Each study uses different approaches to help readers better understand the methods that can be used to map the resistance of the city's transport network. The review of the literature by Zhou et al. (2019) provides a systematic review of metrics and models that measure system vulnerability, resilience, recovery, and adaptation capacity. These include

time-based models, dynamic simulations, stochastic models, mathematical algorithms, and optimising methods to measure or improve system resilience. The above-mentioned articles thoroughly review these models and methods and highlight their benefits and limitations in assessing and improving the resilience of transport systems.

2.5. Budapest's public transport and network analysis

2.5.1. Brief history of Budapest's public transport

To analyse the public transport network in Budapest, it is critical to understand the process of its development and the conditions that shaped it. In this section we go over the most prominent factors that shaped the city's public transport network, based on BKV's two historical overview pages (BKV, n.d. a, n.d. b), the Budapest Urban Development Concept (or Budapest Városfejlesztési Konceptiója, 2011) available on the capital's website, and the thesis of Róbert Agócs (2013) on the cartographic processing of Budapest's tram transport.

From the work of Agócs (2013) and the earlier historical overview of BKV (n.d. a), we can learn that the development of today's transport is not the result of a single periodic planning, rather it has been achieved through more than 150 years of modular development and through several political regimes. This resulted in a constantly renewing concept of the city's future, with some milestones outstanding from the rest. The first milestone consisted of the first tram building wave at the end of the 19th century, along with the construction of Metro line 1 (BKV, n.d. a). The second most important milestone came in the early 1970s, when public transport in Budapest began to develop dynamically, with a newfound focus on reaching the agglomeration. In this period the construction of the second underground railway, Metro line 2 began, connecting Pest and Buda in a new, traffic-free manner 1 (BKV, n.d. b). Lastly, the modern development of transport infrastructure of the early 2000s is worth underlining along with the 1990 opening of Metro line 3 and 2014 opening of Metro line 4 (BKV, n.d. b).

The importance of analysing Budapest's public transport system is emphasised in the Budapest Urban Development Concept (2011) underlines that transport is not only a matter of change of place. It further emphasises the important role of the following dimensions in the complexity of the issue: "national economic sector, social interaction, shaping the built environment, hazardous facility, environmental problem" (Budapest Urban Development Concept, 2011, p. 189). Based on these we concluded that Budapest's public transport could well be an area of interest in the field of network science.

2.5.2. Literature on the Budapest's public transport network

The Budapest public transport network is the subject of several research papers containing information on its operation and performance. One study focused on stochastic route planning in public transport, which was aimed at optimising the route selection process by taking into account uncertainties such as travel time variations and interruptions. The research aims to improve the reliability and efficiency of Budapest's public transportation system, ultimately improving passenger experiences and reducing travel time variability by incorporating stochastic elements into the planning process. The results of this study highlight the potential benefits of using stochastic modelling approaches to plan road maps for public transportation networks, including the Budapest system. (Bérczi et al., 2017)

Another study from Gaal and his co-writers (2015) analysed the performance of the Budapest public transport network, particularly in terms of reliability of services, travel time and passenger satisfaction. By studying data on service frequency, delays, and passenger feedback, we provided a comprehensive assessment of network performance indicators. The research identified areas of improvement and proposed strategies to improve the reliability and efficiency of the Budapest public transport system. The results of this analysis helped to better understand the strengths and weaknesses of the network, enabling policymakers and transport authorities to make informed decisions and implement measures that could optimise the performance and quality of the Budapest public transport network for the benefit of its users.

2.6. *Physarum polycephalum* slime mould-based approach of network development

Based on Treo et al. (2006), *Physarum polycephalum* is a single-cell organism that moves and eats organic matter in a wet environment. Despite the lack of communication between individuals, these slime moulds can navigate efficiently on path networks, for example to food sources and better environmental conditions. The researchers describe a method called the *Physarum* solver, which uses the behaviour of the slime *Physarum polycephalum* to navigate the pathway network. On the basis of the concentration value, the path network is simulated over time, the concentration value changes, and the slime mould-generated network changes. After the road network is formed, the network defined in the *Physarum* Solver method can be used to solve complex traffic problems such as road design, road search, and traffic network planning.

3. Network analysis of Budapest's public transport network

In the forthcoming section of the study, we intend to conduct an analysis of Budapest's public transportation network using a diverse set of methodologies.

3.1. Fundamentals of Budapest's public transport network

In the subsequent section, we will undertake an assessment of Budapest's public transportation system by employing descriptive and resilience-removal techniques.

3.1.1 Descriptives of Budapest's public transport network

We are building Budapest's public transport network by using the general transit feed specification data (bkk.hu, 2023). The data contains Budapest's trains, metros (including HÉVs), buses, and trolleybuses in a daily shift; we have removed the all-night services and ships from the data. Simply put, we use G to denote the undirect graph of the public transport network:

$$G = (V, E)$$

where V is the node set representing the stations and E is the edge set representing at least one line that exists between the node pair. To measure the stops next to each other as one, we have created a threshold that aggregates these stops into one.

We have also added weights to the edges by measuring the frequency and the maximum capacity of the transport vehicles. Unfortunately, we could not find any kind of data regarding the flow of passengers, so we used this measurement as a proxy like Lam and his co-authors (2002) did.

