Q1:

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A stabilizable system will reach the desired state $x_{ref} \equiv 0$ for a control input $u \equiv 0$. If a system is not stabilizable no minimum of the LQR problem can be found since the state cost will increase continuously.

A reachability test could be performed since a reachable system is also stabilizable. Reachability is a necessary condition for stabilizability. Therefore, the full rank of the controllability Matrix or the PBH test is to be checked.

The PBH test for reachability is performed: A system (A, B) is unreachable if and only if there exists a $\lambda_i \in \mathbb{C}$ left hand eigenvector $w \in \mathbb{C}$ with $w \neq 0$ such that:

$$w^T A = \lambda w^T \text{ and } w^T B = 0.$$

The calculation delivers the following eigenvalues:

$$\lambda_i = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$$

$$w^T B \ = [\, 1.03\ 10^{-02} \ -6.25\ 10^{-04} \ \ 8.57\ 10^{-04} \ \ 0] \neq 0$$

As a result, the system is reachable and since the reasoning above it's also stabilizable.

Q2:

Figure 1 shows the original response without modification of Q or R.

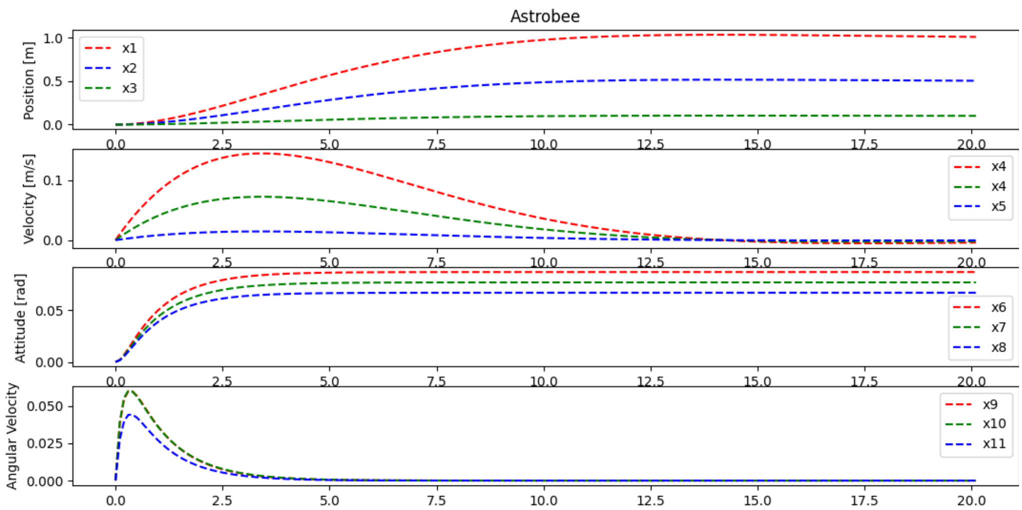

Figure 1: Astrobee states over simulation time with no modified Q or R matrix

R multiplied with 10:

By multiplying the R matrix by factor 10 the control input is penalized stronger so that smaller control values are calculated. As a result, it the time to reach the desired states is increased since the focus is on low control input, see figure 2. The translational and rotational velocity is also lower.

