Monte Carlo Simulation - MA 323

Lab 02 - Report - Lakshya Kohli - 210123077

Answer 1:

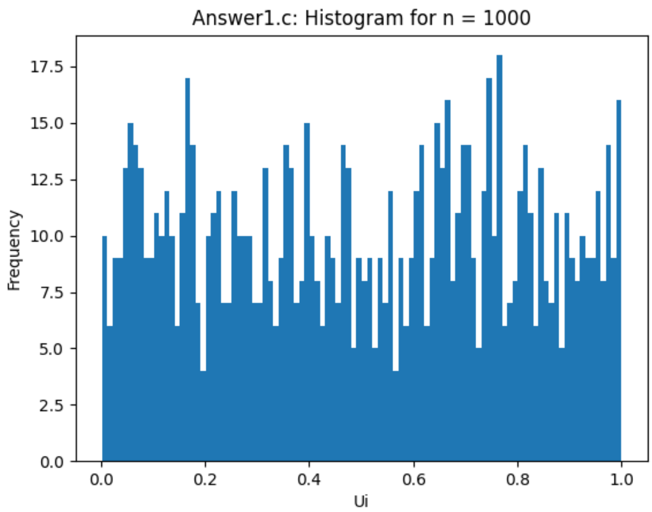
The Linear Congruence Generator used to generate first 17 values of U_i is of the form:

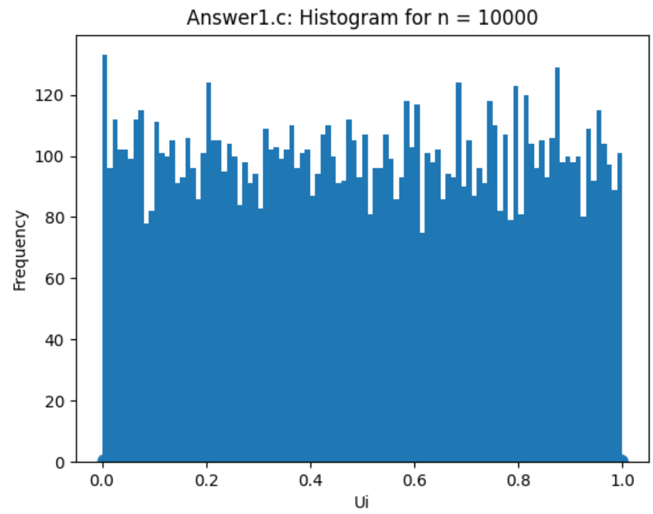
$x_{i+1} = (a*x_i + b) \bmod m$

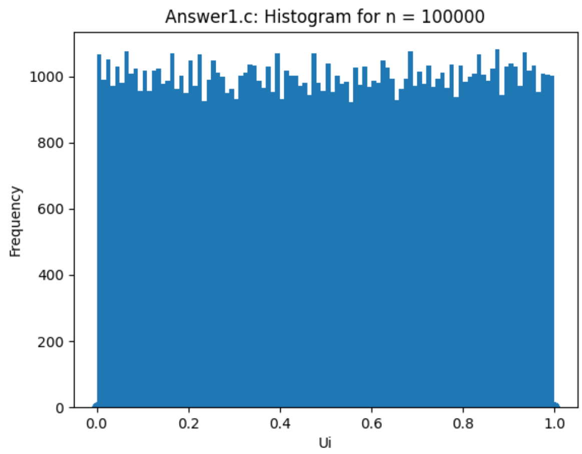
$u_{i+1} = x_{i+1}/m$

with a = 1229, b = 1, m = 2048, x_0 = 1

C) Histograms for sample size in [1000,10000,10000]:

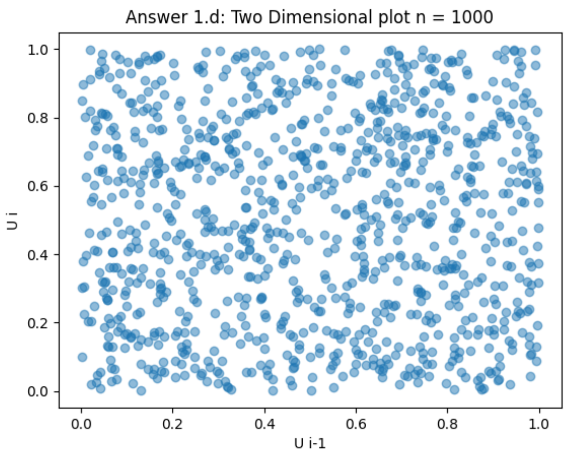


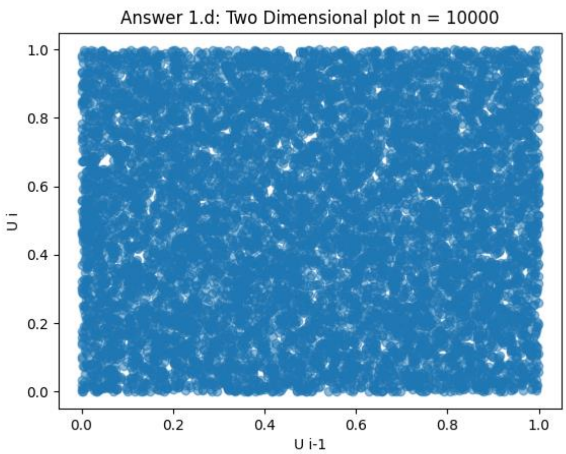


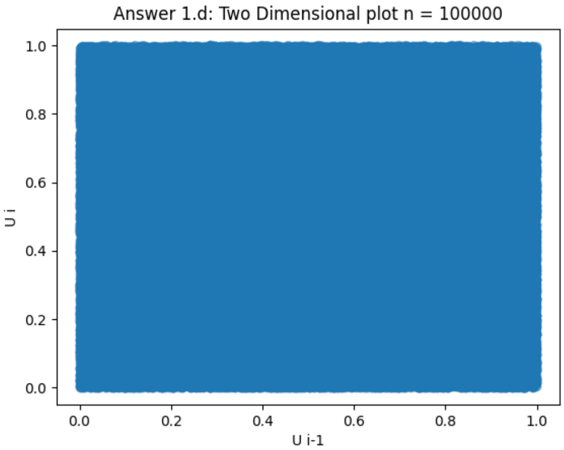


D)

Two Dimensional plot for coordinates (u_{i-1}, u_i) with a = 1229, b = 1, m = 2048 and seed(x0) = 1 :







Observations:

1. The scatter plot suggests that the U_is do not follow any particular pattern, so they are almost completely random.
2. The frequency distribution plots suggest that the random generator follows the two properties of the ideal random generator:

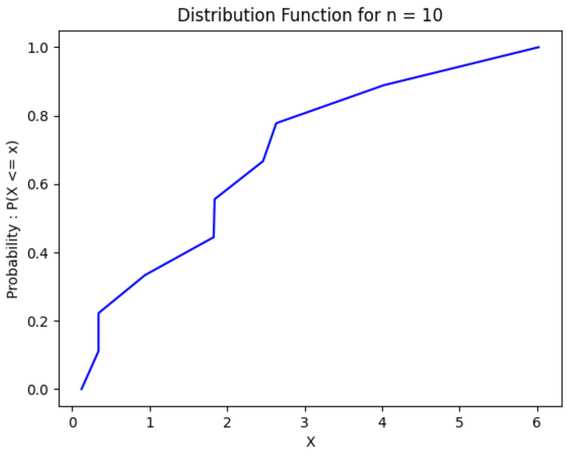
   a) Each U_i is uniformly distributed between 0 and 1, which is quite obvious.

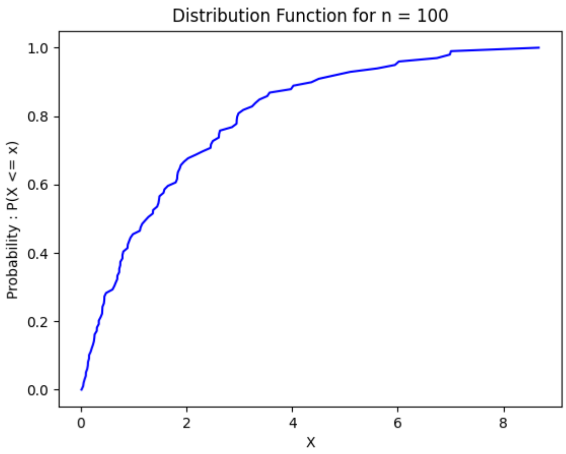
   b) The U_is are mutually independent.

