# Solved tasks

## Task 1

### Dot product

#### Dot product formula

$$1 \cdot 5 + 2 \cdot 4 + 3 \cdot 3 + 4 \cdot 2 + 5 \cdot 1 = 35$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = 35$$

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## Task 2

### 3D cross product

#### Cross product formula

$$\begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

#### X component calculation

$$2 \cdot 1 - 3 \cdot 2 = -4$$

#### Y component calculation

$$3 \cdot 3 - 1 \cdot 1 = 8$$

#### Z component calculation

$$1 \cdot 2 - 2 \cdot 3 = -4$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \\ -4 \end{pmatrix}$$

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# Task 3

## 7D cross product

### 7D Cross Product Formula

$$\begin{pmatrix} a_1b_3 - a_3b_1 + a_2b_6 - a_6b_2 + a_4b_5 - a_5b_4 \\ a_2b_4 - a_4b_2 + a_3b_0 - a_0b_3 + a_5b_6 - a_6b_5 \\ a_3b_5 - a_5b_3 + a_4b_1 - a_1b_4 + a_6b_0 - a_0b_6 \\ a_4b_6 - a_6b_4 + a_5b_2 - a_2b_5 + a_0b_1 - a_1b_0 \\ a_5b_0 - a_0b_5 + a_6b_3 - a_3b_6 + a_1b_2 - a_2b_1 \\ a_6b_1 - a_1b_6 + a_0b_4 - a_4b_0 + a_2b_3 - a_3b_2 \\ a_0b_2 - a_2b_0 + a_1b_5 - a_5b_1 + a_3b_4 - a_4b_3 \end{pmatrix}$$

### Component 0 Calculation

$$2 \cdot 4 - 4 \cdot 6 + 3 \cdot 1 - 7 \cdot 5 + 5 \cdot 2 - 6 \cdot 3 = -56$$

### Component 1 Calculation

$$3 \cdot 3 - 5 \cdot 5 + 4 \cdot 7 - 1 \cdot 4 + 6 \cdot 1 - 7 \cdot 2 = 0$$

### Component 2 Calculation

$$4 \cdot 2 - 6 \cdot 4 + 5 \cdot 6 - 2 \cdot 3 + 7 \cdot 7 - 1 \cdot 1 = 56$$

### Component 3 Calculation

$$5 \cdot 1 - 7 \cdot 3 + 6 \cdot 5 - 3 \cdot 2 + 1 \cdot 6 - 2 \cdot 7 = 0$$

### Component 4 Calculation

$$6 \cdot 7 - 1 \cdot 2 + 7 \cdot 4 - 4 \cdot 1 + 2 \cdot 5 - 3 \cdot 6 = 56$$

### Component 5 Calculation

$$7 \cdot 6 - 2 \cdot 1 + 1 \cdot 3 - 5 \cdot 7 + 3 \cdot 4 - 4 \cdot 5 = 0$$

### Component 6 Calculation

$$1 \cdot 5 - 3 \cdot 7 + 2 \cdot 2 - 6 \cdot 6 + 4 \cdot 3 - 5 \cdot 4 = -56$$

$$\begin{pmatrix}1\\2\\3\\4\\5\\6\\7\end{pmatrix} \times \begin{pmatrix}7\\6\\5\\4\\3\\2\\1\end{pmatrix} = \begin{pmatrix}-56\\0\\56\\0\\56\\0\\-56\end{pmatrix}$$

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# Task 4

## 3D triple product

### Triple product formula

$$(\vec{a} \times \vec{b}) \cdot \vec{c}$$

### Cross product formula

$$\begin{pmatrix}a_y b_z - a_z b_y\\a_z b_x - a_x b_z\\a_x b_y - a_y b_x\end{pmatrix}$$

### X component calculation

$$5 \cdot 155222 - 6 \cdot 8 = 776062$$

### Y component calculation

$$6 \cdot 7 - 4 \cdot 155222 = -620846$$

### Z component calculation

$$4 \cdot 8 - 5 \cdot 7 = -3$$

### Dot product formula

$$1 \cdot 776062 + 2 \cdot -620846 + 3 \cdot -3 = -465639$$
$$\begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}4\\5\\6\end{pmatrix} \times \begin{pmatrix}7\\8\\155222\end{pmatrix} = -465639$$

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# Task 5

## 7D triple product

### Triple product formula (7D)

$$(\vec{a} \cdot (\vec{b} \times \vec{c}))$$

### 7D Cross Product Formula

$$\begin{pmatrix} a_1b_3 - a_3b_1 + a_2b_6 - a_6b_2 + a_4b_5 - a_5b_4 \\ a_2b_4 - a_4b_2 + a_3b_0 - a_0b_3 + a_5b_6 - a_6b_5 \\ a_3b_5 - a_5b_3 + a_4b_1 - a_1b_4 + a_6b_0 - a_0b_6 \\ a_4b_6 - a_6b_4 + a_5b_2 - a_2b_5 + a_0b_1 - a_1b_0 \\ a_5b_0 - a_0b_5 + a_6b_3 - a_3b_6 + a_1b_2 - a_2b_1 \\ a_6b_1 - a_1b_6 + a_0b_4 - a_4b_0 + a_2b_3 - a_3b_2 \\ a_0b_2 - a_2b_0 + a_1b_5 - a_5b_1 + a_3b_4 - a_4b_3 \end{pmatrix}$$