Our results showed that Budapest public transport network consists of 2,249 nodes and 10,641 edges made by over 266 means of transport forms a moderately connected network (bkk.hu, 2023). The average node degree is 9.462872, and the node has multiple connections, facilitating

efficient travel routes. The average shortest path length of 421527.1 indicates a reasonable distance of journey within the system.



Figure 1: Budapest's Public Transport Network, source: own figure

3.1.2. Clustering measures of Budapest's public transport network

The 0.1295936 clustering coefficient quantifies the existence of a tightly connected group within the city. This measurement shows the degree to which nodes tend to cluster and form local communities or transit hubs, which facilitate smoother passenger transfers and shorter travel times. Also, the walk trap algorithm shows the network's modularity, which was determined to be 0.6707785. This indicates the existence of different communities within the system too. The analysis of modularity helps to identify areas where targeted improvement and resource allocation can be made. These areas were Kispest, Újpest and Városmajor.

3.1.3. Network resilience of Budapest's public transport network, node removal

Robustness analysis is crucial to understanding the resilience of transport systems to failures or disruptions of nodes. High-concentration nodes such as Keleti pályaudvar and Kelenföldi pályaudvar play an important role in maintaining network connectivity. According to our measures, these two had the highest connectivity in our analysis. The fragmentation analysis revealed that different centrality measures affect network fragmentation. Removing nodes based on betweenness centrality (13 nodes), closeness centrality (8 nodes), eigenvector centrality (3 nodes), or degree centrality (6 nodes) resulted in network fragmentation. Random

node removal required on average the removal of 56 nodes before the network fragmented. In summary, nodes with high centrality are crucial for maintaining network connectivity.

Upon considering the recalculation of connectivity following the removal of each individual node, it was found that it has no impact on the resulting output. According to our calculations the most important stops and squares by degree centrality are the already mentioned Keleti and Kelenföldi pályaudvar, Déli pályaudvar, Móricz Zsigmond körtér, Deák Ferenc tér, Balahá Lujza tér, Astoria, Örs vezér tere, Kálvin tér, Széll Kálmán tér, Savolyai Jenő tér.

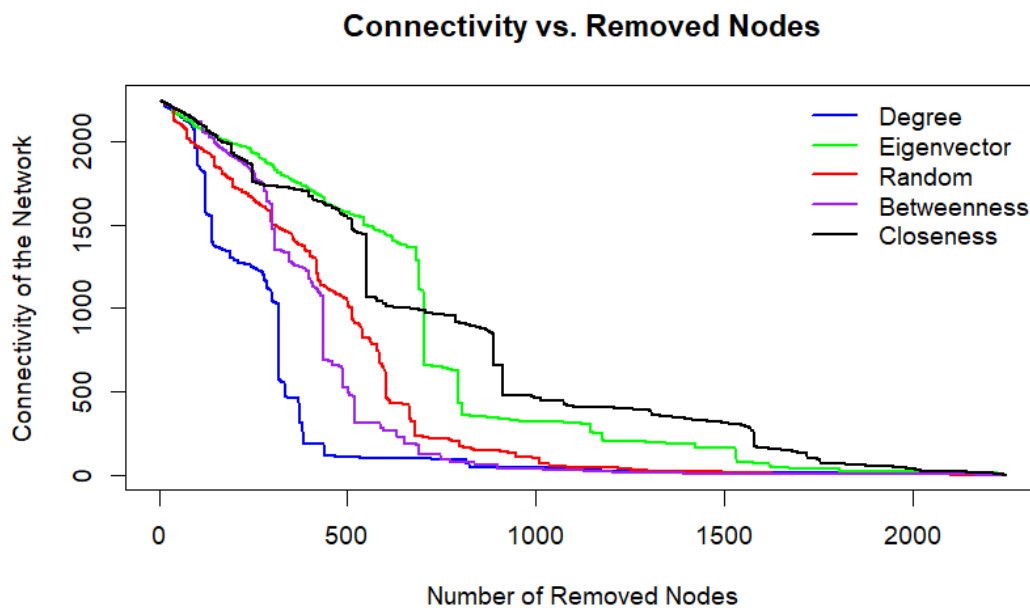


Figure 2: Connectivity vs. Removed nodes, source: own figure

Starting with the degree-centrality measurement, which represents the number of connections the nodes have, we see that the network's connectivity decreases as nodes with high centrality are removed. This indicates that several nodes are essential to maintaining the overall flow and connectivity of the network.

Eliminating these highly connected nodes can disrupt flow patterns and cause fragmentation within the transport system. Moving on to the centrality measure betweenness, which identifies nodes as important intermediaries in traffic flow, we observe a similar trend. With the removal of nodes with high centrality, network connectivity decreases. This indicates that these intermediary nodes play an important role in facilitating the transport of passengers and vehicles between different parts of the network.

The removal of these substances causes congestion or inefficiency in the transport system and disrupts flow patterns. Secondly, considering the random sequence of node removal, we find a

relatively constant decrease in connectivity. This shows that even random deletions of nodes without considering their centrality measures can have a negative impact on the overall network flow. It highlights the interconnectedness of the transport system, which may interfere with the smooth operation of the network by eliminating any node.