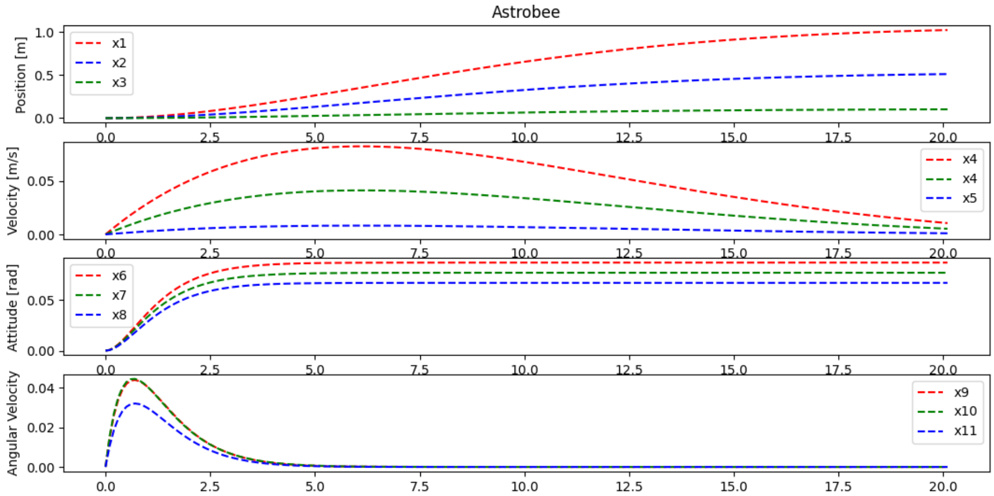

Figure 2: Astrobee with R = 10

Addition of Q[3:6] and Q[9:] with 100:

By increasing the penalty on the translational velocities and rotational velocities by adding 100, the rise time for the position and attitude have been increased. The translational velocities and rotational velocities have also been decreased. The Astrobee doesn't reach its desired positional state or attitude. See the plot in figure 3.

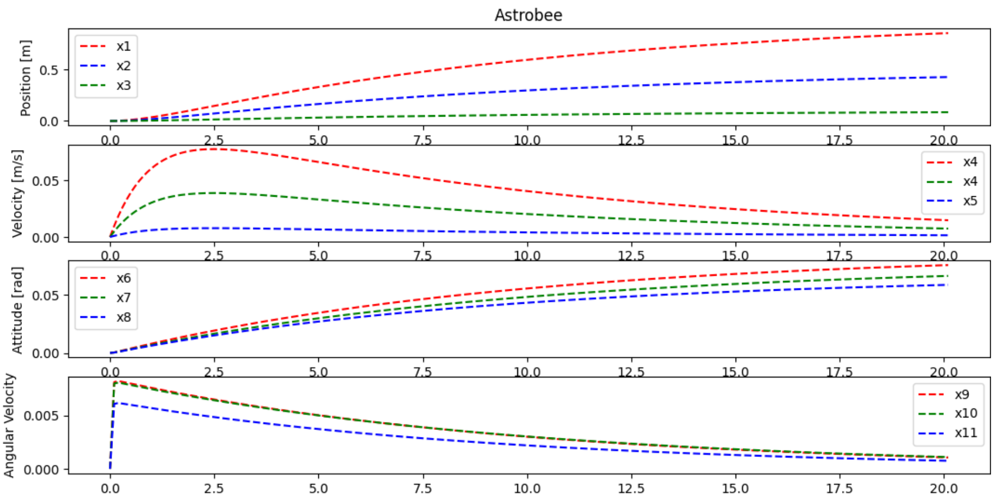

Figure 3: Astrobee with Q[3:6] = 100 and Q[9:] = 100

Addition of Q[0:3] and Q[6:9] with 100:

By high penalization of position and attitude, these state are reach in short time, since its minimum is only found if these variables are reaching their desired state as fast as possible. This leads to an increase of the translational and rotational velocities. See figure 4.

