

Statistical Machine Learning Homework 2

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I. QUESTION 1

A. (Conceptual)

In this conceptual problem, the challenge of predicting autism lies in the different training and testing set. The training set, which comes from the 4 U.S. database, containing genetic, neurologics, environmental risks factors along with ethnicity and demographic info. But, the testing set is in European standard. Given that we cannot mix the training and test sets, I focused on mitigating distributional differences among the data sets.

The first step is to clean the data by handling missing values, either by removing samples with excessive missingness or imputing missing values using the median. Since categorical variables likely contain important demographic information, we should one-hot encode these features to preserve their structure. Also, feature selection is crucial to prevent overfitting and improve model interpretability. We can use Recursive Feature Elimination (RFE), LASSO, or Elastic Net, adjusting the regularization parameter α to retain only the most predictive features.

Given the significant demographic differences between training and test populations, we should ensure our model is exposed to a more diverse representation during training. To improve generalization, we can introduce controlled bias into the training data by identifying and sampling cases from the U.S. dataset that resemble the European population. This subset is then merged with the original training data, ensuring that the model is exposed to patterns more representative of the test set. By doing this, we introduce a slight bias in the training data, which helps reduce variance when making predictions on the European cases.

This technique is beneficial because it prevents the model from being overly optimized for the American population while still respecting the constraint that the training and test sets must remain separate. Additionally, after constructing this adjusted training set, we can apply stratified sampling to maintain balanced class distributions, ensuring that rare cases (e.g., autism-positive samples) are well-represented in training.

As our model selection and sampling, since our data contains both discrete and continuous variables, we should consider models that can handle such a structure effectively. Logistic regression is a strong baseline, providing probabilistic outputs and interpretability. Moreover, Linear SVM with L2 regularization can improve generalizability by preventing overfitting, while Linear Discriminant Analysis (LDA) can be useful if the data follows a normal distribution and class separation is strong.

Given the known differences between the training and test features, we must account for domain shift explicitly. One approach is to scale features based on their distributional differences across the two datasets, making sure that model predictions are not biased toward one population. To capture nonlinear relationships in the data, kernel ridge regression can be introduced, particularly if the feature space exhibits complex interactions.

Next, as the hyper tuning approach, We should use stratified K-fold cross-validation to ensure class balance in each fold, which is particularly important in medical classification problems where class imbalances are common. For logistic regression, tuning α helps control the level of regularization. For SVM, we should tune the regularization parameter to balance bias and variance. In kernel ridge regression, we should optimize the kernel parameter to ensure that the model effectively captures nonlinearity.

Lastly, to evaluate prediction performance, we should use a combination of metrics that account for both overall accuracy and class-specific performance. Accuracy alone can be misleading, especially if there is class imbalance, so we should report the mis classification rate (1accuracy) as a baseline metric. I also believe that precision and recall are critical, particularly in medical applications where false negatives (missed autism cases) are costly. In addition, the F1 score, as the harmonic mean of precision and recall, provides a balanced measure of performance. Lastly, the ROC-AUC curve allows us to assess the model's ability to distinguish between classes across different decision thresholds.

II. QUESTION 2

A. (Data Preprocessing)

We are given Communities and Crime dataset and our goal is to the violent crime rate based upon features of a community.

The data (X) came with a mix of numerical and object columns with some columns having Na values. My approaches of data cleaning is as follows: First, I found columns with '?' values and checked their data types. Since they were all 'object' data types and contained over 1000 Na values, except 'OtherPerCap', which only had just one Na value. So, I removed all columns with '?' values, while keeping 'OtherPerCap' column. I imputed the '?' in 'OtherPerCap' with its median value. Next, I checked the data types of the rest of remaining columns in the original data set. In here, 'state', 'fold', and 'communityname' were non-float data types, so I dropped those three columns. Now, the updated data set (X) had contained 99 columns with zero Na

values and all features were float data types, which matched with the data type of y .

III. CROSS-VALIDATION

A. Manually coded K-fold CV Approach

In order for the K-fold CV function to work for both RBF and Polynomial Kernel, I first created Gaussian kernel, Polynomial Kernel, and Kernel Ridge Regression function. The Gaussian kernel function computes a similarity measure between two sets of input vectors, $X1$ and $X2$, using the Radial Basis Function. It calculates the squared Euclidean distance between each pair of points in $X1$ and $X2$ and applies an exponential transformation controlled by the parameter γ . The resulting kernel matrix represents the non-linear relationships between the data points.

Polynomial kernel function computes a similarity score between two sets of input vectors based on a polynomial transformation. The Kernel Ridge Regression function solves the regularized least squares problem using a precomputed kernel matrix K . Given a kernel matrix K , target values y , and a regularization parameter λ , the function solves the system $(K + \lambda I) \alpha = y$ to obtain the weight vector α . The inclusion of λ helps control overfitting by penalizing large coefficients.

My approach of K-fold CV function was to evaluate Kernel Ridge Regression using different hyperparameter combinations. It partitions the dataset into n folds, where each fold serves as a validation set while the rest act as the training set. For each combination of regularization parameter λ and kernel parameter, the function computes the kernel matrix for both training and validation data. The model is trained using the training kernel matrix, and predictions are made on the validation set. The function calculates the Mean Squared Error for each fold and records the average MSE and its standard error across all folds. This approach helps in selecting the optimal combination of λ and kernel parameters, ensuring the model generalizes well to unseen data.

When performing CV for RBF and Polynomial kernel, I used the following values: λ s = [0.01, 0.1, 1, 10] γ s = [0.01, 0.1, 1] degrees = [2, 3, 4]

B. Results from CV

Below are the best parameters that provided the least MSE loss.

- **RBF Kernel:** ' γ '=0.01, ' α '=1, 'MSE'=0.0203
- **Polynomial Kernel:** 'degree'=2, ' α '=10, 'MSE'=0.0720

After this, I compared the results from the built-in sklearn cv as well as the linear regression model from the previous homework. The results are below:

- **RBF Kernel (sklearn):** ' α ': 1, ' γ ': 0.01, 'MSE': 0.02
- **Polynomial Kernel (sklearn):** ' α ': 10, 'degree': 3, 'MSE': 0.0179
- **Linear Regression in HW1:** 'MSE': 0.192

Comparing the prediction errors from manually coded K-fold CV, the best RBF kernel model achieved an MSE of 0.02035, while the polynomial kernel model had an MSE of 0.07207. This result suggested that the RBF kernel was a much better fit for the data than the polynomial kernel. However, when using the built-in sklearn cross-validation, the polynomial kernel model improved a lot, with an MSE of 0.0179, even outperforming the RBF model (MSE = 0.02053). Interestingly, the linear regression model from Homework 1 had an MSE of 0.0192, which was better than my manually tuned polynomial kernel but slightly worse than the best RBF and sklearn-optimized polynomial kernel models.

These results highlight that non-linear models provide improvements, but their effectiveness depends heavily on careful hyperparameter tuning. While the RBF kernel performed well, it did not significantly outperform linear regression, indicating that the data might not require complex transformations. However, the polynomial kernel, when optimized using sklearn, showed the lowest MSE, suggesting that some non-linear relationships in the data could be captured effectively.

IV. MULTI-CLASS CLASSIFICATION

A. Approaches

The dataset was first filtered to include only the selected digits [3,5,8], and the images were reshaped into a flattened vector format to be compatible with the classifiers.

The One-vs-Rest (OvR) logistic regression model fits separate binary classifiers for each class, treating it as a multi-label problem. In contrast, multinomial logistic regression directly optimizes for multi-class classification by considering all classes simultaneously. Naive Bayes, being a generative model, assumes feature independence and uses Bayes' theorem to compute class probabilities, making it computationally efficient. LDA, another generative model, assumes normally distributed data and maximizes class separability by projecting the data onto lower-dimensional space. Finally, Linear SVM (OvR) finds a linear decision boundary using support vector machines but extends the binary SVM classifier to multi-class classification using the One-vs-Rest strategy.

For evaluation, I trained each model and measured performance using accuracy, classification reports, and confusion matrices. Additionally, I visualized correctly and incorrectly classified images to gain insights into the errors made by each model. The confusion matrices helped identify misclassification patterns, showing which digits were often confused. Furthermore, I randomly selected examples of both correctly and incorrectly classified images to better understand how well each model captured digit variations.

