Solved tasks

Task 1

Dot product

Dot product formula

$$1 \cdot 5 + 2 \cdot 4 + 3 \cdot 3 + 4 \cdot 2 + 5 \cdot 1 = 35$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = 35$$

Task 2

3D cross product

Cross product formula

$$\begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

X component calculation

$$2 \cdot 1 - 3 \cdot 2 = -4$$

Y component calculation

$$3 \cdot 3 - 1 \cdot 1 = 8$$

Z component calculation

$$1 \cdot 2 - 2 \cdot 3 = -4$$

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \times \begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}-4\\8\\-4\end{pmatrix}$$

Task 3

7D cross product

7D Cross Product Formula

$$\begin{pmatrix}
a_1b_3 - a_3b_1 + a_2b_6 - a_6b_2 + a_4b_5 - a_5b_4 \\
a_2b_4 - a_4b_2 + a_3b_0 - a_0b_3 + a_5b_6 - a_6b_5 \\
a_3b_5 - a_5b_3 + a_4b_1 - a_1b_4 + a_6b_0 - a_0b_6 \\
a_4b_6 - a_6b_4 + a_5b_2 - a_2b_5 + a_0b_1 - a_1b_0 \\
a_5b_0 - a_0b_5 + a_6b_3 - a_3b_6 + a_1b_2 - a_2b_1 \\
a_6b_1 - a_1b_6 + a_0b_4 - a_4b_0 + a_2b_3 - a_3b_2 \\
a_0b_2 - a_2b_0 + a_1b_5 - a_5b_1 + a_3b_4 - a_4b_3
\end{pmatrix}$$

Component 0 Calculation

$$2 \cdot 4 - 4 \cdot 6 + 3 \cdot 1 - 7 \cdot 5 + 5 \cdot 2 - 6 \cdot 3 = -56$$

Component 1 Calculation

$$3 \cdot 3 - 5 \cdot 5 + 4 \cdot 7 - 1 \cdot 4 + 6 \cdot 1 - 7 \cdot 2 = 0$$

Component 2 Calculation

$$4 \cdot 2 - 6 \cdot 4 + 5 \cdot 6 - 2 \cdot 3 + 7 \cdot 7 - 1 \cdot 1 = 56$$

Component 3 Calculation

$$5 \cdot 1 - 7 \cdot 3 + 6 \cdot 5 - 3 \cdot 2 + 1 \cdot 6 - 2 \cdot 7 = 0$$

Component 4 Calculation

$$6 \cdot 7 - 1 \cdot 2 + 7 \cdot 4 - 4 \cdot 1 + 2 \cdot 5 - 3 \cdot 6 = 56$$

Component 5 Calculation

$$7 \cdot 6 - 2 \cdot 1 + 1 \cdot 3 - 5 \cdot 7 + 3 \cdot 4 - 4 \cdot 5 = 0$$

Component 6 Calculation

$$1 \cdot 5 - 3 \cdot 7 + 2 \cdot 2 - 6 \cdot 6 + 4 \cdot 3 - 5 \cdot 4 = -56$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \times \begin{pmatrix} 7 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -56 \\ 0 \\ 56 \\ 0 \\ 56 \\ 0 \\ -56 \end{pmatrix}$$

Task 4

3D triple product

Triple product formula

$$(\vec{a} \times \vec{b}) \cdot \vec{c}$$

Cross product formula

$$\begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

X component calculation

$$5 \cdot 155222 - 6 \cdot 8 = 776062$$

Y component calculation

$$6 \cdot 7 - 4 \cdot 155222 = -620846$$

Z component calculation

$$4 \cdot 8 - 5 \cdot 7 = -3$$

Dot product formula

$$1 \cdot 776062 + 2 \cdot -620846 + 3 \cdot -3 = -465639$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \times \begin{pmatrix} 7 \\ 8 \\ 155222 \end{pmatrix} = -465639$$