### Component 0 Calculation

$$9 \cdot 18 - 11 \cdot 16 + 10 \cdot 21 - 14 \cdot 17 + 12 \cdot 20 - 1123124 \cdot 1241215 = -1394038355462$$

### Component 1 Calculation

$$10 \cdot 1241215 - 12 \cdot 17 + 11 \cdot 15 - 8 \cdot 18 + 1123124 \cdot 21 - 14 \cdot 20 = 35997291$$

### Component 2 Calculation

$$11 \cdot 20 - 1123124 \cdot 18 + 12 \cdot 16 - 9 \cdot 1241215 + 14 \cdot 15 - 8 \cdot 21 = -31386713$$

### Component 3 Calculation

$$12 \cdot 21 - 14 \cdot 1241215 + 1123124 \cdot 17 - 10 \cdot 20 + 8 \cdot 16 - 9 \cdot 15 = 1716143$$

### Component 4 Calculation

$$1123124 \cdot 15 - 8 \cdot 20 + 14 \cdot 18 - 11 \cdot 21 + 9 \cdot 17 - 10 \cdot 16 = 16846714$$

### Component 5 Calculation

$$14 \cdot 16 - 9 \cdot 21 + 8 \cdot 1241215 - 12 \cdot 15 + 10 \cdot 18 - 11 \cdot 17 = 9929568$$

### Component 6 Calculation

$$8 \cdot 17 - 10 \cdot 15 + 9 \cdot 20 - 1123124 \cdot 16 + 11 \cdot 1241215 - 12 \cdot 18 = -4316669$$

### Dot product formula

$$1 \cdot -1394038355462 + 2 \cdot 35997291 + 3 \cdot -31386713 + 4 \cdot 1716143 + 5 \cdot 16846714 + 6 \cdot 9929568 + 7 \cdot -4316669 = -1393940062152$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \times \begin{pmatrix} 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 1123124 \\ 14 \end{pmatrix} \times \begin{pmatrix} 15 \\ 16 \\ 17 \\ 18 \\ 1241215 \\ 20 \\ 21 \end{pmatrix} = -1393940062152$$

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# Task 6

## Vector addition

### Vector addition formula

$$(a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$$

### Component 1

$$1 + 4 = 5$$

### Component 2

$$2 + 3 = 5$$

### Component 3

$$3 + 2 = 5$$

### Component 4

$$4 + 1 = 5$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \end{pmatrix}$$

## Task 7

### Vector subtraction

#### Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

#### Component 1

$$10 - 5 = 5$$

#### Component 2

$$20 - 10 = 10$$

#### Component 3

$$30 - 15 = 15$$

$$\begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix} - \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix}$$

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## Task 8

### Scalar multiplication

#### Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

#### Component 1

$$3 \cdot 7 = 21$$

#### Component 2

$$3 \cdot 8 = 24$$

### Component 3

$$3 \cdot 9 = 27$$

$$\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \cdot 3 = \begin{pmatrix} 21 \\ 24 \\ 27 \end{pmatrix}$$

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# Task 9

## Norm

### Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

### Dot product formula

$$3 \cdot 3 + 4 \cdot 4 + 12 \cdot 12 = 169$$

### Norm calculation

$$\|\vec{v}\| = \sqrt{169}$$

### Norm result

$$\|\vec{v}\| = \sqrt{169} = 13.0000000000$$

$$\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix} = \|\vec{v}\| = 13.0000000000$$

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# Task 10

## Gram-Schmidt orthogonalization

### Input vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

**Given set of vectors:**

$$\{\vec{v}_1 = \begin{pmatrix}1\\1\\0\end{pmatrix},\ \vec{v}_2 = \begin{pmatrix}1\\0\\1\end{pmatrix},\ \vec{v}_3 = \begin{pmatrix}0\\1\\1\end{pmatrix}\}$$

**Take vector $\vec{v}_1$:**

$$\begin{pmatrix}1\\1\\0\end{pmatrix}$$

**Norm formula**

$$\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}}$$

**Dot product formula**

$$1\cdot 1 + 1\cdot 1 + 0\cdot 0 = 2$$

**Norm calculation**

$$\|\vec{v}\| = \sqrt{2}$$

**Norm result**

$$\|\vec{v}\| = \sqrt{2} = 1.4142135623$$

**Normalize $\vec{u}_1$ to obtain orthonormal vector:**

$$\begin{pmatrix}0.7071067811\\0.7071067811\\0\end{pmatrix}$$

**Take vector $\vec{v}_2$:**

$$\begin{pmatrix}1\\0\\1\end{pmatrix}$$

**Dot product formula**

$$1\cdot 0.7071067811 + 0\cdot 0.7071067811 + 1\cdot 0 = 0.7071067811$$

### Dot product formula

$$0.7071067811 \cdot 0.7071067811 + 0.7071067811 \cdot 0.7071067811 + 0 \cdot 0 = 0.9999999999$$

