Erasmus
School of
Economics

MOOC Econometrics

Lecture 1.2 on Simple Regression: Representation

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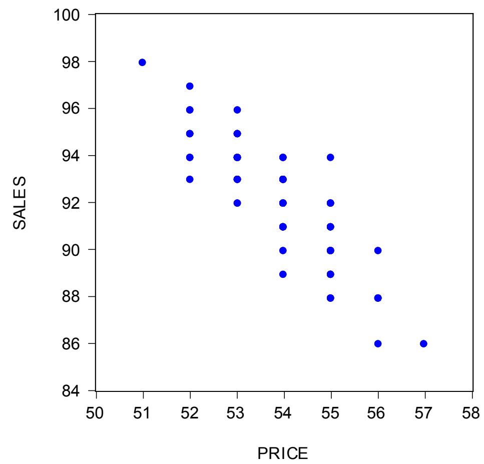
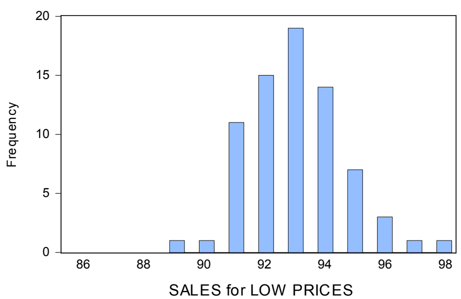
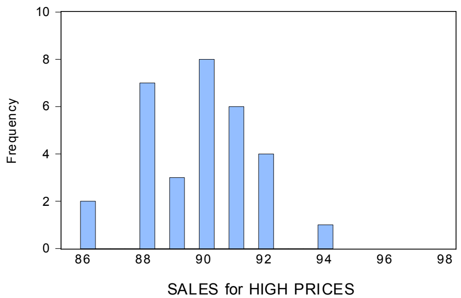
Scatter diagram of sales against price



Two histograms of sales





Simple regression: conditional mean

- y_i Normally Identically Distributed with mean μ and variance σ^2:

  $y_i \sim NID(\mu, \sigma^2)$

- Expected value: $E(y_i) = \mu$

  Variance: $E(y_i - \mu)^2 = \sigma^2$

- Sample estimates: $\hat{\mu} = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

  (see Building Blocks) $\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$

- Simple regression: replace unconditional mean $E(y_i) = \mu$

  by conditional mean $E(y_i) = \alpha + \beta x_i$

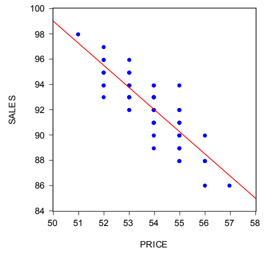
Simple regression equation

- Unconditional prediction with $y_i \sim N(\mu, \sigma^2)$: $\quad E(y_i) = \mu$
- Conditional prediction with $y_i \sim N(\alpha + \beta x_i, \sigma^2)$: $E(y_i) = \alpha + \beta x_i$
- An alternative format is: $y_i = \alpha + \beta x_i + \varepsilon_i$

  $\varepsilon_i \sim N(0, \sigma^2)$
- If x_i is fixed (non-random), $\varepsilon_i \sim N(0, \sigma^2)$, and $y_i = \alpha + \beta x_i + \varepsilon_i$, then y_i has mean $\alpha + \beta x_i$ and variance σ^2. (see Building Blocks)

Prediction

- Consider again scatter plot of sales and price:



Test

Predict sales for a price of 50, and also for a price of 58.

- Answer: Fit straight line to the observed data.
- Price $= 50 \rightarrow$ Sales ≈ 99; $\quad$ Price $= 58 \rightarrow$ Sales ≈ 85.

Absolute and relative changes

Test

Retail store A has a sales level of 150 units and store B of 250 units. The two store managers start an advertising campaign, after which store A sells 225 units and store B sells 400.
Which store has the largest relative increase in sales?

- Answer: Relative change for A: $\frac{225-150}{150} = 0.5$

  Relative change for B: $\frac{400-250}{250} = 0.6 \;\leftarrow$ largest
- If current level x increases by dx, then relative increase is $\frac{dx}{x}$

Elasticity

- For $y = \alpha + \beta x$, the slope (marginal effect) is $\beta = \frac{dy}{dx}$
- Definition of elasticity: $\frac{dy/y}{dx/x}$

  So: relative change in y divided by relative change in x
- Elasticity in $y = \alpha + \beta x$:

  elasticity $= \frac{dy/y}{dx/x} = \frac{dy}{dx} \times \frac{x}{y} = \beta \times \frac{x}{y}$
- Fact: elasticity in $\log(y) = \alpha + \beta \log(x)$ is equal to β.

TRAINING EXERCISE 1.2

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).