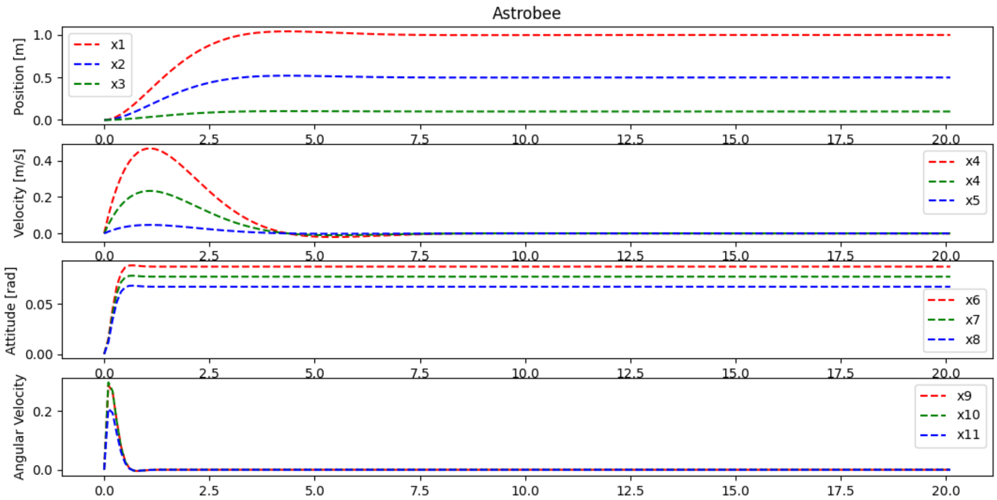

Figure 4: Astrobee with 100 penalized position and attitude

Addition of 100 to all elements of Q:

By increasing all elements of Q with 100, the penalty of the controller becomes decreased, i.e. larger control actions is possible. As seen in figure 5 the rise time for the position and attitude is decreased and translational and rotational velocities are increased which is the result of a more aggressive controller.

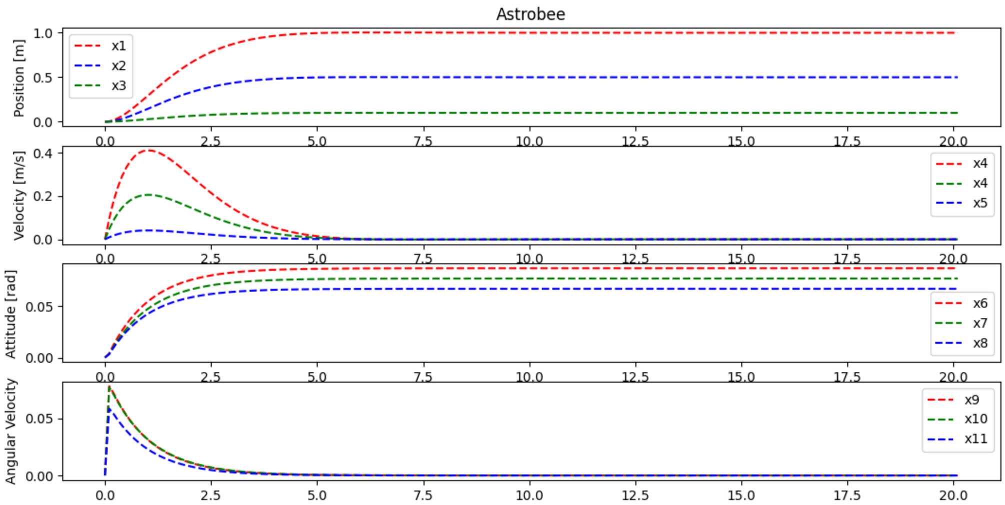

Figure 5 - Astrobee with all Q elements increased with 100

Q3:

The requirements are met by the following weighting matrices:

$$R = \begin{bmatrix} 484.42 & 0 & 0 & 0 & 0 & 0 \\ 0 & 482.42 & 0 & 0 & 0 & 0 \\ 0 & 0 & 482.42 & 0 & 0 & 0 \\ 0 & 0 & 0 & 125000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12500 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12500 \end{bmatrix}$$

For the weights of the Q-Matrix we found:

$$diag(Q) = [416.67 \quad 416.67 \quad 416.67 \quad 1666.67 \quad 1666.67 \quad 1666.67 \quad 3000 \quad 3000 \quad 3000 \quad 1.5 \quad 1.5 \quad 1.5]$$

The following requirements are fulfilled.

