(a) ① $x \equiv 2 \pmod 9$ $\longrightarrow$ $x \equiv 2 \pmod{3^2}$ ...

② $x \equiv 6 \pmod{12}$ $\longrightarrow$ $x \equiv 6 \pmod{2^2}$

$\longrightarrow$ $x \equiv 6 \pmod 3$

($\because$ CRT)

① $x = 9k + 2 \quad (k \in \mathbb{Z})$,

② $9k + 2 \equiv 6 \pmod{12}$

$9k \equiv 4 \pmod{12}$

$\gcd(9, 12) = 3$.

$3 \nmid 4 \quad \longrightarrow$ 해가 없다. ($\because$ Thm 101)

$\therefore$ system은 해가 없다.

Q2 (b) $5, 6, 11, 13 \in \mathbb{N}$ that are pairwise relatively prime,

$1, 3, 5, 10 \in \mathbb{Z}$

$\longrightarrow$ CRT 적용

$x \equiv 1 \pmod 5$	,	$N_1 = 6 \cdot 11 \cdot 13 = 858$,	$x_1 : N_1 x_1 \equiv 1 \pmod 5$ 의 해 --(1)
$x \equiv 3 \pmod 6$	,	$N_2 = 5 \cdot 11 \cdot 13 = 715$	$x_2 : N_2 x_2 \equiv 1 \pmod 6$ 의 해 ...(2)
$x \equiv 5 \pmod{11}$	,	$N_3 = 5 \cdot 6 \cdot 13 = 390$	$x_3 : N_3 x_3 \equiv 1 \pmod{11}$ 의 해 .(3)
$x \equiv 10 \pmod{13}$	,	$N_4 = 5 \cdot 6 \cdot 11 = 330$	$x_4 : N_4 x_4 \equiv 1 \pmod{13}$ 의 해 (4)

$\longrightarrow x \equiv N_1 x_1 \cdot 1 + N_2 x_2 \cdot 3 + N_3 x_3 \cdot 5 + N_4 x_4 \cdot 10 \quad \left(\bmod \dfrac{5 \cdot 6 \cdot 11 \cdot 13}{4290} \right)$

i) $858 x_1 \equiv 1 \pmod 5$, $\emptyset_5 (858) = 3$

$\sim 3 x_1 \equiv 1 \pmod 5$

$5 = 3 \cdot 1 + 2$

$3 = 2 \cdot 1 + 1$

$\rightarrow 1 = 3 - 2$

$= 3 - (5 - 3)$

$= 3 \cdot ② + 5 \cdot (-1)$

$\rightarrow x_1 = 2.$

ii) $715 \, x_2 \equiv 1 \pmod 6$, $\emptyset_6 (715) = 1$

$\rightarrow 1 \cdot x_2 \equiv 1 \pmod 6$

$\rightarrow x_2 = 1$

iii) $390 x_3 \equiv 1 \pmod{11}$, $\emptyset_{11}(390) = 5$

$\sim 5 x_3 \equiv 1 \pmod{11}$

$11 = 5 \cdot 2 + 1$

$1 = 11 \, (1) + 5 \, (-2)$

$x_3 = -2$

iv) $330 \, x_4 \equiv 1 \pmod{13}$, $\emptyset_{13}(330) = 5$

$5 \, x_4 \equiv 1 \pmod{13}$

$13 = 5 \cdot 2 + 3$

$5 = 3 \cdot 1 + 2$

$3 = 2 + ①$

$1 = 3 - 2$

$= 3 - (5 - 3)$

$= 3 \cdot 2 - 5$

$= (13 - 5 \cdot 2) \cdot 2 - 5$

$= 13 (2) + 5 \cdot (-5)$, $x_4 = -5$

∴ System의 $x = 858 \cdot 2 \cdot 1 + 715 \cdot 1 \cdot 3 + 390 \cdot (-2) \cdot 5 + 330(-5) \cdot 10$
general solution $(\bmod\ 4290)$

$\rightarrow x \equiv -16559 \pmod{4290}$
$x \equiv -3669 \pmod{4290}$
$x \equiv 621 \pmod{4290}$

(c) $2x \equiv 3 \pmod 5$ ··· ①
$3x \equiv 4 \pmod 7$ ··· ②
$5x \equiv 7 \pmod{11}$ ··· ③

① 에서

$3 \cdot 2x \equiv 3 \cdot 3 \pmod 5$
$6x \equiv 9 \pmod 5$
$\phi_5(6) x \equiv \phi_5(9) \pmod 5$
$2x \equiv 4 \pmod 5$