Task 5

7D triple product

Triple product formula (7D)

$$(\vec{a} \cdot (\vec{b} \times \vec{c}))$$

7D Cross Product Formula

$$\begin{pmatrix} a_1b_3 - a_3b_1 + a_2b_6 - a_6b_2 + a_4b_5 - a_5b_4 \\ a_2b_4 - a_4b_2 + a_3b_0 - a_0b_3 + a_5b_6 - a_6b_5 \\ a_3b_5 - a_5b_3 + a_4b_1 - a_1b_4 + a_6b_0 - a_0b_6 \\ a_4b_6 - a_6b_4 + a_5b_2 - a_2b_5 + a_0b_1 - a_1b_0 \\ a_5b_0 - a_0b_5 + a_6b_3 - a_3b_6 + a_1b_2 - a_2b_1 \\ a_6b_1 - a_1b_6 + a_0b_4 - a_4b_0 + a_2b_3 - a_3b_2 \\ a_0b_2 - a_2b_0 + a_1b_5 - a_5b_1 + a_3b_4 - a_4b_3 \end{pmatrix}$$

Component 0 Calculation

$$9 \cdot 18 - 11 \cdot 16 + 10 \cdot 21 - 14 \cdot 17 + 12 \cdot 20 - 1123124 \cdot 1241215 = -1394038355462$$

Component 1 Calculation

$$10 \cdot 1241215 - 12 \cdot 17 + 11 \cdot 15 - 8 \cdot 18 + 1123124 \cdot 21 - 14 \cdot 20 = 35997291$$

Component 2 Calculation

$$11 \cdot 20 - 1123124 \cdot 18 + 12 \cdot 16 - 9 \cdot 1241215 + 14 \cdot 15 - 8 \cdot 21 = -31386713$$

Component 3 Calculation

$$12 \cdot 21 - 14 \cdot 1241215 + 1123124 \cdot 17 - 10 \cdot 20 + 8 \cdot 16 - 9 \cdot 15 = 1716143$$

Component 4 Calculation

$$1123124 \cdot 15 - 8 \cdot 20 + 14 \cdot 18 - 11 \cdot 21 + 9 \cdot 17 - 10 \cdot 16 = 16846714$$

Component 5 Calculation

$$14 \cdot 16 - 9 \cdot 21 + 8 \cdot 1241215 - 12 \cdot 15 + 10 \cdot 18 - 11 \cdot 17 = 9929568$$

Component 6 Calculation

$$8 \cdot 17 - 10 \cdot 15 + 9 \cdot 20 - 1123124 \cdot 16 + 11 \cdot 1241215 - 12 \cdot 18 = -4316669$$

Dot product formula

$$1 \cdot -1394038355462 + 2 \cdot 35997291 + 3 \cdot -31386713 + 4 \cdot 1716143 + 5 \cdot 16846714 + 6 \cdot 9929568 + 7 \cdot -4316669 = -1393940062152$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \times \begin{pmatrix} 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 1123124 \\ 14 \end{pmatrix} \times \begin{pmatrix} 15 \\ 16 \\ 17 \\ 18 \\ 1241215 \\ 20 \\ 21 \end{pmatrix} = -1393940062152$$

Task 6

Vector addition

Vector addition formula

$$(a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$$

Component 1

$$1 + 4 = 5$$

Component 2

$$2 + 3 = 5$$

Component 3

$$3 + 2 = 5$$

Component 4

$$4 + 1 = 5$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \end{pmatrix}$$

Task 7

Vector subtraction

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$10 - 5 = 5$$

Component 2

$$20 - 10 = 10$$

Component 3

$$30 - 15 = 15$$

$$\begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix} - \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix}$$

Task 8

Scalar multiplication

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$3 \cdot 7 = 21$$

Component 2

$$3 \cdot 8 = 24$$

Component 3

$$3 \cdot 9 = 27$$

$$\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \cdot 3 = \begin{pmatrix} 21 \\ 24 \\ 27 \end{pmatrix}$$

Task 9

Norm

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$3 \cdot 3 + 4 \cdot 4 + 12 \cdot 12 = 169$$