- Max distance to reference:
  $0.0246 < 0.06\ m$
- Max speed:
  $0.0205 < 0.03\ m/s$
- Max forces: $(< 0.85\ N)$
  x: 0.92
  y: 0.40
  z: 0.12
- Max torques:$(< 0.04\ Nm)$
  x: 0.036
  y: 0.036
  z: 0.027
- Max Euler angle deviations: $(< 10^{-7})$
  $roll$: $1.41e - 08$
  $pitch$: $1.19e - 08$
  yaw: $8.8e - 09$
- Overshoot $(< 0.02m)$: fulfilled.

For further insights have a look on Figure 6.

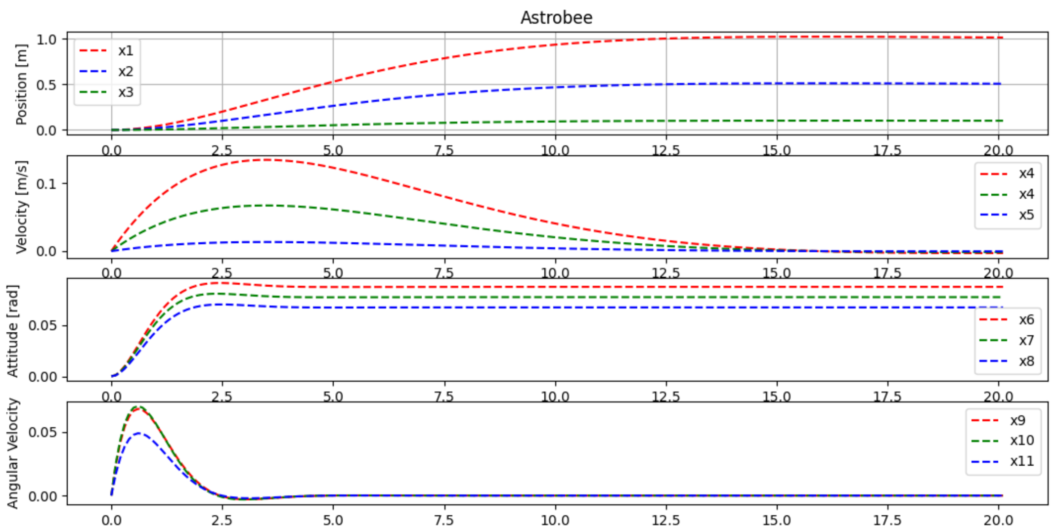

Figure 6: Astrobee with tunes LQR and constraints met

Q4:

The performance of the controller is influenced measurement noise now. The matrix R_n is the input of this measurement noise for the Kalman filter. Small position deviations are detected, see Figure 7.

