Solved tasks

Task 1

Dot product

Dot product formula

$$1 \cdot 5 + 2 \cdot 4 + 3 \cdot 3 + 4 \cdot 2 + 5 \cdot 1 = 35$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = 35$$

Task 2

3D cross product

Cross product formula

$$\begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

X component calculation

$$2 \cdot 1 - 3 \cdot 2 = -4$$

Y component calculation

$$3 \cdot 3 - 1 \cdot 1 = 8$$

Z component calculation

$$1 \cdot 2 - 2 \cdot 3 = -4$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \\ -4 \end{pmatrix}$$

Task 3

7D cross product

7D Cross Product Formula

$$\begin{pmatrix} a_1b_3 - a_3b_1 + a_2b_6 - a_6b_2 + a_4b_5 - a_5b_4 \\ a_2b_4 - a_4b_2 + a_3b_0 - a_0b_3 + a_5b_6 - a_6b_5 \\ a_3b_5 - a_5b_3 + a_4b_1 - a_1b_4 + a_6b_0 - a_0b_6 \\ a_4b_6 - a_6b_4 + a_5b_2 - a_2b_5 + a_0b_1 - a_1b_0 \\ a_5b_0 - a_0b_5 + a_6b_3 - a_3b_6 + a_1b_2 - a_2b_1 \\ a_6b_1 - a_1b_6 + a_0b_4 - a_4b_0 + a_2b_3 - a_3b_2 \\ a_0b_2 - a_2b_0 + a_1b_5 - a_5b_1 + a_3b_4 - a_4b_3 \end{pmatrix}$$

Component 0 Calculation

$$2 \cdot 4 - 4 \cdot 6 + 3 \cdot 1 - 7 \cdot 5 + 5 \cdot 2 - 6 \cdot 3 = -56$$

Component 1 Calculation

$$3 \cdot 3 - 5 \cdot 5 + 4 \cdot 7 - 1 \cdot 4 + 6 \cdot 1 - 7 \cdot 2 = 0$$

Component 2 Calculation

$$4 \cdot 2 - 6 \cdot 4 + 5 \cdot 6 - 2 \cdot 3 + 7 \cdot 7 - 1 \cdot 1 = 56$$

Component 3 Calculation

$$5 \cdot 1 - 7 \cdot 3 + 6 \cdot 5 - 3 \cdot 2 + 1 \cdot 6 - 2 \cdot 7 = 0$$

Component 4 Calculation

$$6 \cdot 7 - 1 \cdot 2 + 7 \cdot 4 - 4 \cdot 1 + 2 \cdot 5 - 3 \cdot 6 = 56$$

Component 5 Calculation

$$7 \cdot 6 - 2 \cdot 1 + 1 \cdot 3 - 5 \cdot 7 + 3 \cdot 4 - 4 \cdot 5 = 0$$

Component 6 Calculation

$$1 \cdot 5 - 3 \cdot 7 + 2 \cdot 2 - 6 \cdot 6 + 4 \cdot 3 - 5 \cdot 4 = -56$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \times \begin{pmatrix} 7 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -56 \\ 0 \\ 56 \\ 0 \\ 56 \\ 0 \\ -56 \end{pmatrix}$$

Task 4

3D triple product

Triple product formula

$$(\vec{a} \times \vec{b}) \cdot \vec{c}$$

Cross product formula

$$\begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

X component calculation

$$5 \cdot 155222 - 6 \cdot 8 = 776062$$

Y component calculation

$$6 \cdot 7 - 4 \cdot 155222 = -620846$$

Z component calculation

$$4 \cdot 8 - 5 \cdot 7 = -3$$

Dot product formula

$$1 \cdot 776062 + 2 \cdot -620846 + 3 \cdot -3 = -465639$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \times \begin{pmatrix} 7 \\ 8 \\ 155222 \end{pmatrix} = -465639$$

Task 5

7D triple product

Triple product formula (7D)

$$(\vec{a} \cdot (\vec{b} \times \vec{c}))$$

7D Cross Product Formula

$$\begin{pmatrix} a_1b_3 - a_3b_1 + a_2b_6 - a_6b_2 + a_4b_5 - a_5b_4 \\ a_2b_4 - a_4b_2 + a_3b_0 - a_0b_3 + a_5b_6 - a_6b_5 \\ a_3b_5 - a_5b_3 + a_4b_1 - a_1b_4 + a_6b_0 - a_0b_6 \\ a_4b_6 - a_6b_4 + a_5b_2 - a_2b_5 + a_0b_1 - a_1b_0 \\ a_5b_0 - a_0b_5 + a_6b_3 - a_3b_6 + a_1b_2 - a_2b_1 \\ a_6b_1 - a_1b_6 + a_0b_4 - a_4b_0 + a_2b_3 - a_3b_2 \\ a_0b_2 - a_2b_0 + a_1b_5 - a_5b_1 + a_3b_4 - a_4b_3 \end{pmatrix}$$

Component 0 Calculation

$$9 \cdot 18 - 11 \cdot 16 + 10 \cdot 21 - 14 \cdot 17 + 12 \cdot 20 - 1123124 \cdot 1241215 = -1394038355462$$

Component 1 Calculation

$$10 \cdot 1241215 - 12 \cdot 17 + 11 \cdot 15 - 8 \cdot 18 + 1123124 \cdot 21 - 14 \cdot 20 = 35997291$$

Component 2 Calculation

$$11 \cdot 20 - 1123124 \cdot 18 + 12 \cdot 16 - 9 \cdot 1241215 + 14 \cdot 15 - 8 \cdot 21 = -31386713$$

Component 3 Calculation

$$12 \cdot 21 - 14 \cdot 1241215 + 1123124 \cdot 17 - 10 \cdot 20 + 8 \cdot 16 - 9 \cdot 15 = 1716143$$

Component 4 Calculation

$$1123124 \cdot 15 - 8 \cdot 20 + 14 \cdot 18 - 11 \cdot 21 + 9 \cdot 17 - 10 \cdot 16 = 16846714$$

Component 5 Calculation

$$14 \cdot 16 - 9 \cdot 21 + 8 \cdot 1241215 - 12 \cdot 15 + 10 \cdot 18 - 11 \cdot 17 = 9929568$$

Component 6 Calculation

$$8 \cdot 17 - 10 \cdot 15 + 9 \cdot 20 - 1123124 \cdot 16 + 11 \cdot 1241215 - 12 \cdot 18 = -4316669$$