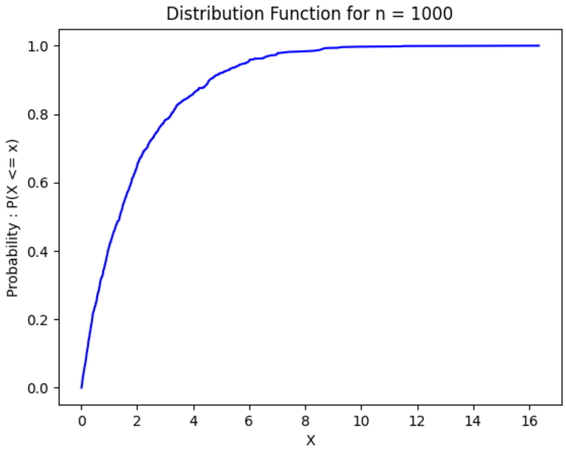
The frequency of different numbers lying in the same length intervals are almost the same. So, the given random generator behaves like a good random generator.

Answer 2:

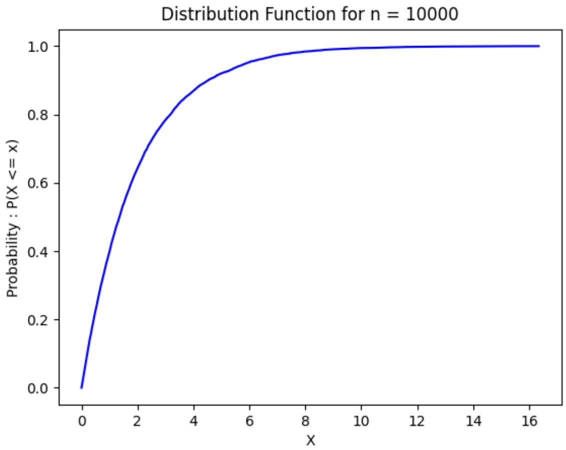
Mean (ɵ) = 2 (assumed)
The graphs(Empirical) for 5 different cases in which the number of values generated are 10, 100, 1000, 10000, 100000 (i.e., sample size).



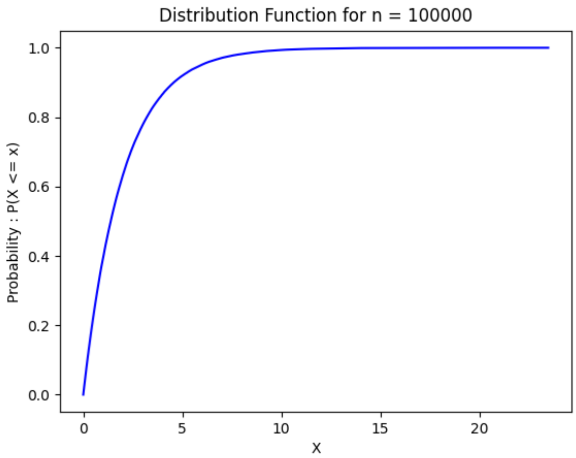




Distribution Function for n = 1000
Probability : P(X <= x)
X

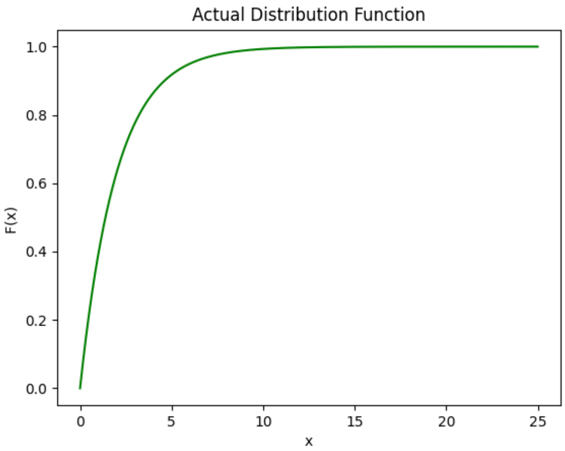


Distribution Function for n = 10000
Probability : P(X <= x)
X



Distribution Function for n = 100000
Probability : P(X <= x)
X

Actual distribution function:



Actual mean = 2 (ɵ)
Actual Variance = 4 (ɵ^2)

S.No	Sample Size	Sample Mean	Sample Variance
1.	10	1.3578136833838736	2.5279620773967872
2.	100	1.947446195731913	3.4054178229048815
3.	1000	1..946986259164634	3.9628595171881633
4.	10000	2.0393413918611705	4.232676382299758
5.	100000	1.9985972350919927	3.977290761274775

Observations:

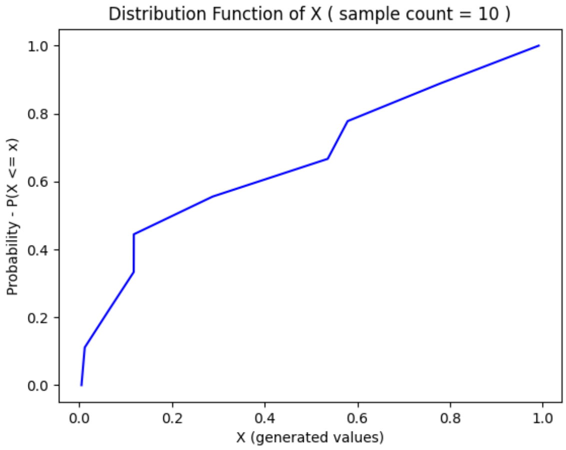
1. As depicted by the Law of Large Numbers and the observation when we increase the number of values generated (Sample Size), the mean and the variance of the generated values (X) converge to the actual mean and variance.

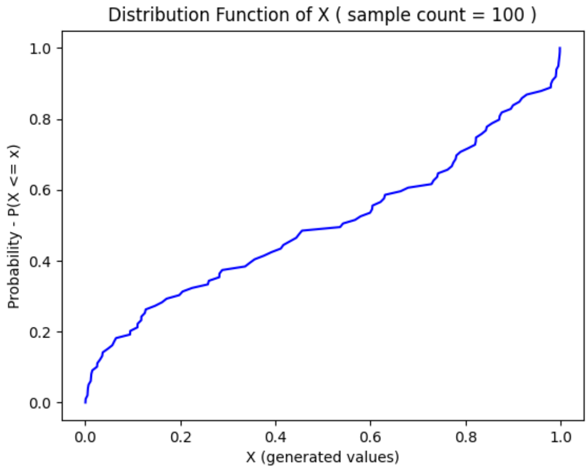
2. It is also evident from the distribution function of the X for different values of sample size which approaches the plot of F(x) as sample count increases.

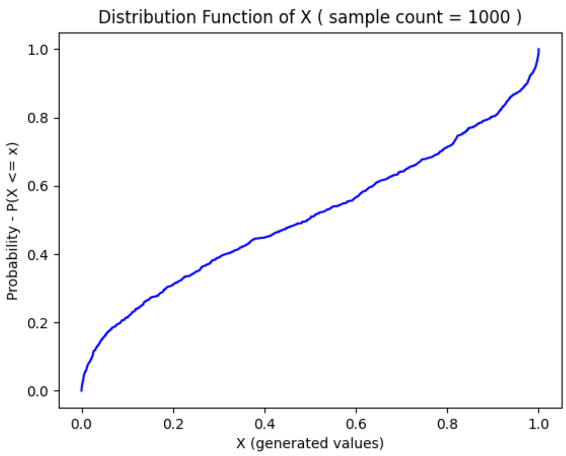
3. The distribution function of X is identical to the CDF F(x) from which random variable X was generated. This is because F(x) is a continuous strictly increasing function and U is a uniform distribution function on [0, 1], so, F^{-1} (U) will be a sample from F. This clearly shows the Inverse Transform Method and the corresponding algorithm.

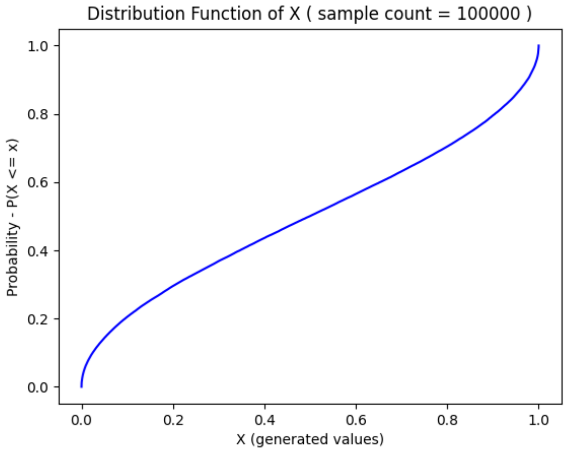
Answer 3:

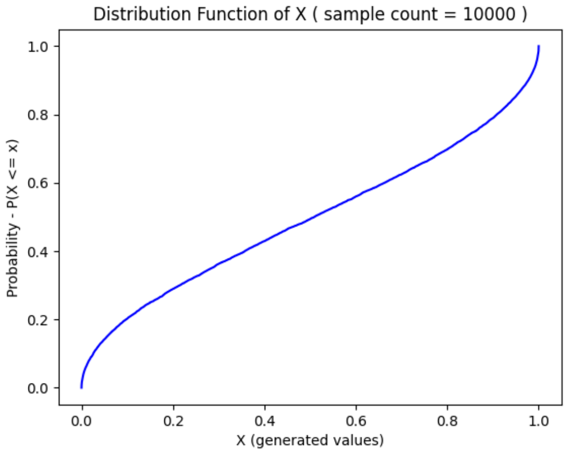
The graphs for 5 different cases in which the number of values generated are 10, 100, 1000, 10000, 100000 (i.e. sample size).