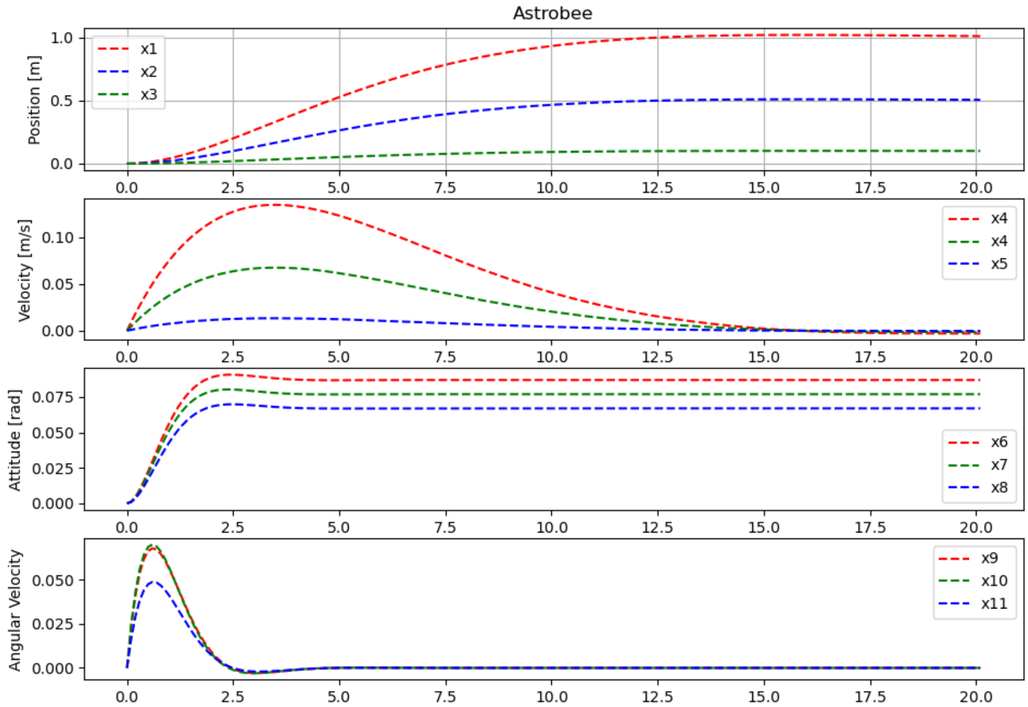

Figure 7: Astrobee with measurement noise: No huge deviations

To find out more about the influence of the measurement noise on the control, the measurement noise is increased from (-0.1, 0.1) to (-1, 1). It results in a highly disturbed position and velocity states that do not meet the requirements anymore. For example, the requirement of a maximum deviation of 2 cm on the position for the time after 12 sec does not hold anymore. Also, the control effort is much higher because the system does not stabilize on one value and must be controlled always to tend to the reference state, see Figure 8.

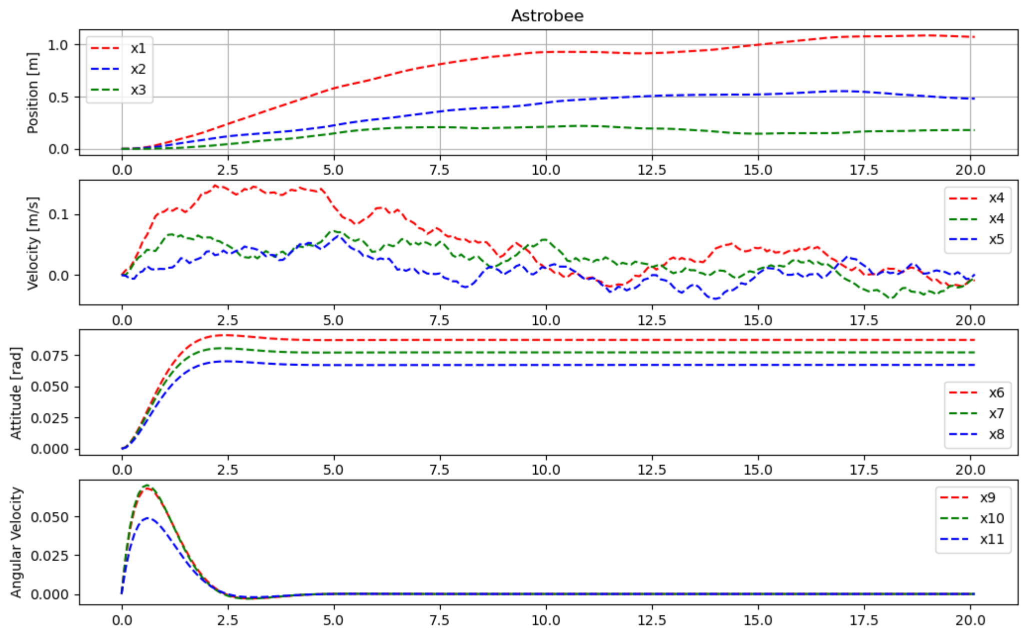

Figure 8: Astrobee with high measurement noise of amount (-1,1): huge deviations in position and speed

Q5:

Now process noise of (-0.05, 0.05) is included to make the system more realistic. The matrix Q_n represents the process noise for the Kalman filter, see Figure 9.