B. Results of 5 Classification Models

- **Logistic Regression OvR: 0.9256**

Confusion Matrix:

947	29	34
34	806	52
28	37	909

- **Multinomial: 0.9221**

Confusion Matrix:

946	29	35
41	803	48
32	39	903

- **Naive Bayes: 0.5035**

Confusion Matrix:

422	16	572
45	90	757
19	19	936

- **Linear Discriminant Analysis: 0.9200**

Confusion Matrix:

933	43	34
33	823	36
30	54	890

- **Linear SVM OvR: 0.8814**

Confusion Matrix:

979	20	11
68	798	26
139	77	758

Across all five models, the accuracy stayed around high 80s percent to low 90s percent range, except Naive Bayes, which produced 50 percent predictive accuracy. Of all the methods, the Logistic Regression (One-vs-Rest) method performs best in terms of test accuracy, achieving an accuracy of 0.9256. I believe the reason for its superior performance lies in its ability to effectively separate the classes by training a binary classifier for each class. This approach works well for the given dataset, as it can capture the distinct features of each digit (3, 5, and 8) and minimize misclassifications. Logistic Regression (OvR) is particularly effective when the classes are well-separated, and it benefits from the regularization applied during training, which helps prevent overfitting.

When we look at Logistic Regression OvR's confusion matrix, the model correctly classified 94 instances of digit 3. However, it misclassified 29 instances as digit 5 and 34 instances as digit 8. For digit 5, the model correctly classified 806 instances of digit 5. and misclassified 52 instances as digit 3 and 28 instances as digit 8. For digit 8, the model correctly classified 909 instances of digit 8 and misclassified 37 instances as digit 3 and 34 instances as digit 5.

The confusion matrices indicates that the digit 3 is the most often misclassified across all models, including the best-performing Logistic Regression (OvR). In the confusion matrix above, digit 3 has a total of 63 misclassifications (29 as digit 5 and 34 as digit 8). This suggests that the features of digit 3 overlap more with those of digits 5 and 8, making it harder for the model to distinguish between them. This could be due to similarities in the shape or structure of these digits, such as curves or strokes that are common across them. According to figure 1 through figure 5, where I randomly selected examples of both correctly and incorrectly classified images, there are many cases where the predicted value was a 5 when the truth was a 3, and vice versa.

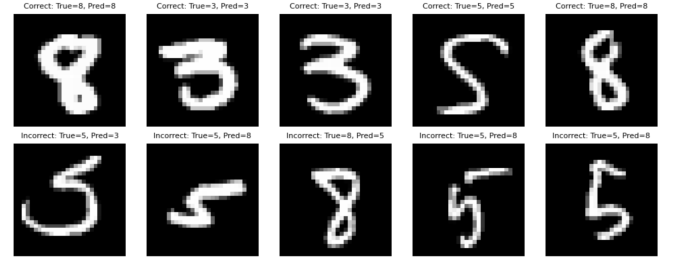


Fig. 1. "Logistic Regression OvR"

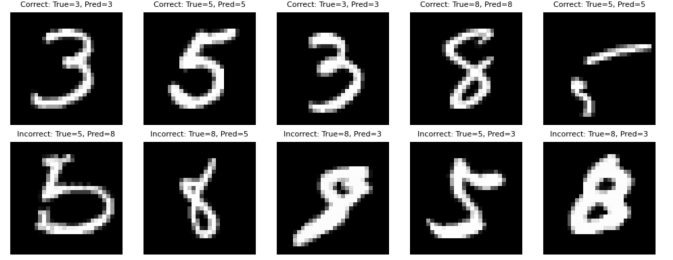


Fig. 2. "Multinomial"

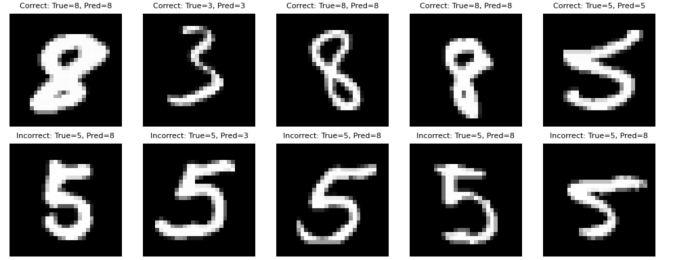


Fig. 3. "Naive Bayes"

C. Regularization

To apply Group-Lasso regularized multinomial logistic regression for feature selection, I first flattened images into 1D vectors, resulting in a feature vector of length 784 (28x28 pixels). With the grouping approach, each pixel was treated as an individual group. This means that the Group-Lasso regularization was applied to each pixel independently, allowing the model to select or discard individual pixels based on their importance for classification.

The model was then trained on the standardized training data and the corresponding labels. During training, the Group-Lasso regularization encouraged sparsity in the feature space, effectively selecting only the most important pixels for classification. After training, the sparsity mask attribute of the GroupLasso model was used to extract the selected features. This mask is a boolean array where True indicates that the corresponding feature (pixel) was selected by the model, and False indicates that it was discarded.

The result provided a sparse selection, selecting only 34 out of 784 total pixels. The visualization in Fig 6 provides insights into which parts of the images are most discriminative

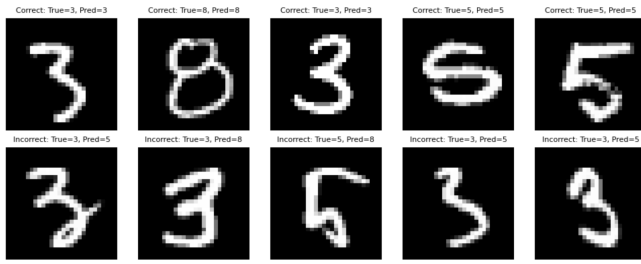


Fig. 4. "Linear Discriminant Analysis"

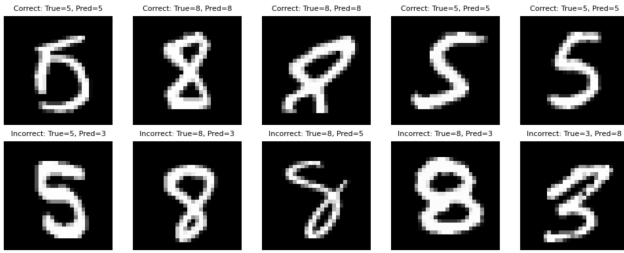


Fig. 5. "SVM OvR"

for classification. The Selected Pixels are in white, meaning these are the pixels that the Group-Lasso model identified as important for distinguishing between the digits. Pixels appear in the upper, middle, and lower regions of the image. This may indicate that the model finds differences across the entire digit structure rather than just in one localized area. The visualization helps us understand which regions of the images (e.g., edges, curves, or specific strokes) are critical for the model to differentiate between the digits.

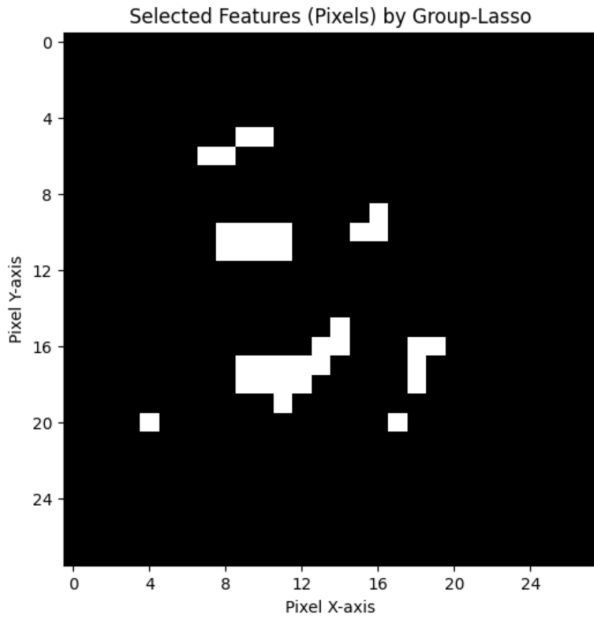


Fig. 6. "Selected Features By Group Lasso"

```
import pandas as pd
import numpy as np
import warnings; warnings.simplefilter('ignore')
```

```
Intel MKL WARNING: Support of Intel(R) Streaming SIMD Extensions 4.2
(Intel(R) SSE4.2) enabled only processors has been deprecated. Intel
oneAPI Math Kernel Library 2025.0 will require Intel(R) Advanced
Vector Extensions (Intel(R) AVX) instructions.
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```

