Rajk College for Advanced Studies

Efficiency Evaluation of Budapest's Public Transport Network Using Network Analysis and Physarum Polycephalum Slime Mould Modelling

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Abstract

This research aims to analyse the current state of Budapest's public transport network and explore avenues to enhance its efficiency. By employing network analysis methods and *Physarum polycephalum* slime mould modelling, the study investigates the system's effectiveness and proposes strategies for its improvement. Targeted attacks on routes are executed to assess the system's ability to withstand disruptions, revealing that while it shows strong resilience against route-focused attacks, it can rapidly collapse if critical stops and important squares are targeted. The study also introduces a novel approach to model the spread of *Physarum polycephalum* slime mould. The networks are analysed using well-known methods such as number of links, number of nodes, diameter, average clustering, mean betweenness, average path length, and more specific metrics such as the beta-index, global efficiency and assortativity. The paper's structure includes discussions on network analysis tools, research execution process, and presentation of key findings and conclusions. The research highlights the significance of an effective and resilient public transport system for well-being, development, and disaster management. Analysis on an alternative public transport network created by imitating the network growth and refinery of the slime mould *Physarum polycephalum* produces better results. These findings suggest that improvements are achievable by redesigning key parts of the current public transport network.

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1. Introduction

In today's world, the creation of a resilient and stable public transport system is of utmost importance for the overall well-being and development of a city. The goal of this research is to analyse the current state of Budapest's public transport network and explore strategies to enhance its efficiency. To achieve this, the study examines the system's resilience and utilises *Physarum polycephalum* slime mould modelling to propose potential improvements.

The research involves combining publicly available data with a manually curated dataset that captures the average capacity of buses, trams, metros, and trolleybuses to construct a comprehensive graph of the Budapest public transport network. Various well-established network analysis methods are then employed to gain insights into the network's characteristics and behaviour. Additionally, a unique modelling approach utilising *Physarum polycephalum* slime mould is employed to further understand potential enhancements for the network.

The study begins by discussing the significance of resilient and stable public transport systems in today's world. It then delves into the methodology, describing how the research data is collected, combined, and analysed. The findings of the analysis provide valuable insights and recommendations for optimising Budapest's public transport network, with the aim of improving its efficiency and resilience.

2. Applications of network science in studying public transport networks

In the following section we introduce the most important and relevant basics that we used in the analysis. First, we introduce the fundamentals of network analysis, expanded by the literature of spatial networks closely followed by the most commonly used measures in the field. We then turn our attention to the literature of network analysis and public transportation, with a special focus on literature concerning Budapest.

2.1. Fundamentals of network analysis

Networks are basic structures used to represent relationships and interactions between entities. They have found widespread applications in various fields, including social sciences, computer science, biology, transport and communications systems. Network science is an interdisciplinary field of research into the properties, structures and dynamics of networks. A network consists of a set of nodes (or vertices) connected to an edge (or link). (Barabási, 2016)

Nodes represent individual entities, while edges represent relationships and connections between entities. Networks can be classified on the basis of various features, including connectivity, size, and edge direction. Networks have certain properties that shape their structure and behaviour. Network analysis gives valuable insight into the structure, dynamics and functions of complex systems. It enables the study of information dissemination, social influence, disease spread, transportation efficiency, and interconnected system resilience. (Barabási, 2016)

According to Barabási (2016), networks provide a powerful framework for understanding and analysing relationships and interactions in different fields. It can discover hidden patterns, identify important entities, and understand the functions and behaviours of complex systems by studying networks. Using network analysis techniques, we can better understand the complexity of social networks, biological systems, technological infrastructures, and many other interconnected phenomena.

2.2. Spatial networks

In addition to the growing importance of network analysis on a broader scale, there is a larger area with untapped potential: spatial networks. Along with many natural networks, there is a plethora of man-made networks that are inherently spatial and thus could be analysed using network analytic tools. Marc Barthélemy (2006, pp. 1) defines spatial networks as “networks for which the nodes are located in a space equipped with a metric,” usually referring to complex networks that are embedded in a physical space and whose connections are dependent on some spatial proximity.

They are usually classified as metric networks and topological networks. Metric networks are based on continuous spatial embedding, which means that it accounts for the distances between the nodes, as well as their spatial location. Topological networks lack this characteristic, their focus is on relative connectivity between nodes, irrespective of the actual distances (Barthélemy, 2006). Budapest's public transport system is fundamentally a metric spatial network, with all the advantages and limitations that this entails. In our paper the main implication of this is observable in the presence of spatial constraints as they lead to increased clustering and shorter average path lengths, indicating a higher degree of local interconnectedness (Barthélemy, 2006).

In the next chapter we introduce three major network analysis tools which are suitable for measuring important aspects of spatial networks: centrality, community detection and network resilience.

2.3. Most widely applied network analysis tools

Our analysis builds on some of the most widely applied network analysis tools. In the next section we introduce the four most common centrality measures (degree, interdependence, closeness and eigenvector centrality), then we lay the foundations of both community detection and network resilience.

2.3.1 Centrality

Centrality is a fundamental concept of network science that quantifies the importance and influence of network nodes. It provides useful insights into the roles and positions of nodes and helps to identify key players, influential individuals, or key elements of a network. Several central measures have been developed to capture various aspects of node importance. A commonly used central measure is degree centrality. The degree of centrality simply counts the number of direct connections to the network node. High-centralised nodes are considered central hubs because they have many connections and can quickly disseminate information and resources. Degree centrality has been widely studied and applied in various fields such as social networks, reference networks, and biological networks. (Newman, 2010)

Newman (2010) says that another important centralization measurement is interdependence centralization, which determines the nodes acting as bridges or intermediaries between other nodes in the network. It measures how many nodes are placed on the shortest paths between pairs of other nodes. High-segment central nodes have important control over information flow because they serve as important connections between various parts of the network. Betweenness centrality is widely used in transport networks, social networks, and organisational networks.

Closeness centrality is a measure of the speed with which a node can reach other nodes on the network. It quantifies the average distance between nodes and all other nodes on the network. (Newman, 2010)

Eigenvector centrality is another widely used centrality measurement that considers the centrality of neighbours of nodes. It considers both the quantity and quality of the connection of a node. A node with a high centrality of a given vector is connected to a dominant node and often has a high influence itself. (Bonacich, 1987)

2.3.2 Community detection

Community, also known as clustering or modules according to Newman (2006) is an essential concept in network science. Community detection involves identifying a dense network of nodes within the network and a few nodes outside the network. Understanding network community structures provides an overview of complex system organisational, functional, and dynamic dynamics. Various methods and algorithms for community detection in networks have been developed. A common approach is modularity optimization. Modularity is a measure of the quality of network partitioning into communities. It compares the actual number of edges within the community with the expected number of such edges in the random network. Maximising the modularity value indicates the optimal separation with the most important community structure.

2.3.3 Network resilience

According to Barabási and his co-writers (2016) the resilience in network contexts refers to the ability of a network to maintain its functionality and structural integrity in the event of disturbances, failures, or attacks. It involves understanding how the network recovers, adapts and continues to provide essential services, even in the event of a disruption. Resilience to networks is a key aspect of the study of the ability to cope with network problems, failures, and attacks while maintaining its functionality and structural integrity. It involves understanding how networks recover from disturbances, adapt to changes in conditions and continue to provide essential services. Network resilience is widely studied in various fields such as transportation systems, communication networks, and social networks. (Barabási et al., 2016).

The researchers developed resilience metrics, modelling techniques, and optimization strategies to improve network resilience and reduce the impact of damage. It is important to analyse the robustness and vulnerability of the network in order to identify crucial elements and design strategies to improve its resilience. The study of the resilience of the network can help to develop effective disaster management plans, optimise resource allocation and improve the overall performance of network systems. (Barabási, 2016)

2.4. Network analysis and public transport systems

In the next section we dive into the intersection of network analysis and public transport systems. We introduce the most relevant parts of the literature on different applications, then explain the importance of resilience in this field.

2.4.1. Introduction of network analysis tools on public transport systems

Network analysis application to public transportation systems has become an important area of research, with several studies that highlight its various applications. A key application is the design of transit networks, in which graph theory and network science techniques are used to optimise the configuration and efficiency of public transport networks. These studies aim to identify the most effective routes, connections and service frequencies to improve accessibility, reduce travel time and improve the overall performance of the transit system. By analysing network connectivity, central measurement, and network flow dynamics, researchers gain insight into the principles and functional characteristics of public transportation networks. (Derrible – Kennedy, 2011)

According to Luo and his co-writers (2019) another important application of network analysis in public transport systems is the assessment of accessibility. Researchers have combined network science with accessibility analysis to evaluate the level of service provided by transit networks. Using travel times, route options and connectivity, these studies assess the ease of reaching different destinations in the network. Comparison assessments are often carried out to identify differences in accessibility between different regions or modes of transport. Such analyses help policymakers and transportation planners identify areas that need improvement, support decision-making processes, and promote equitable access to public transport services.

In addition, Derrible (2012) says that network analysis techniques have been used to examine the structure and topological characteristics of public transport networks. By analysing network metrics such as degree distribution, clustering coefficients and path lengths, researchers gain an idea of network robustness, efficiency, and vulnerability. Understanding network structures helps identify key stations, key connections, and potential areas of network expansion or improvement. These insights help to improve the resilience and efficiency of public transport systems. (Derrible, 2012)

2.4.2. Importance of resilience in public transport networks

In the event of natural disasters and other unexpected disruptions, the resilience of the transport system is crucial to ensuring the stability and reliability of the transport network. To date, several studies have been carried out to investigate the vulnerability of transportation networks in specific cities (see e.g. Berdica and Mattsson, 2007, Rodríguez-Núñez and Palomares, 2014, or Santos et al., 2020). Each study uses different approaches to help readers better understand the methods that can be used to map the resistance of the city's transport network.

