Hypothesis Testing I

ANA 500 – Foundations of Data Analytics

Module 2 - week 3B

Hypothesis testing with one sample

- The average temperature in Maryland in July is 88 degrees.
- A new Honda Accord gets 38 miles per gallon.
- 64% of all college students graduate in four years.
- A new lawyer makes an average of $100,000 per year.
- 70% of all U.S. households have internet access.

- These types of claims can be assessed statistically using a sample of data to determine if there is sufficient evidence to support them.
- First, we will cover hypothesis tests about a single mean or a single proportion.

Hypothesis testing with one sample

- There is a straightforward approach to hypothesis testing, however, remember that we are using a random sample of data, and therefore, we can make errors.

- The approach:
  - 1. State the null hypothesis and alternative hypothesis/hypotheses.
  - 2. Collect a random sample of data
  - 3. Determine the correct distribution to use for the test
  - 4. Analyze/assess the data to determine which hypothesis is supported
  - 5. Reject or fail to reject the null hypothesis

Hypothesis testing with one sample

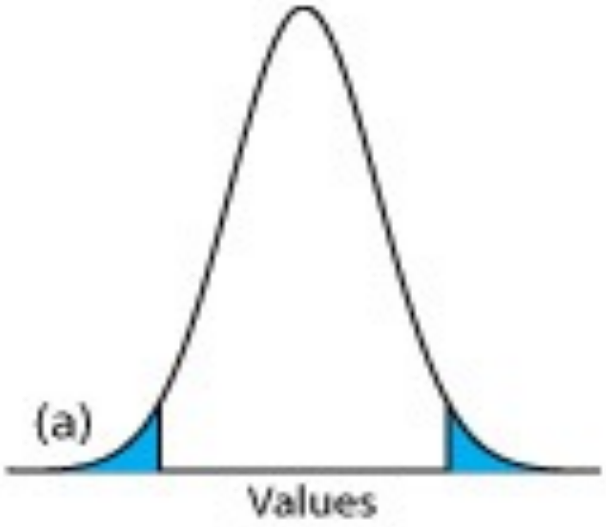
- **Null hypothesis**: H_0 is an assumption about μ or P;
  - It is a statement of no difference or no effect.
- **Alternative hypothesis**: H_a is counter to H_0
  - It is a statement of difference or effect.

- E.g., The average annual salary of a new lawyer is $100,000
- H_0: $\mu = 100,000$
- H_a: $\mu \neq 100,000$

Three Possible forms for Hypothesis Tests

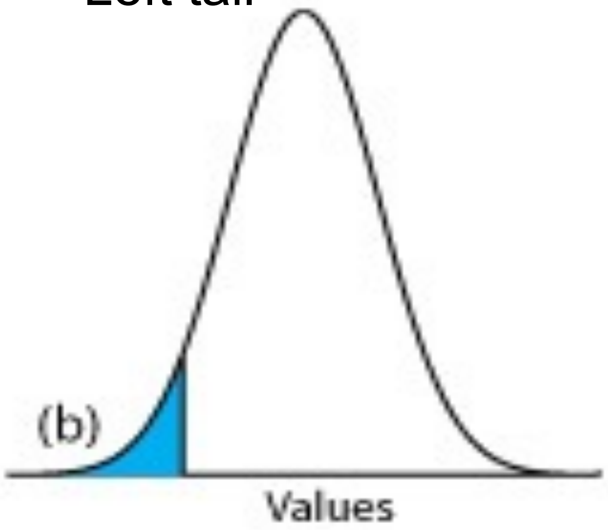
The average annual salary of a new lawyer is \$100,000

H_0: $\mu = 100{,}000$
H_a: $\mu \neq 100{,}000$
Two-tailed test



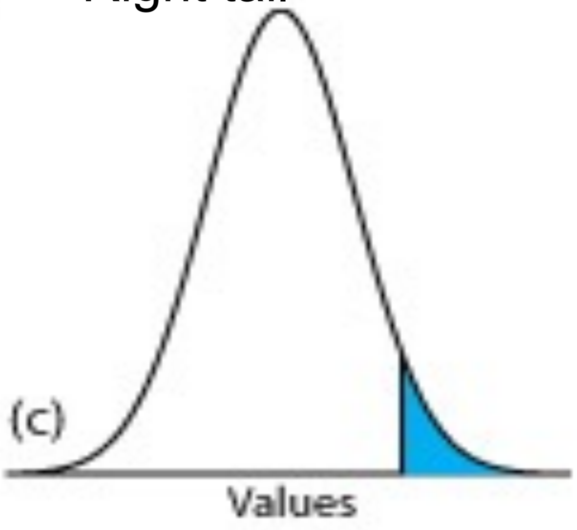
The average annual salary of a new lawyer at least is \$100,000