```
from ucimlrepo import fetch_ucirepo

communities_and_crime = fetch_ucirepo(id=183)
X = communities_and_crime.data.features
y = communities_and_crime.data.targets
```

Data Preprocessing

```
#find columns with '?' values
na_columns = X.loc[:, X.isin(["?"]).any()]

#find number of na values in each columns with '?' values
for col in na_columns:
    occurence = na_columns[col].value_counts().get('?', 0)
    print(col, "has", occurence, "na values")
```

```
county has 1174 na values
community has 1177 na values
OtherPerCap has 1 na values
LemasSwornFT has 1675 na values
LemasSwFTPerPop has 1675 na values
LemasSwFTFieldOps has 1675 na values
LemasSwFTFieldPerPop has 1675 na values
LemasTotalReq has 1675 na values
LemasTotReqPerPop has 1675 na values
PolicReqPerOffic has 1675 na values
PolicPerPop has 1675 na values
RacialMatchCommPol has 1675 na values
PctPolicWhite has 1675 na values
PctPolicBlack has 1675 na values
PctPolicHisp has 1675 na values
PctPolicAsian has 1675 na values
PctPolicMinor has 1675 na values
OfficAssgnDrugUnits has 1675 na values
NumKindsDrugsSeiz has 1675 na values
PolicAveOTWorked has 1675 na values
PolicCars has 1675 na values
```

```
PolicOperBudg has 1675 na values
LemasPctPolicOnPatr has 1675 na values
LemasGangUnitDeploy has 1675 na values
PolicBudgPerPop has 1675 na values
```

```
#Only 'OtherPerCap' col has one na value, so it is not relevant to  
remove the whole column.
```

```
#So, I will impute missing value with the median for just 'OtherPerCap'
```

```
X['OtherPerCap'] = pd.to_numeric(X['OtherPerCap'].replace('?',  
np.nan), errors='coerce')
```

```
median_OtherPerCap = X['OtherPerCap'].median()  
X['OtherPerCap'].fillna(median_OtherPerCap, inplace=True)
```

```
#remove na_columns from original datasets and keep 'OtherPerCap' in X
```

```
X = X.drop(na_columns, axis=1)
```

```
na_columns = na_columns.drop('OtherPerCap', axis=1)
```

```
# Check datatype of columns in X
```

```
# 'state', 'communityname', and 'fold' are non float datatypes, so  
remove these columns.
```

```
for col in X:  
    print(col, "has", X[col].dtypes, "types")
```

```
X = X.drop("communityname", axis=1)
```

```
X = X.drop("state", axis=1)
```

```
X = X.drop("fold", axis=1)
```

```
state has int64 types
```

```
communityname has object types
```

```
fold has int64 types
```

```
population has float64 types
```

```
householdsize has float64 types
```

```
racepctblack has float64 types
```

```
racePctWhite has float64 types
```

```
racePctAsian has float64 types
```

```
racePctHisp has float64 types
```

```
agePct12t21 has float64 types
```

```
agePct12t29 has float64 types
```

```
agePct16t24 has float64 types
```

```
agePct65up has float64 types
```

```
numbUrban has float64 types
```

```
pctUrban has float64 types
```

```
medIncome has float64 types
```

```
pctWWage has float64 types
```

```
pctWFarmSelf has float64 types
```

```
pctWInvInc has float64 types
```

```
pctWSocSec has float64 types
```

```
pctWPubAsst has float64 types
```

pctWRetire has float64 types
medFamInc has float64 types
perCapInc has float64 types
whitePerCap has float64 types
blackPerCap has float64 types
indianPerCap has float64 types
AsianPerCap has float64 types
HispPerCap has float64 types
NumUnderPov has float64 types
PctPopUnderPov has float64 types
PctLess9thGrade has float64 types
PctNotHSGrad has float64 types
PctBSorMore has float64 types
PctUnemployed has float64 types
PctEmploy has float64 types
PctEmplManu has float64 types
PctEmplProfServ has float64 types
PctOccupManu has float64 types
PctOccupMgmtProf has float64 types
MalePctDivorce has float64 types
MalePctNevMarr has float64 types
FemalePctDiv has float64 types
TotalPctDiv has float64 types
PersPerFam has float64 types
PctFam2Par has float64 types
PctKids2Par has float64 types
PctYoungKids2Par has float64 types
PctTeen2Par has float64 types
PctWorkMomYoungKids has float64 types
PctWorkMom has float64 types
NumIlleg has float64 types
PctIlleg has float64 types
NumImmig has float64 types
PctImmigRecent has float64 types
PctImmigRec5 has float64 types
PctImmigRec8 has float64 types
PctImmigRec10 has float64 types
PctRecentImmig has float64 types
PctRecImmig5 has float64 types
PctRecImmig8 has float64 types
PctRecImmig10 has float64 types
PctSpeakEnglOnly has float64 types
PctNotSpeakEnglWell has float64 types
PctLargHouseFam has float64 types
PctLargHouseOccup has float64 types
PersPerOccupHous has float64 types
PersPerOwnOccHous has float64 types
PersPerRentOccHous has float64 types
PctPersOwnOccup has float64 types

```
PctPersDenseHous has float64 types
PctHousLess3BR has float64 types
MedNumBR has float64 types
HousVacant has float64 types
PctHous0ccup has float64 types
PctHousOwn0cc has float64 types
PctVacantBoarded has float64 types
PctVacMore6Mos has float64 types
MedYrHousBuilt has float64 types
PctHousNoPhone has float64 types
PctW0FullPlumb has float64 types
Own0ccLowQuart has float64 types
Own0ccMedVal has float64 types
Own0ccHiQuart has float64 types
RentLowQ has float64 types
RentMedian has float64 types
RentHighQ has float64 types
MedRent has float64 types
MedRentPctHousInc has float64 types
MedOwnCostPctInc has float64 types
MedOwnCostPctIncNoMtg has float64 types
NumInShelters has float64 types
NumStreet has float64 types
PctForeignBorn has float64 types
PctBornSameState has float64 types
PctSameHouse85 has float64 types
PctSameCity85 has float64 types
PctSameState85 has float64 types
LandArea has float64 types
PopDens has float64 types
PctUsePubTrans has float64 types
LemasPctOfficDrugUn has float64 types
```

To recap data cleaning, I found columns with '?' values and checked their datatypes. Since they are all object datatypes and contained over 1000 na values except 'OtherPerCap', I removed all columns with '?' values and kept 'OtherPerCap' column by imputing the '?' with median value. Then, I checked the datatypes of the rest of columns in X. 'state', 'fold', and 'communityname' are non float datatypes, so I dropped those three columns.

#2 a)

```
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import KFold
from sklearn.metrics import mean_squared_error

y = y.values.ravel()
scaler = StandardScaler(with_mean=True, with_std=True)
X_standardized = scaler.fit_transform(X)
```



```

#from lab7
#Generates a RBF Gaussian kernel from training data X
def gaussian_kernel(X_1: np.ndarray, X_2: np.ndarray, gamma: float = 1.0):
    pairwise = np.sum(X_1**2, axis=1).reshape(-1,1) + np.sum(X_2**2, axis=1)-2 * np.dot(X_1,X_2.T)
    return np.exp(-gamma * pairwise)

#Generates a simple polynomial kernel from training data X
def polynomial_kernel(X_1: np.ndarray, X_2: np.ndarray, d: int, r: float = 1.0, gamma: float = 1.0):
    return (r + gamma * np.dot(X_1, X_2.T)) ** d

#Kernel ridge regression function
def kernel_ridge_regression(K: np.ndarray, y: np.ndarray, lambd: float):
    n = K.shape[0]
    return np.linalg.solve(K+lambd*np.eye(n),y)

def cross_val_kernel_ridge(X_data: np.ndarray, y_data: np.ndarray, num_folds: int, kernel_name: str, kernel_vals: list, reg_params: list):
    kfold = KFold(n_splits=num_folds, shuffle=True, random_state=42)
    cv_results = {'reg_param': [], 'kernel_val': [], 'mean_mse': []}

    for reg_param in reg_params:
        for kernel_val in kernel_vals:
            mse_scores = []

            for train_indices, val_indices in kfold.split(X_data):
                X_train, X_valid = X_data[train_indices], X_data[val_indices]
                y_train, y_valid = y_data[train_indices], y_data[val_indices]