Norm calculation

$$\|\vec{v}\| = \sqrt{169}$$

Norm result

$$\|\vec{v}\| = \sqrt{169} = 13.0000000000$$

$$\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix} = \|\vec{v}\| = 13.0000000000$$

Task 10

Gram-Schmidt orthogonalization

Input vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Given set of vectors:

$$\{\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\ \vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\}$$

Take vector $\vec{v}_1$:

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 2$$

Norm calculation

$$\|\vec{v}\| = \sqrt{2}$$

Norm result

$$\|\vec{v}\| = \sqrt{2} = 1.4142135623$$

Normalize $\vec{u}_1$ to obtain orthonormal vector:

$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix}$$

Take vector $\vec{v}_2$:

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Dot product formula

$$1 \cdot 0.7071067811 + 0 \cdot 0.7071067811 + 1 \cdot 0 = 0.7071067811$$

Dot product formula

$$0.7071067811 \cdot 0.7071067811 + 0.7071067811 \cdot 0.7071067811 + 0 \cdot 0 = 0.9999999999$$

Calculate projection coefficient of $\vec{v}_2$ onto orthonormal vector $\vec{u}_1$:

$$\frac{\vec{v}_2 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = 0.7071067811$$

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 2

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 3

$$0.7071067811 \cdot 0 = 0$$

Projection vector:

$$\begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$1 - 0.5 = 0.5$$

Component 2

$$0 - 0.5 = -0.5$$

Component 3

$$1 - 0 = 1$$

Subtract projection from $\vec{u}_2$, resulting in:

$$\begin{pmatrix} 0.5 \\ -0.5 \\ 1 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$0.5 \cdot 0.5 + -0.5 \cdot -0.5 + 1 \cdot 1 = 1.5$$

Norm calculation

$$\|\vec{v}\| = \sqrt{1.5}$$

Norm result

$$\|\vec{v}\| = \sqrt{1.5} = 1.2247448713$$

Normalize $\vec{u}_2$ to obtain orthonormal vector:

$$\begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix}$$

Take vector $\vec{v}_3$:

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Dot product formula

$$0 \cdot 0.7071067811 + 1 \cdot 0.7071067811 + 1 \cdot 0 = 0.7071067811$$

Dot product formula

$$0.7071067811 \cdot 0.7071067811 + 0.7071067811 \cdot 0.7071067811 + 0 \cdot 0 = 0.9999999999$$

Calculate projection coefficient of $\vec{v}_3$ onto orthonormal vector $\vec{u}_1$:

$$\frac{\vec{v}_3 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = 0.7071067811$$

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 2

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 3

$$0.7071067811 \cdot 0 = 0$$

Projection vector:

$$\begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$0 - 0.5 = -0.5$$

Component 2

$$1 - 0.5 = 0.5$$

Component 3

$$1 - 0 = 1$$

Subtract projection from $\vec{u}_3$, resulting in:

$$\begin{pmatrix} -0.5 \\ 0.5 \\ 1 \end{pmatrix}$$

Dot product formula

$$0 \cdot 0.4082482904 + 1 \cdot -0.4082482904 + 1 \cdot 0.8164965809 = 0.4082482904$$

Dot product formula

$$0.4082482904 \cdot 0.4082482904 + -0.4082482904 \cdot -0.4082482904 + 0.8164965809 \cdot 0.8164965809 = 0.9999999999$$

Calculate projection coefficient of $\vec{v}_3$ onto orthonormal vector $\vec{u}_2$:

$$\frac{\vec{v}_3 \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = 0.4082482904$$

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$0.4082482904 \cdot 0.4082482904 = 0.1666666666$$

Component 2

$$0.4082482904 \cdot -0.4082482904 = -0.1666666666$$

Component 3

$$0.4082482904 \cdot 0.8164965809 = 0.3333333333$$

Projection vector:

$$\begin{pmatrix} 0.1666666666 \\ -0.1666666666 \\ 0.3333333333 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$-0.5 - 0.1666666666 = -0.6666666666$$

