DDPM

고홍규

GAN : 빠른 생성 + 고퀄

VAE : 빠른 생성 + 다양한 그림

Diffusion : 다양한 그림 + 고퀄

다양한 그림 = Mode Diversity

$$x_T \rightarrow \cdots \xrightarrow{p_\theta} x_0$$

원본

$$x_T \leftarrow \cdots \xleftarrow{q} x_0$$

q = 전파

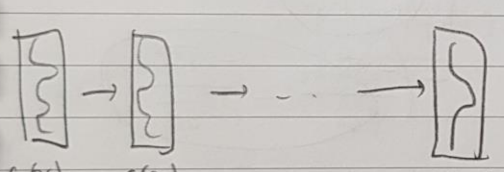
Noising

$$q(x_t) = \int q(x_0) \cdot q(x_t|x_0)\, dx_0$$

$$q(x_t|x_0) = q(x_t|x_{t-1}) \times \cdots \times q(x_2|x_1) \cdot q(x_0)$$

결합분포

$$x_t = \sqrt{1-\beta_t}\, x_{t-1} + \sqrt{\beta_t} \cdot \epsilon_t$$



$q(x_0)$ $q(x_1)$ $q(x_t)$ = 정규분포

f). $q(x_t|x_{t-1}) = \mathcal{N}(x_t ; \sqrt{1-\beta_t} \cdot x_{t-1}, \beta_t)$, $\sqrt{1-\beta_t} \cdot x_{t-1} = \mu$, $\beta_t = \sigma^2$

$$x_t = \sqrt{1-\beta_t}\, x_{t-1} + \sqrt{\beta_t}\, \epsilon_{t-1}$$

$\epsilon_{t-1} \sim N(0,1)$

(Reparameterization trick)

$$= \sqrt{\alpha_t}\, x_{t-1} + \sqrt{1-\alpha_t} \cdot \epsilon_{t-1}$$

$$= \sqrt{\alpha_t}\left(\sqrt{\alpha_{t-1}} \cdot x_{t-2} + \sqrt{1-\alpha_{t-1}} \cdot \epsilon_{t-2}\right) + \sqrt{1-\alpha_t} \cdot \epsilon_{t-1}$$

$$= \sqrt{\alpha_t}\sqrt{\alpha_{t-1}} \cdot x_{t-2} + \underbrace{\sqrt{\alpha_t}\sqrt{1-\alpha_{t-1}}\, \epsilon_{t-2} + \sqrt{1-\alpha_t} \cdot \epsilon_{t-1}}_{= \sqrt{1-\alpha_t \alpha_{t-1}} \cdot \epsilon_{t-2}}$$

!

$$x_t = \sqrt{\alpha_t}\sqrt{\alpha_{t-1}} \times \cdots \sqrt{\alpha_0}\, x_0 + \sqrt{1-\alpha_t \alpha_{t-1} \times \cdots \times \alpha_0} \cdot \epsilon_t$$

$$= \sqrt{\bar{\alpha}_t} \cdot x_0 + \sqrt{1-\bar{\alpha}_t} \cdot \epsilon_t$$

$\epsilon_t = \epsilon_t(t|x_0)$

$\rightarrow q(x_t|x_0) = N\, x_t : (\sqrt{\bar{\alpha}_t}\, x_0, 1-\bar{\alpha}_t)$, $\sqrt{\bar{\alpha}_t}\, x_0 = \mu$, $1-\bar{\alpha}_t = \sigma^2$

$$\rightarrow x_0 = \left(x_t - \sqrt{1-\bar{\alpha}_t} \cdot \epsilon_t\right) \cdot \frac{1}{\sqrt{\bar{\alpha}_t}}$$

f) α_t, β_t : Hyper Parameter.

DDPM $\longleftrightarrow$ Langevin dynamics

$$x_t = \underbrace{\sqrt{1-\beta_t}}_{\text{ss}} \cdot x_{t-1} + \sqrt{\beta_t} \cdot \epsilon_t$$

$$= \left(1 - \frac{1}{2}\beta_t + O(\beta_t)^2\right) \cdot x_{t-1} + \sqrt{\beta_t}\, \epsilon_t$$

$$= x_{t-1} \boxed{-} \frac{1}{2}\beta_t \boxed{x_{t-1}} + \sqrt{\beta_t} \cdot \epsilon_t$$

$-x_{t-1} = \nabla_x \log p(x_{t-1})$ $(\beta_t = \delta)$

$$= x_{t-1} + \frac{\delta}{2} \nabla_x \log p(x_{t-1}) + \sqrt{\delta} \cdot \epsilon_t$$