Moving to the eigenvector centrality measurement, which identifies nodes with connections to other highly connected nodes, we see a similar trend to the previous centrality measurement. The removal of nodes with a high inherent vector slowly reduces network connectivity, indicating that these nodes significantly improve the overall flow and robustness of the transport system.

Finally, when we examine the measure of the closeness of the centrality, we notice that nodes that are easy to reach other nodes in the network have a decrease in connectivity when nodes that are close to the centrality are eliminated. This suggests that central nodes in terms of proximity play an important role in maintaining effective and accessible transport routes. If you remove them, travel time may be longer, or passenger accessibility is limited. Overall, the result of the mapping emphasised the critical nodes of Budapest's public transport network and their importance in maintaining the flow and connectivity of the system.

3.1.4. Network resilience of Budapest's public transport network, edge removal

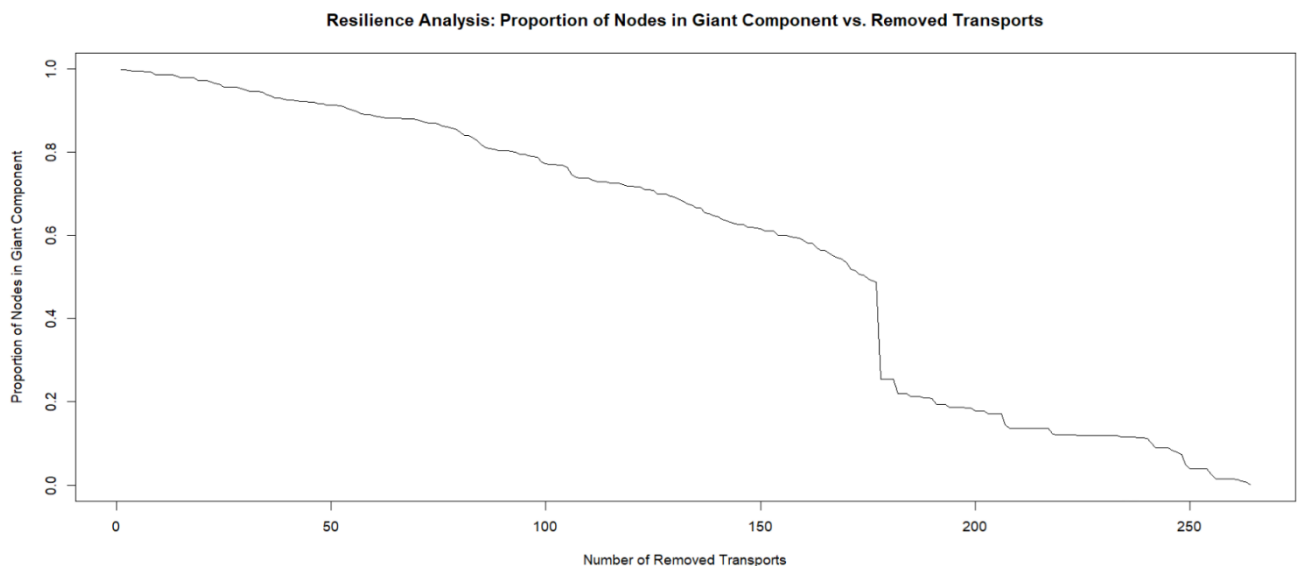


Figure 3: Transports removal, source: own figure

By systematically removing transportation routes, we can assess the network's ability to maintain its structure and functionality in the face of such disruptions. Analysing the network's

response to route removals helps identify critical edges or vulnerable areas that, if disrupted, could have a substantial impact on the network's overall performance or functionality.

We had set the weights of these transports in decreasing order and started to remove the routes one by one, starting with the highest value. According to our calculation, the most valuable routes are the M1, M2, M3, and M4 tram numbers: 1, 4, 6, 3, 17, and bus numbers: 9, 20E, and 7E.

The plot of the proportion of edges in the giant component versus the number of removed transports visualizes the resilience of the network as the transportation infrastructure is gradually disrupted. It shows how the giant component, which represents the largest connected cluster of nodes, evolves as more and more transports are removed from the network. This plot helps to understand the impact of transport removal on the overall connectivity and robustness of the network, providing insights into the network's resilience to transportation disruptions. The analysis of route removal reveals that the public transportation system remains stable with the gradual removal of routes, showing minimal impact on network performance. However, a notable decrease in the proportion of nodes in the giant component occurs after around 150 route removals, indicating a significant decline in network stability beyond this threshold. This highlights the importance of preserving an adequate number of routes to maintain connectivity and ensure the functionality of the public transportation system.

3.2. Resilient network expansion of Budapest's public transport

In addition to analysing the underlying network structure of Budapest's public transport system with a distinct focus on network resilience, we implemented a computer simulation of the spread of the *Physarum polycephalum* mould, to emulate resilient network expansion. First, we make a brief introduction of this imitation model, then we discuss the initial setup, followed by the description of the two stages of the model.