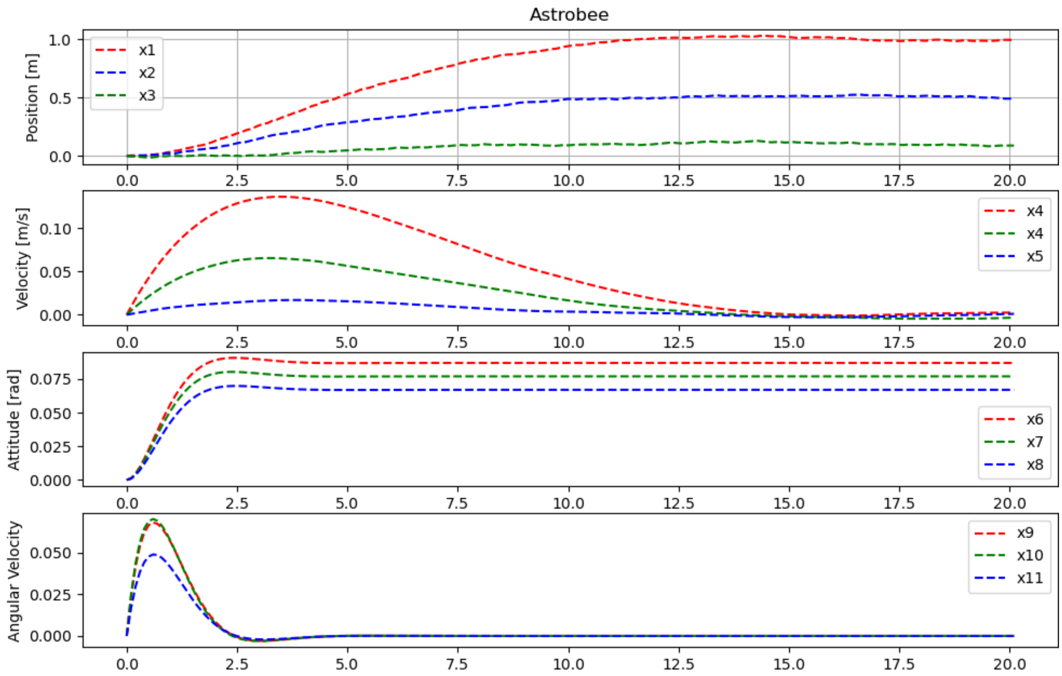

Figure 9: Astrobee with measurement noise of given distribution of (-0.05, 0.05) and Q_n &R_n factorized with 1

By increasing the measurement noise, the deviations in the states, e.g., in the position and attitude get more severe, but the system still stays stable, see Figure 10.