A review of the literature by Zhou et al. (2019) provides a systematic review of metrics and models that measure system vulnerability, resilience, recovery, and adaptation capacity. These include time-based models, dynamic simulations, stochastic models, mathematical algorithms, and optimising methods to measure or improve system resilience. Another comprehensive review is provided by Wan et al. (2017) with a heavier emphasis on the definitions, characteristics, and research methods focusing on resilience. Finally, we relied on the work of Cox et al. (2011) to underline the importance of resilience in public transportation.

The above-mentioned articles thoroughly review these models and methods and highlight their benefits and limitations in assessing and improving the resilience of transport systems. According to their findings the main benefits of working with resilience include referring to an enhanced ability of preparedness and response (Cox et al., 2011), achieving better resource allocation and decision-making (Cox et al., 2011, Zhou et al., 2019), while focusing on long-term adaptability (Wan et al., 2017). Some of these concepts link back to the original idea of preparing and mitigating unforeseen events like natural disasters, road or railway accidents or even terror attacks (Cox et al., 2011).

It is important, however, to get acquainted with the greatest limitations as well. First of all, all three articles address the problem of data availability and quality (Cox et al., 2011, Wan et al., 2017, Zhou et al., 2019), which hinders sufficient analyses in a lot of the cases. This limitation remains one of the greatest setbacks in the efficient network science-based management of resilience in public transport systems. Another important limitation is the rise of interconnected challenges. Wan et al. (2017) underlines the importance of dealing with the problem with greater complexity, broadening the scope to day-to-day weather conditions to larger scale climate conditions, such as addressing the challenges proposed by climate change. Related to this is the limitation of the evolution of threats, or the dynamic nature of threats and disturbances, highlighted in all three articles. Finally, the financial and logistical constraints of the decision-makers present a hard limitation on resilient network redesign or expansion (Cox et al., 2011, Wan et al., 2017, Zhou et al., 2019).

This paper faced most of these challenges as well, starting from data availability to potential resource constraints in drawing conclusions. However, these limitations do not overshadow the potential advantages of resilient network design, merely help to set the direction of research and progress.

2.4.3. Summary of used network science metrics

In transportation studies, we often turn to network theory as a valuable tool for examining and prioritizing actions related to transportation infrastructure. This approach involves representing the transportation infrastructure as a network, where nodes and links form the building blocks. This network perspective allows us to perform crucial analyses with policy implications, like evaluating how new public transportation systems or natural disasters might affect transportation services. A common theme in these studies is the need to prioritize actions that yield the greatest benefits for both users and society. We achieve this by ranking various components of the transportation infrastructure based on their contributions to the overall performance of the system. This process is what we call "transport network criticality analysis." (Sienkiewicz - Hołyst, 2005)

According to Jafino et al. (2020) criticality analysis has two essential characteristics. One of them is the Ranking-Based Approach, which is rather than calculating criticality scores for each individual infrastructure component, the focus is on their relative importance and rank them accordingly. This ranking simplifies decision-making for transportation authorities. On the other hand, the Infrastructure Components Focus is the core of criticality analysis centers on the actual elements of the infrastructure network, such as the links (roads, railways) and nodes (terminals, intersections), rather than concentrating on aspects like user exposure or the network's overall robustness.

Despite the growing interest in transport network criticality analysis, there isn't a universally agreed-upon way to formalize this concept. As a result, there are a wide range of criticality metrics at the disposal, spanning from straightforward measures like road capacity to more complex indicators such as network connectivity measures. Consequently, transportation authorities face the challenge of selecting the most appropriate metrics that align with their unique analytical needs and policy objectives. (Jafino et al., 2020)

The Budapest public transportation network serves as a vital component of the city's infrastructure, facilitating the movement of thousands of commuters every day. In order to gain insights into the efficiency and robustness of this network, an analysis was conducted between June 2020 and June 2021, focusing on several key metrics. Each of these metrics provides a unique perspective on the network's performance and can assist transportation authorities and urban planners in making informed decisions.

The number of links and nodes in the transportation network is a fundamental metric that quantifies the total connections between different transportations. In the context of Budapest, it reveals the complexity and interconnectedness of the public transportation system. The number of nodes directly influences the accessibility and coverage of the public transportation system within Budapest.

The Beta Index is a topological metric that approximates the degree of connectivity within the transportation network. It provides insights into the efficiency of transportation routes, including the ease with which passengers can navigate the system. The diameter of a network signifies the maximum shortest path length between any two nodes within the system. In Budapest, it reflects the longest travel distance a commuter may have to traverse, giving a sense of the network's reach.

Average clustering measures the extent to which nodes in the network tend to cluster together, forming local communities. In the context of public transportation, high clustering can represent well-connected regions within Budapest.

Betweenness centrality is a metric used to identify critical nodes within the transportation network. It calculates the extent to which specific nodes are part of the shortest paths between other nodes. An elevated mean betweenness suggests nodes crucial for efficient transit in Budapest.

Global efficiency quantifies the network's effectiveness in providing alternative routes and redundancy. In the case of Budapest's public transportation, high global efficiency implies that passengers have multiple routes available, reducing the impact of disruptions.

Assortativity measures the tendency of nodes with similar characteristics, such as connectivity or popularity, to connect with one another. A positive assortativity indicates that popular or well-connected nodes tend to connect, which can affect transportation flow.

Average path length calculates the average distance between all pairs of nodes in the network. In Budapest, this metric provides insights into the typical travel distance for commuters, helping to assess convenience and accessibility.

2.5. Budapest’s public transport and network analysis

Apart from an overview of the benefits and limitations of resilience in public transportation in general it is crucial to gain a broader understanding of the specific environment. In this section a brief history and literature review is provided aiming at the public transport system of Budapest and its prior analyses.

2.5.1. Brief history of Budapest’s public transport

To analyse the public transport network in Budapest, it is critical to understand the process of its development and the conditions that shaped it. In this section we go over the most prominent factors that shaped the city’s public transport network, based on BKV's two historical overview pages (BKV, n.d. a, n.d. b), the Budapest Urban Development Concept (or Budapest Városfejlesztési Koncepciója, 2011) available on the capital's website, and the thesis of Róbert Agócs (2013) on the cartographic processing of Budapest's tram transport.

From the work of Agócs (2013) and the earlier historical overview of BKV (n.d. a), we can learn that the development of today's transport is not the result of a single periodic planning, rather it has been achieved through more than 150 years of modular development and through several political regimes. This resulted in a constantly renewing concept of the city’s future, with some milestones outstanding from the rest. The first milestone consisted of the first tram building wave at the end of the 19th century, along with the construction of Metro line 1 (BKV, n.d. a). The second most important milestone came in the early 1970s, when public transport in Budapest began to develop dynamically, with a newfound focus on reaching the agglomeration. In this period the construction of the second underground railway, Metro line 2 began, connecting Pest and Buda in a new, traffic-free manner 1 (BKV, n.d. b). Lastly, the modern development of transport infrastructure of the early 2000s is worth underlining along with the 1990 opening of Metro line 3 and 2014 opening of Metro line 4 (BKV, n.d. b).

The importance of analysing Budapest’s public transport system is emphasised in the Budapest Urban Development Concept (2011) underlines that transport is not only a matter of change of place. It further emphasises the important role of the following dimensions in the complexity of the issue: “national economic sector, social interaction, shaping the built environment, hazardous facility, environmental problem” (Budapest Urban Development Concept, 2011, p. 189). Based on these we concluded that Budapest’s public transport could well be an area of interest in the field of network science.

2.5.2. Literature on the Budapest's public transport network

The Budapest public transport network is the subject of several research papers containing information on its operation and performance. One study focused on stochastic route planning in public transport, which was aimed at optimising the route selection process by taking into account uncertainties such as travel time variations and interruptions. The research aims to improve the reliability and efficiency of Budapest's public transportation system, ultimately improving passenger experiences and reducing travel time variability by incorporating stochastic elements into the planning process. The results of this study highlight the potential benefits of using stochastic modelling approaches to plan road maps for public transportation networks, including the Budapest system. (Bérczi et al., 2017)

Another study from Gaal and his co-writers (2015) analysed the performance of the Budapest public transport network, particularly in terms of reliability of services, travel time and passenger satisfaction. By studying data on service frequency, delays, and passenger feedback, we provided a comprehensive assessment of network performance indicators. The research identified areas of improvement and proposed strategies to improve the reliability and efficiency of the Budapest public transport system (Gaal et al., 2015). The results of this analysis helped to better understand the strengths and weaknesses of the network, enabling policymakers and transport authorities to make informed decisions and implement measures that could optimise the performance and quality of the Budapest public transport network for the benefit of its users.

2.6. *Physarum polycephalum* slime mould-based approach of network development

Following a thorough and comprehensive review of the most commonly used network science tools, the discussion of the most important aspects of resilience in public transportation and the description of the specifics of Budapest's public transport network, this section introduces the methodology at the centre of the research and its benefits.

Physarum polycephalum is a single-cell organism that moves and eats organic matter in a wet environment (Treo et al., 2006, pp. 1). The mould is an ever-changing network of protoplasm, which can navigate efficiently on path networks, for example in the direction of food sources and better environmental conditions, despite the lack of communication between the individual cells (Kay et al., 2022). This is defined as foraging, which encompasses the first stage of the mesh creation of the mould. What makes *Physarum polycephalum* an exceptional network design planner is the second step of the process, the constant feedback loops of network refinery. Via these feedback loops the mould is capable of strengthening the most efficient routes in its network and destroying the least efficient ones (Kay et al., 2022). This creates a well-designed, resilient network, which maximises the efficiency in covering important attractor points (like food sources) and avoiding dangerous environments and threats.

