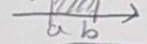
$v \in$ Vector Space $\longrightarrow \|v\| \geq 0$

1. 각 $v \in V$, $\|v\| \geq 0$
2. $\|v\| = 0 \longleftrightarrow v = 0$
3. $\|\beta v\| = |\beta| \|v\|$
4. $\|v+w\| \leq \|v\| + \|w\|$

$f \in C[a,b]$ 

$L^2 : \|f\|_{L^2} = \sqrt{\int_a^b (f(x))^2 dx}$

$L^1 : \|f\|_{L^1} = \int_a^b |f(x)| dx$

$L^\infty : \|f\|_{L^\infty} = \max_{a \leq x \leq b} |f(x)|$

No.

norm. $v \in \mathbb{R}^n$ $\quad v = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$\|v\|_2 = \sqrt{\sum_{i=1}^{N} (x_i)^2}$

$\|v\|_1 = \sum_{i=1}^{N} |x_i|$ $\leftarrow$ Taxicab Distance

$\|v\|_\infty = \max_{i=1,\dots,N} |x_i|$

행렬 곱. $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$f(x, y) = x^T \cdot A \cdot y$

내적) $f(x, y) = x \cdot y = x^T \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot y$

positive define. $\begin{cases} x^T A x \geq 0 \\ x^T A x = 0 \longleftrightarrow x = 0 \end{cases}$

positive semidefinite. $x^T A x \geq 0.$

Rank $\quad y = Ax = \begin{bmatrix} a_1 | a_2 | a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_1 \end{bmatrix} x_1 + \begin{bmatrix} a_2 \end{bmatrix} x_2 + \dots + \begin{bmatrix} a_n \end{bmatrix} x_n$

Column rank = largest subset of linearly independent set.

for $A \in \mathbb{R}^{m \times n}$, $\mathrm{rank}(A) \leq \min(m, n)$

for $A \in \mathbb{R}^{m \times n}$, $\mathrm{rank}(A) = \mathrm{rank}(A^T)$

SVD $\quad A = U \cdot \Sigma \cdot V^T$

$m \times n \quad m \times r \quad r \times r \quad r \times n$

$\begin{bmatrix} \diagdown & 0 \\ 0 & \diagdown \end{bmatrix}$

$A = n \times n$ symmetric $A = A^T$

$U^{-1} A U = D = \begin{bmatrix} \diagdown \end{bmatrix}$

$\rightarrow A = U \cdot D \cdot U^{-1}$ $\quad U = \begin{bmatrix} | & | & | \end{bmatrix}$

$I = \begin{bmatrix} \lambda_1^2 & & 0 \\ & \ddots & \\ 0 & & \lambda_n^2 \end{bmatrix}$

$UDU^T = \underbrace{\begin{bmatrix} \, \end{bmatrix}}_{V} \underbrace{\begin{bmatrix} \, \end{bmatrix}}_{\Sigma^2} \underbrace{\begin{bmatrix} \, \end{bmatrix}}_{V^T}$

Bilinear function

$f: V \times W \to \mathbb{R}$ (or)

$f: V \times W \to \mathbb{F}$

$f(\alpha v_1 + \beta v_2, w) = \alpha f(v_1, w) + \beta f(v_2, w) \quad \forall v_1, v_2 \in V$

$f(v, r w_1 + s w_2) = r f(v, w_1) + s f(v, w_2)$

Quadratic form

$x^T A x = \sum_{i=1}^{n} x_i (Ax)_i = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} x_i x_j$

Eigenvalue

$Ax = \underbrace{\lambda}_{\text{eigen value}} x \quad (x \neq 0) \longrightarrow$ eigenvector

$(\lambda I - A) x = 0 \quad (x \neq 0)$

$|(\lambda I - A)| = 0$

QR algorithm

1. 초기값 $A_0 = A$
2. 반복과정 $A_k = Q_k R_k$

   $A_{k+1} = R_k Q_k$
3. 수렴

$$\lim_{k \to \infty} A_k = \begin{bmatrix} \lambda_1 & * & \cdots & * \\ 0 & \lambda_2 & & * \\ \vdots & & \ddots & \vdots \\ 0 & 0 & & \lambda_n \end{bmatrix}$$

eigen value.

Eigenvector Symmetric Matrix

A: a symmetric $n \times n$ matrix

$\rightarrow$ A는 실수 $\lambda_1\ \lambda_2 \cdots \lambda_n$ 존재.

$\rightarrow \lambda_i \neq \lambda_j \rightarrow v_i \perp v_j$

$\rightarrow \exists U$ s.t. $U^T A U = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$

Here $U = [v_1\ v_2 \cdots v_n] \leftarrow$ orthogonal

$\|v_i\| = 1.$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{U} \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{A} 3 \cdot \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{U^{-1}} 3\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \longrightarrow \frac{1}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \longrightarrow (-1)\frac{1}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \longrightarrow -1\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$