H_0: $\mu \geq 100{,}000$
H_a: $\mu < 100{,}000$
One-tailed test
Left tail



The average annual salary of a new lawyer at most is \$100,000

H_0: $\mu \leq 100{,}000$
H_a: $\mu > 100{,}000$
One-tailed test
Right tail



Type I and Type II errors

Table of error types		Null hypothesis (H_0) is	
		True	**False**
Decision about null hypothesis (H_0)	**Don't reject**	Correct inference (true negative) (probability = $1-\alpha$)	Type II error (false negative) (probability = β)
	Reject	Type I error (false positive) (probability = α)	Correct inference (true positive) (probability = $1-\beta$)

Hypothesis testing for the Population mean

E.g., The manufacturer of light bulbs wants to determine whether his light bulbs last for 1050 hours

- **H_0: μ = 1050**
- **H_a: $\mu \neq$ 1050**

- n = 50;
- $\sigma = 65;$
- $\bar{x}$=1045

Hypothesis testing for the Population mean

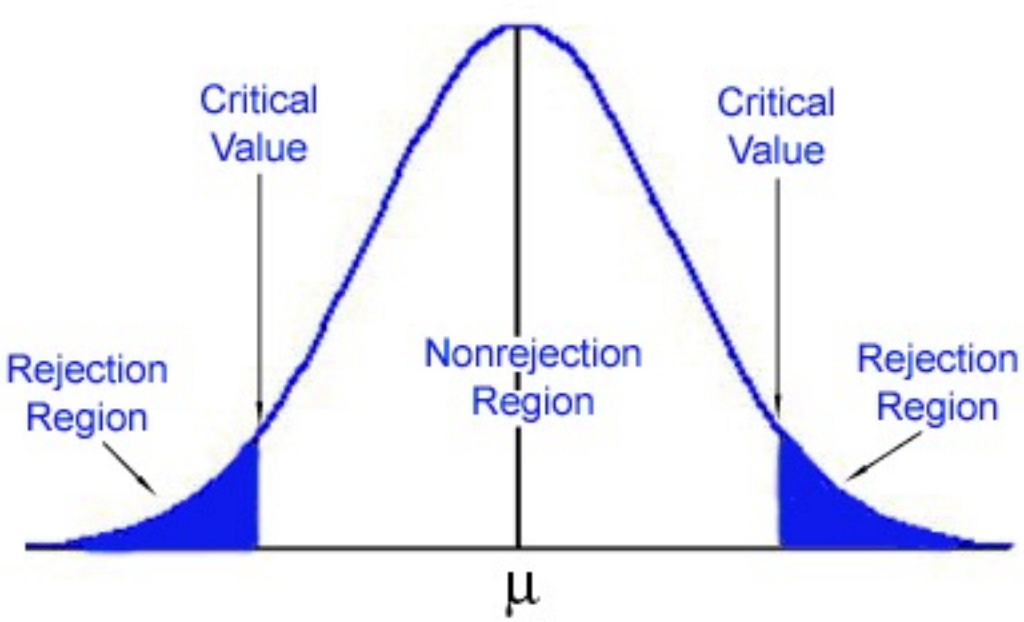
- Calculate Test Statistics: (n = 50; $\sigma = 65$; $\bar{x}$=1045)

- Z = $\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$ (μ_0 means the hypothesized value of the mean)