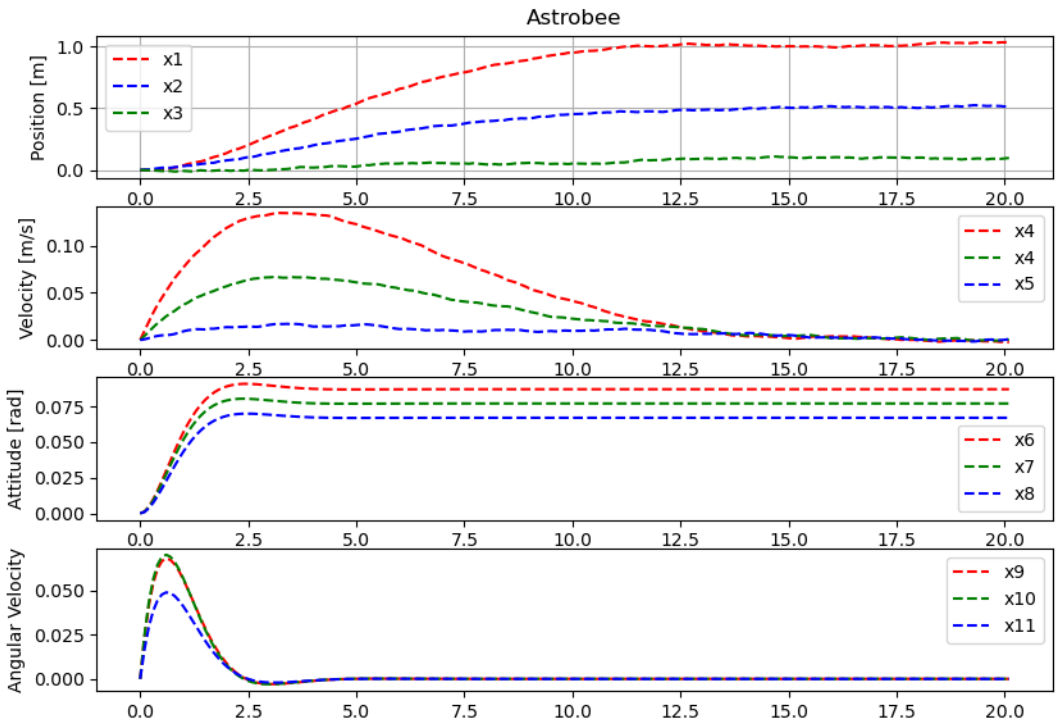

Figure 10: Astrobee with estimated position, measurement noise of (-0.1, 0.1), process noise (-0.005, 0.005) and Q_n &R_n factorized with 10

By adding high measurement noise (-1,1) and process noise (-0.005, 0.005) the performance of the system decreases more. The x_1-position of the astrobee is for example deviating more than $0.3\ m$, so the constraint of < 0.02 m is not fulfilled anymore. Same holds for deviations of the other position states, see Figure 11.

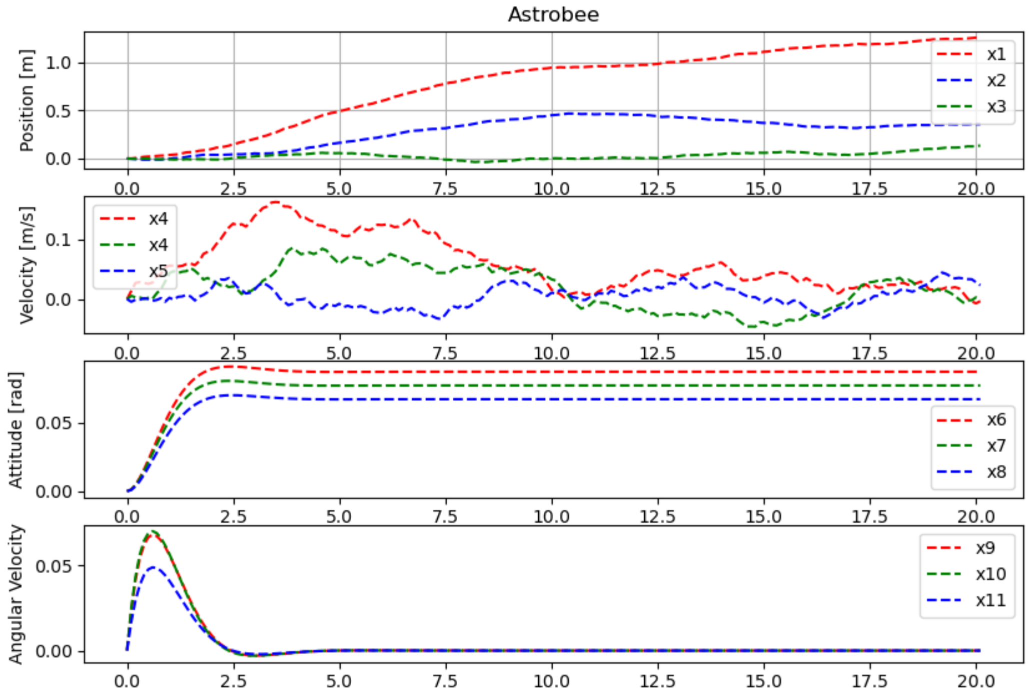

Figure 11: Astrobee with high measurement noise (-1,1) and process noise (-0.05, 0.05)