Component 2

$$0.5 - -0.1666666666 = 0.6666666666$$

Component 3

$$1 - 0.3333333333 = 0.6666666666$$

Subtract projection from $\vec{u}_3$, resulting in:

$$\begin{pmatrix} -0.6666666666 \\ 0.6666666666 \\ 0.6666666666 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-0.6666666666 \cdot -0.6666666666 + 0.6666666666 \cdot 0.6666666666 + 0.6666666666 \cdot 0.6666666666 = 1.3333333333$$

Norm calculation

$$\|\vec{v}\| = \sqrt{1.3333333333}$$

Norm result

$$\|\vec{v}\| = \sqrt{1.3333333333} = 1.1547005383$$

Normalize $\vec{u}_3$ to obtain orthonormal vector:

$$\begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

Orthogonal basis

$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix}, \begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$
$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix} \begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix} \begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

Task 11

Matrix addition

Matrix addition formula

$$\text{For all } i, j: \quad C_{ij} = A_{ij} + B_{ij}$$

Adding elements at position (0,0)

$$1 + 124 = 125$$

Adding elements at position (0,1)

$$2 + 14 = 16$$

Adding elements at position (0,2)

$$3 + 156 = 159$$

Adding elements at position (1,0)

$$4 + 152 = 156$$

Adding elements at position (1,1)

$$5 + 62236 = 62241$$

Adding elements at position (1,2)

$$6 + 2 = 8$$

Adding elements at position (2,0)

$$7 + 1262 = 1269$$

Adding elements at position (2,1)

$$8 + 23623 = 23631$$

Adding elements at position (2,2)

$$9 + 347 = 356$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 124 & 14 & 156 \\ 152 & 62236 & 2 \\ 1262 & 23623 & 347 \end{pmatrix} = \begin{pmatrix} 125 & 16 & 159 \\ 156 & 62241 & 8 \\ 1269 & 23631 & 356 \end{pmatrix}$$

Task 12

Matrix multiplication

Matrix multiplication operation

$$\text{Computing } A \times B \text{ using compound assignment}$$

Matrix multiplication formula

$$\text{For all } i, j: \quad C_{ij} = \sum_{k=0}^{n-1} A_{ik} \times B_{kj}$$

Computing element at position (0,0)

$$C_{0,0} = (1 \times 124) + (2 \times 152) + (3 \times 1262) = 4214$$

Computing element at position (0,1)

$$C_{0,1} = (1 \times 14) + (2 \times 62236) + (3 \times 23623) = 195355$$

Computing element at position (0,2)

$$C_{0,2} = (1 \times 156) + (2 \times 2) + (3 \times 347) = 1201$$

Computing element at position (1,0)

$$C_{1,0} = (4 \times 124) + (5 \times 152) + (6 \times 1262) = 8828$$

Computing element at position (1,1)

$$C_{1,1} = (4 \times 14) + (5 \times 62236) + (6 \times 23623) = 452974$$

Computing element at position (1,2)

$$C_{1,2} = (4 \times 156) + (5 \times 2) + (6 \times 347) = 2716$$

Computing element at position (2,0)

$$C_{2,0} = (7 \times 124) + (8 \times 152) + (9 \times 1262) = 13442$$

Computing element at position (2,1)

$$C_{2,1} = (7 \times 14) + (8 \times 62236) + (9 \times 23623) = 710593$$

Computing element at position (2,2)

$$C_{2,2} = (7 \times 156) + (8 \times 2) + (9 \times 347) = 4231$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \cdot \begin{pmatrix} 124 & 14 & 156 \\ 152 & 62236 & 2 \\ 1262 & 23623 & 347 \end{pmatrix} = \begin{pmatrix} 4214 & 195355 & 1201 \\ 8828 & 452974 & 2716 \\ 13442 & 710593 & 4231 \end{pmatrix}$$

