a) $\log(y_i) = \alpha + \beta \log(x_i) + \varepsilon_i$

$$\text{elasticity} = \frac{\frac{\partial y}{y}}{\frac{\partial x}{x}} = \frac{\partial y}{\partial x} \times \frac{x}{y}$$

$\exp(\log(y)) = y$

$\exp(\alpha + \beta) = \exp(\alpha) \times \exp(\beta)$

$\beta \log(x) = \log(x^{\beta})$

$y_i = \exp(\alpha + \beta \log(x_i) + \varepsilon_i) = \exp(\alpha + \varepsilon_i) \times \exp(\beta \log(x_i))$

$= \exp(\alpha + \varepsilon_i) \times \exp(\log(x_i^{\beta})) = \exp(\alpha + \varepsilon_i) \times x_i^{\beta}$

$\frac{\partial y_i}{\partial x_i} = \beta \exp(\alpha + \varepsilon_i) x^{\beta - 1} = \frac{\beta}{x_i} \cdot \exp(\alpha + \varepsilon_i) x_i^{\beta} = \beta \frac{y_i}{x_i}$

$\text{elasticity} = \frac{\partial y_i}{\partial x_i} \times \frac{x_i}{y_i} = \beta \frac{y_i}{x_i} \times \frac{x_i}{y_i} = \beta$

b) $y_i = \alpha + \beta \log(x_i) + \varepsilon_i$

$$\frac{\partial y_i}{\partial x_i} = \beta \frac{\partial \log(x_i)}{\partial x_i} = \beta \frac{1}{x_i}$$

$$\text{elasticity} = \frac{\partial y_i}{\partial x_i} \times \frac{x_i}{y_i} = \beta \frac{1}{x_i} \times \frac{x_i}{y_i} = \frac{\beta}{y_i}$$

c) $\log(y_i) = \alpha + \beta x_i + \varepsilon_i$

$\exp(\log(y)) = y$

$\exp(\alpha + \beta) = \exp(\alpha) \times \exp(\beta)$

$\beta \log(x) = \log(x^{\beta})$

$$y_i = \exp(\alpha + \beta x_i + \varepsilon_i) = \exp(\alpha + \varepsilon_i) \times \exp(\beta x_i)$$

$$\frac{\partial y_i}{\partial x_i} = \exp(\alpha + \varepsilon_i)\beta \exp(\beta x_i) = \beta y_i$$

$$\text{elasticity} = \frac{\partial y_i}{\partial x_i} \times \frac{x_i}{y_i} = \beta y_i \times \frac{x_i}{y_i} = \beta x_i$$