즉, $(T \rightarrow \infty) \rightarrow (\delta \rightarrow 0)$ $(\sqrt{\beta_t} \rightarrow 0)$ $\rightarrow (x_t \rightarrow$ 정규분포$)$

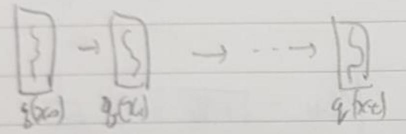
정규분포 = 노이즈 그 자체 $N(0, I)$

Date

Denoising

DDPM = Anderson's reverse time SDE

- 그림의 다양성 → 돌아올 때
- 속도 느림 확산위로 확률성추가
- 각 단계마다 분포변



이건 DDIM과 같다.

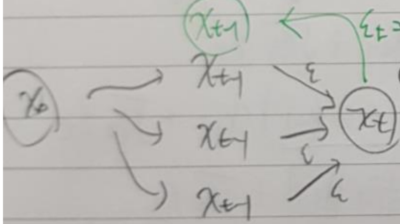
DDIM = Fokker Planck equation

- 돌아올 때 확산이 안전짐
- 미분방정식 풂.

$\nabla_{x_t} \log q(x_0)$에 쓰임.

$\epsilon_\theta(x_t, t)$

목표 ≤≤



$\epsilon_t = \epsilon_t(x_{t-1}|x_0)$ 쓰임

⊕ 새로운 ϵ 쓰임.

$$q(x_{t-1}|x_t, x_0) = \mathcal{N}(x_{t-1}; \underbrace{\tilde{\mu}(x_t, x_0)}_{\mu}, \underbrace{\tilde{\beta}_t I}_{\sigma^2}) = \mathcal{N}(x_{t-1}; \tilde{\mu}(x_t, x_0), \tilde{\beta}_t I)$$

$$= C \cdot \exp -\frac{1}{2}\left(\frac{(x_{t-1} - \tilde{\mu}(x_t, x_0))^2}{\tilde{\beta}_t}\right)$$

$$\| $$

$$\frac{x_{t-1}^2 - 2\tilde{\mu}(x_t, x_0)x_{t-1} + \tilde{\mu}(x_t, x_0)^2}{\tilde{\beta}_t}$$

1) $q(x_{t-1}|x_t)$: 어렵다.

$$q(x_{t-1}|x_t, x_0) = q(x_t|x_{t-1}, \cancel{x_0}) \cdot \frac{q(x_{t-1}|x_0)}{q(x_t|x_0)}$$

2) $q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t}\, x_{t-1}, \beta_t I)$

$\mathcal{N}(x_t; \sqrt{\alpha_t}\, x_{t-1}, (1-\alpha_t) I)$

$$q(x_t|x_0) = \mathcal{N}(x_t; \sqrt{\bar{\alpha}_t} \cdot x_0, (1-\bar{\alpha}_t) \cdot I)$$

$$q(x_{t-1}|x_t, x_0) = q(x_t|x_{t-1}) \cdot \frac{q(x_{t-1}|x_0)}{q(x_t|x_0)}$$

$$= C\exp\left(-\frac{1}{2}\frac{(x_t - \sqrt{\alpha_t}x_{t-1})^2}{\beta_t} + -\frac{1}{2}\frac{(x_{t-1} - \sqrt{\bar{\alpha}_{t-1}} \cdot x_0)^2}{(1-\bar{\alpha}_{t-1})} + \frac{1}{2}\frac{(x_t - \sqrt{\bar{\alpha}_t} \cdot x_0)^2}{1-\bar{\alpha}_t}\right)$$

$$= C\exp -\frac{1}{2}\left(\frac{x_t^2 - 2\sqrt{\alpha_t}\, x_t x_{t-1} + x_{t-1}^2 \cdot \alpha_t}{\beta_t} + \frac{x_{t-1}^2 - 2\sqrt{\bar{\alpha}_{t-1}} \cdot x_0 x_{t-1} + x_0^2 \cdot \bar{\alpha}_{t-1}}{1-\bar{\alpha}_{t-1}} - \frac{x_t^2 - 2\sqrt{\bar{\alpha}_t} \cdot x_0 x_t + \bar{\alpha}_t x_0^2}{1-\bar{\alpha}_t}\right)$$

$$= C\exp -\frac{1}{2}\left(\left(\frac{\alpha_t}{\beta_t} + \frac{1}{1-\bar{\alpha}_{t-1}}\right)x_{t-1}^2 + \left(\frac{-2\sqrt{\alpha_t}\, x_t}{\beta_t} + \frac{-2\sqrt{\bar{\alpha}_{t-1}}\, x_0}{1-\bar{\alpha}_{t-1}}\right)x_{t-1} + C\right)$$

