Q8. (a) $\left(\frac{777}{887}\right) = \left(\frac{7 \cdot 111}{887}\right)$

$= \left(\frac{7}{887}\right) \cdot \left(\frac{111}{887}\right)$ (by Thm 13.7)

$= (-1)^{\frac{7-1}{2}\cdot\frac{887-1}{2}} \left(\frac{887}{7}\right) \cdot (-1)^{\frac{111-1}{2}\cdot\frac{887-1}{2}} \left(\frac{887}{111}\right)$ (by 가우스 상호법칙)

$(-1)(-1) \cdot \left(\frac{5}{7}\right)\left(\frac{110}{111}\right)$ (by remark 11.4, $\phi_7(887)=5$, $\phi_{111}(887)=110$)

$= (-1)^{\frac{5-1}{2}\cdot\frac{7-1}{2}} \cdot \left(\frac{7}{5}\right) \cdot \left(\frac{2\cdot 5\cdot 11}{111}\right)$ (by 가우스 상호법칙)

$= \left(\frac{2}{5}\right)\left(\frac{2}{111}\right) \cdot \left(\frac{5}{111}\right)\left(\frac{11}{111}\right)$ (by remark 11.4, by Thm 13.4)

$= (-1)(1) \cdot \left(\frac{111}{5}\right) (-1)^{\frac{5-1}{2}\cdot\frac{111-1}{2}} \cdot \left(\frac{111}{11}\right) \cdot (-1)^{\frac{11-1}{2}\cdot\frac{111-1}{2}}$ (by Thm 14.0 가우스상호법칙)

$= (-1) \cdot (-1) \cdot \left(\frac{1}{5}\right) \cdot \left(\frac{1}{11}\right)$ (by remark 11.4)

$= 1 \cdot 1 \cdot 1$ (1은 quadratic residue modulo 5 or 11)

$= 1$.

(b) $\left(\frac{12345}{65537}\right) = \left(\frac{3 \cdot 5 \cdot 823}{65537}\right)$

$= \left(\frac{3}{65537}\right)\left(\frac{5}{65537}\right)\left(\frac{823}{65537}\right)$ (by Thm 13.1)

$= (-1)^{\frac{3-1}{2}\cdot\frac{65537-1}{2}} \cdot \left(\frac{65537}{3}\right) \cdot (-1)^{\frac{5-1}{2}\cdot\frac{65537-1}{2}} \cdot \left(\frac{65537}{5}\right) \cdot (-1)^{\frac{823-1}{2}\cdot\frac{65537-1}{2}} \cdot \left(\frac{65537}{823}\right)$

$= \left(\frac{2}{3}\right) \cdot \left(\frac{2}{5}\right)\left(\frac{520}{823}\right)$ (by remark 11.4)

$= (-1)(-1)\left(\frac{2^3 \cdot 5 \cdot 13}{823}\right)$ (by Thm 14.0)

$= \left(\frac{2}{823}\right)\left(\frac{2}{823}\right)\left(\frac{2}{823}\right)\left(\frac{5}{823}\right)\left(\frac{13}{823}\right)$ (by Thm 13.1)

$\phi_8(823) = 7 \rightarrow \left(\frac{2}{823}\right) = 1$

$= (-1)^{\frac{5-1}{2}\cdot\frac{823-1}{2}} \cdot \left(\frac{823}{5}\right) \cdot (-1)^{\frac{13-1}{2}\cdot\frac{823-1}{2}} \left(\frac{823}{13}\right)$ (가우스 상호법칙)

$= \left(\frac{3}{5}\right) \cdot \left(\frac{4}{13}\right)$ (by Remark 11.4)

$= (-1)^{\frac{3-1}{2}\cdot\frac{5-1}{2}} \cdot \left(\frac{5}{3}\right)\left(\frac{2}{13}\right)\left(\frac{2}{13}\right)$ (가우스 상호법칙) (Thm 13.1)

$= \left(\frac{2}{3}\right)(-1)(-1)$ $\phi_8(13) = 5$

$= (-1)$ $\phi_8(3) = 3$

$= -1$