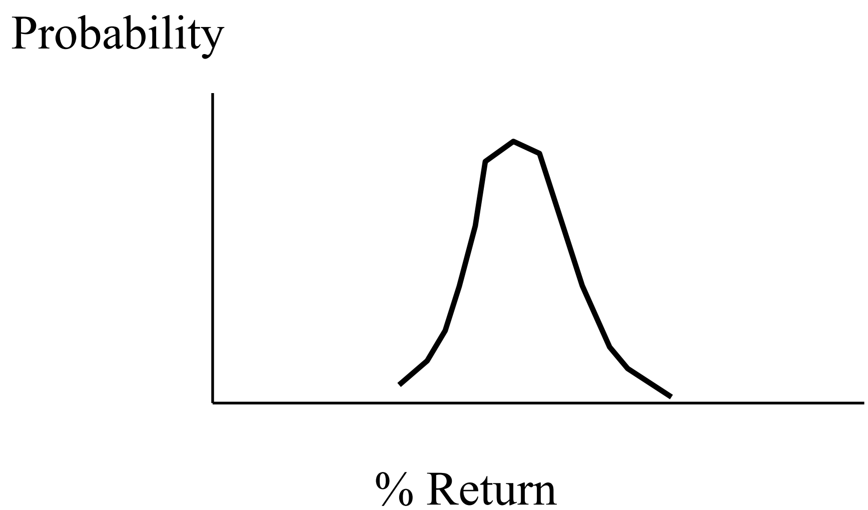
INTRODUCTION TO FINANCE

REVIEW OF STATISTICS

A Review of Statistics

◆In a world with **uncertainty**, we typically describe possible future values or rates of return of a security and the relative likelihoods of these values by means of a **probability distribution** over future values or rates of returns.

◆For example, a **continuous distribution** is given by:



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Discrete distribution - states of the world model

		Security Return in %			
State	**Prob.**	**1**	**2**	**3**	**4**
Expansion	0.10	5.00	3.50	6.00	2.50
Normal	0.40	7.00	4.50	5.00	-0.50
Recession	0.30	6.00	4.00	7.00	1.00
Depression	0.20	-2.00	0.00	4.00	13.00

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① Expected Return:

$$E(\tilde{R}_i) = \overline{R} = \sum_{s=1}^{S} p_s \tilde{R}_{i,s}$$

where $E(\tilde{R}_i) = \overline{R}$ = mean (expected) return

$\tilde{R}_{i,s}$ = unknown return of security i in state s

p_s = probability of state s

S = total number of states occurring.

Examples:

$E(\tilde{R}_1)$ = 0.10 (5.0) + 0.40 (7.0) + 0.30 (6.0) + 0.20 (-2.0)
= 4.70%

$E(\tilde{R}_2)$ = 0.10 (3.5) + 0.40 (4.5) + 0.30 (4.0) + 0.20 (0)
= 3.35%

$E(\tilde{R}_3)$ = 0.10 (6.0) + 0.40 (5.0) + 0.30 (7.0) + 0.20 (4.0)
= 5.50%

$E(\tilde{R}_4)$ = 0.10 (2.5) + 0.40 (-0.5) + 0.30 (1.0) + 0.20 (13.0)
= 2.95%.

② Spread of distribution of outcomes:

$$\mathrm{Std}(\tilde{R}_i) = \left\{ \sum_{s=1}^{S} p_s (\tilde{R}_{i,s} - \bar{R})^2 \right\}^{1/2} = \sigma_i .$$

Note: $\mathrm{Var}(\tilde{R}_i) = \left\{ \mathrm{Std}(\tilde{R}_i) \right\}^2 = \sigma_i^2 .$

Examples:

$$\sigma_1^2 = 0.10\,(5.0 - 4.7)^2 + 0.40\,(7.0 - 4.7)^2$$
$$+\ 0.30\,(6.0 - 4.7)^2 + 0.20\,(-2.0 - 4.7)^2$$
$$= 11.61$$

