Q5

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{24}=\frac{n}{24!}$$

양변에 24! 곱하자.

$$\frac{24!}{1}+\frac{24!}{2}+\frac{24!}{3}+\cdots+\frac{24!}{24}=n$$

$$\rightarrow \frac{24!}{1}+\frac{24!}{2}+\frac{24!}{3}+\cdots+\frac{24!}{24}\equiv n \pmod{13} \quad \cdots\cdots ①$$

$24! = 1\times2\times3\times4\times5\times6\times7\times\cdots\times13\times\cdots\times24$ 이므로

$$\frac{24!}{1}\equiv 0 \pmod{13}$$

$$\frac{24!}{2}\equiv 0 \pmod{13}$$

$\vdots$

$$\frac{24!}{13}\not\equiv 0 \pmod{13}$$

$\vdots$

$$\frac{24!}{24}\equiv 0 \pmod{13}$$

, 따라서 식 ①은

$$\rightarrow \frac{24!}{13}\equiv n \pmod{13}$$

이 식이 된다.

by Wilson's Theorem,

13은 홀수 소수이므로

$$(13-1)!\equiv -1 \pmod{13}$$

$$12!\equiv -1 \pmod{13}$$

$\rightarrow \frac{24!}{13} \equiv n \pmod{13} \rightarrow 12! \cdot \frac{\cancel{13}}{\cancel{13}} \cdot 14 \cdot 15 \cdot 16 \cdot 17 \cdot 18 \cdot 19 \cdot 20 \cdot 21 \cdot 22 \cdot 23 \cdot 24$

$\equiv n \pmod{13}$

$\rightarrow \phi_{13}(12!) \cdot \phi_{13}(14) \cdot \phi_{13}(15) \times \cdots \times \phi_{13}(24) \equiv n \pmod{13}$

$\rightarrow -1 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \times \cdots \times 11 \equiv n \pmod{13}$

$\rightarrow -11! \equiv n \pmod{13}$

$\rightarrow -11! \equiv n \left(\bmod \frac{13}{\gcd(12,13)} \right)$ $\because \gcd(12,13)=1$

$\Longleftrightarrow (-11!) \cdot 12 \equiv n \cdot 12 \pmod{13}$ ($\because$ by Thm 72)

$\rightarrow -12! \equiv 12n \pmod{13}$

$-\phi_{13}(12!) \equiv 12n \pmod{13}$

$(-1)(-1) \equiv 12n \pmod{13}$

$\rightarrow 1 \equiv 12n \pmod{13}$

$\rightarrow 1 \equiv \phi_{13}(12)\, n \pmod{13}$

$1 \equiv -n \pmod{13}$

$\rightarrow \therefore n \equiv -1 \pmod{13}$

$\rightarrow n \equiv 12 \pmod{13}$