3.2.1 Introducing the *Physarum polycephalum* imitation model

The second part of our analysis focused on resilient network expansion. As previously emphasised resilient network expansion is a crucial tool of urban planning which helps to confront unforeseen problems and events in the future. City planning can span from a few years to whole decades ahead, creating a long-lasting and stable public transport network requires thorough preparation and large financial commitment. Thus, abiding by the concept of a flexible, and preferably polycentric public transport network is an important investment in the

lifetime of a major city, such as Budapest. The next section of our analysis takes a leap from the current public transport network and focuses on a possible future development of Budapest's most essential public transport asset, the metro network.

Resilient network expansion was performed by mimicking the growth patterns of the slime mould *Physarum polycephalum*, based on the foundations laid by Kay et al. (2022). After a brief collaboration with the authors of the article, we attempted to recreate the growth and expansion of *Physarum polycephalum* via a two-stage imitation. The first stage consisted of modelling the biased foraging of the mould and its biased meshing, while in the second stage, we implemented a simplified network refinery.

3.2.2. Setting up the first stage of the *Physarum polycephalum* imitation model

In the first stage, we closely followed the work of Kay et al. (2022) and applied a modified version of their agent-based model aimed at modelling the organic network creation of the mould. In the first step two important preliminary elements of the simulation have been defined: the starting point of the simulated mould and a predefined set of attractor points. The first element is substituting the nucleus of the mould from which the biased foraging would start at time zero, and the second element substitutes the oats or other food sources to which the mould is attracted in order to create their mesh.

The starting point of the simulated mould for our experiment was selected by our descriptive network analysis. We choose the eigenvector centrality in the preexisting public transport network to capture the characteristics of the network to the greatest extent. Here it is worth mentioning that the simulation was almost identical for the possible starting points based on all three centrality measures.

Attraction points were selected based on two main sources. First, we selected the 20 largest public transport hubs based on weight derived from vehicle capacity and volume. Then, we supplemented this list with the further, most prominent hubs listed on the BKK website. The first step was calculated after removing the weights of the metro lines, so that only those lines that are associated with metro congestion are included with a high weight. This was implemented as metro lines as clear outliers would have biased the selection of nodes towards less frequented, and thus, less important public transport hubs, with metro network coverage.