                # Compute kernel matrices
                if kernel_name == 'rbf':
                    K_train = gaussian_kernel(X_train, X_train, gamma=kernel_val)
                    K_valid = gaussian_kernel(X_valid, X_train, gamma=kernel_val)
                elif kernel_name == 'poly':
                    K_train = polynomial_kernel(X_train, X_train, d=kernel_val)
                    K_valid = polynomial_kernel(X_valid, X_train, d=kernel_val)

                # Train using Kernel Ridge Regression
                alpha_coeffs = kernel_ridge_regression(K_train, y_train, reg_param)

```

```

        # Predict and compute MSE
        y_predictions = np.dot(K_valid, alpha_coeffs)
        mse_scores.append(mean_squared_error(y_valid,
y_predictions))

    # Store results
    cv_results['reg_param'].append(reg_param)
    cv_results['kernel_val'].append(kernel_val)
    cv_results['mean_mse'].append(np.mean(mse_scores))

    return cv_results

```

b)

```

gammas = [0.001, 0.01, 0.1, 1, 10]
lambdas = [0.001, 0.01, 0.1, 1, 10]
degrees = [2, 3, 4, 5, 6]

# RBF kernel
results_rbf = cross_val_kernel_ridge(X_standardized, y, num_folds=5,
kernel_name='rbf', kernel_vals=gammas, reg_params=lambdas)

# polynomial kernel
results_poly = cross_val_kernel_ridge(X_standardized, y, num_folds=5,
kernel_name='poly', kernel_vals=degrees, reg_params=lambdas)

```

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best_mse_rbf = results_rbf['mean_mse'][best_idx_rbf]

best_idx_poly = np.argmin(results_poly['mean_mse'])
best_lambda_poly = results_poly['reg_param'][best_idx_poly]
best_degree_poly = results_poly['kernel_val'][best_idx_poly]
best_mse_poly = results_poly['mean_mse'][best_idx_poly]

print(f"Best RBF Kernel: gamma={best_gamma_rbf},
lambda={best_lambda_rbf}, MSE={best_mse_rbf}")
print(f"Best Polynomial Kernel: degree={best_degree_poly},
lambda={best_lambda_poly}, MSE={best_mse_poly}")

Best RBF Kernel: gamma=0.001, lambda=0.1, MSE=0.017821125476743072
Best Polynomial Kernel: degree=2, lambda=10, MSE=0.07207397797866742

```

Running with Sklearn

```

from sklearn.kernel_ridge import KernelRidge
from sklearn.model_selection import GridSearchCV

param_grid_rbf = {
    'alpha': [0.001, 0.01, 0.1, 1, 10],
    'gamma': [0.001, 0.01, 0.1, 1]
}

param_grid_poly = {
    'alpha': [0.001, 0.01, 0.1, 1, 10],
    'degree': [2, 3, 4, 5, 6]
}

kr_rbf = KernelRidge(kernel='rbf')
gscv_rbf = GridSearchCV(kr_rbf, param_grid_rbf, cv=5,
scoring='neg_mean_squared_error')
gscv_rbf.fit(X_standardized, y)

kr_poly = KernelRidge(kernel='poly')
gscv_poly = GridSearchCV(kr_poly, param_grid_poly, cv=5,
scoring='neg_mean_squared_error')
gscv_poly.fit(X_standardized, y)

```

Intel MKL WARNING: Support of Intel(R) Streaming SIMD Extensions 4.2 (Intel(R) SSE4.2) enabled only processors has been deprecated. Intel oneAPI Math Kernel Library 2025.0 will require Intel(R) Advanced Vector Extensions (Intel(R) AVX) instructions.

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```

[illegible]

```
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```
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```
GridSearchCV(cv=5, estimator=KernelRidge(kernel='poly'),
             param_grid={'alpha': [0.001, 0.01, 0.1, 1, 10],
                          'degree': [2, 3, 4, 5, 6]},
             scoring='neg_mean_squared_error')
```

```
best_rbf_params = gscv_rbf.best_params_
best_poly_params = gscv_poly.best_params_
best_mse_rbf = -gscv_rbf.best_score_
best_mse_poly = -gscv_poly.best_score_
```

```
print("Best RBF Kernel (sklearn):", best_rbf_params, "MSE:",
      best_mse_rbf)
print("Best Polynomial Kernel (sklearn):", best_poly_params, "MSE:",
      best_mse_poly)
```

```
Best RBF Kernel (sklearn): {'alpha': 0.1, 'gamma': 0.001} MSE:
0.017709797036128978
Best Polynomial Kernel (sklearn): {'alpha': 10, 'degree': 3} MSE:
0.01790170219061269
```

#3

```
import struct
from array import array
from os.path import join

#from kaggle, loading data
class MnistDataLoader(object):
    def __init__(self,
                 training_images_filepath, training_labels_filepath,
                 test_images_filepath, test_labels_filepath):
        self.training_images_filepath = training_images_filepath
        self.training_labels_filepath = training_labels_filepath
```

```

self.test_images_filepath = test_images_filepath
self.test_labels_filepath = test_labels_filepath

def read_images_labels(self, images_filepath, labels_filepath):
    labels = []
    with open(labels_filepath, 'rb') as file:
        magic, size = struct.unpack(">II", file.read(8))
        if magic != 2049:
            raise ValueError('Magic number mismatch, expected
2049, got {}'.format(magic))
        labels = array("B", file.read())

    with open(images_filepath, 'rb') as file:
        magic, size, rows, cols = struct.unpack(">IIII",
file.read(16))
        if magic != 2051:
            raise ValueError('Magic number mismatch, expected
2051, got {}'.format(magic))
        image_data = array("B", file.read())

    images = []
    for i in range(size):
        images.append([0] * rows * cols)
    for i in range(size):
        img = np.array(image_data[i * rows * cols:(i + 1) * rows *
cols])
        img = img.reshape(28, 28)
        images[i][:] = img

    return images, labels

def load_data(self):
    x_train, y_train =
self.read_images_labels(self.training_images_filepath,
self.training_labels_filepath)
    x_test, y_test =
self.read_images_labels(self.test_images_filepath,
self.test_labels_filepath)
    return (x_train, y_train), (x_test, y_test)

%matplotlib inline
import random
import matplotlib.pyplot as plt

input_path = '/Users/kei/Desktop/Spring2025/Statistical Machine
Learning/HW2/'
training_images_filepath = join(input_path,
'train-images-idx3-ubyte/train-images-idx3-ubyte')
training_labels_filepath = join(input_path,

```

```

'train-labels-idx1-ubyte/train-labels-idx1-ubyte')
test_images_filepath = join(input_path, 't10k-images-idx3-ubyte/t10k-
images-idx3-ubyte')
test_labels_filepath = join(input_path, 't10k-labels-idx1-ubyte/t10k-
labels-idx1-ubyte')

#from kaggle, displaying images
def show_images(images, title_texts):
    cols = 5
    rows = int(len(images)/cols) + 1
    plt.figure(figsize=(30,20))
    index = 1
    for x in zip(images, title_texts):
        image = x[0]
        title_text = x[1]
        plt.subplot(rows, cols, index)
        plt.imshow(image, cmap=plt.cm.gray)
        if (title_text != ''):
            plt.title(title_text, fontsize = 15);
        index += 1

mnist_dataloader = MnistDataloader(training_images_filepath,
training_labels_filepath, test_images_filepath, test_labels_filepath)
(x_train, y_train), (x_test, y_test) = mnist_dataloader.load_data()

```

a)

```

#filter dataset to 3, 5, 8
digits = [3, 5, 8]
train_mask = np.isin(y_train, digits)
test_mask = np.isin(y_test, digits)

#using masks to select 3,5,8 from training and testing set.
X_train, Y_train = np.array(x_train)[train_mask], np.array(y_train)
[train_mask]
X_test, Y_test = np.array(x_test)[test_mask], np.array(y_test)
[test_mask]

#flatten each sample into a 1D vector
X_train = X_train.reshape(X_train.shape[0], -1)
X_test = X_test.reshape(X_test.shape[0], -1)

from sklearn.linear_model import LogisticRegression
from sklearn.naive_bayes import GaussianNB
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.svm import LinearSVC

models = {

