基礎数値解析

Fundamental numeric Analysis

第5回講義資料

Lecture notes 5

求根法の基礎

Fundamental of root-finding algorithms

豊橋技術科学大学

Toyohashi University of Technology

電気・電子情報工学系

Department of Electrical and Electronic Information Engineering

准教授　ショウ シュン

Associate Professor　Xun Shao

アクティブラーニング５(Active Learning 5)

３次方程式$f(x) = x^3 - x = 0$を満たす三つの解を小さい順に、$x_1 = -1$、$x_2 = 0$、$x_3 = 1$とする。

We write the three solutions to the cubic equation $f(x) = 0$ as $x_1 = -1$, $x_2 = 0$, and $x_3 = 1$ in ascending order.

解x_2を二分法によって近似計算する場合に、二分法の初期値が満たすべき条件を述べよ。さらに、真の解と近似解との絶対誤差が反復とともにどのように変化するかを数値で答えよ。

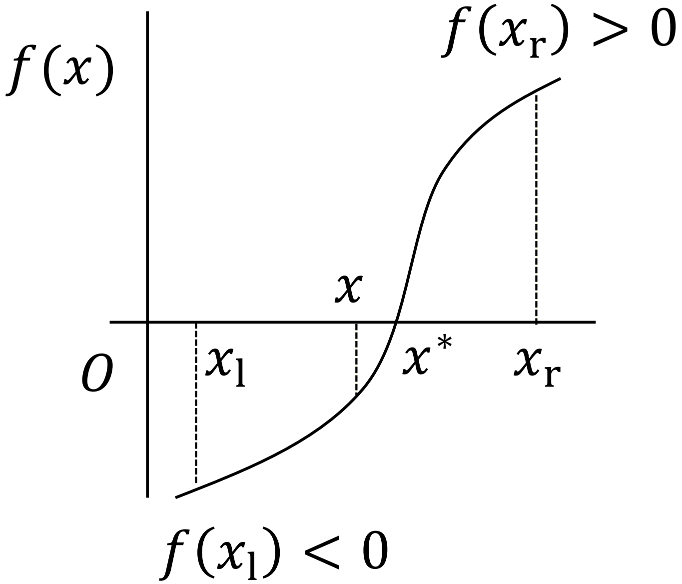
In approximately computing the maximum solution x_2 via bisection, answer what conditions the initial values in the bisection method should satisfy. Furthermore, provide a quantitative answer to the question “how absolute errors between the true and approximate solutions change as the iterations proceed.”

二分法(Bisection Method)

連続関数の性質(Property of continuous functions)

$f(x_\mathrm{l})f(x_\mathrm{r}) < 0$ならば、$x_\mathrm{l}$と$x_\mathrm{r}$の間に$f(x^*) = 0$を満たす解$x^*$が存在する。

There is a solution x^* to $f(x^*) = 0$ between x_l and x_r if $f(x_\mathrm{l})f(x_\mathrm{r}) < 0$ holds.



性質（Property）

二分点$x = (x_\mathrm{l} + x_\mathrm{r})/2$で$f(x_\mathrm{l})f(x) > 0$のとき、解$x^*$は区間$(x, x_r)$に含まれる。

If $f(x_\mathrm{l})f(x) > 0$ holds at the middle point $x = (x_\mathrm{l} + x_\mathrm{r})/2$, the interval (x, x_r) includes a solution x^*.

二分法(Bisection Method)

```
#include<stdio.h>
#include<math.h>

#define EPS 1.0e-15

void Bisection(double xl0, double xr0)
{
  int i,number;
  double x,xl,xr,xs,yl_sign;

  number = 10000; xl = xl0; xr = xr0; xs = 0.0;
  yl_sign = Function(xl0);
  for(i=1;i<=number && xr-xl>EPS;i++) {
    x = (xl+xr)/2.0;
    printf("Iteration=%d, solution=%g, error=%g¥n",i,x,fabs(x-xs));
    if(yl_sign*Function(x)>0.0) xl = x;
    else xr = x;
  }
}
```