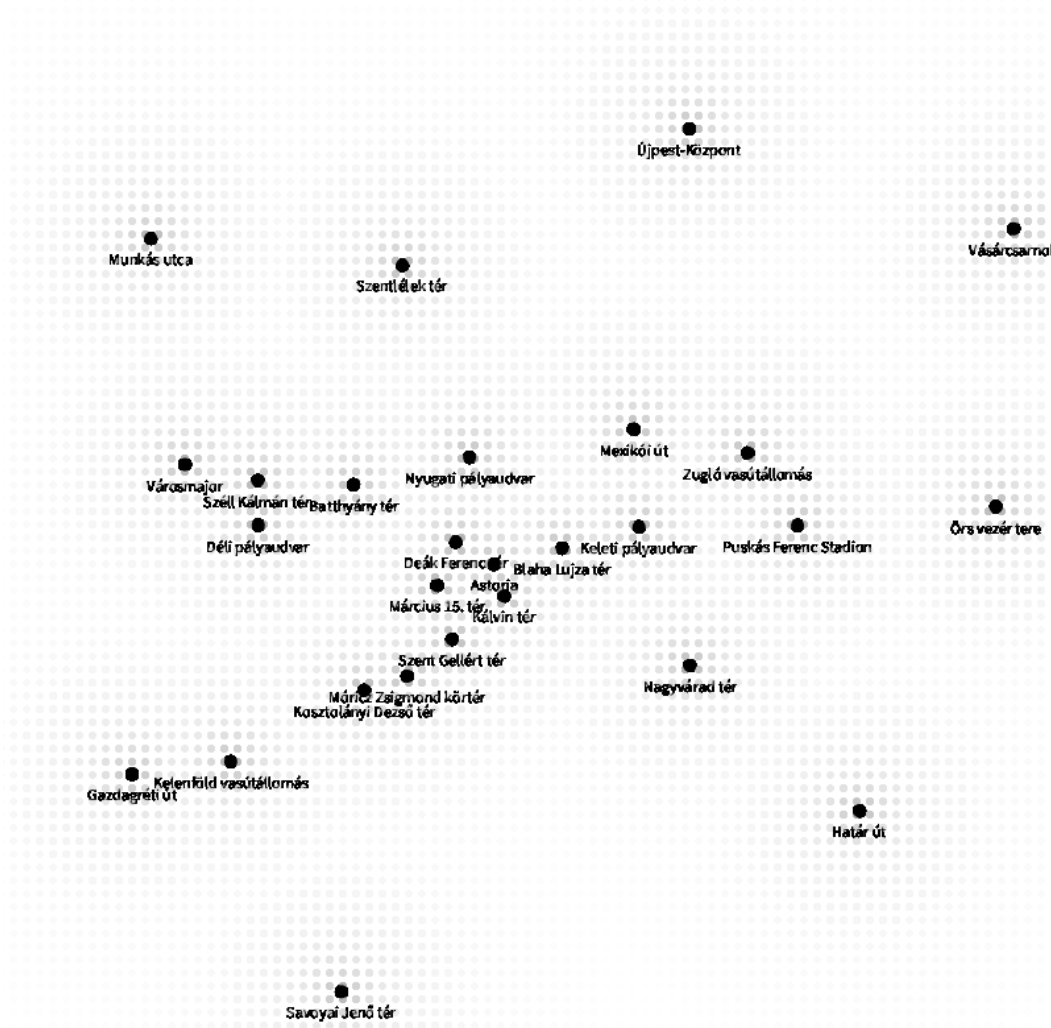


Figure 4: The starting nodes of the simulation, source: own figure

However, this step of removing the weight of metros can introduce a counter-bias into the system through the fact that parts of Budapest were natural hubs even before the great waves of metro construction in the 20th century. To avoid this bias, the nodes with the highest unique line counts, which became underweighted with the removal of the metro system, were introduced into our analysis through this second step. Figure 4 illustrates the starting point and the selected attraction points for our simulation.

3.2.3. Simulating the first stage

As discussed above, the *Physarum polycephalum* is a naturally excellent network creator in terms of network resilience, and part of this success lies in the primer foraging mechanisms behind its growth. As nothing in nature is truly deterministic by only observable factors, the model starts by defining a stochastic element which counts for the factors one cannot control for, and a more tangible deterministic component (Kay et al, 2022).

The stochastic component was created by a one-dimensional Perlin noise algorithm. The algorithm's main advantage lies in making the random walk simulation smoother. It works by generating a value from a predefined range while also storing data on previously generated numbers (Kay et al, 2022). This results in a time-dependent number generation which leads to a visually more appealing, smoother outcome. As the number generation is affected by non-random components (time and previous location), this algorithm defines a pseudo-random walk. The deterministic component is computed based on the square of the attractiveness of every attractor point predefined at the beginning of the simulation, inversely proportional to the distance between the ever-growing mould and the attractor points (Kay et al, 2022).

We used an equiaxial 800×800 -pixel simulation space with a total of 27 attractor points, with Blaha Lujza tér as the chosen centre for the nucleus. The attractor points' distances were normalised to fit between 0-800, maintaining a relative distance from each other and a close approximation of the real-life location of the chosen transportation hubs. This created a denser point concentration in the central regions and a more sparse point concentration in the peripheral regions of Budapest, stemming from the method we used to determine the attractor points themselves in the preparation phase.

As to catch the characteristics of a *Physarum polycephalum* mesh creation more realistically we also followed the approach of Kay et al. (2022) in making the attractor points colonizable for the mould. This meant that after a certain threshold was met, the attractor point lost its ability to attract other branches of the mesh but started to emulate the original nucleus in its fundamentals. The colonised attractor points became new starting points of foraging and further mesh creation, inherently starting from and supplementing the preexisting network. The simulation ended when the last attractor point had been colonised by the network. Figure 5 shows the progression of the mesh creation. The different snapshots were taken every 20 seconds until the simulation ended, starting from the 10th second.