```

```

    "Logistic Regression (OvR)": LogisticRegression(multi_class='ovr',
max_iter=1000),
    "Multinomial Logistic Regression":
LogisticRegression(multi_class='multinomial', solver='lbfgs',
max_iter=1000),
    "Gaussian Naive Bayes": GaussianNB(),
    "Linear Discriminant Analysis": LinearDiscriminantAnalysis(),
    "Linear SVM (OvR)": LinearSVC(multi_class='ovr', max_iter=10000)
}

def show_images(images, titles, n_rows, n_cols, fontsize=10):
    plt.figure(figsize=(n_cols * 2, n_rows * 2))
    for i in range(len(images)):
        plt.subplot(n_rows, n_cols, i + 1)
        plt.imshow(images[i], cmap='gray')
        plt.title(titles[i], fontsize=8)
        plt.axis('off')
    plt.tight_layout()
    plt.show()

from sklearn.metrics import accuracy_score, classification_report,
confusion_matrix
import seaborn as sns

for model_name, model in models.items():
    print(f"\n=== {model_name} ===")

    model.fit(X_train, Y_train)

    Y_pred = model.predict(X_test)

    accuracy = accuracy_score(Y_test, Y_pred)
    print(f"Accuracy: {accuracy:.4f}")

    print("\nClassification Report:")
    print(classification_report(Y_test, Y_pred, target_names=[str(d)
for d in digits]))

    conf_matrix = confusion_matrix(Y_test, Y_pred)
    print("\nConfusion Matrix:")
    print(conf_matrix)

    # Plot confusion matrix
    plt.figure(figsize=(6, 6))
    sns.heatmap(conf_matrix, annot=True, fmt='d', cmap='Blues',
xticklabels=digits, yticklabels=digits)
    plt.xlabel('Predicted Labels')
    plt.ylabel('True Labels')
    plt.title(f'Confusion Matrix for {model_name}')
    plt.show()

```

```

# Find correctly and incorrectly classified images
correct_indices = []
incorrect_indices = []

for i in range(len(Y_test)):
    if Y_test[i] == Y_pred[i]:
        correct_indices.append(i)
    else:
        incorrect_indices.append(i)

num_samples = 5 # Number of samples to display for each case

# Ensure we don't sample more than available
num_correct_samples = min(num_samples, len(correct_indices))
num_incorrect_samples = min(num_samples, len(incorrect_indices))

# Randomly select some correctly and incorrectly classified images
correct_samples = random.sample(correct_indices,
num_correct_samples)
incorrect_samples = random.sample(incorrect_indices,
num_incorrect_samples)

images_2_show = []
titles_2_show = []

# add correctly classified images
for i in correct_samples:
    images_2_show.append(X_test[i].reshape(28, 28))
    titles_2_show.append(f'Correct: True={Y_test[i]},
Pred={Y_pred[i]}')

# Add incorrectly classified images
for i in incorrect_samples:
    images_2_show.append(X_test[i].reshape(28, 28))
    titles_2_show.append(f'Incorrect: True={Y_test[i]},
Pred={Y_pred[i]}')

show_images(images_2_show, titles_2_show, n_rows=2,
n_cols=max(num_correct_samples, num_incorrect_samples), fontsize=8)

```

=== Logistic Regression (0vR) ===
Accuracy: 0.9256

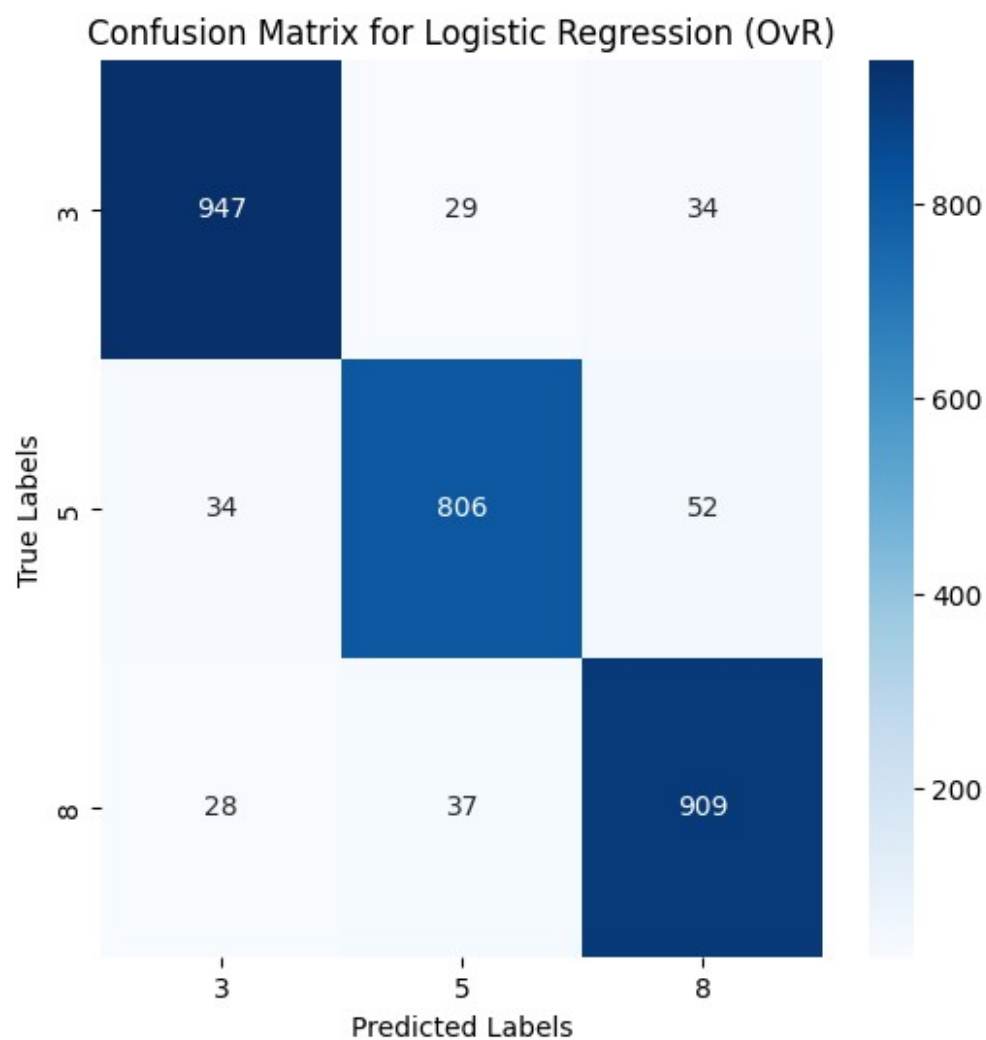
Classification Report:

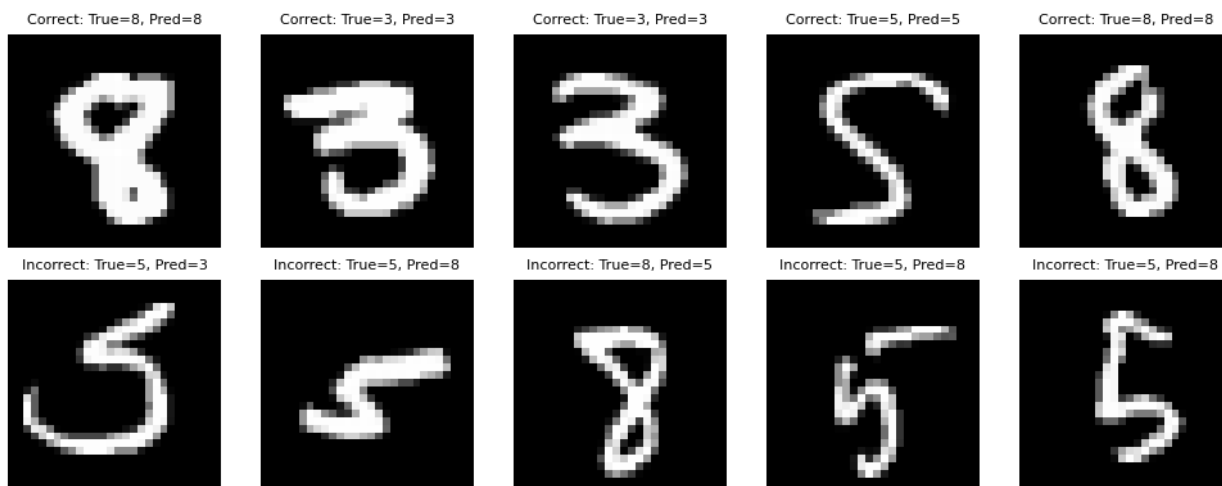
	precision	recall	f1-score	support
3	0.94	0.94	0.94	1010
5	0.92	0.90	0.91	892
8	0.91	0.93	0.92	974

accuracy				0.93	2876
macro avg	0.93	0.92	0.93	0.93	2876
weighted avg	0.93	0.93	0.93	0.93	2876