Task 13

Matrix scalar multiplication

Scalar multiplication operation

$$\text{Computing } A \times \lambda \text{ using compound assignment}$$

Scalar multiplication formula

$$\text{For all } i, j: \quad C_{ij} = A_{ij} \times \lambda$$

Multiplying element at position (0,0)

$$1 \times 52 = 52$$

Multiplying element at position (0,1)

$$2 \times 52 = 104$$

Multiplying element at position (0,2)

$$3 \times 52 = 156$$

Multiplying element at position (1,0)

$$4 \times 52 = 208$$

Multiplying element at position (1,1)

$$5 \times 52 = 260$$

Multiplying element at position (1,2)

$$6 \times 52 = 312$$

Multiplying element at position (2,0)

$$7 \times 52 = 364$$

Multiplying element at position (2,1)

$$8 \times 52 = 416$$

Multiplying element at position (2,2)

$$9 \times 52 = 468$$

$$52 \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 52 & 104 & 156 \\ 208 & 260 & 312 \\ 364 & 416 & 468 \end{pmatrix}$$

Task 14

Determinant

Matrix determinant calculation

Starting determinant calculation by Gaussian elimination with pivot selection.

Pivot element chosen

$$\text{Pivot element } (0,0) = 111, \text{ current determinant: } 111$$

Eliminating element

$$\text{Calculating factor for row 1 : } 4236 \div 111 = 38.1621621621$$

Updating matrix element

$$\text{temp}(1,0) : 4236 - 38.1621621621 \times 111 = 0$$

Updating matrix element

$$\text{temp}(1,1) : 1512512 - 38.1621621621 \times 266 = 1502360.8648648648$$

Updating matrix element

$$\text{temp}(1,2) : 1224 - 38.1621621621 \times 233 = -7667.7837837837$$

Eliminating element

$$\text{Calculating factor for row 2}: \ 1337 \div 111 = 12.0450450450$$

Updating matrix element

$$\text{temp}(2,0) : 1337 - 12.0450450450 \times 111 = 0$$

Updating matrix element

$$\text{temp}(2,1) : 48 - 12.0450450450 \times 266 = -3155.9819819819$$

Updating matrix element

$$\text{temp}(2,2) : 52 - 12.0450450450 \times 233 = -2754.4954954954$$

Pivot element chosen

$$\text{Pivot element } (1,1) = 1502360.8648648648, \ \text{current determinant: } 166762056$$

Eliminating element

$$\text{Calculating factor for row 2}: \ -3155.9819819819 \div 1502360.8648648648 = -0.0021006817$$

Updating matrix element

$$\text{temp}(2,1) : -3155.9819819819 - -0.0021006817 \times 1502360.8648648648 = 0$$

Updating matrix element

$$\text{temp}(2,2) : -2754.4954954954 - -0.0021006817 \times -7667.7837837837 = -2770.6030686021$$

Pivot element chosen

$$\text{Pivot element } (2,2) = -2770.6030686021, \ \text{current determinant: } -462031464080$$

$$\det(\begin{pmatrix} 111 & 266 & 233 \\ 4236 & 1512512 & 1224 \\ 1337 & 48 & 52 \end{pmatrix}) = -462031464080$$

Task 15

Inverse matrix

Matrix inversion

Starting matrix inversion using Gauss-Jordan elimination. The identity matrix is augmented and operations are applied

Normalize pivot row

Dividing row 0 by pivot 1 to make pivot 1.

Eliminate column entry

$$\text{Row 1} \leftarrow \text{Row 1} - 48 \times \text{Row 0}.$$

Eliminate column entry

$$\text{Row 2} \leftarrow \text{Row 2} - 7 \times \text{Row 0}.$$

Normalize pivot row

Dividing row 1 by pivot -91 to make pivot 1.

Eliminate column entry

$$\text{Row 0} \leftarrow \text{Row 0} - 2 \times \text{Row 1}.$$

Eliminate column entry

$$\text{Row 2} \leftarrow \text{Row 2} - -12529 \times \text{Row 1}.$$

Normalize pivot row

Dividing row 2 by pivot 342471.4835164835 to make pivot 1.