Dot product formula

$$1 \cdot -1394038355462 + 2 \cdot 35997291 + 3 \cdot -31386713 + 4 \cdot 1716143 + 5 \cdot 16846714 + 6 \cdot 9929568 + 7 \cdot -4316669 = -1393940062152$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \times \begin{pmatrix} 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 1123124 \\ 14 \end{pmatrix} \times \begin{pmatrix} 15 \\ 16 \\ 17 \\ 18 \\ 1241215 \\ 20 \\ 21 \end{pmatrix} = -1393940062152$$

Task 6

Vector addition

Vector addition formula

$$(a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$$

Component 1

$$1 + 4 = 5$$

Component 2

$$2 + 3 = 5$$

Component 3

$$3 + 2 = 5$$

Component 4

$$4 + 1 = 5$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \end{pmatrix}$$

Task 7

Vector subtraction

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$10 - 5 = 5$$

Component 2

$$20 - 10 = 10$$

Component 3

$$30 - 15 = 15$$

$$\begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix} - \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix}$$

Task 8

Scalar multiplication

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$3 \cdot 7 = 21$$

Component 2

$$3 \cdot 8 = 24$$

Component 3

$$3 \cdot 9 = 27$$

$$\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \cdot 3 = \begin{pmatrix} 21 \\ 24 \\ 27 \end{pmatrix}$$

Task 9

Norm

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$3 \cdot 3 + 4 \cdot 4 + 12 \cdot 12 = 169$$

Norm calculation

$$\|\vec{v}\| = \sqrt{169}$$

Norm result

$$\|\vec{v}\| = \sqrt{169} = 13.0000000000$$

$$\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix} = \|\vec{v}\| = 13.0000000000$$

Task 10

Gram-Schmidt orthogonalization

Input vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Given set of vectors:

$$\{\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\ \vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\}$$

Take vector $\vec{v}_1$:

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 2$$

Norm calculation

$$\|\vec{v}\| = \sqrt{2}$$

Norm result

$$\|\vec{v}\| = \sqrt{2} = 1.4142135623$$

Normalize $\vec{u}_1$ to obtain orthonormal vector:

$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix}$$

Take vector $\vec{v}_2$:

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Dot product formula

$$1 \cdot 0.7071067811 + 0 \cdot 0.7071067811 + 1 \cdot 0 = 0.7071067811$$

Dot product formula

$$0.7071067811 \cdot 0.7071067811 + 0.7071067811 \cdot 0.7071067811 + 0 \cdot 0 = 0.9999999999$$

Calculate projection coefficient of $\vec{v}_2$ onto orthonormal vector $\vec{u}_1$:

$$\frac{\vec{v}_2 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = 0.7071067811$$

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 2

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 3

$$0.7071067811 \cdot 0 = 0$$

Projection vector:

$$\begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$1 - 0.5 = 0.5$$

Component 2

$$0 - 0.5 = -0.5$$

Component 3

$$1 - 0 = 1$$

Subtract projection from $\vec{u}_2$, resulting in:

$$\begin{pmatrix} 0.5 \\ -0.5 \\ 1 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$0.5 \cdot 0.5 + -0.5 \cdot -0.5 + 1 \cdot 1 = 1.5$$

Norm calculation

$$\|\vec{v}\| = \sqrt{1.5}$$

Norm result

$$\|\vec{v}\| = \sqrt{1.5} = 1.2247448713$$

Normalize $\vec{u}_2$ to obtain orthonormal vector:

$$\begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix}$$

Take vector $\vec{v}_3$:

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Dot product formula

$$0 \cdot 0.7071067811 + 1 \cdot 0.7071067811 + 1 \cdot 0 = 0.7071067811$$

Dot product formula

$$0.7071067811 \cdot 0.7071067811 + 0.7071067811 \cdot 0.7071067811 + 0 \cdot 0 = 0.9999999999$$

Calculate projection coefficient of $\vec{v}_3$ onto orthonormal vector $\vec{u}_1$:

$$\frac{\vec{v}_3 \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = 0.7071067811$$

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 2

$$0.7071067811 \cdot 0.7071067811 = 0.5$$

Component 3

$$0.7071067811 \cdot 0 = 0$$

Projection vector:

$$\begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$0 - 0.5 = -0.5$$

Component 2

$$1 - 0.5 = 0.5$$

Component 3

$$1 - 0 = 1$$

Subtract projection from $\vec{u}_3$, resulting in:

$$\begin{pmatrix} -0.5 \\ 0.5 \\ 1 \end{pmatrix}$$

Dot product formula

$$0 \cdot 0.4082482904 + 1 \cdot -0.4082482904 + 1 \cdot 0.8164965809 = 0.4082482904$$

Dot product formula

$$0.4082482904 \cdot 0.4082482904 + -0.4082482904 \cdot -0.4082482904 + 0.8164965809 \cdot 0.8164965809 = 0.9999999999$$

Calculate projection coefficient of $\vec{v}_3$ onto orthonormal vector $\vec{u}_2$:

$$\frac{\vec{v}_3 \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = 0.4082482904$$

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$0.4082482904 \cdot 0.4082482904 = 0.1666666666$$

Component 2

$$0.4082482904 \cdot -0.4082482904 = -0.1666666666$$

Component 3

$$0.4082482904 \cdot 0.8164965809 = 0.3333333333$$

Projection vector:

$$\begin{pmatrix} 0.1666666666 \\ -0.1666666666 \\ 0.3333333333 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$-0.5 - 0.1666666666 = -0.6666666666$$

Component 2

$$0.5 - -0.1666666666 = 0.6666666666$$

Component 3

$$1 - 0.3333333333 = 0.6666666666$$

Subtract projection from $\vec{u}_3$, resulting in:

$$\begin{pmatrix} -0.6666666666 \\ 0.6666666666 \\ 0.6666666666 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-0.6666666666 \cdot -0.6666666666 + 0.6666666666 \cdot 0.6666666666 + 0.6666666666 \cdot 0.6666666666 = 1.3333333333$$

Norm calculation

$$\|\vec{v}\| = \sqrt{1.3333333333}$$

Norm result

$$\|\vec{v}\| = \sqrt{1.3333333333} = 1.1547005383$$

Normalize $\vec{u}_3$ to obtain orthonormal vector:

$$\begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

Orthogonal basis

$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix}, \begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

$$\begin{pmatrix} 0.7071067811 \\ 0.7071067811 \\ 0 \end{pmatrix} \begin{pmatrix} 0.4082482904 \\ -0.4082482904 \\ 0.8164965809 \end{pmatrix} \begin{pmatrix} -0.5773502691 \\ 0.5773502691 \\ 0.5773502691 \end{pmatrix}$$

