

Lab 06 - Report - Lakshya Kohli - 210123077

Answer 1.

M	I_M (estimated value of I)	95% Confidence Interval
100	1.9947105074120772	[1.90764626644459, 2.0817747483795648]
1000	1.9854670058109818	[1.9581113983963325, 2.012822613225631]
10000	2.0068336334099994	[1.998195161598582, 2.015472105221417]
100000	2.000019269038806	[1.99728645453245, 2.0027520835451615]

Observations:

1. The actual (exact) value of I is equal to 2 (calculated by integration shown below)

Handwritten mathematical derivation showing the calculation of $I = E[\exp(\sqrt{u})]$ using two methods: direct integration and substitution.

Given:
 $g(x) = \exp(\sqrt{x})$
 $x \sim U(0,1)$ pdf
 $f(x) = 1$ for $0 < x < 1$

Method 1 (Direct Integration):
$$I = \int_0^1 g(t) f(t) dt = \int_0^1 e^{\sqrt{t}} dt$$

Method 2 (Substitution):
Let $\sqrt{t} = p$
 $\frac{dt}{2\sqrt{t}} = dp$
 $dt = 2p dp$

Then:
$$I = \int_0^1 2p e^p dp = 2[p e^p - \int e^p dp]_0^1 = 2[p e^p - e^p]_0^1 = 2$$

Final result: $I = 2$

2. The value of I_M converges to the actual value of I , that is, 2 as M increases.