Utilising the advantages of its network design researchers describe a method called the Physarum solver (Tero et al., 2006), which uses the behaviour of the slime *Physarum polycephalum* to navigate pathway networks. On the basis of the concentration value, the path network is simulated over time, the concentration value changes, and the slime mould-generated network changes. After the road network is formed, the network defined in the Physarum Solver method can be used to solve complex traffic problems such as road design, road search, and traffic network planning (Tero et al., 2006).

Physarum polycephalum is also used in the field of urban design. Papers like Adamatzky and Alonso-Sanz (2011), Adamatzky and Jones (2010) or Adamatzky et al. (2011) discusses cases of applications and potential applications of the mould in redesigning motorway networks in Spain and Portugal, in England and in Mexico, respectively. These studies collectively conclude that the use of these algorithms inspired by slime moulds offers unconventional, yet innovative solutions to optimise road planning (Adamatzky and Alonso-Sanz, 2011, Adamatzky and Jones, 2010 or Adamatzky et al., 2011). This in turn can lead to more economical, resilient and efficient road designs, underlining the potential of using *Physarum polycephalum* in the network design in transportation systems.

3. Network analysis of Budapest’s public transport network

In the forthcoming section of the study, we intend to conduct an analysis of Budapest's public transportation network using a diverse set of methodologies.

3.1. Fundamentals of Budapest’s public transport network

In the subsequent section, we will undertake an assessment of Budapest's public transportation system by employing descriptive and resilience-removal techniques.

3.1.1 Descriptives of Budapest’s public transport network

We are building Budapest’s public transport network by using the general transit feed specification data (bkk.hu, 2023). The data contains Budapest’s trains, metros (including

HÉVs), buses, and trolleybuses in a daily shift; we have removed the all-night services and ships from the data. Simply put, we use G to denote the indirect graph of the public transport network:

$$G = (V, E)$$

where V is the node set representing the stations and E is the edge set representing at least one line that exists between the node pair. To measure the stops next to each other as one, we have created a threshold that aggregates these stops into one.

We have also added weights to the edges by measuring the frequency and the maximum capacity of the transport vehicles. Unfortunately, we could not find any kind of data regarding the flow of passengers, so we used this measurement as a proxy like Lam and his co-authors (2002) did. The created network can be seen on Figure 1.

Figure 1: Budapest's public transport network, source: own figure

The most interesting aspect of this analysis lies in the stability and resilience of Budapest's public transportation network over a 40-week period. This stability is particularly remarkable because it reflects the network's ability to withstand various challenges, such as changes in ridership patterns, external disruptions, and potential service adjustments, without experiencing significant structural or operational fluctuations.

One of the most compelling findings from this analysis is the network's remarkable resilience and robustness. The consistent Beta Index, a key metric for measuring network robustness, remained steady throughout the 40-week period. This implies that the network can withstand changes and disruptions while retaining its core structure. Such resilience is vital for delivering reliable public transportation services, especially during unexpected events or emergencies. The ability to adapt without compromising service quality is a testament to the network's robustness and its capacity to cater to the dynamic needs of the city's inhabitants.

The stability in the number of links and nodes, remaining around 2500 and 5000 during these weeks, is another noteworthy aspect of this analysis. These metrics indicate that the network's fundamental structure, comprising stops and connections, remained consistent. Passengers could rely on a predictable and unchanging network, ensuring that they had access to the same number of stops throughout the observation period. This stability in connectivity is fundamental to public transportation, as it guarantees a level of predictability that is essential for planning journeys and ensuring accessibility for all passengers.

Efficiency and consistency in public transportation are paramount for user satisfaction. Metrics such as Global Efficiency and Average Path Length demonstrated remarkable stability across the 40 weeks. This constancy implies that passengers could travel efficiently between stops without experiencing significant fluctuations in travel distances or times. Consistency in these metrics is crucial for ensuring a dependable and reliable transportation experience. It underlines the network's ability to provide efficient routes regardless of the week, enhancing passenger confidence in the system's reliability.

While the analysis showcased remarkable stability in most metrics, there were minor fluctuations in some indicators. These fluctuations might invite further scrutiny, particularly regarding specific weeks with noticeable changes.

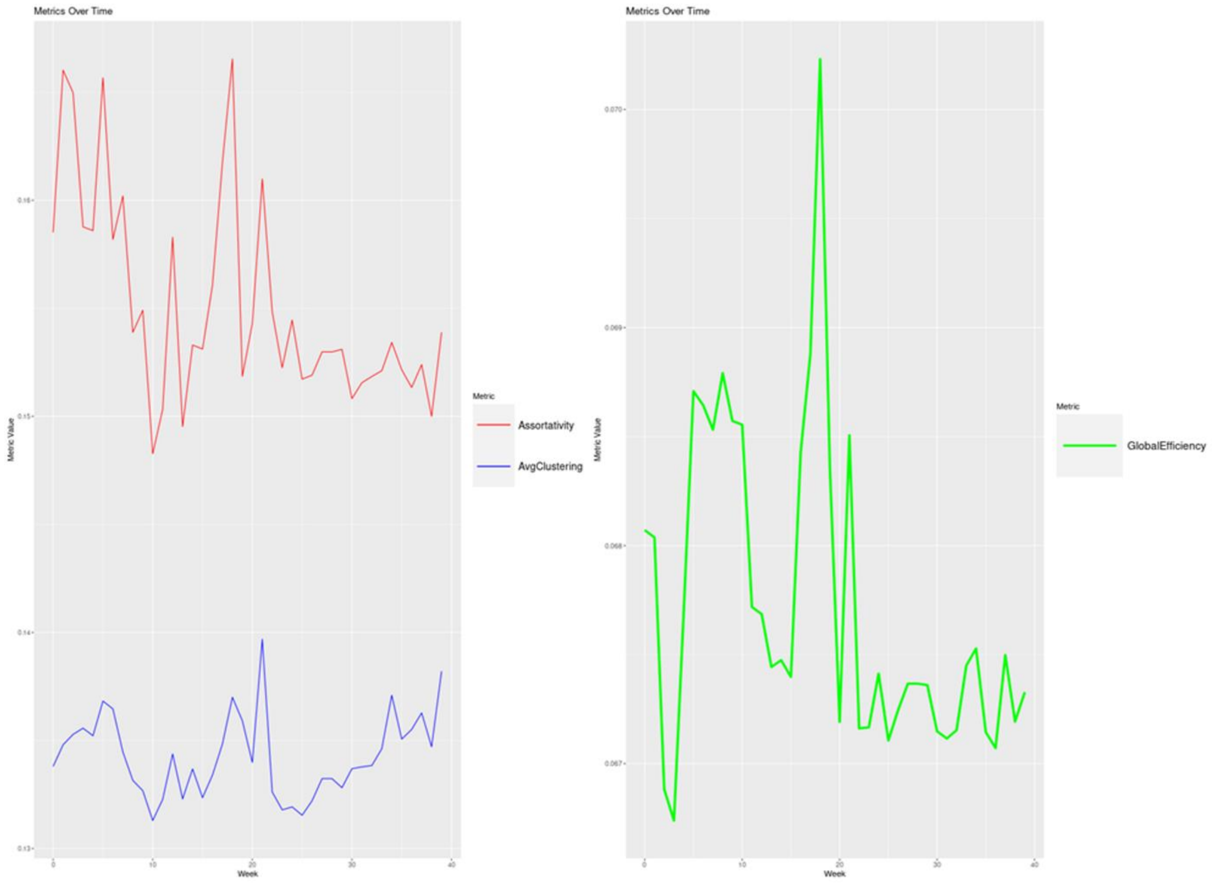

Figure 2: Assortativity, Average Clustering and Global Efficiency measuring during the 40 days, source: own figure

In conclusion, this analysis of Budapest's public transportation network over 40 weeks underscores the network's remarkable capacity to maintain stability, resilience, and service quality. It is a testament to the importance of a robust public transportation system that can adapt to changing circumstances without compromising the quality of service. This bodes well for Budapest's ability to meet the transportation needs of its residents and visitors, even when faced with unexpected challenges. The study also highlights the significance of consistent connectivity, efficient routes, and dependable service in ensuring that public transportation remains the backbone of urban mobility. As cities continue to evolve, the ability to maintain a resilient and stable public transportation network becomes increasingly essential in providing a high quality of life for residents and an excellent experience for tourists.

3.1.2. Clustering measures of Budapest's public transport network

The 0.1295936 clustering coefficient quantifies the existence of a tightly connected group within the city. This measurement shows the degree to which nodes tend to cluster and form local communities or transit hubs, which facilitate smoother passenger transfers and shorter travel times. Also, the walk trap algorithm shows the network's modularity, which was determined to be 0.6707785. This indicates the existence of different communities within the system too. The analysis of modularity helps to identify areas where targeted improvement and resource allocation can be made. These areas were Kispest, Újpest and Városmajor.

3.1.3. Network resilience of Budapest's public transport network, node removal

Robustness analysis is crucial to understanding the resilience of transport systems to failures or disruptions of nodes. High-concentration nodes such as Keleti pályaudvar and Kelenföldi pályaudvar play an important role in maintaining network connectivity. According to our measures, these two had the highest connectivity in our analysis. The fragmentation analysis revealed that different centrality measures affect network fragmentation. Removing nodes based on betweenness centrality (13 nodes), closeness centrality (8 nodes), eigenvector centrality (3 nodes), or degree centrality (6 nodes) resulted in network fragmentation. Random node removal required on average the removal of 56 nodes before the network fragmented. In summary, nodes with high centrality are crucial for maintaining network connectivity.