Task 11

Matrix addition

Matrix addition formula

$$\text{For all } i, j: \quad C_{ij} = A_{ij} + B_{ij}$$

Adding elements at position (0,0)

$$1 + 124 = 125$$

Adding elements at position (0,1)

$$2 + 14 = 16$$

Adding elements at position (0,2)

$$3 + 156 = 159$$

Adding elements at position (1,0)

$$4 + 152 = 156$$

Adding elements at position (1,1)

$$5 + 62236 = 62241$$

Adding elements at position (1,2)

$$6 + 2 = 8$$

Adding elements at position (2,0)

$$7 + 1262 = 1269$$

Adding elements at position (2,1)

$$8 + 23623 = 23631$$

Adding elements at position (2,2)

$$9 + 347 = 356$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 124 & 14 & 156 \\ 152 & 62236 & 2 \\ 1262 & 23623 & 347 \end{pmatrix} = \begin{pmatrix} 125 & 16 & 159 \\ 156 & 62241 & 8 \\ 1269 & 23631 & 356 \end{pmatrix}$$

Task 12

Matrix multiplication

Matrix multiplication operation

Computing $A \times B$ using compound assignment

Matrix multiplication formula

$$\text{For all } i, j : \quad C_{ij} = \sum_{k=0}^{n-1} A_{ik} \times B_{kj}$$

Computing element at position (0,0)

$$C_{0,0} = (1 \times 124) + (2 \times 152) + (3 \times 1262) = 4214$$

Computing element at position (0,1)

$$C_{0,1} = (1 \times 14) + (2 \times 62236) + (3 \times 23623) = 195355$$

Computing element at position (0,2)

$$C_{0,2} = (1 \times 156) + (2 \times 2) + (3 \times 347) = 1201$$

Computing element at position (1,0)

$$C_{1,0} = (4 \times 124) + (5 \times 152) + (6 \times 1262) = 8828$$

Computing element at position (1,1)

$$C_{1,1} = (4 \times 14) + (5 \times 62236) + (6 \times 23623) = 452974$$

Computing element at position (1,2)

$$C_{1,2} = (4 \times 156) + (5 \times 2) + (6 \times 347) = 2716$$

Computing element at position (2,0)

$$C_{2,0} = (7 \times 124) + (8 \times 152) + (9 \times 1262) = 13442$$

Computing element at position (2,1)

$$C_{2,1} = (7 \times 14) + (8 \times 62236) + (9 \times 23623) = 710593$$

Computing element at position (2,2)

$$C_{2,2} = (7 \times 156) + (8 \times 2) + (9 \times 347) = 4231$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \cdot \begin{pmatrix} 124 & 14 & 156 \\ 152 & 62236 & 2 \\ 1262 & 23623 & 347 \end{pmatrix} = \begin{pmatrix} 4214 & 195355 & 1201 \\ 8828 & 452974 & 2716 \\ 13442 & 710593 & 4231 \end{pmatrix}$$

Task 13

Matrix scalar multiplication

Scalar multiplication operation

$$\text{Computing } A \times \lambda \text{ using compound assignment}$$

Scalar multiplication formula

$$\text{For all } i, j: \quad C_{ij} = A_{ij} \times \lambda$$

Multiplying element at position (0,0)

$$1 \times 52 = 52$$

Multiplying element at position (0,1)

$$2 \times 52 = 104$$

Multiplying element at position (0,2)

$$3 \times 52 = 156$$

Multiplying element at position (1,0)

$$4 \times 52 = 208$$

Multiplying element at position (1,1)

$$5 \times 52 = 260$$

Multiplying element at position (1,2)

$$6 \times 52 = 312$$

Multiplying element at position (2,0)

$$7 \times 52 = 364$$

Multiplying element at position (2,1)

$$8 \times 52 = 416$$

Multiplying element at position (2,2)

$$9 \times 52 = 468$$

$$52 \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 52 & 104 & 156 \\ 208 & 260 & 312 \\ 364 & 416 & 468 \end{pmatrix}$$

Task 14

Determinant

Matrix determinant calculation

Starting determinant calculation by Gaussian elimination with pivot selection.

Pivot element chosen

$$\text{Pivot element } (0,0) = 111, \text{ current determinant: } 111$$

Eliminating element

$$\text{Calculating factor for row } 1: \ 4236 \div 111 = 38.1621621621$$

Updating matrix element

$$\text{temp}(1,0) : 4236 - 38.1621621621 \times 111 = 0$$

Updating matrix element

$$\text{temp}(1,1) : 1512512 - 38.1621621621 \times 266 = 1502360.8648648648$$

Updating matrix element

$$\text{temp}(1,2) : 1224 - 38.1621621621 \times 233 = -7667.7837837837$$

Eliminating element

$$\text{Calculating factor for row 2 : } 1337 \div 111 = 12.0450450450$$

Updating matrix element

$$\text{temp}(2,0) : 1337 - 12.0450450450 \times 111 = 0$$

Updating matrix element

$$\text{temp}(2,1) : 48 - 12.0450450450 \times 266 = -3155.9819819819$$

Updating matrix element

$$\text{temp}(2,2) : 52 - 12.0450450450 \times 233 = -2754.4954954954$$

Pivot element chosen

$$\text{Pivot element } (1,1) = 1502360.8648648648, \text{ current determinant: } 166762056$$

Eliminating element

$$\text{Calculating factor for row 2 : } -3155.9819819819 \div 1502360.8648648648 = -0.0021006817$$

Updating matrix element

$$\text{temp}(2,1) : -3155.9819819819 - -0.0021006817 \times 1502360.8648648648 = 0$$

Updating matrix element

$$\text{temp}(2,2) : -2754.4954954954 - -0.0021006817 \times -7667.7837837837 = -2770.6030686021$$

Pivot element chosen

$$\text{Pivot element } (2,2) = -2770.6030686021, \text{ current determinant: } -462031464080$$

$$\det(\begin{pmatrix} 111 & 266 & 233 \\ 4236 & 1512512 & 1224 \\ 1337 & 48 & 52 \end{pmatrix}) = -462031464080$$

Task 15

Inverse matrix

Matrix inversion

Starting matrix inversion using Gauss-Jordan elimination. The identity matrix is augmented and operations are applied

Normalize pivot row

Dividing row 0 by pivot 1 to make pivot 1.

Eliminate column entry

$$\text{Row } 1 \leftarrow \text{Row } 1 - 48 \times \text{Row } 0.$$

Eliminate column entry

$$\text{Row } 2 \leftarrow \text{Row } 2 - 7 \times \text{Row } 0.$$

Normalize pivot row

Dividing row 1 by pivot -91 to make pivot 1.