$$\therefore \frac{1}{\tilde{\beta}_t} = \frac{\alpha_t}{\beta_t} + \frac{1}{1-\bar{\alpha}_{t-1}}$$

$$\frac{-2\tilde{\mu}(x_t, x_0)}{\tilde{\beta}_t} = \frac{-2\sqrt{\alpha_t}\, x_t}{\beta_t} + \frac{-2\sqrt{\bar{\alpha}_{t-1}}\, x_0}{1-\bar{\alpha}_{t-1}}$$

$$\tilde{\beta}_t = \left(\frac{\alpha_t (1-\bar{\alpha}_{t-1}) + \overset{1-\alpha_t}{\overset{\parallel}{\beta_t}}}{(\beta_t)(1-\bar{\alpha}_{t-1})} \right)^{-1}$$

$$= \frac{(\beta_t)((1-\bar{\alpha}_{t-1}))}{1 - \underbrace{\alpha_t \bar{\alpha}_{t-1}}_{\bar{\alpha}_t}}$$

$$= \left(\frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t} \right) \cdot \beta_t$$

$$\tilde{\mu}(x_t, x_0) = \frac{\tilde{\beta}_t}{-2} \cdot \left(\frac{-2\sqrt{\alpha_t}\, x_t}{\beta_t} + \frac{-2\sqrt{\bar{\alpha}_{t-1}} \cdot x_0}{1-\bar{\alpha}_{t-1}} \right)$$

$$= \frac{1}{-2} \cdot \frac{\cancel{1-\bar{\alpha}_{t-1}}}{1-\bar{\alpha}_t} \cancel{\beta_t}$$

$$\times \frac{-2 x_t \sqrt{\alpha_t} (1-\bar{\alpha}_{t-1}) + \beta_t x_0 \sqrt{\bar{\alpha}_{t-1}} (-2)}{\cancel{\beta_t} (\cancel{1-\bar{\alpha}_{t-1}})}$$

$$= \frac{1}{1-\bar{\alpha}_t} \left(x_t \sqrt{\alpha_t} (1-\bar{\alpha}_{t-1}) + \beta_t x_0 \sqrt{\bar{\alpha}_{t-1}} \right)$$

$$\vdots$$

$$=$$

$$\tilde{\mu}(x_t, x_0) = \frac{1}{\sqrt{\alpha_t}} \left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \varepsilon_t \right)$$

Date

$$\therefore x_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(x_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \varepsilon_t \right) + \frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t} \cdot \beta_t \, \varepsilon$$

$\varepsilon_t \overset{\parallel}{=} \varepsilon_t(x_t|x_0)$

$\downarrow$

의미 ① $x_t \to x_0$ 방향

② $x_t \to x_{t-1}$ 의

평균값

$$\varepsilon_\theta(x_t, t) = \int \varepsilon_t(x_t|x_0)\, q(x_0|x_t) \cdot dx_0$$

목표) $\left| \varepsilon_t(x_t|x_0) - \varepsilon_\theta(x_t, t) \right|^2 \Downarrow$

$\rightarrow$ $\varepsilon_\theta(x_t, t)$ 가 다양한 x_0의 영향 골고루 받게됨.

Date

Loss function

$$-\log p_\theta(x_0) \le -\log p_\theta(x_0) + D_{KL}\left(q(x_{1:T}|x_0) \| p_\theta(x_{1:T}|x_0)\right)$$

$$= E_{x_{1:T} \sim q\, x(1:T|x_0)}\left[\log \frac{q(x_{1:T}|x_0)}{p_\theta(x_{0:T})}\right]$$

$$-E_{q(x_0)} \log p_\theta(x_0) \le \underbrace{E_{q(x_{0:T})}\left[\log \frac{q(x_{1:T}|x_0)}{p_\theta(x_{0:T})}\right]}_{= L_{VLB} \downarrow}$$

최소화

의미.

$p_\theta \approx q$.

$$L_{VLB} = E_{q(x_0)}\left[D_{KL}\, q(x_t|x_0) \| p_\theta(x_t)\right]$$

$$+ \sum_{t=2}^{T} E_{q(x_t, x_0)}\left[D_{KL}\, q(x_{t-1}|x_t, x_0) \| p_\theta(x_{t-1}|x_t)\right]$$

$$- E_{q(x_0, x_1)}\left[\log p_\theta(x_0|x_1)\right]$$