

# Bayesian Model for LTRC Data using Splines: Implementation and Explanation

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This document presents the implementation and explanation of a Bayesian model for Left-Truncated and Right-Censored (LTRC) data using splines. The model is based on the likelihood and posterior distributions described in the accompanying paper. The Channing House dataset is used to demonstrate the model.

## 1. Likelihood Function

The likelihood function for the Bayesian model is derived based on different types of observations, including censored and truncated data. The baseline hazard function is approximated using M-splines, and the cumulative hazard function (CHF) is represented using I-splines. The likelihood contributions are split into four categories depending on whether the observation is censored or truncated.

## 2. Posterior Distribution

The posterior distribution is obtained by combining the likelihood function with the prior distributions for the spline parameters and covariate coefficients. The model assumes exponential priors for the spline parameters ( $\gamma$ ) and normal priors for the covariate coefficients ( $\beta$ ).

## 3. Metropolis-Hastings Algorithm

The Metropolis-Hastings algorithm is used to sample from the posterior distribution. In each iteration, new proposals for the spline parameters ( $\gamma$ ) and covariate coefficients ( $\beta$ ) are generated. The acceptance of these proposals is based on the ratio of the posterior probabilities of the proposed and current values.

## 4. Code Explanation

The code provided implements the Bayesian model using Python, specifically NumPy and SciPy for numerical operations, and the Metropolis-Hastings algorithm for MCMC sampling. Below is a breakdown of the key components.

#### 4.1 Likelihood Function

The likelihood function is computed based on the different types of observations in the dataset. The Channing House data provides covariates such as age at entry, age at exit, and death status, which are used to calculate the likelihood contribution for each observation.

#### 4.2 Posterior Distribution

The posterior distribution combines the likelihood function and the priors for the spline parameters and covariates. The spline parameters ( $\gamma$ ) follow an exponential prior, and the covariate coefficients ( $\beta$ ) follow a normal prior.

#### 4.3 Metropolis-Hastings Algorithm

The Metropolis-Hastings algorithm is used to sample from the posterior distribution. In each iteration, a new set of  $\gamma$  and  $\beta$  values are proposed, and the acceptance of these values depends on the posterior probability ratio.