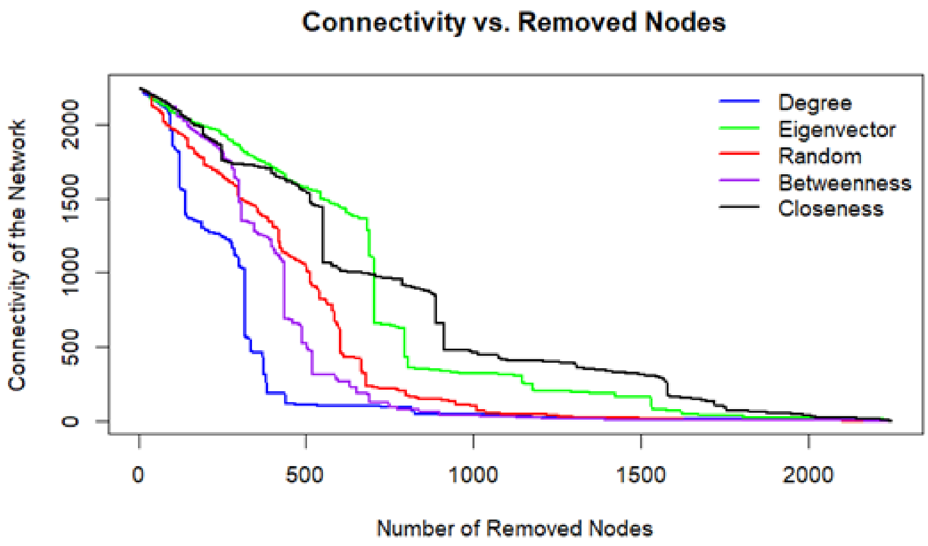

Figure 3: Connectivity vs. removed nodes, source: own figure

Upon considering the recalculation of connectivity following the removal of each individual node, it was found that it has no impact on the resulting output According to our calculations the most important stops and squares by degree centrality are the already mentioned Keleti and Kelenföldi pályaudvar, Déli pályaudvar, Móricz Zsigmond körtér, Deák Ferenc tér, Blaha Lujza tér, Astoria, Örs vezér tere, Kálvin tér, Széll Kálmán tér, Savolyai jenő tér.

Starting with the degree-centrality measurement, which represents the number of connections the nodes have, we see that the network's connectivity decreases as nodes with high centrality are removed. This indicates that several nodes are essential to maintaining the overall flow and connectivity of the network.

Eliminating these highly connected nodes can disrupt flow patterns and cause fragmentation within the transport system. Moving on to the centrality measure betweenness, which identifies nodes as important intermediaries in traffic flow, we observe a similar trend. With the removal of nodes with high centrality, network connectivity decreases. This indicates that these intermediary nodes play an important role in facilitating the transport of passengers and vehicles between different parts of the network.

The removal of these substances causes congestion or inefficiency in the transport system and disrupts flow patterns. Secondly, considering the random sequence of node removal, we find a relatively constant decrease in connectivity. This shows that even random deletions of nodes without considering their centrality measures can have a negative impact on the overall network flow. It highlights the interconnectedness of the transport system, which may interfere with the smooth operation of the network by eliminating any node.

Moving to the eigenvector centrality measurement, which identifies nodes with connections to other highly connected nodes, we see a similar trend to the previous centrality measurement. The removal of nodes with a high inherent vector slowly reduces network connectivity, indicating that these nodes significantly improve the overall flow and robustness of the transport system.

Finally, when we examine the measure of the closeness of the centrality, we notice that nodes that are easy to reach other nodes in the network have a decrease in connectivity when nodes that are close to the centrality are eliminated. This suggests that central nodes in terms of proximity play an important role in maintaining effective and accessible transport routes. If you remove them, travel time may be longer, or passenger accessibility is limited. Overall, the result of the mapping emphasised the critical nodes of Budapest's public transport network and their importance in maintaining the flow and connectivity of the system.

3.1.4. Network resilience of Budapest’s public transport network, edge removal

By systematically removing transportation routes, we can assess the network's ability to maintain its structure and functionality in the face of such disruptions. Analysing the network's response to route removals helps identify critical edges or vulnerable areas that, if disrupted, could have a substantial impact on the network's overall performance or functionality.

We had set the weights of these transports in decreasing order and started to remove the routes one by one, starting with the highest value. According to our calculation, the most valuable routes are the M1, M2, M3, and M4 tram numbers: 1, 4, 6, 3, 17, and bus numbers: 9, 20E, and 7E.

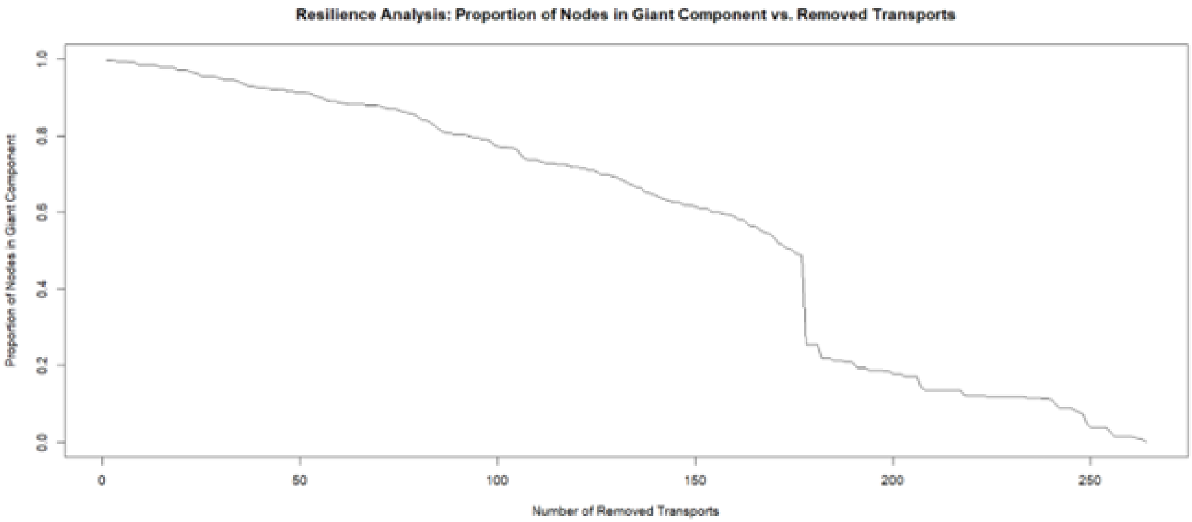

Figure 4: Resilience analysis, source: own figure

The plot of the proportion of edges in the giant component versus the number of removed transports visualises the resilience of the network as the transportation infrastructure is gradually disrupted. It shows how the giant component, which represents the largest connected cluster of nodes, evolves as more and more transports are removed from the network. This plot helps to understand the impact of transport removal on the overall connectivity and robustness of the network, providing insights into the network's resilience to transportation disruptions. The analysis of route removal reveals that the public transportation system remains stable with the gradual removal of routes, showing minimal impact on network performance. However, a notable decrease in the proportion of nodes in the giant component occurs after around 150 route removals, indicating a significant decline in network stability beyond this threshold. This highlights the importance of preserving an adequate number of routes to maintain connectivity and ensure the functionality of the public transportation system.

3.2. *Physarum polycephalum* mould-based network in Budapest

In addition to analysing the underlying network structure of Budapest's public transport system with a distinct focus on network resilience, we implemented a computer simulation of the spread of the *Physarum polycephalum* mould to test the efficiency of the current network against the mould-based network. First, we make a brief introduction of this imitation model, then we discuss the initial setup, followed by the description of implementation of the two stages of the model, ending with a summary of our main findings.

3.2.1 Introducing the *Physarum polycephalum* imitation model

The second part of our analysis focused on an alternative network for the public transport system of Budapest. As previously emphasised network analysis is an effective, but under-used tool of urban planning, which helps to confront unforeseen problems and events in the future. City planning can span from a few years to whole decades ahead, creating a long-lasting and stable public transport network requires thorough preparation and large financial commitment. Thus, abiding by the concept of a flexible, and preferably polycentric public transport network is an important investment in the lifetime of a major city, such as Budapest. The next section of our analysis takes a leap from the current public transport network and focuses on a possible future development of Budapest's most essential public transport asset, the metro network.

Creating an alternative network was performed by mimicking the growth patterns of the slime mould *Physarum polycephalum*, based on the foundations laid by Kay et al. (2022). After a brief collaboration with the authors of the article, we recreated a modified version of the growth and expansion of *Physarum polycephalum* via a two-stage imitation. The first stage consisted of modelling the biased foraging of the mould and its biased meshing, while in the second stage, we implemented network refinery.

3.2.2. Setting up the first stage of the *Physarum polycephalum* imitation model

In the first stage, we closely followed the work of Kay et al. (2022) and applied a modified version of their agent-based model aimed at modelling the organic network creation of the mould. In the first step two important preliminary elements of the simulation have been defined: the starting point of the simulated mould and a predefined set of attractor points. The first element is substituting the nucleus of the mould from which the biased foraging would start at time zero, and the second element substitutes the oats or other food sources to which the mould is attracted in order to create their mesh.

The starting point of the simulated mould for our experiment was selected by our descriptive network analysis. We choose eigenvector centrality in the preexisting public transport network to capture the characteristics of the network to the greatest extent. The centremost point was Blaha Lujza tér, which became the centre of the foraging of the simulated mould. Here it is worth mentioning that the simulation was almost identical for the possible starting points based on all three centrality measures.

We had access to confidential, anonymized, and aggregated data pertaining to individuals' mobility patterns within the urban landscape of Budapest from Telekom. The city has been subdivided into hexagonal regions, each spanning 100 square metres. These hexagons enable us to monitor and analyse individuals' activities within these areas, specifically focusing on those who spend a duration exceeding 20 minutes within a given hexagon.

The set of attractor points were defined with the help of this dataset. We selected the 100 most frequently visited hexagons then narrowed down the sample to meet the base criteria of the attractor points: they should emulate potential public transport hubs that cover the most frequently visited places. Figure 5 displays the centre of the 100 most frequented hexagons by our sample population between July 2020 and June 2021. We decided to include this wide range of frequently visited places in order to acquire a reliable pool of potential public transport hubs after cleaning out the irrelevant points. The used dataset contains geographic information on the places most visited by the sample population, stored in hexagons.

