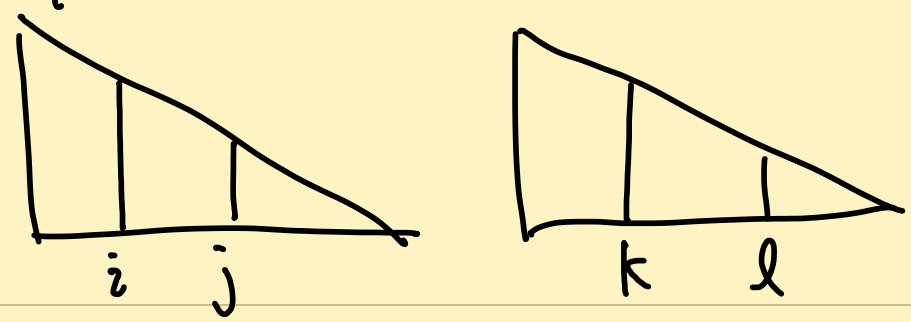
2017 WF D / BOJ 14636



$(y_j - x_i)(j-i)$

$(y_l - x_i)(l-i)$: max

$\rightarrow (y_l - x_i)(l-i) - (y_k - x_i)(k-i) \geq 0$

$\rightarrow l y_l - l x_i - i y_l + i x_i - k y_k + k x_i + i y_k - i x_i$

$= (l-i) y_l + (k-l) x_i + (i-k) y_k \geq 0$

$(y_l - x_j)(l-j) - (y_k - x_j)(k-j)$

$= l y_l - l x_j - j y_l + j x_j - k y_k + k x_j + j y_k - y x_j$

$= (l-j) y_l + (k-l) x_j + (j-k) y_k$

$\geq \frac{l-j}{l-i}(l-k) x_i + \frac{l-j}{l-i}(k-i) y_k + (k-l) x_j$

$+ (j-k) y_k$

$= (l-k)(x_i - x_j) - \frac{j-i}{l-i}(l-k) x_i + (j-i) y_k$

$- \frac{j-i}{l-i}(k-i) y_k$

$\geq (l-k)(x_i - x_j) + (j-i) y_k + \frac{j-i}{l-i}(i-l) y_l$

$= (l-k)(x_i - x_j) + (j-i)(y_k - y_l) \geq 0$

$$(y_k - x_i)(k-i) + (y_l - x_j)(l-j)$$

$$= ky_k - kx_i - iy_k + ix_i + ly_l - lx_j - jy_l + jx_j$$

$$(y_l - x_i)(l-i) + (y_k - x_j)(k-j)$$

$$= ly_l - lx_i - iy_l + ix_i + ky_k - kx_j - jy_k + jx_j$$

$$(l-k)x_j + (i-j)y_k + (j-i)y_l + (k-l)x_i$$

$$= (l-k)(x_j - x_i) + (j-i)(y_l - y_k) \leq 0$$

$$f(i, opt_i) \geq f(i, opt_j)$$

$$f(j, opt_j) \geq f(j, opt_i)$$

$$\rightarrow f(i, opt_i) + f(j, opt_j) \geq f(i, opt_j) + f(j, opt_i)$$

if $opt_i \geq opt_j$

$$f(i, l) + f(j, k) \geq f(i, k) + f(j, l) \quad (*)$$

$\rightarrow opt_i \leq opt_j$