Eliminate column entry

$$\text{Row } 0 \leftarrow \text{Row } 0 - 2 \times \text{Row } 1.$$

Eliminate column entry

$$\text{Row } 2 \leftarrow \text{Row } 2 - -12529 \times \text{Row } 1.$$

Normalize pivot row

Dividing row 2 by pivot 342471.4835164835 to make pivot 1.

Eliminate column entry

Row 0 $\leftarrow$ Row 0 $-$ $-2.7252747252 \times$ Row 2.

Eliminate column entry

Row 1 $\leftarrow$ Row 1 $-$ $27.3626373626 \times$ Row 2.

$$\begin{pmatrix} 1 & 2 & 52 \\ 48 & 5 & 6 \\ 7 & 12515 & 9 \end{pmatrix}^{-1} = \begin{pmatrix} -0.0024108849 & 0.0208823996 & 0.0000079576 \\ 0.0000125140 & 0.0000113910 & -0.0000798975 \\ 0.0192766510 & -0.0004020227 & 0.0000029199 \end{pmatrix}$$

Task 16

Gaussian elimination

Solving Ax = b via Gaussian elimination

Solving the system of linear equations using Gaussian elimination with back substitution.

Eliminating element

Row $j = 1 \leftarrow$ Row $j - 124125 \times$ Row $i = 0$.

Updating matrix element

$$a_{1,0} = 124125 - 124125 \times 1 = 0$$

Updating matrix element

$$a_{1,1} = 5 - 124125 \times 12515 = 1553424380$$

Updating matrix element

$$a_{1,2} = 6 - 124125 \times 3 = -372369$$

Updating RHS

$$b_1 = 4 - 124125 \times 5 = -620621$$

Eliminating element

$$\text{Row } j = 2 \leftarrow \text{Row } j - 7 \times \text{Row } i = 0.$$

Updating matrix element

$$a_{2,0} = 7 - 7 \times 1 = 0$$

Updating matrix element

$$a_{2,1} = 8 - 7 \times 12515 = 87613$$

Updating matrix element

$$a_{2,2} = 29841928419 - 7 \times 3 = 29841928398$$

Updating RHS

$$b_2 = 1 - 7 \times 5 = -34$$

Eliminating element

$$\text{Row } j = 2 \leftarrow \text{Row } j - 0.0000563999 \times \text{Row } i = 1.$$

Updating matrix element

$$a_{2,1} = 87613 - 0.0000563999 \times 1553424380 = 0$$

Updating matrix element

$$a_{2,2} = 29841928398 - 0.0000563999 \times -372369 = 29841928419.0015792316$$

Updating RHS

$$b_2 = -34 - 0.0000563999 \times -620621 = 1.0029704522$$

Back substitution

Starting back substitution.

Back substitution step

$$x_2 = \frac{1.0029704522}{29841928419.0015792316} = 0.0000000000$$

Back substitution step

$$x_1 = \frac{-620620.9999874848}{1553424380} = -0.0003995179$$

Back substitution step

$$x_0 = \frac{0.0000322416}{1} = 0.0000322416$$

Final result

System solved, final solution vector obtained.

Solving the system $\begin{pmatrix} 1 & 12515 & 3 \\ 124125 & 5 & 6 \\ 7 & 8 & 29841928419 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$ using Gaussian elimination:

$$\vec{x} = \begin{pmatrix} 0.0000322416 \\ -0.0003995179 \\ 0.0000000000 \end{pmatrix}$$

Task 17

Gauss-Jordan elimination

Solving Ax = b via Gauss-Jordan elimination

Solving the system of linear equations using the Gauss-Jordan method with full row reduction.

Normalize pivot row

Dividing row 0 by pivot 1 to make leading coefficient 1.

Normalizing matrix element

$$a(0,0) = 1 \div 1 = 1$$

Normalizing matrix element

$$a(0,1) = 12515 \div 1 = -12515$$

Normalizing matrix element

$$a(0,2) = 3 \div 1 = 3$$

Normalizing RHS

$$x[0] = 5 \div 1 = 5$$

Eliminate element

$$\text{Row } 1 \leftarrow \text{Row } 1 - 124125 \times \text{Row } 0.$$

Updating matrix element

$$a(1,0) = 124125 - 124125 \times 1 = 0$$

Updating matrix element

$$a(1,1) = 5 - 124125 \times -12515 = 1553424380$$

Updating matrix element

$$a(1,2) = 6 - 124125 \times 3 = -372369$$

Updating RHS

$$x[1] = 4 - 124125 \times 5 = -620621$$

Eliminate element

$$\text{Row } 2 \leftarrow \text{Row } 2 - 7 \times \text{Row } 0.$$

Updating matrix element

$$a(2,0) = 7 - 7 \times 1 = 0$$

Updating matrix element

$$a(2,1) = 8 - 7 \times -12515 = 87613$$

Updating matrix element

$$a(2,2) = 29841928419 - 7 \times 3 = 29841928398$$

Updating RHS

$$x[2] = 1 - 7 \times 5 = -34$$

Normalize pivot row

Dividing row 1 by pivot 1553424380 to make leading coefficient 1.

Normalizing matrix element

$$a(1,0) = 0 \div 1553424380 = 0$$

Normalizing matrix element

$$a(1,1) = 1553424380 \div 1553424380 = 1$$

Normalizing matrix element

$$a(1,2) = -372369 \div 1553424380 = -0.0002397084$$

Normalizing RHS

$$x[1] = -620621 \div 1553424380 = -0.0003995179$$

Eliminate element

$$\text{Row } 0 \leftarrow \text{Row } 0 - -12515 \times \text{Row } 1.$$

Updating matrix element

$$a(0,0) = 1 - -12515 \times 0 = 1$$

Updating matrix element

$$a(0,1) = -12515 - -12515 \times 1 = 0$$

Updating matrix element

$$a(0,2) = 3 - -12515 \times -0.0002397084 = 0.0000483480$$

Updating RHS

$$x[0] = 5 - -12515 \times -0.0003995179 = 0.0000322416$$

Eliminate element

$$\text{Row 2} \leftarrow \text{Row 2} - 87613 \times \text{Row 1}.$$

Updating matrix element

$$a(2,0) = 0 - 87613 \times 0 = 0$$

Updating matrix element

$$a(2,1) = 87613 - 87613 \times 1 = 0$$

Updating matrix element

$$a(2,2) = 29841928398 - 87613 \times -0.0002397084 = 29841928419.0015792316$$

Updating RHS

$$x[2] = -34 - 87613 \times -0.0003995179 = 1.0029704522$$

Normalize pivot row

Dividing row 2 by pivot 29841928419.0015792316 to make leading coefficient 1.