② 에서

$5 \cdot 3x \equiv 5 \cdot 4 \pmod 7$
$\phi_7(15) x \equiv \phi_7(20) \pmod 7$
$1 x \equiv 6 \pmod 7$

③ 에서

$9 \cdot 5x \equiv 9 \cdot 7 \pmod{11}$
$\phi_{11}(45) x \equiv \phi_{11}(63) \pmod{11}$
$1 \cdot x \equiv 8 \pmod{11}$

즉 (c)의 성질

$$x \equiv 4 \quad (mod\ 5)$$
$$x \equiv 6 \quad (mod\ 7)$$
$$x \equiv 8 \quad (mod\ 11)$$

의 system으로 만든다.

$5, 7, 11 \in \mathbb{N}$ that are pairwise relatively prime,
$4, 6, 8 \in \mathbb{Z}$

→ CRT 적용

$N_1 = 7 \cdot 11 = 77 \qquad X_1 : N_1 X_1 \equiv 1 \ (mod\ 5)$ 의 해
$N_2 = 5 \cdot 11 = 55 \qquad X_2 : N_2 X_2 \equiv 1 \ (mod\ 7)$ 의 해
$N_3 = 5 \cdot 7 = 35 \qquad X_3 : N_3 X_3 \equiv 1 \ (mod\ 11)$ 의 해

$\hookrightarrow x = N_1 X_1 \cdot 4 + N_2 X_2 \cdot 6 + N_3 X_3 \cdot 8 \quad (mod\ 5 \cdot 7 \cdot 11)$

ⅰ) $77 X_1 \equiv 1 \quad (mod\ 5) \qquad , \quad \phi_5(77) = 2$
$2 X_1 \equiv 1 \ (mod\ 5)$
$5 = 2 \cdot 2 + 1$
$2 = 1 \cdot 2 + 0$
$1 = 2(\boxed{-2}) + 5(1)$
→ $X_1 = -2$

ⅱ) $55 X_2 \equiv 1 \quad (mod\ 7) \qquad , \quad \phi_7(55) = 6$
$6 X_2 \equiv 1 \ (mod\ 7)$
$7 = 6 \cdot 1 + 1$
$1 = 7(1) + 6(\boxed{-1})$
→ $X_2 = -1$

(iii) $35 X_3 \equiv 1 \pmod{11}$, $\phi_{11}(35) = 2$

$2X_3 \equiv 1 \pmod{11}$

$11 = 2 \cdot 5 + 1$

$1 = 11(1) + 2(-5)$

$\rightarrow X_3 = -5$

∴ System의 general solution $\Big)$ $x \equiv 77 \cdot (-2) \cdot 4 + 55 \cdot (-1) \cdot 6 + 35(-5) \cdot 8$

$\pmod{385}$

$\rightarrow x \equiv -2346 \pmod{385}$

$x \equiv -36 \pmod{385}$

∴ $x \equiv 349 \pmod{385}$

(d) $12X \equiv 18 \pmod{24}$ --- ①

$15X \equiv 20 \pmod{30}$ --- ②

$10X \equiv 25 \pmod{50}$ --- ③.

① $12X \equiv 18 \pmod{24}$

$6 \cdot 2X \equiv 6 \cdot 3 \pmod{24}$

$2X \equiv 3 \left(\bmod \dfrac{24}{\gcd(6,24)} \right)$, $\gcd(6,24) = 6$.

$2X \equiv 3 \pmod 4$

$\rightarrow 2X - 3 = 4k \qquad (k \in \mathbb{Z})$

$2X - 3 : 홀수$

$4k : 짝수$

$\rightarrow 0은 해가 없다.$

② $15X \equiv 20 \pmod{30}$

$3X \equiv 4 \left(mod \; \dfrac{30}{gcd(5,30)} \right)$, $gcd(5,30) = 5$

$3X \equiv 4 \pmod 6$

$6 = 3 \cdot 2 + 0$

$gcd(3,6) = 3$

$3 \nmid 4 \longrightarrow$ 해가 없다. $(\because Thm \; 10)$

③ $10X \equiv 25 \pmod{50}$

$2X \equiv 5 \left(mod \; \dfrac{50}{gcd(5,50)} \right)$, $gcd(5,50) = 5$.

$2X \equiv 5 \pmod{10}$

$gcd(2,10) = 2$.

$2 \nmid 5$

$\longrightarrow$ 해가 없다. $(\because Thm \; 10)$

①, ②, ③ 에 의해 $\therefore$ System은 해가 없다.