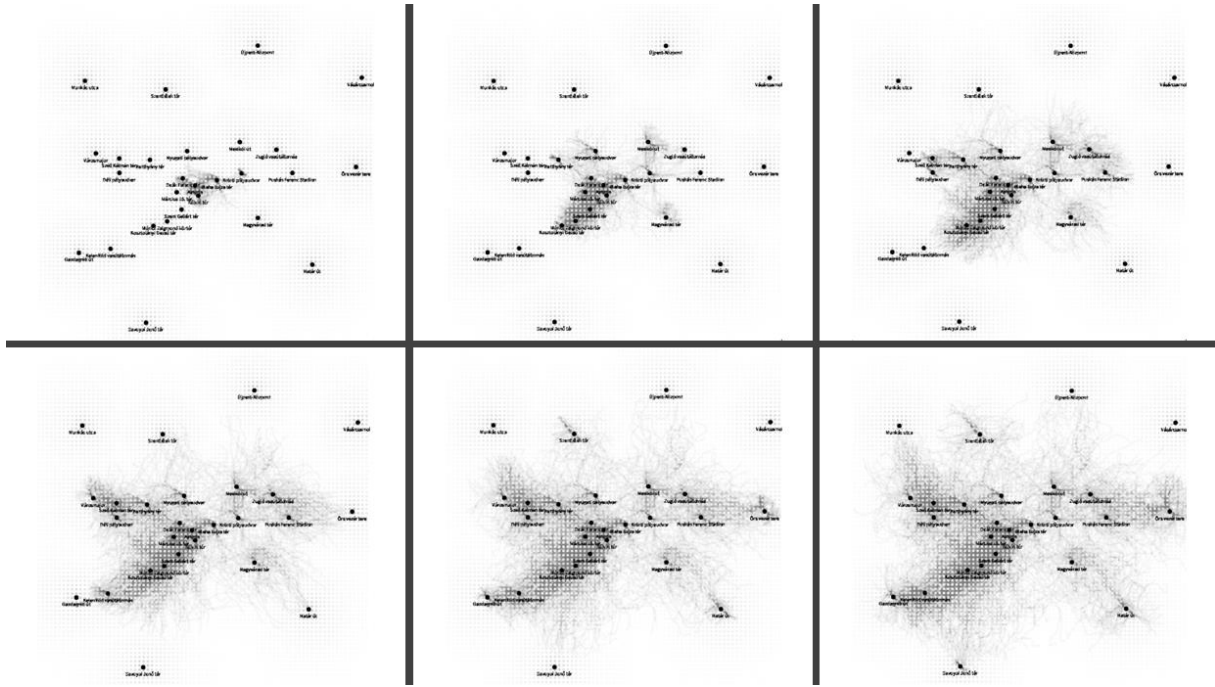


Figure 5: Mesh creation, source: own figure

3.2.4. The second stage of the *Physarum polycephalum* imitation model

After the simulation ended, we created a proximity table from the cellular morphology of the last stage of the meshing. We recorded the x and y coordinates of the simulated mould at every three frames it travelled throughout the 1 minute and 55 second the simulation took place. This resulted in a node list of over 1.29 million nodes, with three variables: an id, x coordinates of the nodes (the latest point the mould reached through foraging), and their respective y coordinates, in the uniaxial 800 x 800-pixel space. We created a proximity table on a subsample of 100,000 elements using the 10 nearest neighbours of the nodes, for computational purposes. (A basic visualisation of the proximity table in graph form on the whole data, with 2 nearest neighbours is shown in Appendix Figure 1.)

At this point, our analysis deteriorated from the work of Kay et al. (2022), from here on our analysis integrates a more descriptive analysis. As the resilient nature of *Physarum polycephalum* and the implications it could provide for urban planning have already been introduced, in the last section of our analysis we provide some of our main insights based on the simulations and the real-life data.

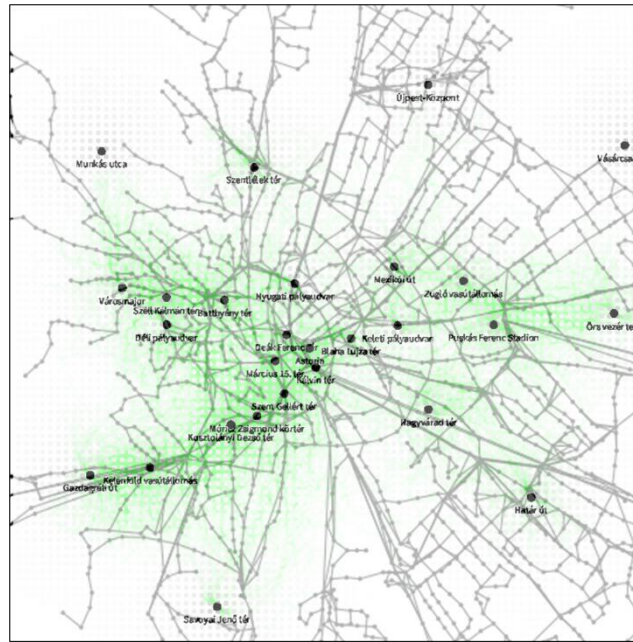


Figure 6: The simulation of the mesh of *Physarum polycephalum* with an underlying map of Budapest's public transport network, source: own figure

Figure 6 portrays an image of the simulated *Physarum polycephalum* after 90 seconds on top of the map of Budapest's public transport network. When the weights of the current metro-system were removed, most of the busiest transportation hubs came to be located in the city centre and on the Buda side of the Danube. These hubs greatly coincide with the main station of Metro line 2 and 4. However, both lines run under the Danube, rendering the transportation implications for Buda much lesser.

Based on these, we concluded that constructing a solely or mainly Buda-side metro line could improve the network resilience and quality of Budapest's public transport system, connecting the northern parts of Buda (like Szentlélek tér) with the southern parts (like Kelenföld vasútállomás).

4. Discussion and research restraints

Our results suggest that the public transport network in Budapest should be investigated using network science tools. Based on our results the analysis of Budapest's public transport network reveals important insights into its structure, connectivity, and resilience. The network consists of a moderately connected system with a large number of nodes and edges, facilitating efficient travel routes and shorter travel times. The clustering coefficient and modularity measurements indicate the presence of tightly connected groups and different communities within the network, suggesting the existence of transit hubs and areas for targeted improvements.

When examining the network's resilience through node removal, high-concentration nodes such as Keleti pályaudvar and Kelenföldi pályaudvar are found to play a crucial role in maintaining network connectivity. Removing nodes based on different centrality measures leads to network fragmentation, emphasising the importance of highly connected nodes for overall flow and connectivity.

Analysing edge removal in the network reveals the most valuable routes in terms of weights. The gradual removal of routes has minimal impact on the network's performance, indicating its stability and resilience to transportation disruptions.

Based on our simulations regarding resilient network growth, building a 5th metro line on only the Buda-side would be advantageous in terms of network resilience and quality. As we have paid meticulous attention to the placement of attractor points, and have filtered out the possibly harmful, distorting effects of the weight of the metro network, meanwhile also controlling for the possible causes of hub creation with the second step of the hub selection, we have obtained a self-sufficient analysis result. As exploring the overall quality of the public transport system was not in the scope of this analysis, these statements should be further investigated before actionable policy implications could be drawn.

