

# Assignment 1

**Q1.** Performed in `astrobee_1d one_axis_ground_dynamics()`.

**Q2.** The numerical values returned from `casadi_c2d` correspond to the matrices:

$$A = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.00024 \\ 0.0048 \end{bmatrix}$$

The numerical values from analytically derived model in exercise 13 c gives the following matrices:

$$A = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix}$$

The discrete A-matrices is the same for both cases because the B matrix in general doesn't affect the A matrix during the transition. Since the continuous B-matrices differ they are also not the same in the discrete time domain. See the equations below:

$$A_d = e^{A_c h} \text{ and } B_d = \int_{s=0}^h e^{A_c s} B_c ds$$

**Q3.** The continuous system has the following transfer function:

$$G(s) = \frac{1}{s^2 \cdot m_G}$$

There are two poles at origin, i.e. 0 and no zeros in the nominator of the transferfunction. Our intuition was that there are two poles at 0 since matrix A's characteristic polynomial will be  $\lambda^2 = 0$ . For verification, see figure 1 below.

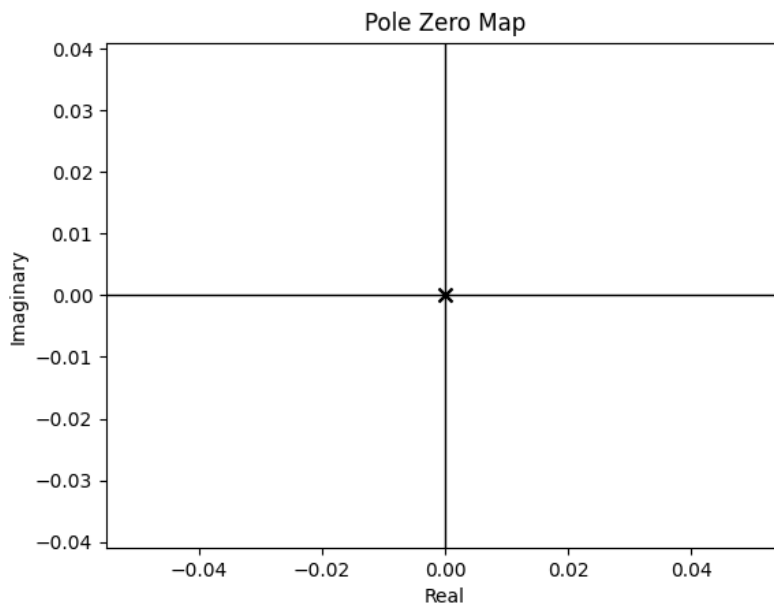


Figure 1 - Poles for continuous system

Regarding the discrete system we expected 2 poles at 1 since the following expression:

$$\lambda(A_d) = e^{\lambda(A_c) \cdot h}$$

Since  $e^0 = 1$  this assumption holds. See the figure 2 below for verification.

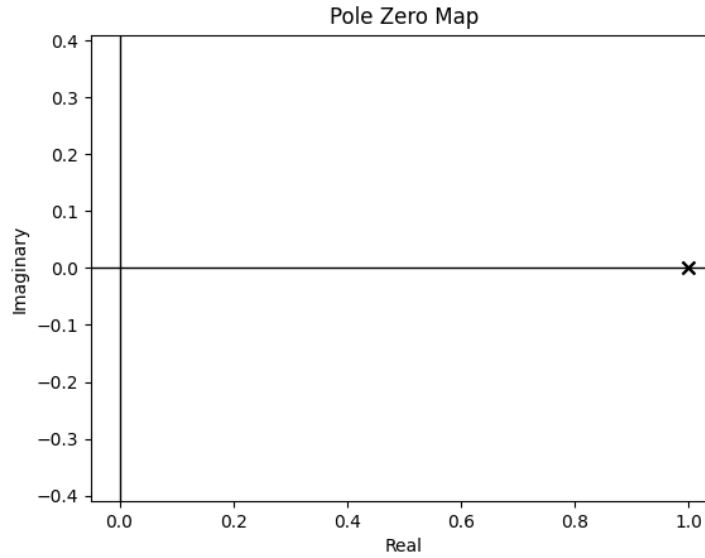


Figure 2 - poles for discrete system

**Q4.** If the control gain  $L$  for the state feedback controller is designed with the two desired poles at  $\lambda_1 = 0.975$  and  $\lambda_2 = 0.985$  the requirements  $x_T = [0 \ 0]^T$ ,  $|u| < 0.85 \text{ N}$  and  $x_{1,25s} < 0.9 \cdot |x_0 - x_{ref}|$ , see figure 3.