Eliminate column entry

$$\text{Row 0} \leftarrow \text{Row 0} - -2.7252747252 \times \text{Row 2}.$$

Eliminate column entry

$$\text{Row } 1 \leftarrow \text{Row } 1 - 27.3626373626 \times \text{Row } 2.$$

$$\begin{pmatrix} 1 & 2 & 52 \\ 48 & 5 & 6 \\ 7 & -12515 & 9 \end{pmatrix}^{-1} = \begin{pmatrix} -0.0024108849 & 0.0208823996 & 0.0000079576 \\ 0.0000125140 & 0.0000113910 & -0.0000798975 \\ 0.0192766510 & -0.0004020227 & 0.0000029199 \end{pmatrix}$$

Task 16

Gaussian elimination

Solving Ax = b via Gaussian elimination

Solving the system of linear equations using Gaussian elimination with back substitution.

Eliminating element

$$\text{Row } j = 1 \leftarrow \text{Row } j - 124125 \times \text{Row } i = 0.$$

Updating matrix element

$$a_{1,0} = 124125 - 124125 \times 1 = 0$$

Updating matrix element

$$a_{1,1} = 5 - 124125 \times -12515 = 1553424380$$

Updating matrix element

$$a_{1,2} = 6 - 124125 \times 3 = -372369$$

Updating RHS

$$b_1 = 4 - 124125 \times 5 = -620621$$

Eliminating element

$$\text{Row } j = 2 \leftarrow \text{Row } j - 7 \times \text{Row } i = 0.$$

Updating matrix element

$$a_{2,0} = 7 - 7 \times 1 = 0$$

Updating matrix element

$$a_{2,1} = 8 - 7 \times -12515 = 87613$$

Updating matrix element

$$a_{2,2} = 29841928419 - 7 \times 3 = 29841928398$$

Updating RHS

$$b_2 = 1 - 7 \times 5 = -34$$

Eliminating element

$$\text{Row } j = 2 \leftarrow \text{Row } j - 0.0000563999 \times \text{Row } i = 1.$$

Updating matrix element

$$a_{2,1} = 87613 - 0.0000563999 \times 1553424380 = 0$$

Updating matrix element

$$a_{2,2} = 29841928398 - 0.0000563999 \times -372369 = 29841928419.0015792316$$

Updating RHS

$$b_2 = -34 - 0.0000563999 \times -620621 = 1.0029704522$$

Back substitution

Starting back substitution.

Back substitution step

$$x_2 = \frac{1.0029704522}{29841928419.0015792316} = 0.0000000000$$

Back substitution step

$$x_1 = \frac{-620620.9999874848}{1553424380} = -0.0003995179$$

Back substitution step

$$x_0 = \frac{0.0000322416}{1} = 0.0000322416$$

Final result

System solved, final solution vector obtained.

$$\text{Solving the system } \begin{pmatrix} 1 & -12515 & 3 \\ 124125 & 5 & 6 \\ 7 & 8 & 29841928419 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \text{ using Gaussian elimination:}$$

$$\vec{x} = \begin{pmatrix} 0.0000322416 \\ -0.0003995179 \\ 0.0000000000 \end{pmatrix}$$

Task 17

Gauss-Jordan elimination

Solving Ax = b via Gauss-Jordan elimination

Solving the system of linear equations using the Gauss-Jordan method with full row reduction.

Normalize pivot row

Dividing row 0 by pivot 1 to make leading coefficient 1.

Normalizing matrix element

$$a(0,0) = 1 \div 1 = 1$$

Normalizing matrix element

$$a(0,1) = -12515 \div 1 = -12515$$

Normalizing matrix element

$$a(0,2) = 3 \div 1 = 3$$

Normalizing RHS

$$x[0] = 5 \div 1 = 5$$

Eliminate element

$$\text{Row 1} \leftarrow \text{Row 1} - 124125 \times \text{Row 0}.$$

Updating matrix element

$$a(1,0) = 124125 - 124125 \times 1 = 0$$

Updating matrix element

$$a(1,1) = 5 - 124125 \times -12515 = 1553424380$$

Updating matrix element

$$a(1,2) = 6 - 124125 \times 3 = -372369$$

Updating RHS

$$x[1] = 4 - 124125 \times 5 = -620621$$

Eliminate element

$$\text{Row 2} \leftarrow \text{Row 2} - 7 \times \text{Row 0}.$$

Updating matrix element

$$a(2,0) = 7 - 7 \times 1 = 0$$

Updating matrix element

$$a(2,1) = 8 - 7 \times -12515 = 87613$$

Updating matrix element

$$a(2,2) = 29841928419 - 7 \times 3 = 29841928398$$

Updating RHS

$$x[2] = 1 - 7 \times 5 = -34$$

Normalize pivot row

Dividing row 1 by pivot 1553424380 to make leading coefficient 1.