While this gives a picture of both the public transport network in Budapest and the potential for the development of the metro network, the current analysis has certain limitations and possible future expansions. The first potential future improvement lies in the shortest distance calculation in stage two of the simulations, applied by Kay et al. (2022), which could not be recreated due to computational and technical limitations. Furthermore, live mould-growing experiments could not be performed as on top of a great delivery time the mould did not arrive to the date of writing with a substantial delay.

All this suggests that further investigation of the objectives of our research plan can be further developed with more advanced *Physarum polycephalum* modelling. Furthermore, the addition of population demographic and other passenger data to the initial database may open exciting areas for further research development. A possible collaboration with BKK could produce fruitful results that could greatly help urban planning in Budapest for the coming decades.

5. Conclusion

Our paper concludes that applying network analysis tools to Budapest's public transport system yields interesting results. In our paper, we summarised the fundamentals of network analysis as

well as the most widely used analytical tools for spatial networks. Based on the preexisting literature we highlighted the importance of managing public transport systems with tools provided by the field of network science, through identifying key stations, connections or even potential areas of network expansion and improvement. Our paper also contains a brief theoretical background behind the *Physarum* solver, a network pathway optimization tool based on the behaviour of the slime *Physarum polycephalum*.

In our study we showed that Budapest's public transport system is a metric spatial network, thus interconnectedness in the central parts of the city is inherently higher, resulting in increased clustering and shorter average path lengths. The importance of measuring and improving resilience in such a system lies in the increased effectiveness in pre-emptive disaster management, optimal resource allocation, and the overall performance of the network. After a descriptive analysis on Budapest's public transport system, we also conducted a simulation of *Physarum polycephalum* mesh creation. Based on these analyses our paper concludes that a fifth metro line connecting the northern parts of Buda with the southern parts could best improve the network resilience of Budapest's public transport system.

Further research in *Physarum polycephalum* modelling and resilience could greatly enhance the findings of this paper, providing valuable insights for improving Budapest's public transport network. Additionally, conducting similar studies across multiple European cities and comparing the results would offer a more comprehensive understanding of network resilience and enable effective strategies for optimising public transportation systems in urban areas.

6. Citations

Agócs, R. (2013). A budapesti közlekedési hálózat fejlesztésének lehetőségei (Unpublished bachelor's thesis). *Eötvös Loránd University*, Budapest, Hungary, 10-42. Retrieved April 9, from http://lazarus.elte.hu/hun/digkonyv/szakdolg/agocs_robert.pdf

Barabási, A.-L. (2016). Network Science. *Cambridge University Press*, 1-318.

Barthelemy, M. (2011). Spatial networks. *Physics reports*, 499(1-3), 1-101.

Bérczi, K., Jüttner, A., Laumanns, M., & Szabó, J. (2017). Stochastic route planning in public transport. *Transportation Research Procedia*, 27, 1080-1087.

Blondel, V. D., Guillaume, J.-L., Lambiotte, R., & Lefebvre, E. (2008). Fast Unfolding of Communities in Large Networks. *Journal of Statistical Mechanics: Theory and Experiment*, 2008(10), 1008.

Bonacich, P. (1987). Power and Centrality: A Family of Measures. *American Journal of Sociology*, 92(5), 1170-1182

Bruneau, M., Chang, S. E., Eguchi, R. T., Lee, G. C., O'Rourke, T. D., Reinhorn, A. M., ... & Von Winterfeldt, D. (2003). A framework to quantitatively assess and enhance the seismic resilience of communities. *Earthquake spectra*, 19(4), 733-752.

Budapest Városfejlesztési Konceptió. (2011). Közlekedési infrastruktúra. Retrieved April 10, 2023, from: https://budapest.hu/Documents/varosfejlesztesi_koncepcio_2011dec/10_Kozlekedesi_infrastruktura.pdf

Budapesti Közlekedési Vállalat. (n.d a.). A BKV története: A kezdetektől 1968-ig. Retrieved April 10, 2023, from https://www.bkv.hu/hu/a_bkv_tortenete/a_kezdetektol_1968ig

Budapesti Közlekedési Vállalat. (n.d. b). A BKV története: 1968-tól napjainkig. Retrieved April 10, 2023, from https://www.bkv.hu/hu/a_bkv_tortenete/1968tol_napjainkig

Colizza, V., Barrat, A., Barthélemy, M., & Vespignani, A. (2006). The role of the airline transportation network in the prediction and predictability of global epidemics. *Proceedings of the National Academy of Sciences*, 103(7), 2015-2020.

Derrible, S. (2012). Network centrality of metro systems. *PloS one*, 7(7), e40575.

Derrible, S., & Kennedy, C. (2011). Applications of graph theory and network science to transit network design. *Transport reviews*, 31(4), 495-519.

Fejlesztőknek, bkk (2023), Retrieved May 28, from: <https://bkk.hu/fejlesztések/fejlesztoknek/>

Fortunato, S. (2010). Community Detection in Graphs. *Physics Reports*, 486(3-5), 75-174.

Freeman, L. C. (1977). A Set of Measures of Centrality Based on Betweenness. *Sociometry*, 40(1), 35-41.

Gaal, G., Horváth, E., Török, Á., & Csete, M. (2015). Analysis of public transport performance in Budapest, Hungary. *Periodica Polytechnica Social and Management Sciences*, 23(1), 68-72.