Confusion Matrix:

```
[[947  29  34]
 [ 34 806  52]
 [ 28  37 909]]
```





=== Multinomial Logistic Regression ===
 Accuracy: 0.9221

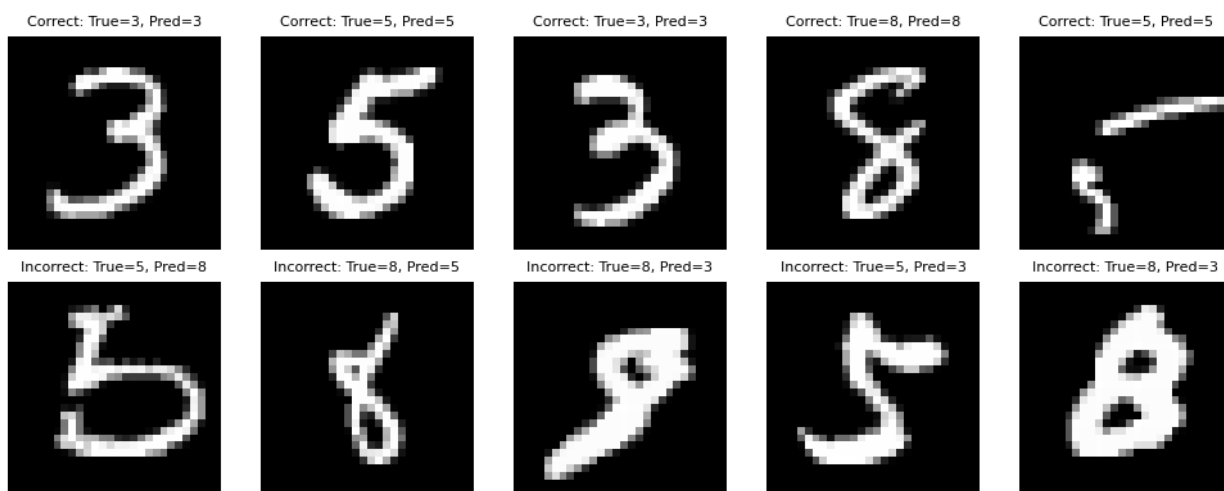
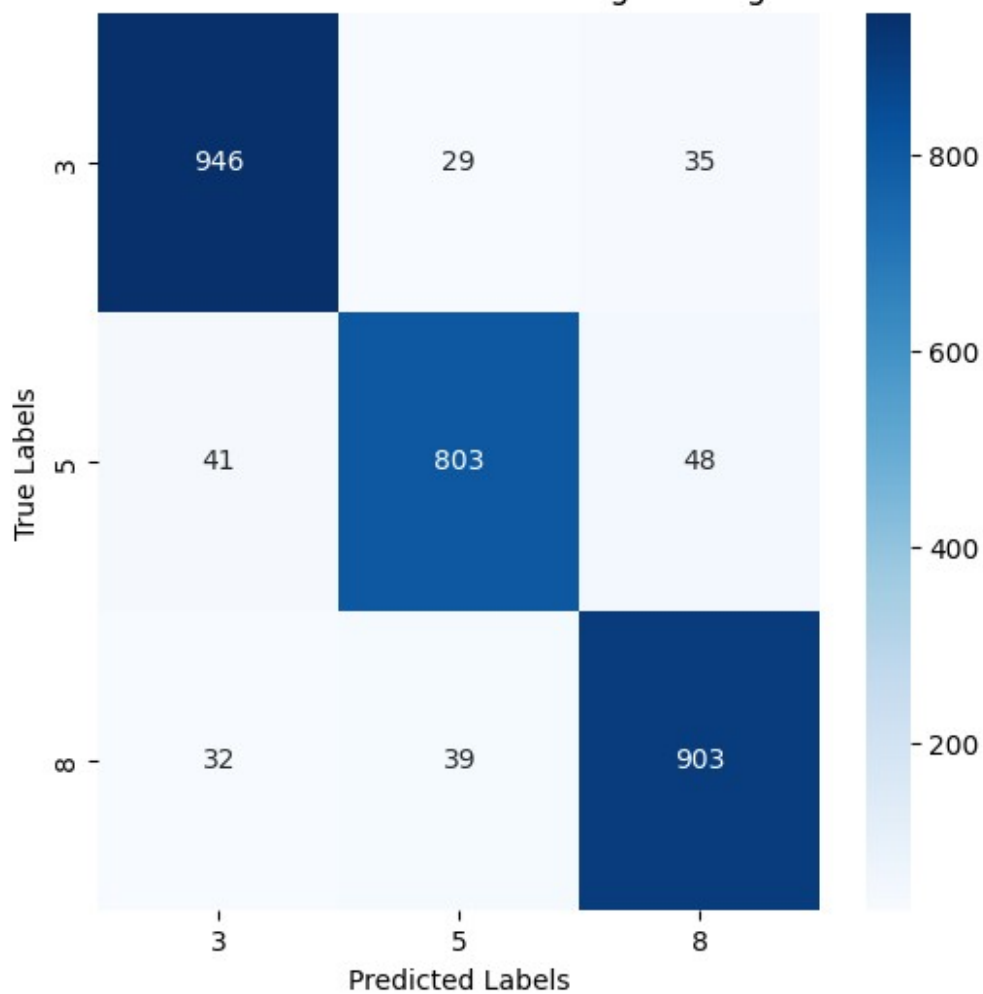
Classification Report:

	precision	recall	f1-score	support
3	0.93	0.94	0.93	1010
5	0.92	0.90	0.91	892
8	0.92	0.93	0.92	974
accuracy			0.92	2876
macro avg	0.92	0.92	0.92	2876
weighted avg	0.92	0.92	0.92	2876