$\Rightarrow \sigma_1 = 3.41\%.$

$$\sigma_2^2 = 0.10\,(3.5 - 3.35)^2 + 0.40\,(4.5 - 3.35)^2$$
$$+\ 0.30\,(4.0 - 3.35)^2 + 0.20\,(0.0 - 3.35)^2$$
$$= 2.90.$$

$\Rightarrow \sigma_2 = 1.70\%.$

σ_3= Calculate

σ_4= Calculate

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③ Covariance:

$$\mathrm{Cov}(\tilde{R}_i, \tilde{R}_j) = \left\{ \sum_{s=1}^{S} p_s (\tilde{R}_{i,s} - \overline{R}_i)(\tilde{R}_{j,s} - \overline{R}_j) \right\} = \sigma_{i,j}.$$

◆When the returns of securities i and j tend to deviate from their means in the same direction they are called **positively correlated** or they **covary positively** [that is, $\mathrm{Cov}(\tilde{R}_i, \tilde{R}_j) > 0$].

◆When they tend to deviate from their means in opposite directions, they are **negatively correlated** or they **covary negatively** [that is, $\mathrm{Cov}(\tilde{R}_i, \tilde{R}_j) < 0$].

◆When the returns of securities i and j are unrelated with each other, then $\mathrm{Cov}(\tilde{R}_i, \tilde{R}_j) = 0$ and they are **uncorrelated**.

Examples:

$$\sigma_{1,2} = 0.10\,(5.0 - 4.7)\,(3.5 - 3.35)$$
$$+ 0.40\,(7.0 - 4.7)\,(4.5 - 3.35)$$
$$+ 0.30\,(6.0 - 4.7)\,(4.0 - 3.35)$$
$$+ 0.20\,(-2.0 - 4.7)\,(0.0 - 3.35)$$
$$= 5.81.$$

$$\sigma_{1,3} = 0.10\,(5.0 - 4.7)\,(6.0 - 5.5)$$
$$+ 0.40\,(7.0 - 4.7)\,(5.0 - 5.5)$$
$$+ 0.30\,(6.0 - 4.7)\,(7.0 - 5.5)$$
$$+ 0.20\,(-2.0 - 4.7)\,(4.0 - 5.5)$$
$$= 2.15.$$

$\sigma_{1,4}$ = Calculate

$\sigma_{2,3}$ = Calculate

$\sigma_{2,4}$ = Calculate

$\sigma_{3,4}$ = Calculate

④ Correlation--A "standardized" Covariance:

$$\rho_{i,j} = \frac{Cov(\tilde{R}_i,\tilde{R}_j)}{Std(\tilde{R}_i)Std(\tilde{R}_j)} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j},$$

$$where\ -1 \le \rho_{i,j} \le +1.$$

- Since $\mathrm{Std}(\tilde{R}_i)$ and $\mathrm{Std}(\tilde{R}_j)$ are always positive, the sign of $\rho_{i,j}$ depends on the sign of $\sigma_{i,j}$.
- What is the relation between $\rho_{i,j}$ and $\rho_{j,i}$?

⑤ Regression Analysis:

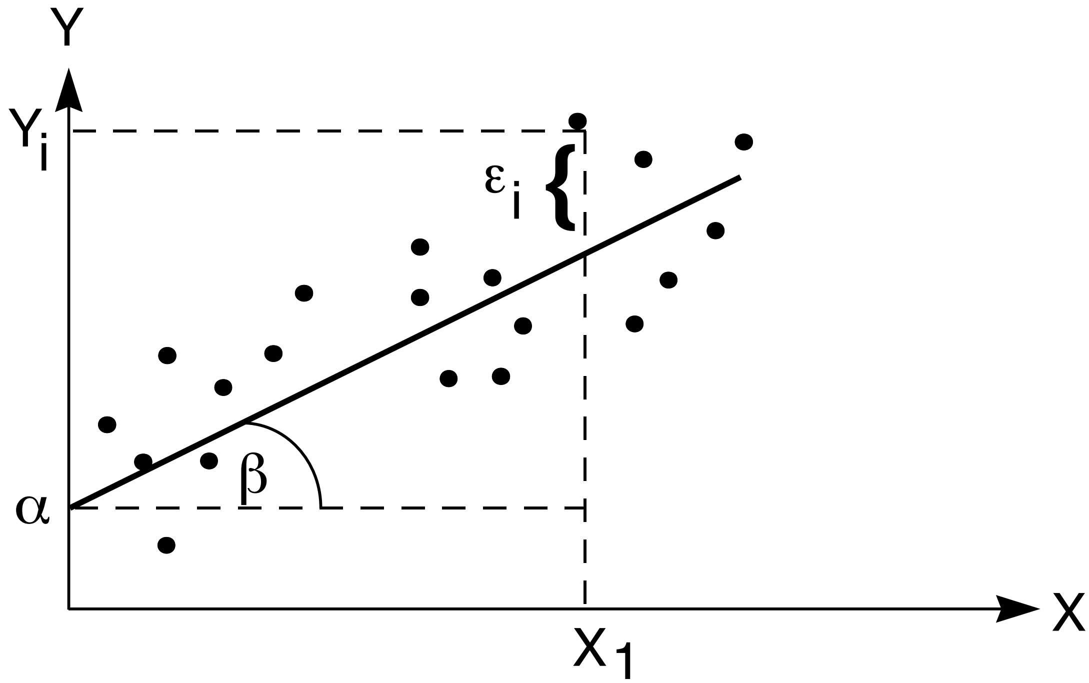
- The general idea of a regression is that it involves fitting a line through a set of points. In particular, consider the following regression model:

$$\tilde{y} = \alpha + \beta\tilde{x} + \tilde{\varepsilon}$$

where

$\tilde{y}$ = the value of the dependent variable
$\tilde{x}$ = the value of the independent variable
$\tilde{\varepsilon}$ = a random error with $E(\tilde{\varepsilon}) = 0$
α = the Y axis intercept
β = the slope of the line.

The picture that we are trying to capture is as follows:



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- In general the idea is that there is a linear relationship between y and x which is not quite exact.

  Since we do not know the relationship we want to estimate α and β that minimize the sum of the deviations squared from the fitted line.

  The best fit is given by the slope:

$$\beta_{y,x} = \frac{Cov(\tilde{y},\tilde{x})}{\sigma_x^2} = \frac{\sigma_{yx}}{\sigma_x^2}.$$

- How would you write the relation between security 1 and

  2?
- What is the relation between $\beta_{2,1}$ and $\beta_{1,2}$?

- What is the relation between $\beta_{2,1}$ and $\rho_{1,2}$?

Examples of Correlation and Regression:

Perfect Positive Correlation

$$\rho_{1,2} = \frac{\sigma_{1,2}}{\sigma_1 \quad \sigma_2} = \frac{5.81}{(3.41)(1.70)} = 1.00.$$

Note: $\tilde{R}_2 = 1.00 + 0.50\tilde{R}_1$.

Less Than Perfect Correlation:

$$\rho_{1,3} = \frac{\sigma_{1,3}}{\sigma_1 \quad \sigma_3} = \frac{2.15}{(3.41)(1.12)} = 0.563.$$

Note: $\tilde{R}_3 = 4.63 + 0.185\tilde{R}_1 + \tilde{e}_3$.

Perfect Negative Correlation

$$\rho_{1,4} = \frac{\sigma_{1,4}}{\sigma_1 \quad \sigma_4} = \frac{-17.42}{(3.41)(5.11)} = -1.00.$$

Note: $\tilde{R}_4 = 10.00 - 1.50\tilde{R}_1$.