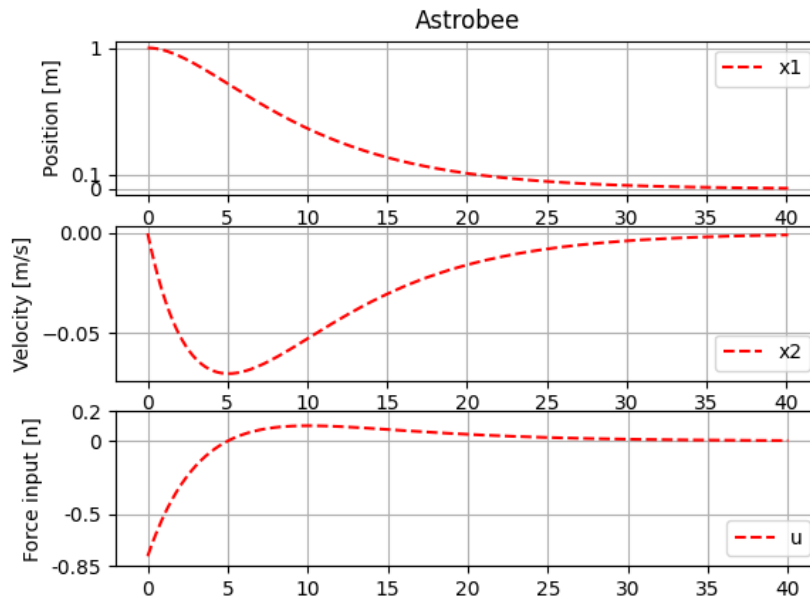


Figure 3: Control of the astrobee - Linear control without disturbance and without feedback

Including disturbances, the terminal condition couldn't be reached, see picture 4.

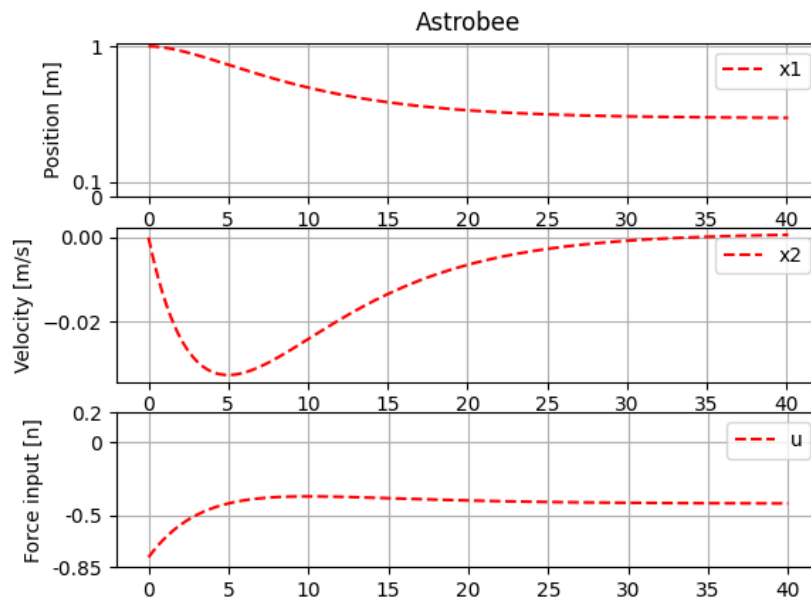


Figure 4: Control of the astrobee - Linear control with disturbance and without feedback

**Q5.** Design of Feedback loop with integral part needed to deal with disturbances. For reaching the requirement 90 % within 30 seconds, the Linear feedback gain  $L$  is to be manipulated to accelerate the control with desired poles  $\lambda_1 = 0.974$  and  $\lambda_2 = 0.984$  for calculation. By implementing the integral state feedback with an integral gain  $k_i = 0.028$  the disturbance is controlled without overshooting, see figure 5.

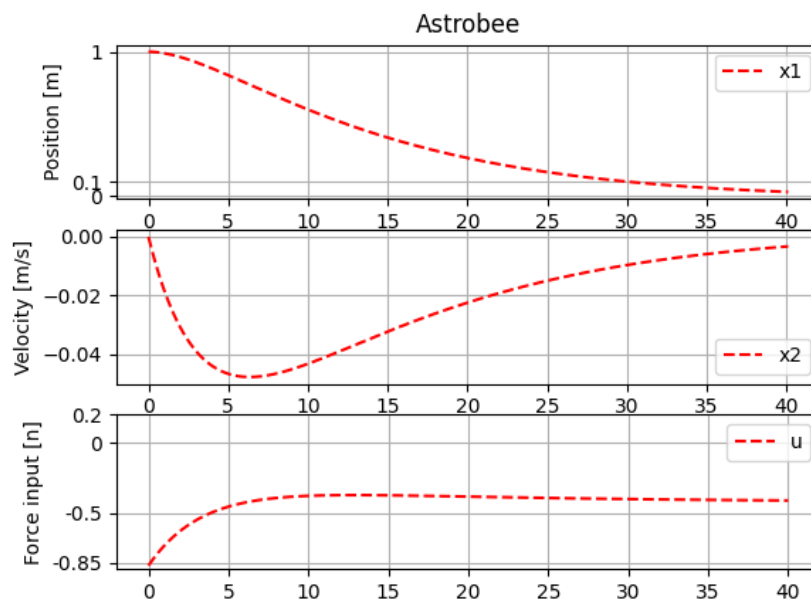


Figure 5: Control of Astrobee – Linear control, with disturbance and integral feedback control