Normalizing matrix element

$$a(2,0) = 0 \div 29841928419.0015792316 = 0$$

Normalizing matrix element

$$a(2,1) = 0 \div 29841928419.0015792316 = 0$$

Normalizing matrix element

$$a(2,2) = 29841928419.0015792316 \div 29841928419.0015792316 = 1$$

Normalizing RHS

$$x[2] = 1.0029704522 \div 29841928419.0015792316 = 0.0000000000$$

Eliminate element

$$\text{Row } 0 \leftarrow \text{Row } 0 - 0.0000483480 \times \text{Row } 2.$$

Updating matrix element

$$a(0,0) = 1 - 0.0000483480 \times 0 = 1$$

Updating matrix element

$$a(0,1) = 0 - 0.0000483480 \times 0 = 0$$

Updating matrix element

$$a(0,2) = 0.0000483480 - 0.0000483480 \times 1 = 0$$

Updating RHS

$$x[0] = 0.0000322416 - 0.0000483480 \times 0.0000000000 = 0.0000322416$$

Eliminate element

$$\text{Row } 1 \leftarrow \text{Row } 1 - -0.0002397084 \times \text{Row } 2.$$

Updating matrix element

$$a(1,0) = 0 - -0.0002397084 \times 0 = 0$$

Updating matrix element

$$a(1,1) = 1 - -0.0002397084 \times 0 = 1$$

Updating matrix element

$$a(1,2) = -0.0002397084 - -0.0002397084 \times 1 = 0$$

Updating RHS

$$x[1] = -0.0003995179 - -0.0002397084 \times 0.0000000000 = -0.0003995179$$

Solving the system $\begin{pmatrix} 1 & 12515 & 3 \\ 124125 & 5 & 6 \\ 7 & 8 & 29841928419 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$ using Gauss-Jordan elimination:

$$\vec{x} = \begin{pmatrix} 0.0000322416 \\ -0.0003995179 \\ 0.0000000000 \end{pmatrix}$$

Task 18

Matrix rank

Rank calculation

Starting rank calculation by Gaussian elimination.

Row swap

Swapped rows 0 and 2 to bring pivot into position.

Pivot selected

Pivot at position $(0, 0) = 7$

Eliminating row

Row 1 -= $0.5714285714 \times$ row 0

Updating element

$$\text{temp}(1, 0) = 4 - 0.5714285714 \times 7 = 0$$

Updating element

$$\text{temp}(1, 1) = 5 - 0.5714285714 \times 8 = 0.4285714285$$

Updating element

$$\text{temp}(1,2) = 6 - 0.5714285714 \times 9 = 0.8571428571$$

Eliminating row

$$\text{Row 2 -= } 0.1428571428 \times \text{row 0}$$

Updating element

$$\text{temp}(2,0) = 1 - 0.1428571428 \times 7 = 0$$

Updating element

$$\text{temp}(2,1) = 2 - 0.1428571428 \times 8 = 0.8571428571$$

Updating element

$$\text{temp}(2,2) = 3 - 0.1428571428 \times 9 = 1.7142857142$$

Rank incremented

Current rank: 1

Row swap

Swapped rows 1 and 2 to bring pivot into position.

Pivot selected

Pivot at position $(1,1) = 0.8571428571$

Eliminating row

$$\text{Row 2 -= } 0.5 \times \text{row 1}$$

Updating element

$$\text{temp}(2,1) = 0.4285714285 - 0.5 \times 0.8571428571 = 0$$

Updating element

$$\text{temp}(2,2) = 0.8571428571 - 0.5 \times 1.7142857142 = 0$$

Rank incremented

Current rank: 2

Skipping column

All elements below row 2 in column 2 are zero, skipping.

$$\text{rank}(\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}) = 2$$

Task 19

Span size

Span dimension

Constructing a matrix from input vectors to compute the dimension of their span.

Inserting vector

$$\text{Row } 0 = [1, 2, 3]$$

Inserting vector

$$\text{Row } 1 = [4, 5, 6]$$

Inserting vector

$$\text{Row } 2 = [7, 8, 9]$$

Rank calculation

Starting rank calculation by Gaussian elimination.

Row swap

Swapped rows 0 and 2 to bring pivot into position.

Pivot selected

$$\text{Pivot at position } (0,0) = 7$$

Eliminating row

$$\text{Row 1 -= } 0.5714285714 \times \text{row } 0$$

Updating element

$$\text{temp}(1,0) = 4 - 0.5714285714 \times 7 = 0$$

Updating element

$$\text{temp}(1,1) = 5 - 0.5714285714 \times 8 = 0.4285714285$$

Updating element

$$\text{temp}(1,2) = 6 - 0.5714285714 \times 9 = 0.8571428571$$

Eliminating row

$$\text{Row 2 -= } 0.1428571428 \times \text{row } 0$$

Updating element

$$\text{temp}(2,0) = 1 - 0.1428571428 \times 7 = 0$$

Updating element

$$\text{temp}(2,1) = 2 - 0.1428571428 \times 8 = 0.8571428571$$

Updating element

$$\text{temp}(2,2) = 3 - 0.1428571428 \times 9 = 1.7142857142$$

Rank incremented

Current rank: 1

Row swap

Swapped rows 1 and 2 to bring pivot into position.

Pivot selected

$$\text{Pivot at position } (1,1) = 0.8571428571$$

Eliminating row

$$\text{Row 2 -= } 0.5 \times \text{row 1}$$

Updating element

$$\text{temp}(2,1) = 0.4285714285 - 0.5 \times 0.8571428571 = 0$$

Updating element

$$\text{temp}(2,2) = 0.8571428571 - 0.5 \times 1.7142857142 = 0$$

Rank incremented

Current rank: 2

Skipping column

All elements below row 2 in column 2 are zero, skipping.

Dimension of the span of the given vectors:

$$\dim(\text{span}) = 2$$

Task 20

Membership in span

Span membership check

Checking if the given vector is in the span of the basis.

