Q3 $n \geq 2$, $n \in \mathbb{Z}$

n is a composite number $\longleftrightarrow$ There is a prime factor p of n s.t. $p \leq \sqrt{n}$

pf) ($\Longleftarrow$) There is a prime factor p of n s.t. $p \leq \sqrt{n}$ $\Longrightarrow$ $p \leq \sqrt{n}$, $p \mid n$

$p \leq \sqrt{n}$

$n \geq 2 \rightarrow \sqrt{n} < n$ $(\sqrt{n} \neq n)$

$2 \leq p \leq \sqrt{n} < n$

$\rightarrow 2 \leq p < n$ ---- ①

$p \mid n$

$\rightarrow n = pq$ $(q \in \mathbb{Z})$

$\rightarrow \frac{n}{p} = q$ $(q \in \mathbb{Z})$ ---- ② $\longrightarrow p \geq 2$, $q < n$ $(q \in \mathbb{Z})$ ---- ①

$q > 0$ ---- ② (by ①: $n \geq 2$, $p \geq 2 \rightarrow \frac{n}{p} = q > 0$)

$q = 1$ 이라 가정하자.

$\rightarrow n = p$ ↯ (모순 by ①)

$\rightarrow q \neq 1$ ---- ③

①+②+③ 에 의해 $2 \leq q < n$ $(q \in \mathbb{Z})$ ---- ③

$n = p\left(\frac{n}{p}\right) = pq$

$2 \leq p < n$ (by ①)

$2 \leq q < n$ (by ③)

$\rightarrow n$ 은 합성수이다. ■

n is a composite number

pf) $\rightarrow$ $n = d\left(\frac{n}{d}\right)$ for some divisor d of n s.t. $1 < d < n$

$1 < \frac{n}{d} < n$

d의 소인수 중 하나를 P_1

$\frac{n}{d}$의 소인수 중 하나를 P_2 라 하자.

$1 < P_1 < n$
$1 < P_2 < n$ $\rightarrow P_1 P_2 \leq n$ ... ①

Let $p = \min\{P_1, P_2\}$

$\rightarrow P_1 P_2 \geq p \cdot p = p^2$ ..... ②

①+②에 의해

$p^2 \leq P_1 P_2 \leq n$

$\rightarrow p^2 \leq n$

$\rightarrow p \leq \sqrt{n}$ $(p \geq 2,\ n \geq 2)$

∴ There is a prime factor p of n s.t. $p \leq \sqrt{n}$

■