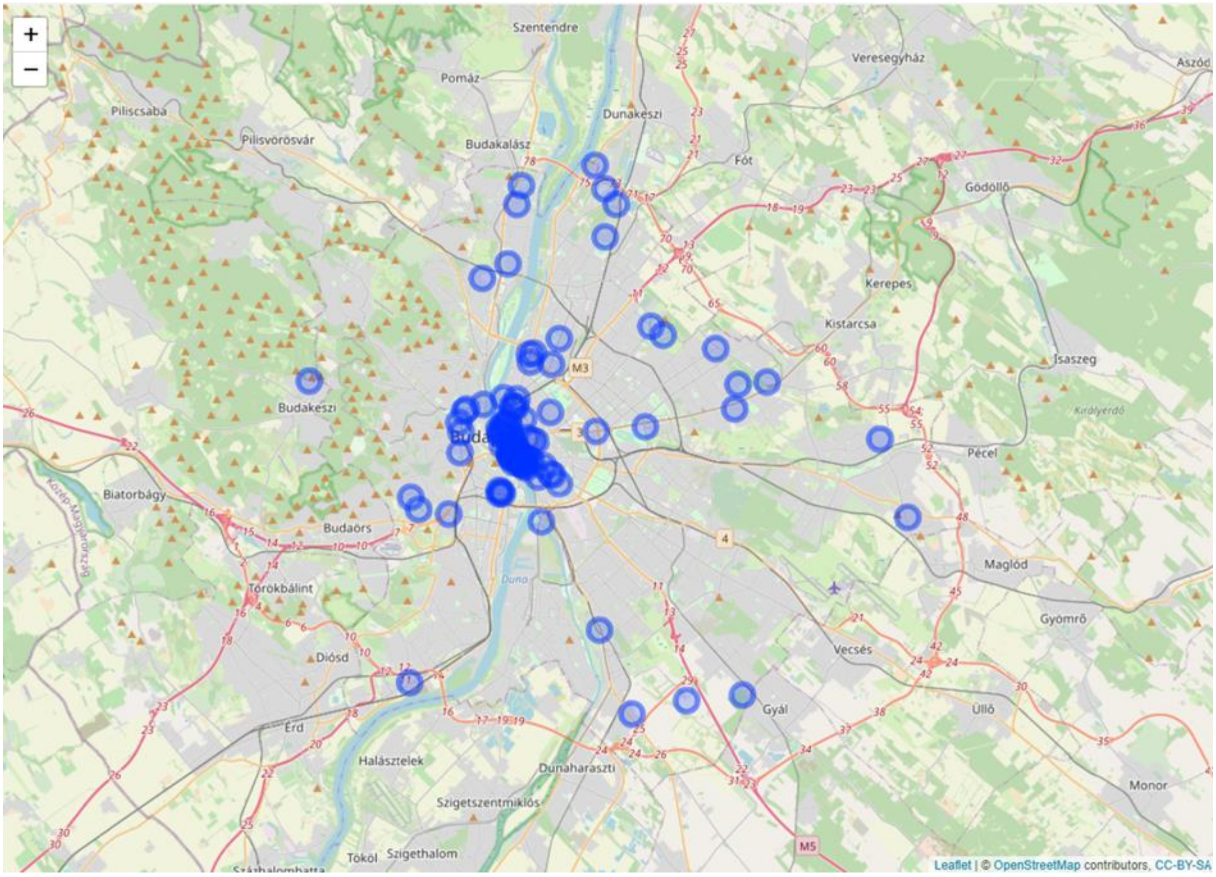

Figure 5: The coordinates of the centre of the 100 most frequented hexagons, source: Telekom data, own figure

Data cleaning for our project served three main purposes. Purpose 1 was to ensure that the selected public transport hubs embody the most sought-after destinations to make the network of the *Physarum Polycephalum* as relevant as possible. Purpose 2 was to merge popular hexagons in close proximity to each other. Purpose 3 was to refine the network by removing sample-specific hexagons and manually adjusting hexagons to match their closest current public transport hub coordinates to ensure a better comparability between the current public transport network and the *Physarum Polycephalum* network. Sample-specific hexagons include hospitals and other medical institutions with similar activities during the pandemic, which are overrepresented in our sample as the Covid-19 epidemic was present throughout the entire period, with differing intensity.

We narrowed down the pool of hexagons in 3 main steps, each step aiming to achieve a purpose described above, with their respective numbers. Step 1 consisted of throwing out hexagons located on the peripheral parts of Budapest. This step was crucial to filter out every hexagon that while technically belonging to Budapest is irrelevant to the scope of the current study. In this step we filtered out every hexagon with latitudes south of 47.42 or north of 47.56, and with longitudes west of 19.00 or east of 19.14. After this step 76 out of the 100 hexagons remained, these can be seen on figure 6.

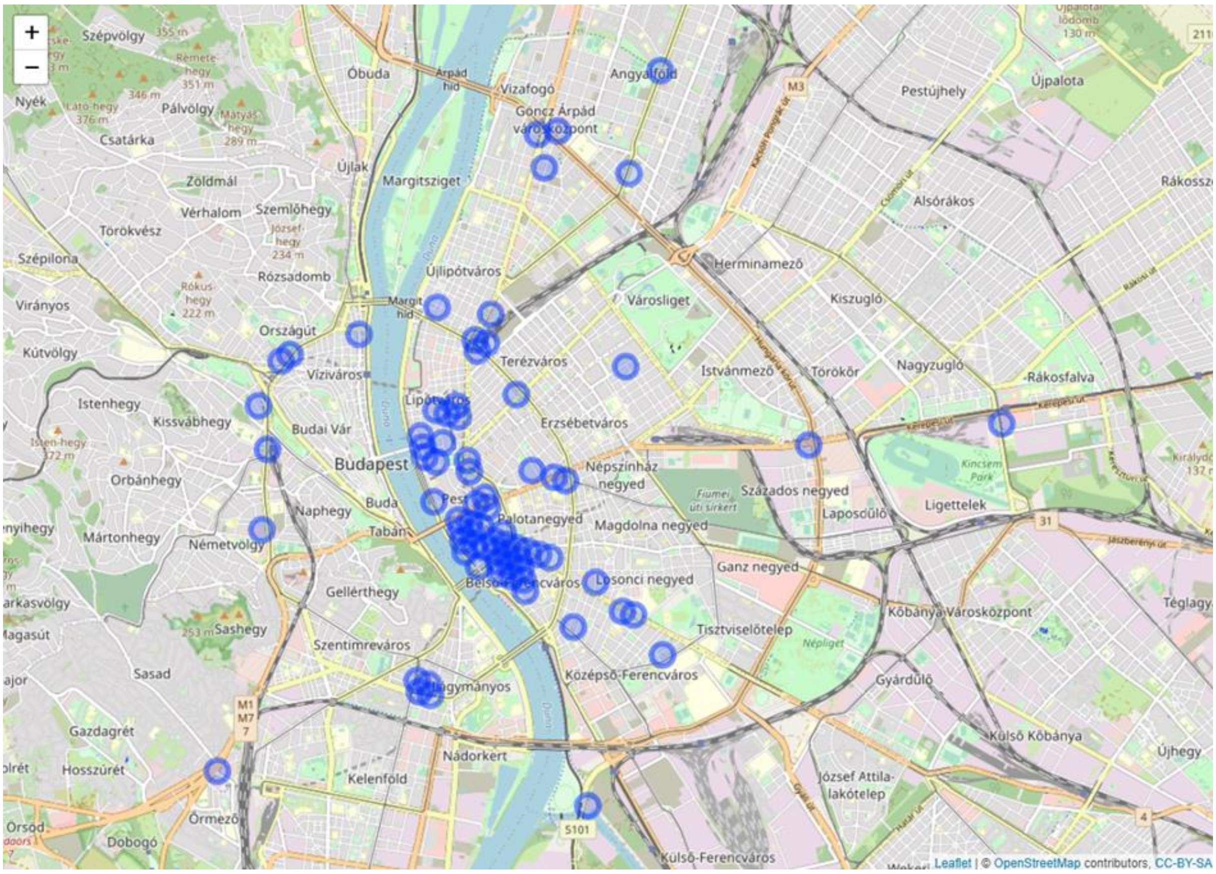

Figure 6: The coordinates of the centre of the most frequented hexagons (76) after the first step of data filtering, source: Telekom data, own figure

Step 2 consisted of cleaning the pool of the most frequented hexagons by merging stations that were close to each other and made up one main potential transportation hub. In this step we deleted unnecessary hexagons in close proximity to at least another one, keeping only one of them in the dataset. We defined close proximity as hexagons being in a 0.5-kilometre radius around each other, creating clusters containing up to 9 hexagons. This created 30 clusters with an average of 4.6 hexagons in them. The most frequented hexagons became the new pool of possible public transport hubs after step 2, visualised on figure 7.