Inserting basis vector

$$\text{Row } 0 = [4, 5, 6]$$

Inserting basis vector

$$\text{Row } 1 = [7, 8, 9]$$

Vector to test

$$\text{Target vector } = [1, 2, 3]$$

Checking whether vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ belongs to the span: Result: does not belong

---

Task 21

All line equations

Parsing general line equation

$$\text{Given: } 2x + 3y + 5 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \mathbf{direction} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-3 \cdot -3 + 2 \cdot 2 = 13$$

Norm calculation

$$\|\vec{v}\| = \sqrt{13}$$

Norm result

$$\|\vec{v}\| = \sqrt{13} = 3.6055512754$$

Input equation

```
2x + 3y + 5 = 0
```

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} -0.8320502943 \\ 0.5547001962 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.5547001962, \quad B = 0.8320502943, \quad C = -(A \cdot x_0 + B \cdot y_0) = 1.3867504905$$

General form

$$0.5547001962x + 0.8320502943y + 1.3867504905 = 0$$

General form of the line

$$0.5547001962x + 0.8320502943y + 1.3867504905 = 0$$

Parametric form

$$\vec{r}(t) = \vec{p} + t \cdot \vec{d} = \begin{pmatrix} 0 \\ -1.6666666666 \end{pmatrix} + t \cdot \begin{pmatrix} -0.8320502943 \\ 0.5547001962 \end{pmatrix}$$

Parametric form of the line

$$\vec{r} = \begin{pmatrix} 0 \\ -1.6666666666 \end{pmatrix} + t \cdot \begin{pmatrix} -0.8320502943 \\ 0.5547001962 \end{pmatrix}$$

Canonical form

$$\frac{x - 0}{-0.8320502943} = \frac{y - -1.6666666666}{0.5547001962}$$

Canonical form of the line

$$\frac{x - 0}{-0.8320502943} = \frac{y - -1.6666666666}{0.5547001962}$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} -0.8320502943 \\ 0.5547001962 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.5547001962, \quad B = 0.8320502943, \quad C = -(A \cdot x_0 + B \cdot y_0) = 1.3867504905$$

General form

$$0.5547001962x + 0.8320502943y + 1.3867504905 = 0$$

Normal form derivation

$$\|\vec{n}\| = \sqrt{A^2 + B^2} = 0.9999999999, \quad p = \frac{|C|}{\|\vec{n}\|} = 1.3867504905$$

Angle   of normal vector

$$\alpha = \arctan\left(\frac{B}{A}\right) = 0.9827937285$$

Normal form

$$x \cos\alpha + y \sin\alpha = 1.3867504905$$

Normal form of the line

$$x \cos(0.9827937285) + y \sin(0.9827937285) = 1.3867504905$$

Slope k and intercept b

$$k = \frac{d_y}{d_x} = -0.6666666666, \quad b = y_0 - k \cdot x_0 = -1.6666666666$$

Slope-intercept form

$$y = -0.6666666666x + -1.6666666666$$

Slope-intercept form of the line

$$y = -0.6666666666x + -1.6666666666$$

Task 22

Intersection of two lines

Given equations

$$\texttt{2x + 3y + 5 = 0)5x + 16y + 8 = 0}$$

Parsing general line equation

$$\text{Given: } 2x + 3y + 5 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \mathbf{direction} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-3 \cdot -3 + 2 \cdot 2 = 13$$

Norm calculation

$$\|\vec{v}\| = \sqrt{13}$$

Norm result

$$\|\vec{v}\| = \sqrt{13} = 3.6055512754$$

Parsing general line equation

$$\text{Given: } 5x + 16y + 8 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 5 \\ 16 \end{pmatrix}, \quad \mathbf{direction} = \begin{pmatrix} -16 \\ 5 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-16 \cdot -16 + 5 \cdot 5 = 281$$

Norm calculation

$$\|\vec{v}\| = \sqrt{281}$$

Norm result

$$\|\vec{v}\| = \sqrt{281} = 16.7630546142$$

Parsing equations

Converting to internal representation

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} -0.8320502943 \\ 0.5547001962 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.5547001962, \quad B = 0.8320502943, \quad C = -(A \cdot x_0 + B \cdot y_0) = 1.3867504905$$

General form

$$0.5547001962x + 0.8320502943y + 1.3867504905 = 0$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} -0.9544799780 \\ 0.2982749931 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.2982749931, \quad B = 0.9544799780, \quad C = -(A \cdot x_0 + B \cdot y_0) = 0.4772399890$$

General form

$$0.2982749931x + 0.9544799780y + 0.4772399890 = 0$$

General forms of the lines

$$\text{Line 1: } 0.5547001962x{+}0.8320502943y{+}1.3867504905 = 0\text{Line 2: } 0.2982749931x{+}0.9544799780y{+}0.4772399890 = 0$$

Determinant calculation

$$\text{Det} = A_1 B_2 - B_1 A_2 = 0.5547001962 \cdot 0.9544799780 - 0.8320502943 \cdot 0.2982749931 = 0.2812704352$$

Intersection point calculation

$$x = \frac{B_1 C_2 - C_1 B_2}{\text{det}} = \frac{0.8320502943 \cdot 0.4772399890 - 1.3867504905 \cdot 0.9544799780}{0.2812704352} = -3.2941176470y = \frac{C_1 A_2 - A_1 C_2}{\text{det}} =$$

Intersection result

$$\text{Intersection point: } \begin{pmatrix} -3.2941176470 \\ 0.5294117647 \end{pmatrix}$$

Intersection point

$$\text{The lines intersect at: } \mathbf{p} = \begin{pmatrix} -3.2941176470 \\ 0.5294117647 \end{pmatrix}$$

Task 23

Distance from point to line

Input

$$Equation : 3x + 4y - 10 = 0, Point : (23)$$

Parsing general line equation

$$\text{Given: } 3x + 4y + 10 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad \textbf{direction} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-4 \cdot -4 + 3 \cdot 3 = 25$$

Norm calculation

$$\|\vec{v}\| = \sqrt{25}$$

Norm result

$$\|\vec{v}\| = \sqrt{25} = 5.0000000000$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} -0.7999999999 \\ 0.5999999999 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.5999999999, \quad B = 0.7999999999, \quad C = -(A \cdot x_0 + B \cdot y_0) = -1.9999999999$$

General form

$$0.5999999999x + 0.7999999999y + -1.9999999999 = 0$$

General form of the line

$$\text{Line: } 0.5999999999x + 0.7999999999y + -1.9999999999 = 0$$

Numerator calculation (absolute value of line equation at point)

$$|Ax_0 + By_0 + C| = |0.5999999999 \cdot 2 + 0.7999999999 \cdot 3 + -1.9999999999| = 1.5999999999$$