$=\frac{1045-1050}{\frac{65}{\sqrt{50}}}$ = -.5441

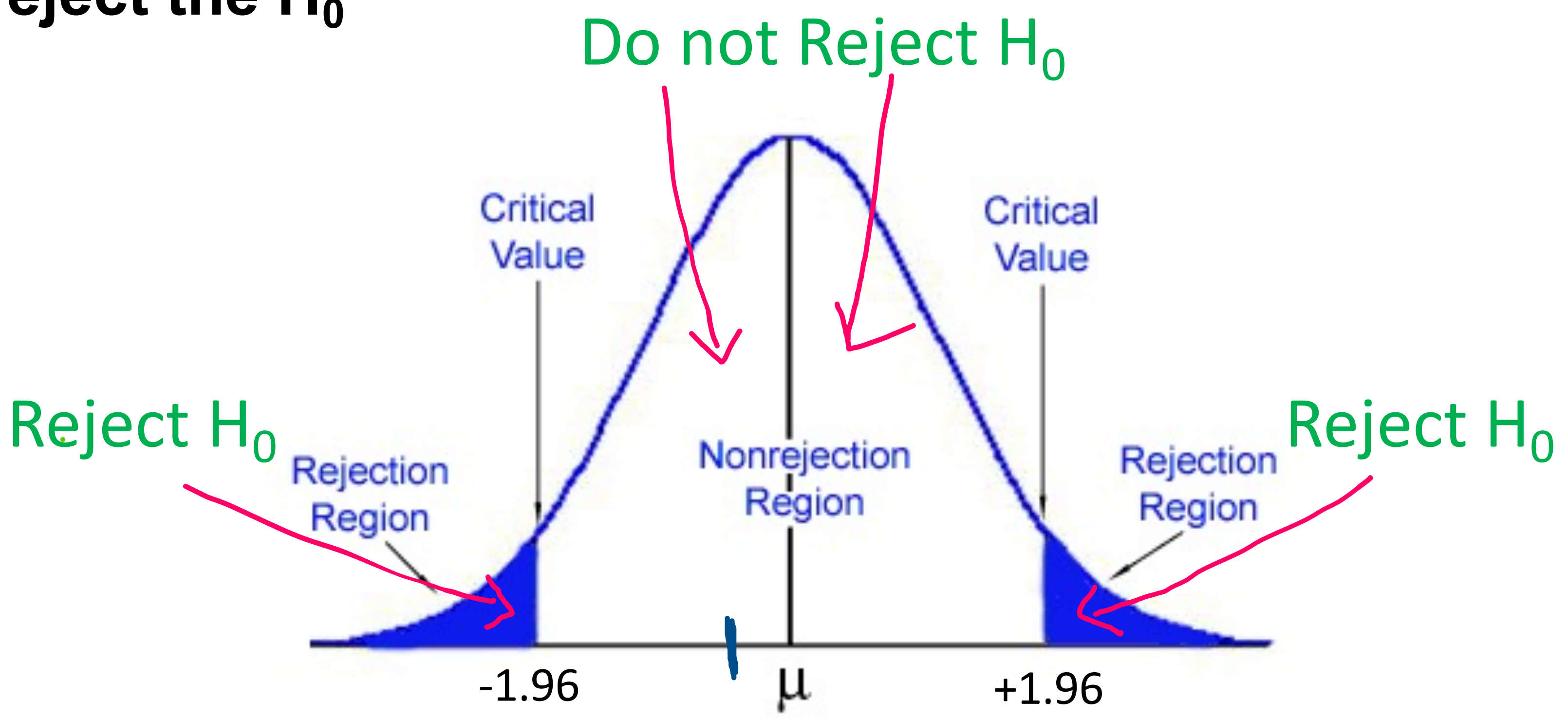
Hypothesis testing for the Population mean

- α = 0.05; $\alpha/2$ =0.025; z= ± 1.96 (critical value)



Hypothesis testing for the Population mean

- α = 0.05; $\alpha/2$ =0.025; z= ± 1.96 (critical value)
- test statistic = -0.5441
- **Fail to reject the H_0**



Hypothesis testing with one sample

- Now, we will use a p-value to make our decision (reject or fail to reject H_o).
- A p-value is the probability, under the assumption that H_o is true, that another random sample would produce results as extreme or more extreme as the results obtained in the given sample.
- E.g. p-value = 0.12
  - The probability of getting results as adverse to H_o as the ones obtained in the sample used is 0.12.
- A large p-value supports H_o
- A small p-value does not support H_o

Hypothesis testing with one sample

- P (z ≤ -.5441)=0.2946

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Hypothesis testing with one sample

For a two tailed test:

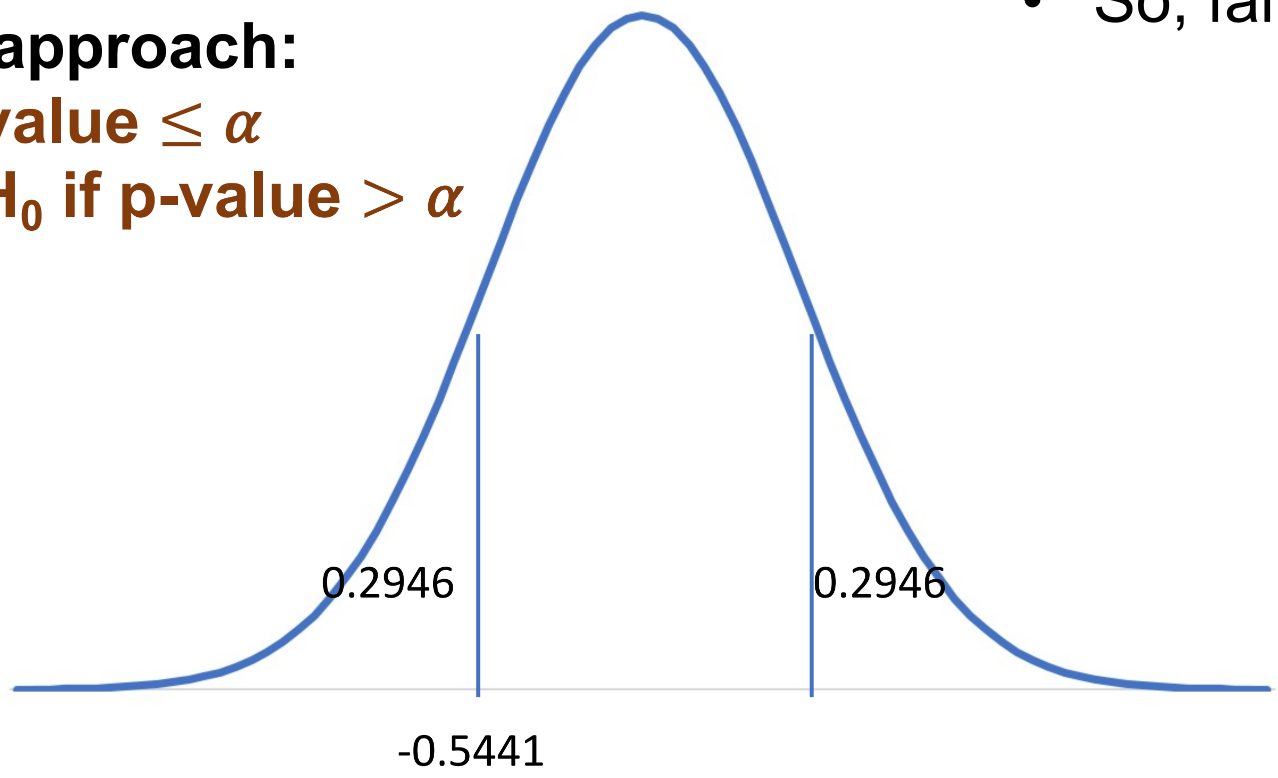
- Double the p value and compare to α
    - 0.2946*2=0.5892

Rule for p-value approach:

- **Reject H_0 if p-value $\leq \alpha$**
- **Fail to reject H_0 if p-value $> \alpha$**

In this example,

- α =0.05
- 0.5892>0.05
- So, fail to reject H_0



Hypothesis testing for the Population mean

- σ unknown cases:
  - use s as an estimator
  - use t distribution instead of Z

- t = $\dfrac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ (μ_0 means the hypothesized value of the mean)