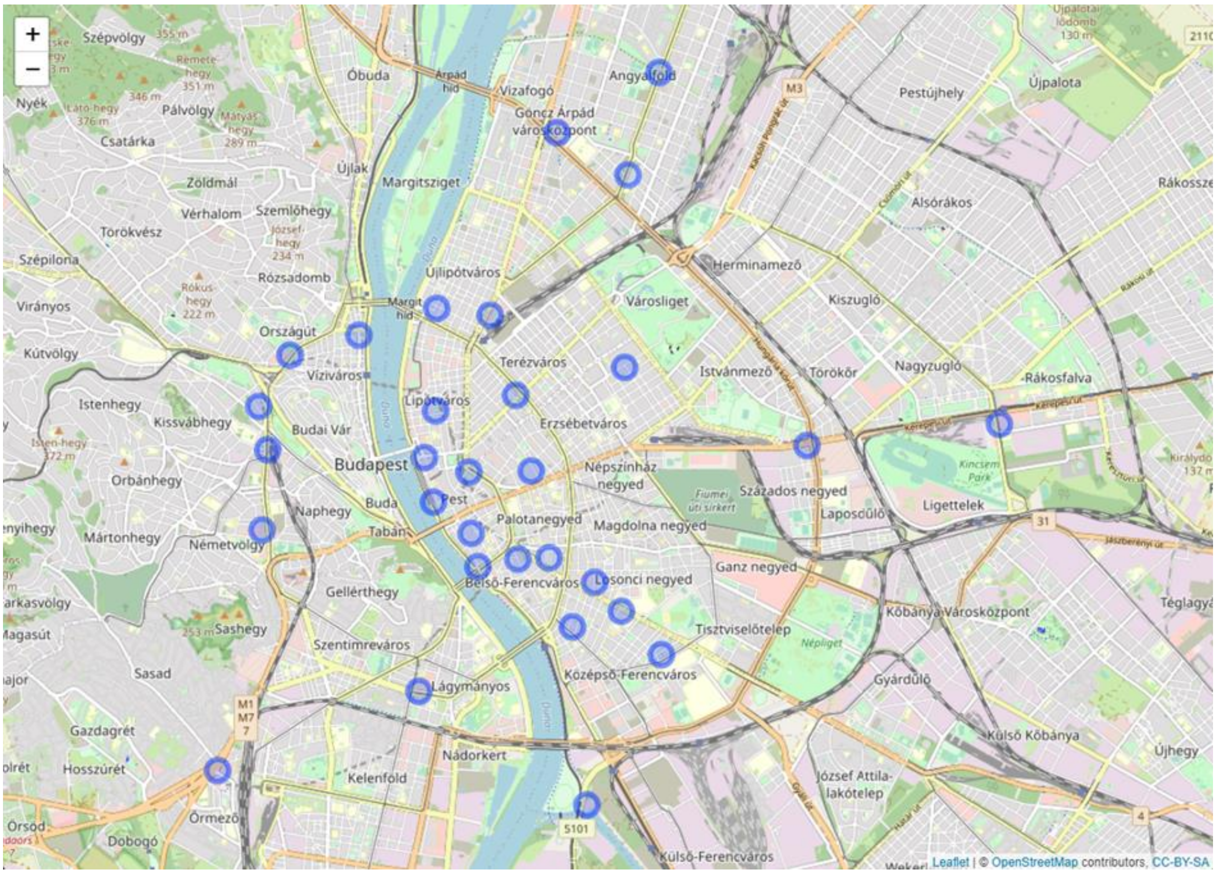

Figure 7: The coordinates of the centre of the most frequented hexagons (30) after the second step of data filtering, source: Telekom data, own figure

Finally, step 3 consisted of a manual data refinery by locating and removing hexagons that appeared in our data with high probability due to sampling bias and adjusting hexagon coordinates to better represent the centres of the potential public transport hubs. In this step we filtered out 4 hexagons, all containing or in the proximity of hospitals or other medical institutions. After that we made greater adjustments to the position of 15 hexagons, and some smaller adjustments to a further 4. After step 3 of our data cleaning only the most relevant frequented hexagons remained, which now provide an adequate basis for the mesh creation of the *Physarum Polycephalum* model. The finalised pool of potential public transport hubs is shown on figure 8, with the remaining 26 points of interest.

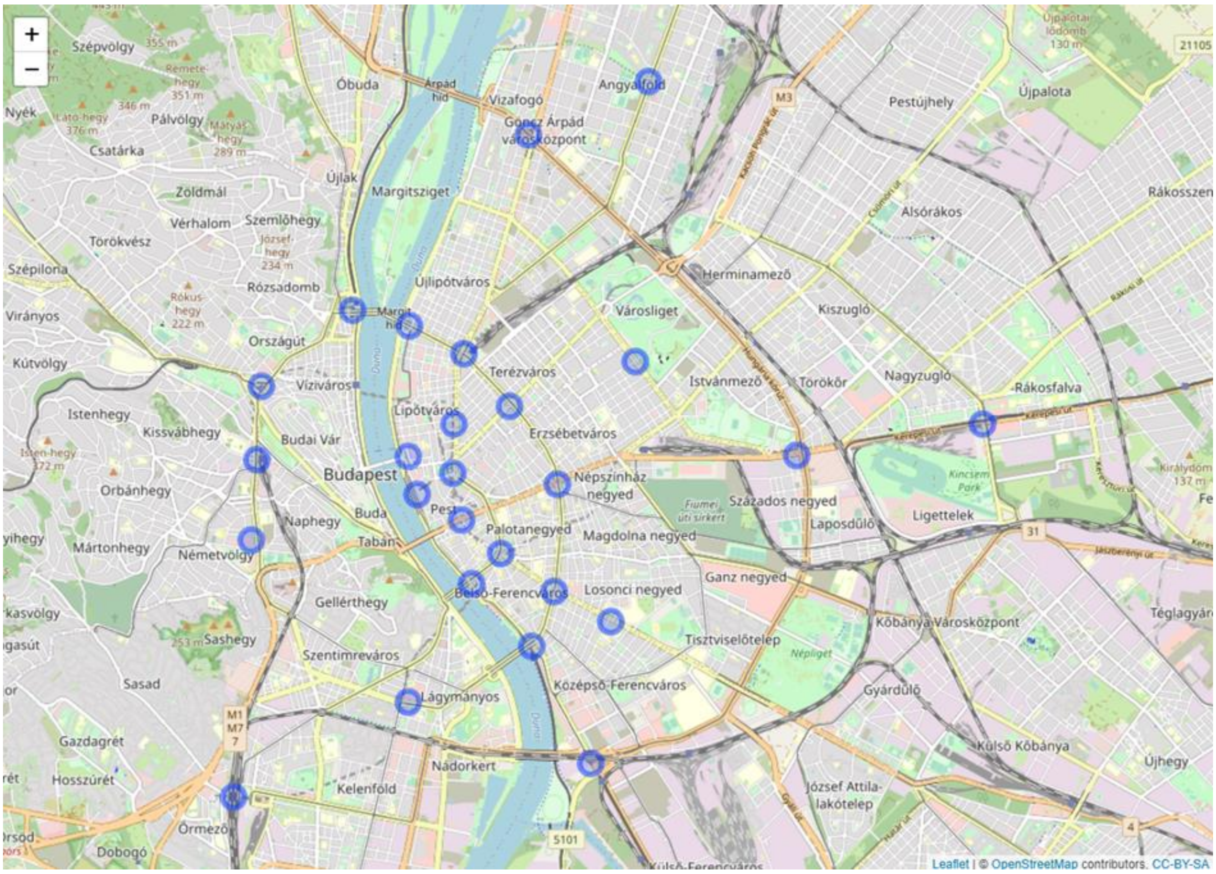

Figure 8: The coordinates of the centre of the most frequented hexagons (26) after the third step of data filtering, source: Telekom data, own figure

3.2.3. Simulating the first stage: biased meshing

As discussed above, the *Physarum polycephalum* is a naturally excellent network creator in terms of network resilience, and part of this success lies in the primer foraging mechanisms behind its growth. As nothing in nature is truly deterministic by only observable factors, the model starts by defining a stochastic element which counts for the factors one cannot control for, and a more tangible deterministic component (Kay et al, 2022).

The stochastic component was created by a one-dimensional Perlin noise algorithm. The algorithm's main advantage lies in making the random walk simulation smoother. It works by generating a value from a predefined range while also storing data on previously generated numbers (Kay et al, 2022). This results in a time-dependent number generation which leads to a visually more appealing, smoother outcome. As the number generation is affected by non-random components (time and previous location), this algorithm defines a pseudo-random walk. The deterministic component is computed based on the square of the attractiveness of every attractor point predefined at the beginning of the simulation, inversely proportional to the distance between the ever-growing mould and the attractor points (Kay et al, 2022).

We used an equiaxial 800 × 800-pixel simulation space with a total of 26 attractor points, with Blaha Lujza tér as the chosen centre for the nucleus. The attractor points' distances were normalised to fit between 0-800, maintaining a relative distance from each other and a close approximation of the real-life location of the chosen transportation hubs. This created a denser point concentration in the central regions and a sparser point concentration in the peripheral regions of Budapest, stemming from the method we used to determine the attractor points themselves in the preparation phase.

As to catch the characteristics of a *Physarum polycephalum* mesh creation more realistically we also followed the approach of Kay et al. (2022) in making the attractor points colonizable for the mould. This meant that after a certain threshold was met, the attractor point lost its ability to attract other branches of the mesh and started to emulate the original nucleus in its fundamentals. The colonised attractor points became new starting points of foraging and further mesh creation, inherently starting from and supplementing the preexisting network. The simulation ended when the last attractor point had been colonised by the network. Figure 9 shows the progression of the mesh creation. The different snapshots were taken every 20 seconds starting from the 20th second, until the simulation ended after 3 minutes and 52 seconds. The figure displays the first five and the final state of the process. Appendix figure 1 shows an overlap of the *Physarum polycephalum* network and Budapest's public transport network.

