Q2. $(a,b) \neq (0,0)$ c는 정수

$c = ax+by$ x y는 정수

$\longleftrightarrow \gcd(a,b) \mid c$

$(\rightarrow)$ $c = ax+by$ (To show that $\gcd(a,b) \mid c$.)

$d = \gcd(a,b) \rightarrow d \mid a$ and $d \mid b$

$\rightarrow d \mid ax+by$ $x, y \in \mathbb{Z}$ ($\because$ Thm 17)

$\rightarrow d \mid c \rightarrow$

$\rightarrow \gcd(a,b) \mid c$.

Q2. $(\leftarrow)$ $\gcd(a,b) \mid c$ (To show that $c = ax+by$ for some $x, y \in \mathbb{Z}$)

$d = \gcd(a,b)$

$d \mid c$

$\begin{cases} c \neq 0 \\ c = dq \quad (q \in \mathbb{Z}) \quad \text{--(1)} \end{cases}$

$(a,b) \neq (0,0)$ 이므로

$S = \{ n \in \mathbb{N} \mid n = ax+by \text{ for some } x, y \in \mathbb{Z} \}$ 라 하면

S는 최솟값 m을 가지고, ($\because$ Well-Ordering Principle)

$m = \gcd(a,b)$ ($\because$ Thm 36)

--(2)

즉 $d = \gcd(a,b) = m = ax_0 + by_0$ ($\because$ (2))

(1)에서 $c = dq$ 이므로

$c = q(ax_0 + by_0)$

$= a(qx_0) + b(qy_0)$

$= ax + by$ for some $x, y \in \mathbb{Z}$ ▨