# Programming Refresher Workshop

Session 4 Exercises

**Learning objective:**

- Problem-solving on 2-dimensional arrays

**Exercise 11 (ex11): Convex Polygon**

A polygon is **convex** if it contains all line segments connecting any pair of its vertices. A polygon that is not convex is a concave polygon. Figure 2(a) below shows a convex polygon and Figure 2(b) shows a concave one.
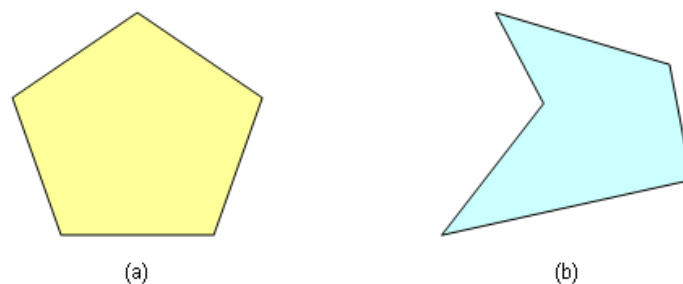


**Figure 2. (a)** A convex polygon.   **(b)** A concave polygon.

A simple way to determine whether a polygon is convex is to conduct a *walk* along the boundary of the polygon, going from one vertex to the next. If we *walk* in the clockwise direction, as shown in Figure 3(a) below, then every turn is a **right turn**. For example, *A-B-C* is a right turn, so is *B-C-D*, and so on.

On the other hand, if we *walk* in the counter-clockwise direction, as shown in Figure 3(b) below, then every turn is a **left turn**.
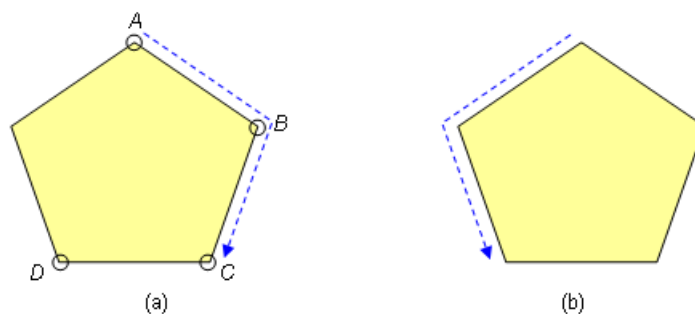


**Figure 3. (a)** Walking along the boundary of a convex polygon in the clockwise direction, every turn is a right turn.  **(b)** In the counter-clockwise direction, every turn is a left turn.

As expected, a concave polygon will have a mix of right turn(s) and left turn(s) in either direction.

Given three consecutive vertices *A*, *B*, and *C* of a polygon, to determine if *A-B-C* is a right turn or a left turn, we compute the determinant of this 3×3 matrix:

$$\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix}$$

where $A = (x_A, y_A)$, $B = (x_B, y_B)$, and $C = (x_C, y_C)$. If the determinant is a negative value, then *A-B-C* is a right turn; if the determinant is positive, then it is a left turn. (What if the determinant is zero?)

The determinant of a general 3×3 matrix is computed as follows:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1) + a_3(b_1 c_2 - b_2 c_1)$$

$$= a_1 b_2 c_3 - a_1 b_3 c_2 - a_2 b_1 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1$$

You are to write a program to read in a polygon represented by a number of (at least 3 and at most 10) vertices with integer coordinates, and determine if the polygon is convex or not by printing "Yes" or "No". You may assume that no three consecutive vertices are co-linear (i.e. they are on a straight line).

**Sample runs**

```
Enter number of vertices: 5
Enter vertices:
0 10
5 20
10 10
8 0
2 0
Yes
```

```
Enter number of vertices: 4
Enter vertices:
0 0
-6 -2
4 -2
0 5
No
```