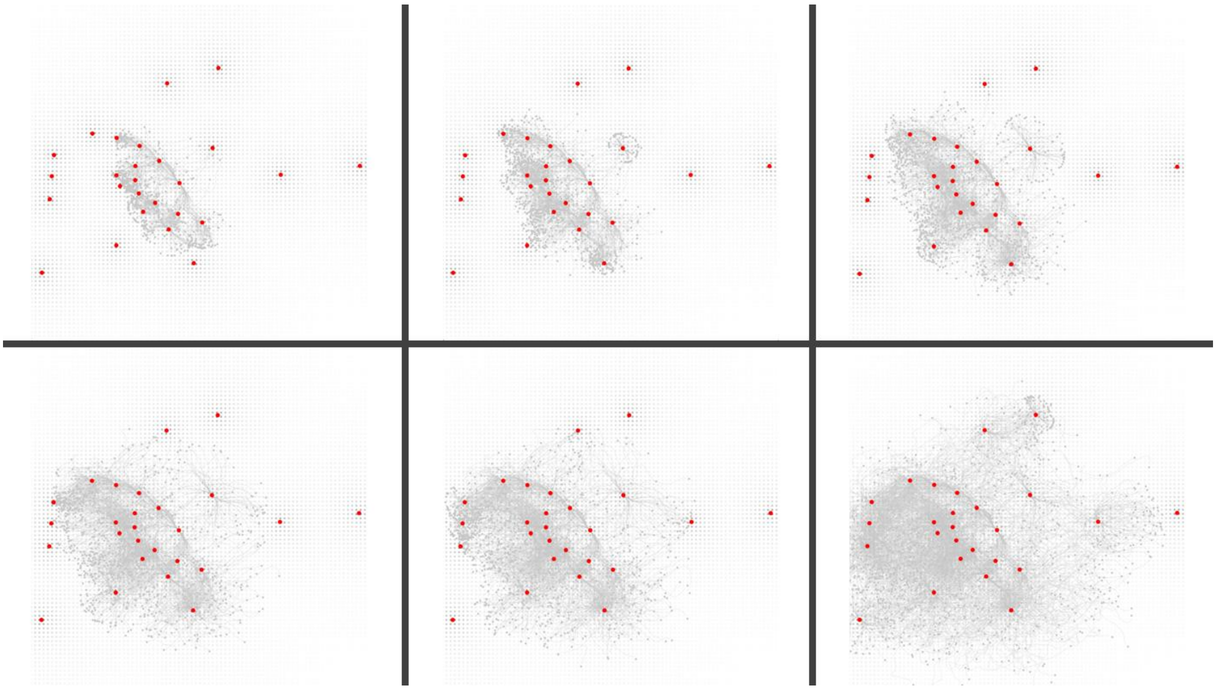

Figure 9: Mesh creation process. The panel displays screenshots after 20, 40, 60, 80, 100 and 232 seconds from left to right, from the upper, then the lower row. Source: own figure

3.2.4. The second stage: adaptive network refinery

The second stage of the model encompassed an adaptive network refinery, which corresponds to the mould's ability to strengthen the most efficient routes and destroy the least efficient ones.

After the simulation ended, we created a proximity table from the cellular morphology of the last stage of the imitated meshing. We recorded the x and y coordinates of the simulated mould at every three frames it travelled throughout the 232 seconds the simulation took place, these were labelled as trail nodes. This resulted in a list of over 350 thousand trail nodes, which, in addition to the 26 attractor points labelled as food nodes, created a total of 350,782 nodes. Each node had three corresponding variables: an id, x coordinates of the nodes (the latest point the mould reached through foraging), and their respective y coordinates, in the uniaxial 800 x 800-pixel space.

We then visualised these points in a 3D modelling program, in a Rhino 7 environment and computed a shortest walk-calculation from each attractor point. A shortest-walk network is the most cost-effective network that can connect the main potential transportation hubs or attractor points). It can be calculated using a pre-specified set of edges, defined as the connectivity parameter of the model (Kay et al., 2022). We computed shortest walk-calculations with differing connectivity parameters, from 1 to 6, displayed on Figure 10. After comparing the results with the density of Budapest's public transport network, the network created with the connectivity parameter set to 5 was chosen for the most relevant comparison.

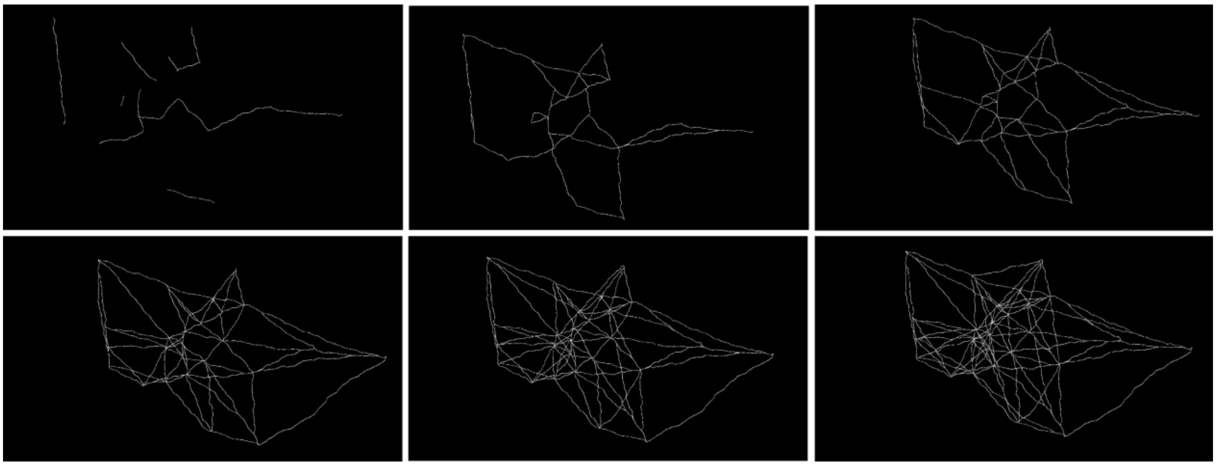

Figure 10: Refined networks. The panel displays refined networks with connectivity parameters of 1, 2, 3, 4, 5 and 6 from left to right, from the upper, then the lower row. Source: own figure

Finally, we extracted a two-dimensional network from the 3D modelling environment, with an extra step to ensure comparability. The final shortest-walk network consisted of the 26 food nodes and 130 polylines. A polyline is an object that "consists of line segments that are joined end to end" (*Polyline | Rhino 3-D Modeling*, n.d., pp. 1). An exemplary representation of the polylines is depicted on Appendix figure 2. Here an alteration in the network was needed as the network analysis packages, we used could not run analyses on polylines; we needed to deconstruct polylines to lines.

This created a trade-off between including many dividend points that are connected to only two other points but can most effectively imitate the shape of the different polylines, or include only a smaller set of dividends points, but risk losing comparability with the more detailed Budapest data. In order to maintain the highest possible comparability between the created shortest-walk network and Budapest's public transport network and to not lose the initial curve of the refined networks at the same time, every polyline was divided into 4 equal-distance lines. Figure 11 displays the final network created by the two stages of *Physarum polycephalum* modelling.

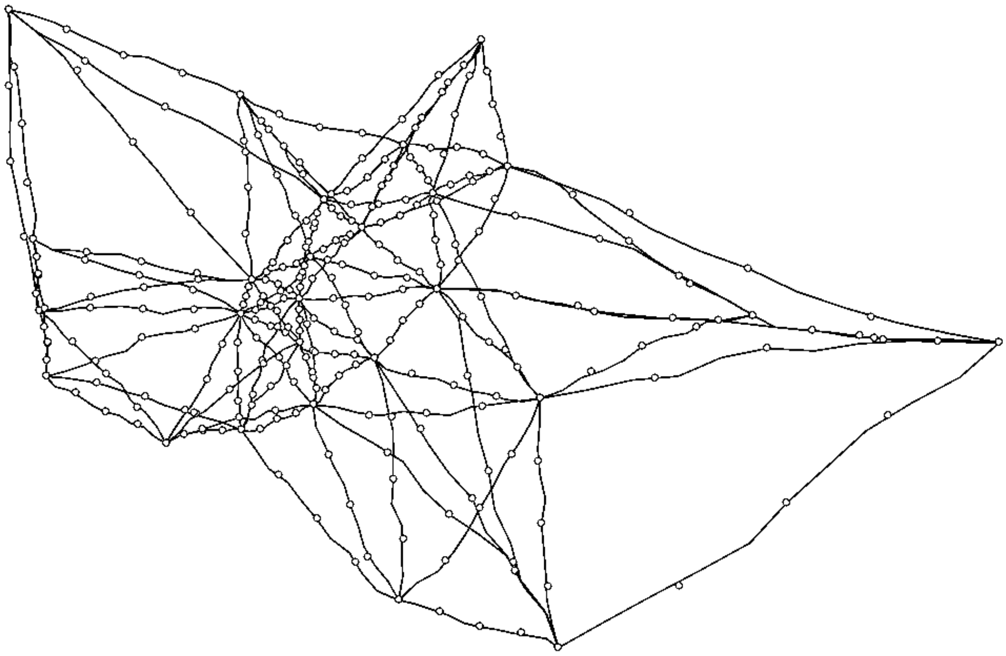

Figure 11: The final *Physarum polycephalum* network, source: own figure

3.2.5. Comprehensive analysis of the *Physarum polycephalum* network

In the last section of our analysis, we provide some of our main insights based on the comparison of the simulations and the real-life data. The simulation of the spread of *Physarum Polycephalum* allowed us to create a network with traditional edges and nodes. In this context, edges represent connections within the simulated *Physarum* network, akin to paths for resource or information flow. Nodes symbolise key locations where these connections converge, or critical resource allocation decisions occur.

The mould-inspired network showcases an extraordinarily low Beta-index (0.0003820725), emphasising its remarkable efficiency in resource distribution. A low Beta Index suggests that this network excels at balancing loads, a quality that public transportation networks can aspire to achieve. Further emphasising its efficiency, the mould network consistently maintains a low diameter (20), signifying short path lengths. These short paths underscore the network's ability to swiftly respond to changing conditions and optimise resource allocation.