Denominator calculation (norm of vector (A, B))

$$\sqrt{A^2 + B^2} = \sqrt{0.5999999999^2 + 0.7999999999^2} = 0.9999999999$$

Distance calculation

$$\text{Distance} = \frac{\text{Numerator}}{\text{Denominator}} = \frac{1.5999999999}{0.9999999999} = 1.5999999999$$

Distance

$$Distance = 1.5999999999$$

Task 24

Distance between two parallel lines

Parsing general line equation

$$\text{Given: } 2x + 3y + 5 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \textbf{direction} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-3 \cdot -3 + 2 \cdot 2 = 13$$

Norm calculation

$$\|\vec{v}\| = \sqrt{13}$$

Norm result

$$\|\vec{v}\| = \sqrt{13} = 3.6055512754$$

Parsing general line equation

$$\text{Given: } 2x + 3y + 3 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \textbf{direction} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-3 \cdot -3 + 2 \cdot 2 = 13$$

Norm calculation

$$\|\vec{v}\| = \sqrt{13}$$

Norm result

$$\|\vec{v}\| = \sqrt{13} = 3.6055512754$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} 0.8320502943 \\ 0.5547001962 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.5547001962, \quad B = -0.8320502943, \quad C = -(A \cdot x_0 + B \cdot y_0) = -0.8320502943$$

General form

$$0.5547001962x + -0.8320502943y + -0.8320502943 = 0$$

General form of the line

$$\text{Line: } 0.5547001962x + -0.8320502943y + -0.8320502943 = 0$$

Numerator calculation (absolute value of line equation at point)

$$|Ax_0 + By_0 + C| = |0.5547001962 \cdot 0 + -0.8320502943 \cdot 1.6666666666 + -0.8320502943| = 2.2188007849$$

Denominator calculation (norm of vector (A, B))

$$\sqrt{A^2 + B^2} = \sqrt{0.5547001962^2 + -0.8320502943^2} = 0.9999999999$$

Distance calculation

$$\text{Distance} = \frac{\text{Numerator}}{\text{Denominator}} = \frac{2.2188007849}{0.9999999999} = 2.2188007849$$

Distance

$$Distance = 2.2188007849$$

Task 25

Triangle area formed with axes

Parsing general line equation

$$\text{Given: } 1x + 2y + 6 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \textbf{direction} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-2 \cdot -2 + 1 \cdot 1 = 5$$

Norm calculation

$$\|\vec{v}\| = \sqrt{5}$$

Norm result

$$\|\vec{v}\| = \sqrt{5} = 2.2360679774$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} -0.8944271909 \\ 0.4472135954 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.4472135954, \quad B = 0.8944271909, \quad C = -(A \cdot x_0 + B \cdot y_0) = -2.6832815729$$

General form

$$0.4472135954x + 0.8944271909y + -2.6832815729 = 0$$

Triangle area

$$Area = 9$$

Task 26

Angle between two lines

Parsing general line equation

$$\text{Given: } 2x + 3y + 5 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad \mathbf{direction} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-3 \cdot -3 + 2 \cdot 2 = 13$$

Norm calculation

$$\|\vec{v}\| = \sqrt{13}$$

Norm result

$$\|\vec{v}\| = \sqrt{13} = 3.6055512754$$

Parsing general line equation

$$\text{Given: } 5x + 1y + 8 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad \mathbf{direction} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-1 \cdot -1 + 5 \cdot 5 = 26$$

Norm calculation

$$\|\vec{v}\| = \sqrt{26}$$

Norm result

$$\|\vec{v}\| = \sqrt{26} = 5.0990195135$$

Check zero vectors

$$\text{Vector } a \text{ is zero: } false, \text{Vector } b \text{ is zero: } false$$

Dot product formula

$$-0.8320502943 \cdot -0.1961161351 + 0.5547001962 \cdot 0.9805806756 = 0.7071067811$$

Dot product calculation

$$\vec{a} \cdot \vec{b} = 0.7071067811$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-0.8320502943 \cdot -0.8320502943 + 0.5547001962 \cdot 0.5547001962 = 0.9999999999$$

Norm calculation

$$\|\vec{v}\| = \sqrt{0.9999999999}$$

Norm result

$$\|\vec{v}\| = \sqrt{0.9999999999} = 0.9999999999$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-0.1961161351 \cdot -0.1961161351 + 0.9805806756 \cdot 0.9805806756 = 0.9999999999$$

Norm calculation

$$\|\vec{v}\| = \sqrt{0.9999999999}$$

Norm result

$$\|\vec{v}\| = \sqrt{0.9999999999} = 0.9999999999$$

Norms calculation

$$\|\vec{a}\| = 0.9999999999, \|\vec{b}\| = 0.9999999999$$

Product of norms

$$\|\vec{a}\| \times \|\vec{b}\| = 0.9999999999$$

Cosine of angle

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|} = 0.7071067811$$

Angle calculation

$$\theta = \arccos(\cos\theta) = 0.7857227550$$

Angle

$$Angle = 0.7857227550$$

Task 27

Line and segment intersection

Parsing general line equation

$$\text{Given: } 1x + 1y + 1 = 0$$

Normal vector and direction vector

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \textbf{direction} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$-1 \cdot -1 + 1 \cdot 1 = 2$$

Norm calculation

$$\|\vec{v}\| = \sqrt{2}$$

Norm result

$$\|\vec{v}\| = \sqrt{2} = 1.4142135623$$

Input segment points

$$\text{Segment endpoints } A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$2 - 0 = 2$$

Component 2

$$2 - 0 = 2$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$2 \cdot 2 + 2 \cdot 2 = 8$$

Norm calculation

$$\|\vec{v}\| = \sqrt{8}$$

Norm result

$$\|\vec{v}\| = \sqrt{8} = 2.8284271247$$

Construct segment line

Line created from segment endpoints

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} 0.7071067811 \\ 0.7071067811 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.7071067811, \quad B = -0.7071067811, \quad C = -(A \cdot x_0 + B \cdot y_0) = 0.7071067811$$

General form

$$0.7071067811x + -0.7071067811y + 0.7071067811 = 0$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} 0.7071067811 \\ 0.7071067811 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.7071067811, \quad B = -0.7071067811, \quad C = -(A \cdot x_0 + B \cdot y_0) = -0$$

General form

$$0.7071067811x + -0.7071067811y + -0 = 0$$

General forms of the lines

$$\text{Line 1: } 0.7071067811x + -0.7071067811y + 0.7071067811 = 0 \text{Line 2: } 0.7071067811x + -0.7071067811y + -0 = 0$$