Gao, J., Barzel, B., & Barabási, A. L. (2016). Universal resilience patterns in complex networks. *Nature*, 530(7590), 307-312.

Kay, R., Mattacchione, A., Katrycz, C., & Hatton, B. D. (2022). Stepwise slime mould growth as a template for urban design. *Scientific Reports*, 12(1), 1322.

Lam, W. H., Zhou, J., & Sheng, Z. H. (2002). A capacity restraint transit assignment with elastic line frequency. *Transportation Research Part B: Methodological*, 36(10), 919-938.

Luo, D., Cats, O., van Lint, H., & Currie, G. (2019). Integrating network science and public transport accessibility analysis for comparative assessment. *Journal of Transport Geography*, 80, 102505.

Newman, M. E. J. (2006). Modularity and Community Structure in Networks. *Proceedings of the National Academy of Sciences*, 103(23), 8577-8582.

Newman, M. E. J. (2010). *Networks: An Introduction*. Oxford University Press, 167-240.

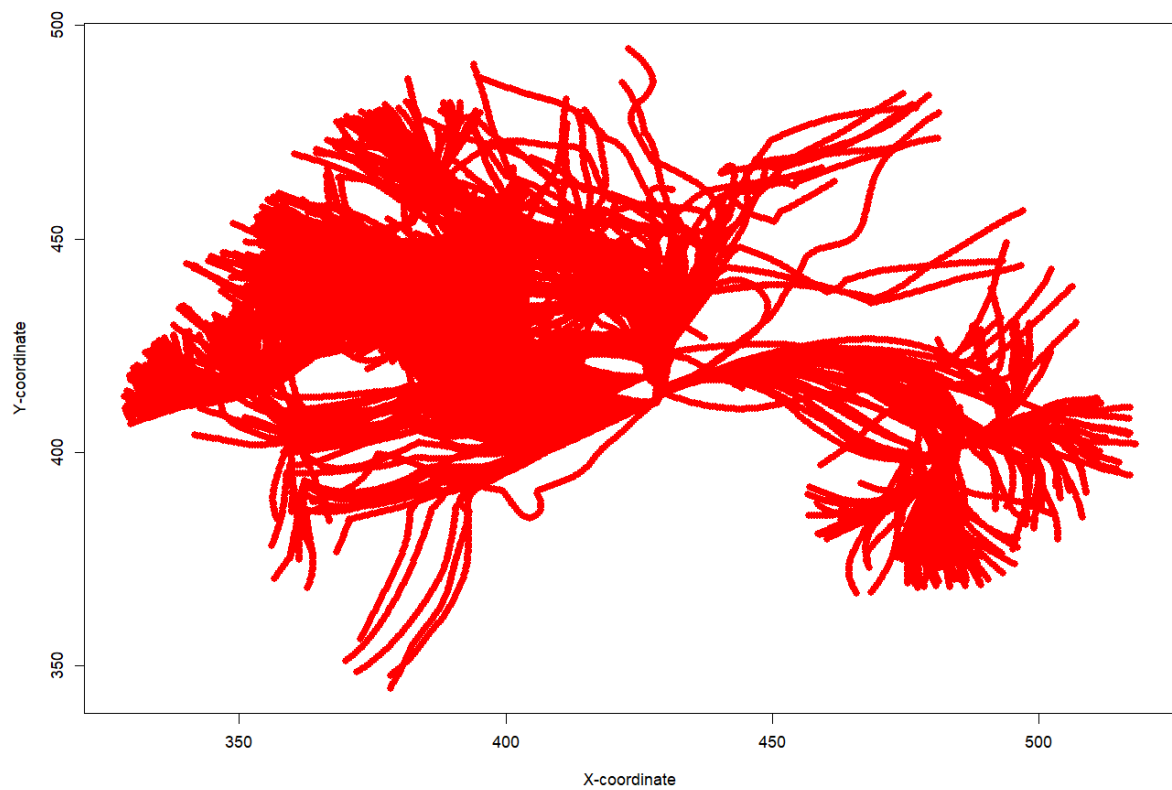
Rosvall, M., & Bergstrom, C. T. (2008). Maps of Random Walks on Complex Networks Reveal Community Structure. *Proceedings of the National Academy of Sciences*, 105(4), 1118-1123.

Sabidussi, G. (1966). The Centrality Index of a Graph. *Psychometrika*, 31(4), 581-603.

Treo, E. F., Felice, C. J., & Madrid, R. M. (2006, January). Non linear dielectric properties of microbiological suspensions at electrode-electrolyte interfaces. *In 2005 IEEE Engineering in Medicine and Biology 27th Annual Conference*, 4588-4591.

Zhou, Y., Wang, J., & Yang, H. (2019). Resilience of transportation systems: concepts and comprehensive review. *IEEE Transactions on Intelligent Transportation Systems*, 20(12), 4262-4276. Retrieved May 10, from: <https://ieeexplore.ieee.org/abstract/document/8602445>

Appendix



Appendix Figure 1: The visualisation of the proximity table in graph form on the whole data, with 2 nearest neighbours, source: own figure