Hypothesis testing for the Population mean

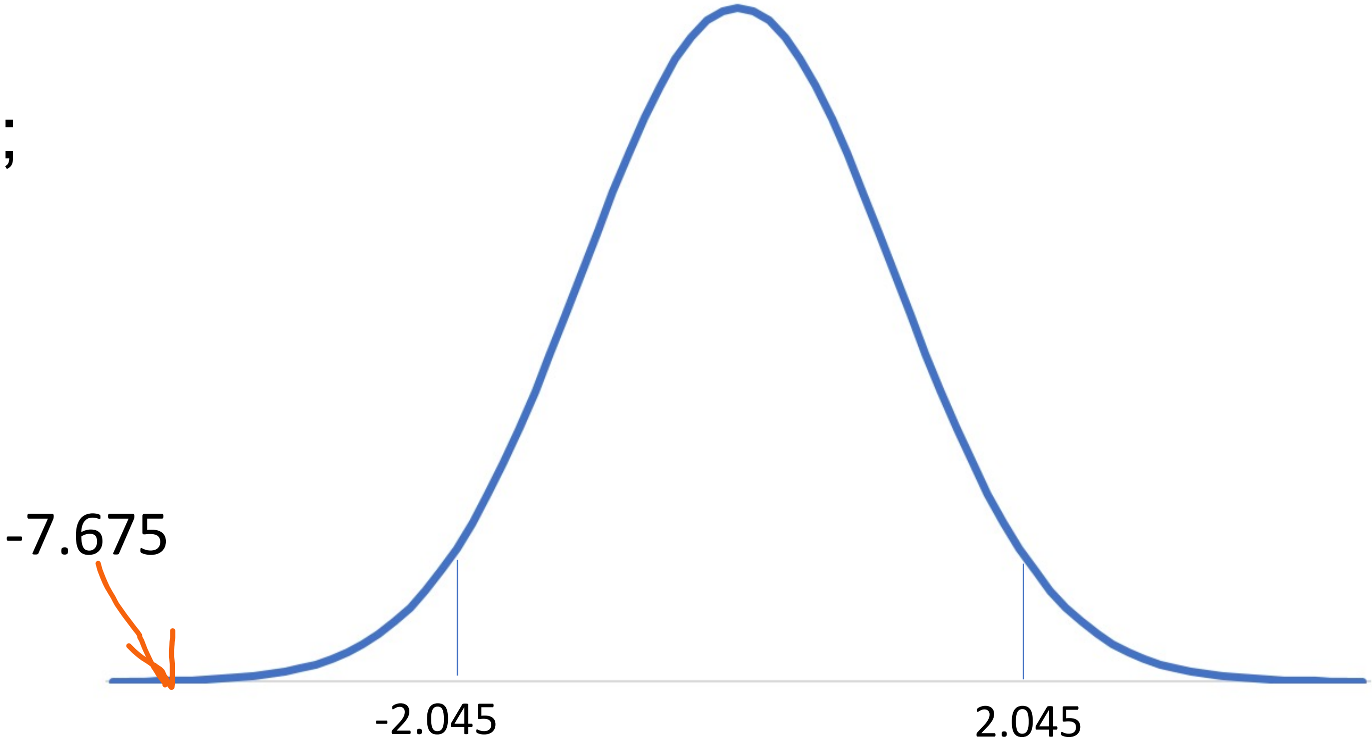
- Example: Is the mean fill of cereal boxes 16 ounces?
  - H_0: $\mu = 16$
  - H_a: $\mu \neq 16$

- n = 30; s $= 0.5$; $\bar{x}$=15.3
- t = $\dfrac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{15.3-16}{\frac{0.5}{\sqrt{30}}}$ = -7.675

Hypothesis testing for the Population mean

- $t = -7.675$
- Level of Significance = $\alpha = 0.05$;
- $\alpha/2 = 0.025$
- df = n -1 = 30-1 =29 ;

- Reject H_0



Hypothesis testing for the Population proportion

Example: 70% of U.S. households have internet access

$$\mathbf{H_0}: p = 0.70$$
$$\mathbf{H_a}: p \neq 0.70$$

- Suppose we collect a random sample of 81 households and calculate a sample proportion of 0.66.
- This is a test about a proportion, therefore, the appropriate distribution is the normal distribution.
- If Ho is true (i.e. p = 0.70), how many standard deviations is our sample result (i.e. p = 0.66) from the population mean?

Hypothesis testing for the Population proportion

Test-statistic (z-stat)= $\dfrac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$ = $\dfrac{0.66-0.70}{\sqrt{0.70(1-0.70)/81}}$ = $\frac{-0.04}{0.0509}$=-.786

The p-Value is 0.431. α =0.05

0.43 >.05.

Fail to reject the H_0