### Calculate projection coefficient of $\vec{v}_2$ onto orthonormal vector $\vec{u}_1$:

$$\frac{\vec{v}_2 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = 0.7071067811$$

### Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

### Component 1

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

### Component 2

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

### Component 3

$$0.7071067811 \cdot 0 = 0$$

### Projection vector:

$$\begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

### Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

### Component 1

$$1 - 0.5 = 0.5$$

### Component 2

$$0 - 0.5 = -0.5$$

**Component 3**

$$1 - 0 = 1$$

**Subtract projection from $\vec{u}_2$, resulting in:**

$$\begin{pmatrix} 0.5 \\ -0.5 \\ 1 \end{pmatrix}$$

**Norm formula**

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

**Dot product formula**

$$0.5 \cdot 0.5 + -0.5 \cdot -0.5 + 1 \cdot 1 = 1.5$$

**Norm calculation**

$$\|\vec{v}\| = \sqrt{1.5}$$

**Norm result**

$$\|\vec{v}\| = \sqrt{1.5} = 1.2247448713$$

**Normalize $\vec{u}_2$ to obtain orthonormal vector:**

$$\begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix}$$

**Take vector $\vec{v}_3$:**

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

**Dot product formula**

$$0 \cdot 0.7071067811 + 1 \cdot 0.7071067811 + 1 \cdot 0 = 0.7071067811$$

### Dot product formula

$$0.7071067811 \cdot 0.7071067811 + 0.7071067811 \cdot 0.7071067811 + 0 \cdot 0 = 0.9999999999$$

### Calculate projection coefficient of $\vec{v}_3$ onto orthonormal vector $\vec{u}_1$:

$$\frac{\vec{v}_3 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = 0.7071067811$$

### Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

### Component 1

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

### Component 2

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

### Component 3

$$0.7071067811 \cdot 0 = 0$$

### Projection vector:

$$\begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

### Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

### Component 1

$$0 - 0.5 = -0.5$$

### Component 2

$$1 - 0.5 = 0.5$$

### Component 3

$$1 - 0 = 1$$

### Subtract projection from $\vec{u}_3$, resulting in:

$$\begin{pmatrix} -0.5 \\ 0.5 \\ 1 \end{pmatrix}$$

### Dot product formula

$$0 \cdot 0.4082482904 + 1 \cdot -0.4082482904 + 1 \cdot 0.8164965809 = 0.4082482904$$

### Dot product formula

$$0.4082482904 \cdot 0.4082482904 + -0.4082482904 \cdot -0.4082482904 + 0.8164965809 \cdot 0.8164965809 = 0.9999999999$$

### Calculate projection coefficient of $\vec{v}_3$ onto orthonormal vector $\vec{u}_2$:

$$\frac{\vec{v}_3 \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = 0.4082482904$$

### Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

### Component 1

$$0.4082482904 \cdot 0.4082482904 = 0.1666666666$$

### Component 2

$$0.4082482904 \cdot -0.4082482904 = -0.1666666666$$

### Component 3

$$0.4082482904 \cdot 0.8164965809 = 0.3333333333$$

**Projection vector:**

$$\begin{pmatrix} 0.1666666666 \\ -0.1666666666 \\ 0.3333333333 \end{pmatrix}$$

**Vector subtraction formula**

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

**Component 1**

$$-0.5 - 0.1666666666 = -0.6666666666$$

**Component 2**

$$0.5 - -0.1666666666 = 0.6666666666$$

**Component 3**

$$1 - 0.3333333333 = 0.6666666666$$

**Subtract projection from $\vec{u}_3$, resulting in:**

$$\begin{pmatrix} -0.6666666666 \\ 0.6666666666 \\ 0.6666666666 \end{pmatrix}$$

**Norm formula**

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

**Dot product formula**

$$-0.6666666666 \cdot -0.6666666666 + 0.6666666666 \cdot 0.6666666666 + 0.6666666666 \cdot 0.6666666666 = 1.3333333333$$

**Norm calculation**

$$\|\vec{v}\| = \sqrt{1.3333333333}$$

**Norm result**

$$\|\vec{v}\| = \sqrt{1.3333333333} = 1.1547005383$$

**Normalize $\vec{u}_3$ to obtain orthonormal vector:**

$$\begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

**Orthogonal basis**

$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix}, \begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix} \begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix} \begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

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