Normalizing matrix element

$$a(1,0) = 0 \div 1553424380 = 0$$

Normalizing matrix element

$$a(1,1) = 1553424380 \div 1553424380 = 1$$

Normalizing matrix element

$$a(1,2) = -372369 \div 1553424380 = -0.0002397084$$

Normalizing RHS

$$x[1] = -620621 \div 1553424380 = -0.0003995179$$

Eliminate element

Row 0 $\leftarrow$ Row 0 $- -12515 \times$ Row 1.

Updating matrix element

$$a(0,0) = 1 - -12515 \times 0 = 1$$

Updating matrix element

$$a(0,1) = -12515 - -12515 \times 1 = 0$$

Updating matrix element

$$a(0,2) = 3 - -12515 \times -0.0002397084 = 0.0000483480$$

Updating RHS

$$x[0] = 5 - -12515 \times -0.0003995179 = 0.0000322416$$

Eliminate element

$$\text{Row } 2 \leftarrow \text{Row } 2 - 87613 \times \text{Row } 1.$$

Updating matrix element

$$a(2,0) = 0 - 87613 \times 0 = 0$$

Updating matrix element

$$a(2,1) = 87613 - 87613 \times 1 = 0$$

Updating matrix element

$$a(2,2) = 29841928398 - 87613 \times -0.0002397084 = 29841928419.0015792316$$

Updating RHS

$$x[2] = -34 - 87613 \times -0.0003995179 = 1.0029704522$$

Normalize pivot row

Dividing row 2 by pivot 29841928419.0015792316 to make leading coefficient 1.

Normalizing matrix element

$$a(2,0) = 0 \div 29841928419.0015792316 = 0$$

Normalizing matrix element

$$a(2,1) = 0 \div 29841928419.0015792316 = 0$$

Normalizing matrix element

$$a(2,2) = 29841928419.0015792316 \div 29841928419.0015792316 = 1$$

Normalizing RHS

$$x[2] = 1.0029704522 \div 29841928419.0015792316 = 0.0000000000$$

Eliminate element

$$\text{Row } 0 \leftarrow \text{Row } 0 - 0.0000483480 \times \text{Row } 2.$$

Updating matrix element

$$a(0,0) = 1 - 0.0000483480 \times 0 = 1$$

Updating matrix element

$$a(0,1) = 0 - 0.0000483480 \times 0 = 0$$

Updating matrix element

$$a(0,2) = 0.0000483480 - 0.0000483480 \times 1 = 0$$

Updating RHS

$$x[0] = 0.0000322416 - 0.0000483480 \times 0.0000000000 = 0.0000322416$$

Eliminate element

$$\text{Row } 1 \leftarrow \text{Row } 1 - -0.0002397084 \times \text{Row } 2.$$

Updating matrix element

$$a(1,0) = 0 - -0.0002397084 \times 0 = 0$$

Updating matrix element

$$a(1,1) = 1 - -0.0002397084 \times 0 = 1$$

Updating matrix element

$$a(1,2) = -0.0002397084 - -0.0002397084 \times 1 = 0$$

Updating RHS

$$x[1] = -0.0003995179 - -0.0002397084 \times 0.0000000000 = -0.0003995179$$

Solving the system $\begin{pmatrix} 1 & -12515 & 3 \\ 124125 & 5 & 6 \\ 7 & 8 & 29841928419 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$ using Gauss-Jordan elimination:

$$\vec{x} = \begin{pmatrix} 0.0000322416 \\ -0.0003995179 \\ 0.0000000000 \end{pmatrix}$$