Confusion Matrix:

```
[[946 29 35]
 [ 41 803 48]
 [ 32 39 903]]
```


Confusion Matrix for Multinomial Logistic Regression



=== Gaussian Naive Bayes ===

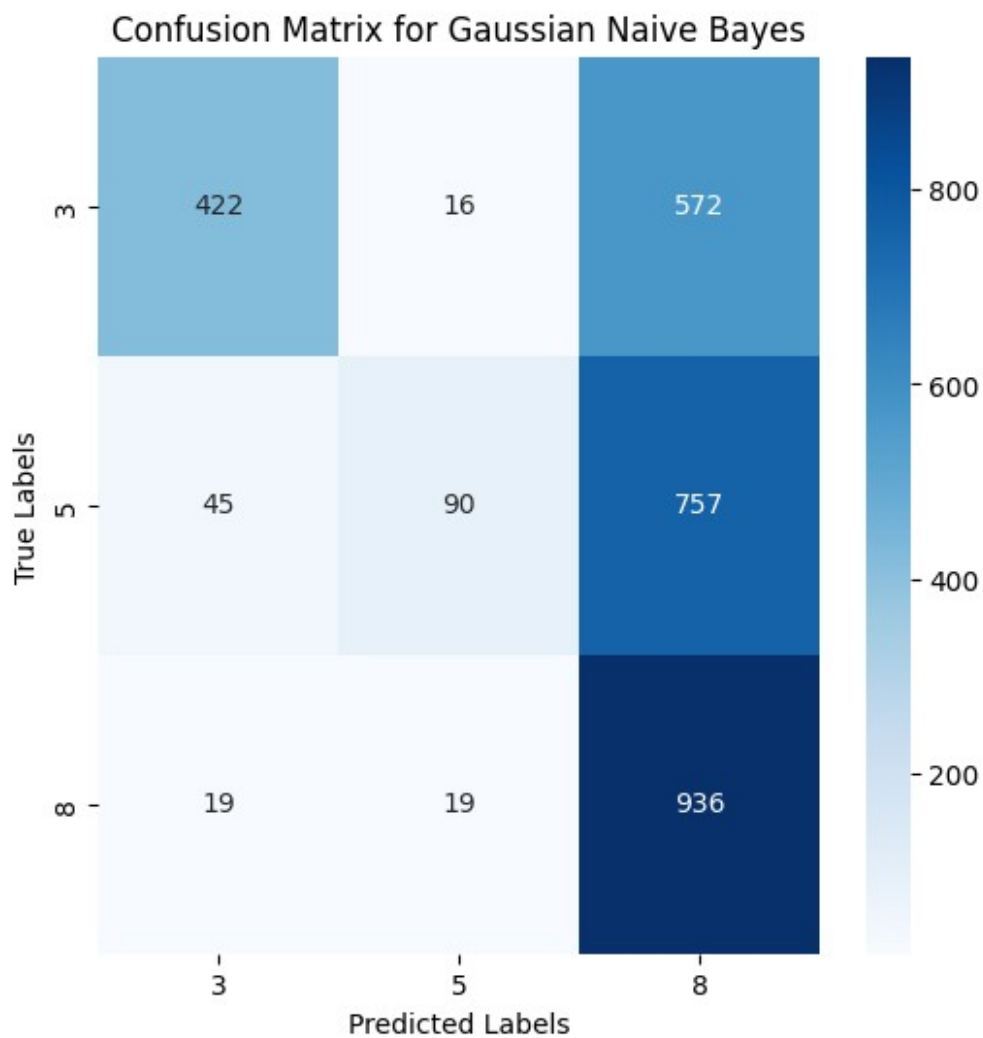
Accuracy: 0.5035

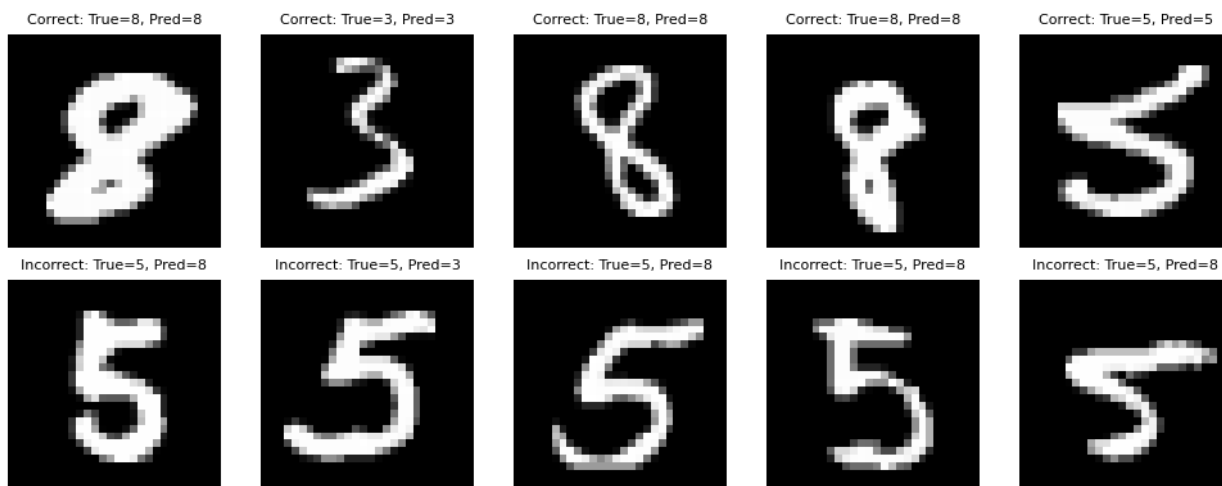
Classification Report:

	precision	recall	f1-score	support
3	0.87	0.42	0.56	1010
5	0.72	0.10	0.18	892
8	0.41	0.96	0.58	974
accuracy			0.50	2876
macro avg	0.67	0.49	0.44	2876
weighted avg	0.67	0.50	0.45	2876

Confusion Matrix:

```
[[422  16 572]
 [ 45  90 757]
 [ 19  19 936]]
```





=== Linear Discriminant Analysis ===
Accuracy: 0.9200

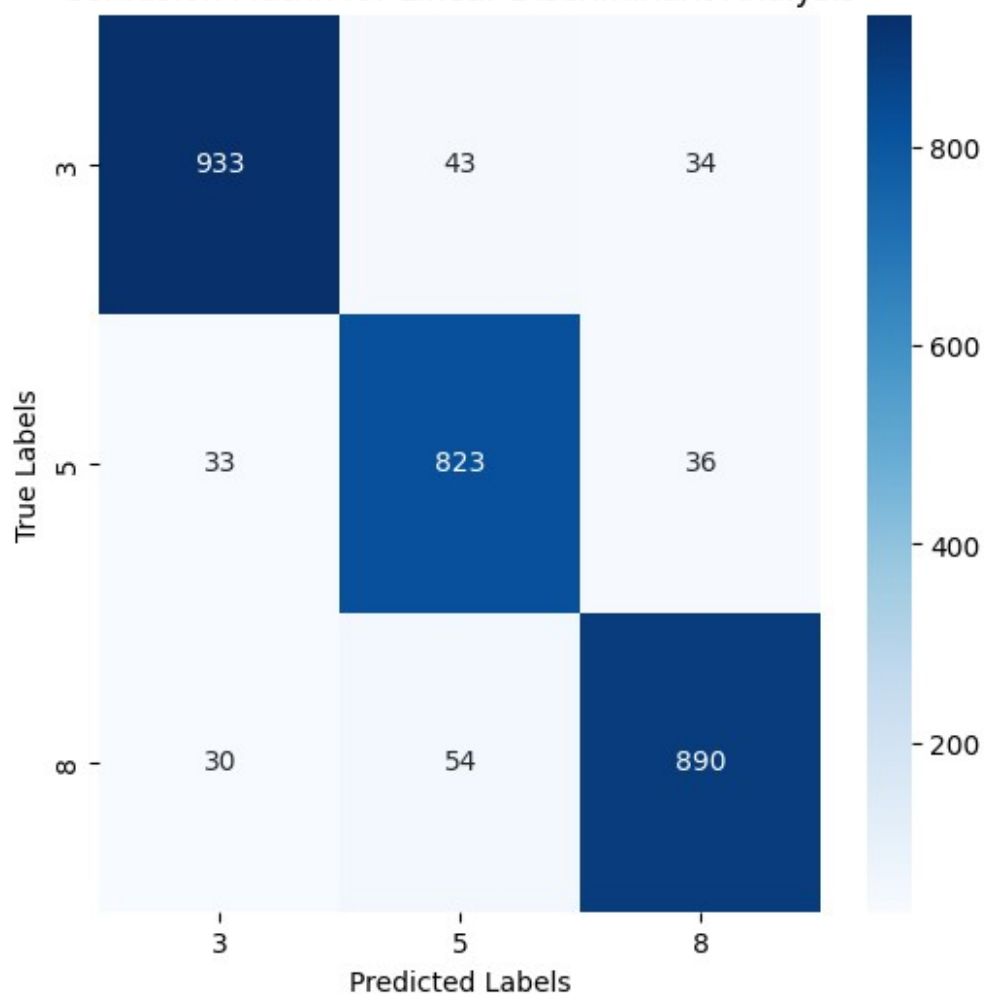
Classification Report:

	precision	recall	f1-score	support
3	0.94	0.92	0.93	1010
5	0.89	0.92	0.91	892
8	0.93	0.91	0.92	974
accuracy			0.92	2876
macro avg	0.92	0.92	0.92	2876
weighted avg	0.92	0.92	0.92	2876