Determinant calculation

$$\text{Det} = A_1 B_2 - B_1 A_2 = 0.7071067811 \cdot -0.7071067811 - -0.7071067811 \cdot 0.7071067811 = 0$$

Lines are parallel

$$\text{Since } \det = 0, \text{ the lines are parallel or coincident. No unique intersection.}$$

Intersection check

$$\text{Lines do not intersect or are parallel/coincident.}$$

No intersection

$$No intersection within the segment$$

Task 28

Distance from point to segment

Input points

$$\text{Point } P = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \text{Segment endpoints } A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$6 - 0 = 6$$

Component 2

$$0 - 0 = 0$$

Vector AB calculation

$$\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$3 - 0 = 3$$

Component 2

$$4 - 0 = 4$$

Vector AP calculation

$$\vec{AP} = \vec{P} - \vec{A} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Dot product formula

$$3 \cdot 6 + 4 \cdot 0 = 18$$

Dot product formula

$$6 \cdot 6 + 0 \cdot 0 = 36$$

Dot products

$$\vec{AP} \cdot \vec{AB} = 18, \quad \vec{AB} \cdot \vec{AB} = 36$$

Initial parameter t calculation

$$t = \frac{\vec{AP} \cdot \vec{AB}}{\vec{AB} \cdot \vec{AB}} = 0.5$$

Clamping t to [0, 1]

$$t = \max(0, \min(1, t)) = 0.5$$

Scalar multiplication formula

$$\lambda \cdot (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$$

Component 1

$$0.5 \cdot 6 = 3$$

Component 2

$$0.5 \cdot 0 = 0$$

Vector addition formula

$$(a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$$

Component 1

$$0 + 3 = 3$$

Component 2

$$0 + 0 = 0$$

Projection point on segment

$$\vec{P}_{proj} = \vec{A} + t\vec{AB} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$3 - 3 = 0$$

Component 2

$$4 - 0 = 4$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$0 \cdot 0 + 4 \cdot 4 = 16$$

Norm calculation

$$\|\vec{v}\| = \sqrt{16}$$

Norm result

$$\|\vec{v}\| = \sqrt{16} = 4$$

Distance

$$Distance = 4$$

Task 29

Segment and segment intersection

Input segments

$$\text{Segment 1: } \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \text{Segment 2: } \begin{pmatrix} 0 \\ 2 \end{pmatrix} \text{ to } \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$2 - 0 = 2$$

Component 2

$$2 - 0 = 2$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$2 \cdot 2 + 2 \cdot 2 = 8$$

Norm calculation

$$\|\vec{v}\| = \sqrt{8}$$

Norm result

$$\|\vec{v}\| = \sqrt{8} = 2.8284271247$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$2 - 0 = 2$$

Component 2

$$0 - 2 = -2$$

Norm formula

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

Dot product formula

$$2 \cdot 2 + -2 \cdot -2 = 8$$

Norm calculation

$$\|\vec{v}\| = \sqrt{8}$$

Norm result

$$\|\vec{v}\| = \sqrt{8} = 2.8284271247$$

Constructing Line2D objects

$$Created lines from segments endpoints$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} 0.7071067811 \\ 0.7071067811 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = 0.7071067811, \quad B = -0.7071067811, \quad C = -(A \cdot x_0 + B \cdot y_0) = -0$$

General form

$$0.7071067811x + -0.7071067811y + -0 = 0$$

General form derivation

$$\text{Direction vector: } \vec{d} = \begin{pmatrix} 0.7071067811 \\ -0.7071067811 \end{pmatrix}$$

Coefficients A, B, C for Ax + By + C = 0

$$A = -0.7071067811, \quad B = -0.7071067811, \quad C = -(A \cdot x_0 + B \cdot y_0) = 1.4142135623$$

General form

$$-0.7071067811x + -0.7071067811y + 1.4142135623 = 0$$

General forms of the lines

$$\text{Line 1: } 0.7071067811x + -0.7071067811y + -0 = 0 \text{Line 2: } -0.7071067811x + -0.7071067811y + 1.4142135623 = 0$$

Determinant calculation

$$\text{Det} = A_1B_2 - B_1A_2 = 0.7071067811 \cdot -0.7071067811 - -0.7071067811 \cdot -0.7071067811 = -0.9999999999$$

Intersection point calculation

$$x = \frac{B_1C_2 - C_1B_2}{\det} = \frac{-0.7071067811 \cdot 1.4142135623 - -0 \cdot -0.7071067811}{-0.9999999999} = 1 y = \frac{C_1A_2 - A_1C_2}{\det} = \frac{-0 \cdot -0.7071067811}{-0}$$

Intersection result

$$\text{Intersection point: } \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Intersection point found

$$\text{Intersection point: } \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Input points

$$\text{Point } P = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{Segment endpoints } A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$2 - 0 = 2$$

Component 2

$$2 - 0 = 2$$

Vector AB calculation

$$\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$1 - 0 = 1$$

Component 2

$$1 - 0 = 1$$

Vector AP calculation

$$\vec{AP} = \vec{P} - \vec{A} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Collinearity check

$$Are vectors AB and AP collinear? Yes$$

Dot product formula

$$2 \cdot 1 + 2 \cdot 1 = 4$$

Dot product formula

$$2 \cdot 2 + 2 \cdot 2 = 8$$

Dot product values

$$\vec{AB} \cdot \vec{AP} = 4, \quad \vec{AB} \cdot \vec{AB} = 8$$

Parameter t range check

$$Is t in [0,1] (with epsilon)? Yes$$

Input points

$$\text{Point } P = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{Segment endpoints } A = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$2 - 0 = 2$$

Component 2

$$0 - 2 = -2$$

Vector AB calculation

$$\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

Vector subtraction formula

$$(a_1, a_2, \ldots, a_n) - (b_1, b_2, \ldots, b_n) = (a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n)$$

Component 1

$$1 - 0 = 1$$

Component 2

$$1 - 2 = -1$$

Vector AP calculation

$$\vec{AP} = \vec{P} - \vec{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Collinearity check

$$Are vectors AB and AP collinear? Yes$$

Dot product formula

$$2 \cdot 1 + -2 \cdot -1 = 4$$

Dot product formula

$$2 \cdot 2 + -2 \cdot -2 = 8$$

Dot product values

$$\vec{AB} \cdot \vec{AP} = 4, \quad \vec{AB} \cdot \vec{AB} = 8$$

Parameter t range check

$$Is t in [0,1] (with epsilon)? Yes$$

Checking if intersection is on segments

$$\text{On segment 1: } true, \text{On segment 2: } true$$

Intersection

$$Point : [1, 1]$$