Global Efficiency (0.1443908), a crucial metric in network analysis, stands out in the mould-inspired network. This measure reflects the network's proficiency in efficient information transfer. With a high Global Efficiency value, the mould network excels in transmitting data or resources across its network, underscoring the efficiency of the network. Intriguingly, the mould network exhibits negative assortativity (-0.2130905), suggesting adaptability in node connections. This flexibility in forming connections based on need rather than uniformity mirrors the slime mould's adaptability and resource efficiency, a characteristic that public transportation networks could harness.

The importance of certain nodes in the mould network is highlighted by its high Mean Betweenness (1173.528). These nodes act as critical hubs, facilitating the efficient flow of resources or information. This critical node identification is vital for the network's smooth functioning. Lastly, the mould network maintains a notably short average path length (8.720578), reflecting its efficiency in path traversal.

In summary, the mould-inspired network exemplifies the incredible capabilities of nature's design. Its resource efficiency, adaptability, negative assortativity, critical node identification, and path traversal efficiency provide a more cost-efficient way than that of the original public transport network of Budapest.

4. Discussion

Our findings suggest that the public transport network in Budapest should be investigated using network science tools. Based on our results the analysis of Budapest's public transport network reveals important insights into its structure, connectivity, and resilience. The network consists of a moderately connected system with a large number of nodes and edges, facilitating efficient travel routes and shorter travel times. The clustering coefficient and modularity measurements indicate the presence of tightly connected groups and different communities within the network, suggesting the existence of transit hubs and areas for targeted improvements.

When examining the network's resilience through node removal, high-concentration nodes such as Keleti pályaudvar and Kelenföldi pályaudvar are found to play a crucial role in maintaining network connectivity. Removing nodes based on different centrality measures leads to network fragmentation, emphasising the importance of highly connected nodes for overall flow and connectivity. Analysing edge removal in the network reveals the most valuable routes in terms of weights. The gradual removal of routes has minimal impact on the network's performance, indicating its stability and resilience to transportation disruptions.

The *Physarum polycephalum* network formation provides a unique way to understand resource optimization, adaptability, and hub identification in complex systems. The resilient nature of *Physarum polycephalum* and the previously introduced implications it provides for urban planning suggest that the insights from the simulation can be applied to real-world scenarios, such as improving public transportation networks or enhancing supply chain efficiency in Budapest. The network performed very highly in the area of resource efficiency, adaptability, negative assortativity, critical node identification, and path traversal efficiency. These attributes suggest that implementing redesign choices inspired by the mould-based network could improve the efficiency of the current public transport system.

There are some key differences in the two networks that should be taken into account. Budapest's network operates in a dynamic real-world urban setting, making it susceptible to changing conditions, while the nature-inspired network exists in controlled conditions. Furthermore, the simulated conditions are independent of the geographical and existing infrastructural differences of different parts of Budapest. Robust testing and adaptation of insights are essential to apply findings from one network to the other.

This analysis provides a deeper understanding of network principles, which extends beyond public transportation to various domains. By embracing shared principles while acknowledging differences, we can create more robust and adaptable networks in an ever-evolving world. A collaboration with BKK could produce mutually beneficial results that could both greatly help urban planning in Budapest for the coming decades and improve *Physarum polycephalum*-based research. Additionally, conducting similar studies across multiple European cities and comparing the results would offer a more comprehensive understanding of efficient mould-based network-planning and enable more effective strategies for optimising public transportation systems in urban areas.

5. Conclusion

Our paper concludes that applying network analysis tools to Budapest’s public transport system yields interesting results. In our paper, we summarised the fundamentals of network analysis as well as the most widely used analytical tools for spatial networks. Based on the pre existing literature we highlighted the importance of managing public transport systems with tools provided by the field of network science, through identifying key stations, connections or even potential areas of network expansion and improvement. Our paper also contains a brief theoretical background behind the *Physarum* solver, a network pathway optimization tool based on the behaviour of the slime mould *Physarum polycephalum*.

In our study we showed that Budapest’s public transport system is a metric spatial network, thus interconnectedness in the central parts of the city is inherently higher, resulting in increased clustering and shorter average path lengths. The importance of measuring and improving resilience in such a system lies in the increased effectiveness in pre-emptive disaster management, optimal resource allocation, and the overall performance of the network. After a descriptive analysis on Budapest's public transport system a simulation of *Physarum polycephalum* mesh creation and network refinery was conducted, creating an alternative network of public transportation for the most frequented transportation hubs in Budapest.

Our analysis comparing Budapest's public transportation network with a nature-inspired network derived from *Physarum Polycephalum* revealed valuable insights for enhancing the city's public transportation system. Redesigning the current network in favour of a more polycentric public transport network more similar to the mould-based one would benefit the overall effectiveness of the network in terms of resource efficiency, adaptability, negative assortativity, critical node identification, and path traversal efficiency. Our paper concludes that the usage of *Physarum Polycephalum* modelling in urban design is an effective way to increase the efficiency of current public transport networks similar to Budapest's.

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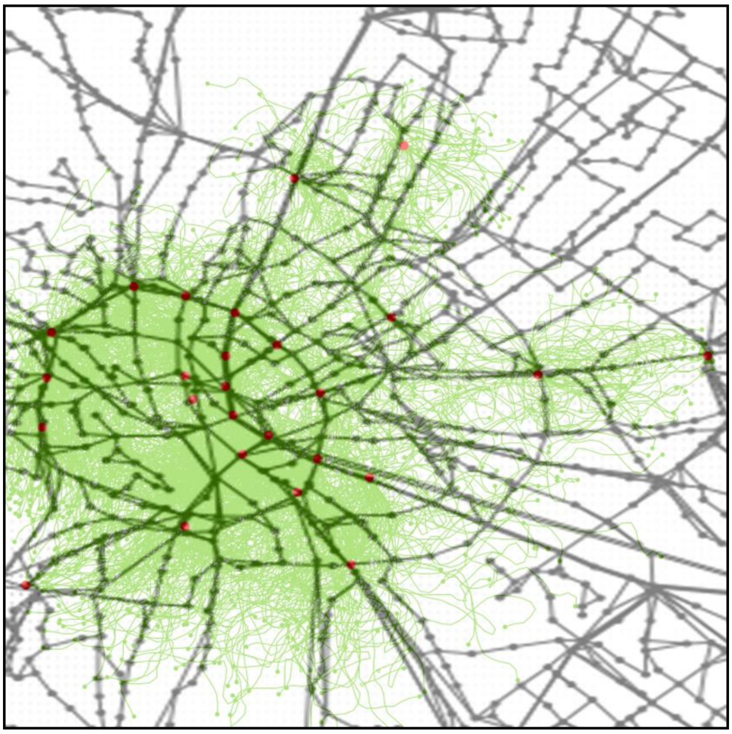
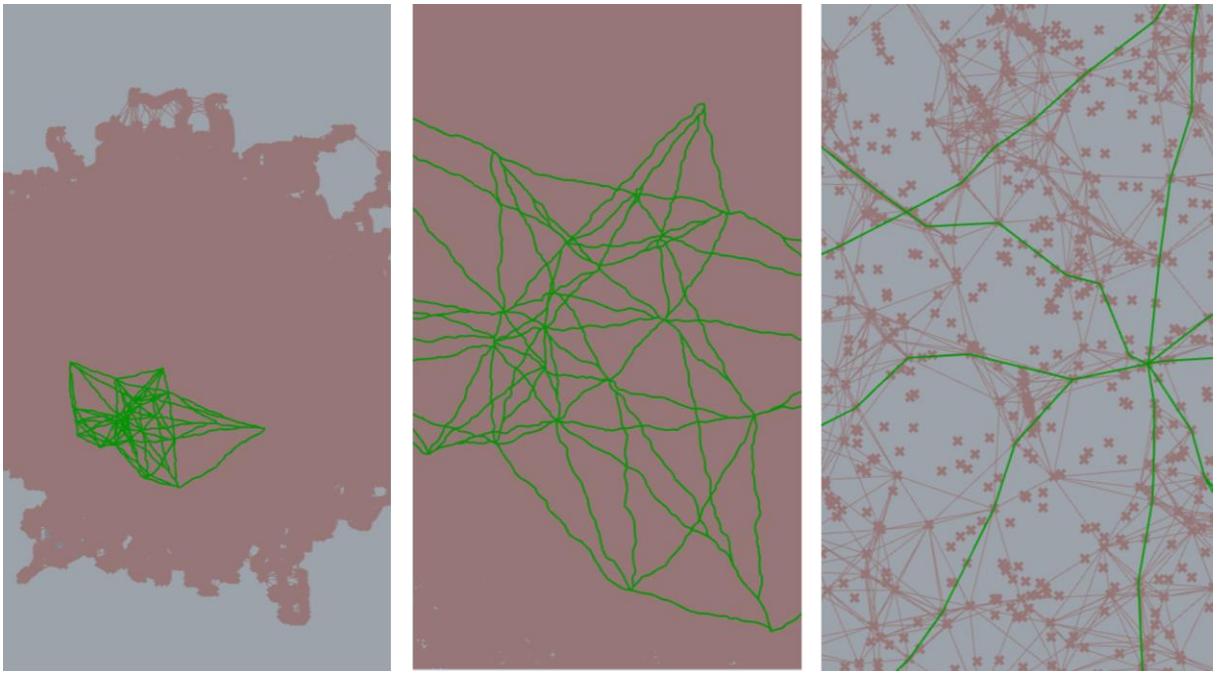
Appendix



Appendix figure 1: The simulation of the mesh of *Physarum polycephalum (green)* with an underlying map of Budapest's public transport network (black), source: own figure



Appendix figure 2: Visualisation of the mesh created in stage one, composed of polylines. The panel shows the mesh created (pink points and connecting lines) and the shortest-walk network with connectivity parameter set to 5 (green points and connecting polylines), increasingly zoomed in from left to right

source: own figure