Confusion Matrix:

```
[[933  43  34]
 [ 33 823  36]
 [ 30  54 890]]
```

Confusion Matrix for Linear Discriminant Analysis



Correct: True=3, Pred=3



Correct: True=8, Pred=8



Correct: True=3, Pred=3



Correct: True=5, Pred=5



Correct: True=5, Pred=5



Incorrect: True=3, Pred=5



Incorrect: True=3, Pred=8



Incorrect: True=5, Pred=8



Incorrect: True=3, Pred=5



Incorrect: True=3, Pred=5



=== Linear SVM (OvR) ===

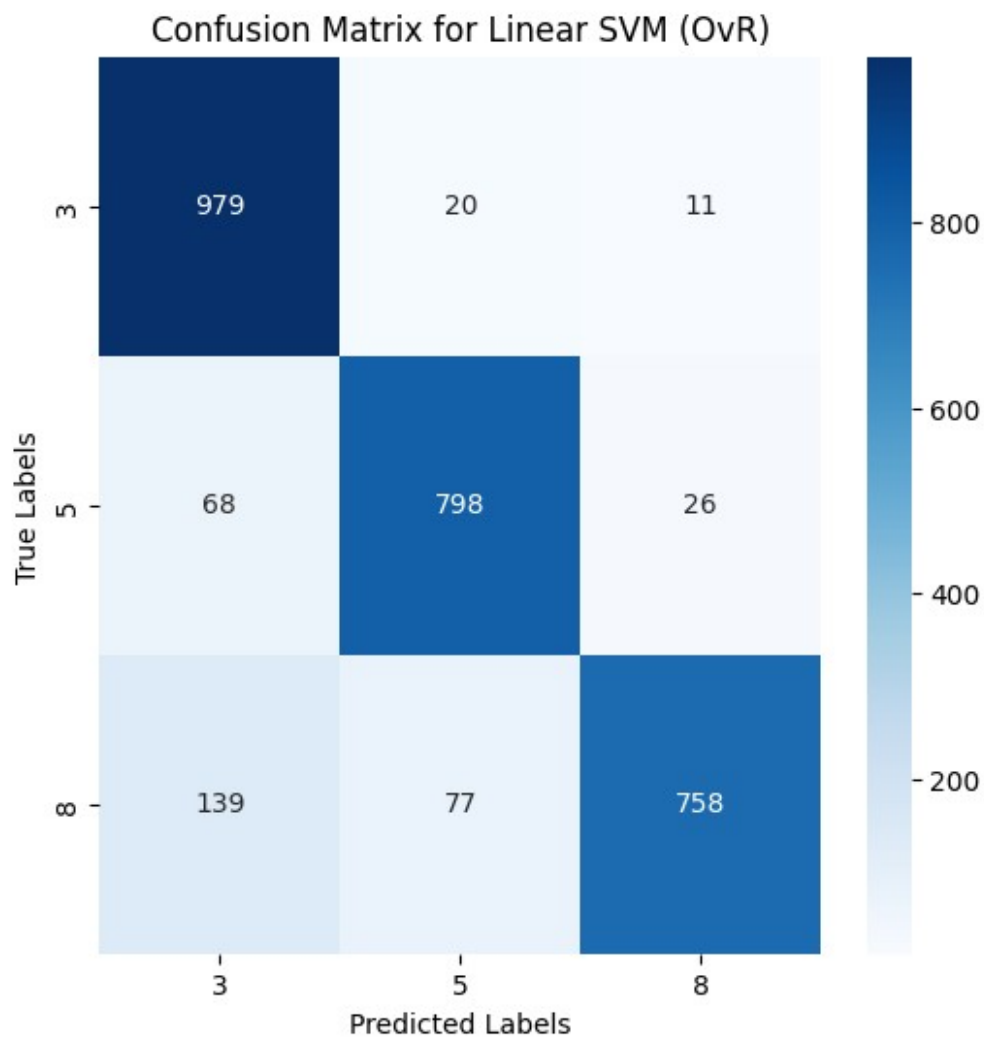
Accuracy: 0.8814

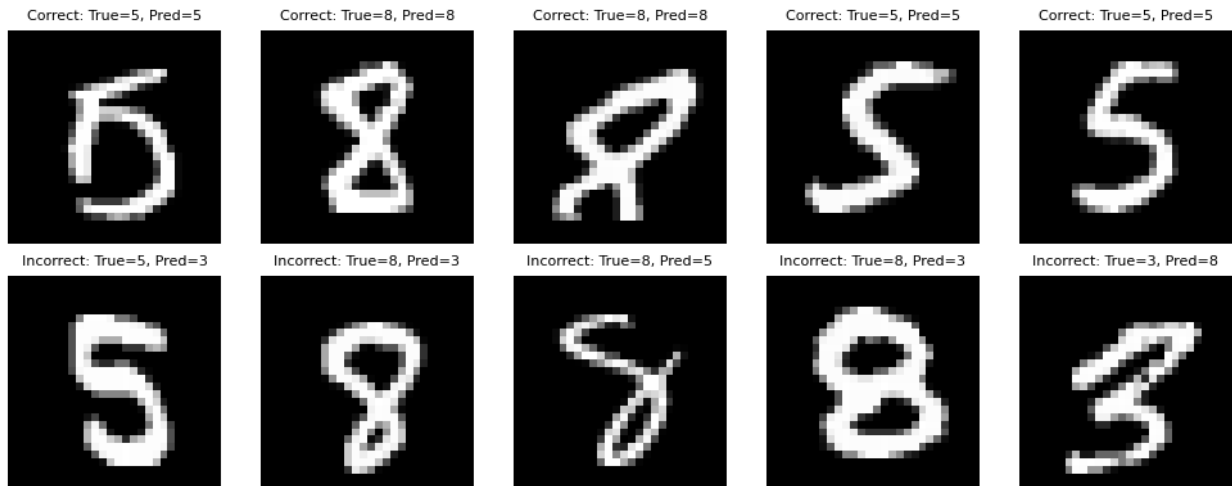
Classification Report:

	precision	recall	f1-score	support
3	0.83	0.97	0.89	1010
5	0.89	0.89	0.89	892
8	0.95	0.78	0.86	974
accuracy			0.88	2876
macro avg	0.89	0.88	0.88	2876
weighted avg	0.89	0.88	0.88	2876

Confusion Matrix:

```
[[979 20 11]
 [ 68 798 26]
 [139 77 758]]
```





Regularization

```
pip install group_lasso
```

```
Requirement already satisfied: group_lasso in
/Users/kei/anaconda3/envs/NewPython/lib/python3.11/site-packages
(1.5.0)
```

```
Requirement already satisfied: numpy in
/Users/kei/anaconda3/envs/NewPython/lib/python3.11/site-packages (from
group_lasso) (1.26.0)
```

```
Requirement already satisfied: scikit-learn in
/Users/kei/anaconda3/envs/NewPython/lib/python3.11/site-packages (from
group_lasso) (1.3.1)
```

```
Requirement already satisfied: scipy>=1.5.0 in
/Users/kei/anaconda3/envs/NewPython/lib/python3.11/site-packages (from
scikit-learn->group_lasso) (1.11.3)
```

```
Requirement already satisfied: joblib>=1.1.1 in
/Users/kei/anaconda3/envs/NewPython/lib/python3.11/site-packages (from
scikit-learn->group_lasso) (1.3.2)
```

```
Requirement already satisfied: threadpoolctl>=2.0.0 in
/Users/kei/anaconda3/envs/NewPython/lib/python3.11/site-packages (from
scikit-learn->group_lasso) (3.2.0)
```

Note: you may need to restart the kernel to use updated packages.

```
from group_lasso import GroupLasso
```

```
scaler = StandardScaler()
```

```
X_train_scaled = scaler.fit_transform(X_train)
```

```
X_test_scaled = scaler.transform(X_test)
```

```
groups = np.arange(X_train_scaled.shape[1])
```

```
group_lasso_model = GroupLasso(groups=groups, group_reg=0.1, l1_reg=0,
supress_warning=True, n_iter=1000, tol=1e-4)
```

```

group_lasso_model.fit(X_train_scaled, Y_train)

selected_features = group_lasso_model.sparsity_mask_

selected_feature_indices = np.where(selected_features)[0]
print(f"\nSelected feature indices: {selected_feature_indices}")
print(f"Number of selected features: {len(selected_feature_indices)}")

plt.figure(figsize=(6, 6))
plt.imshow(selected_features_image, cmap='gray')
plt.title("Selected Features (Pixels) by Group-Lasso")
plt.xlabel("Pixel X-axis")
plt.ylabel("Pixel Y-axis")

plt.xticks(range(0, 28, 4))
plt.yticks(range(0, 28, 4))
plt.show()

```

```

Selected feature indices: [149 150 175 176 268 288 289 290 291 295 296
316 317 318 319 434 461 462
466 467 485 486 487 488 489 494 513 514 515 516 522 543 564 577]
Number of selected features: 34

```