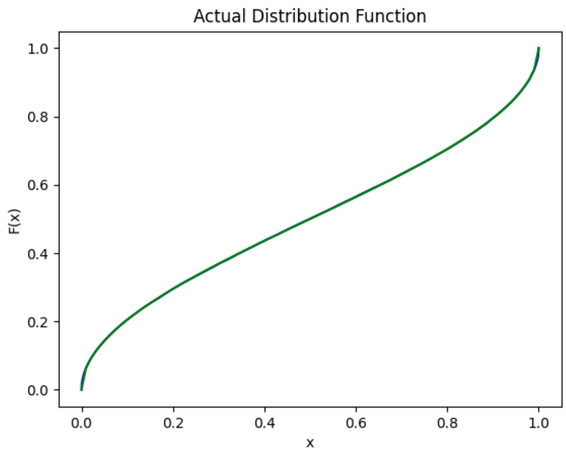








Actual distribution function:



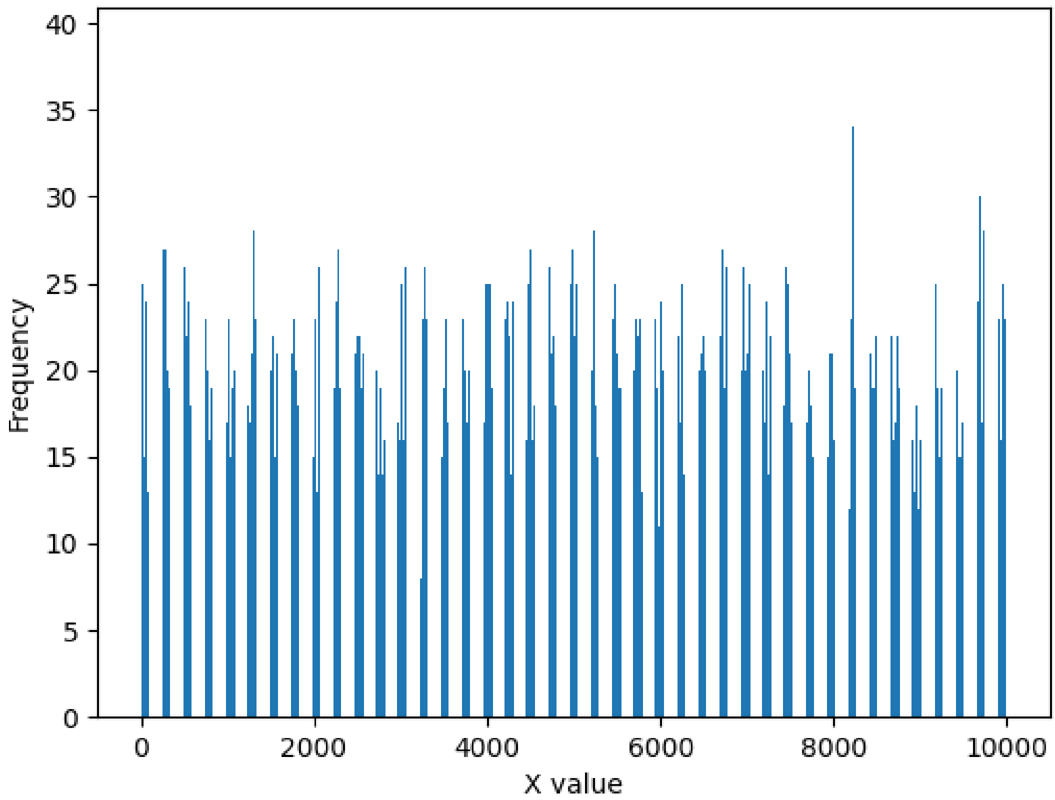
S.No	Sample Size	Sample Mean	Sample Variance
1.	10	0.34925093420024206	0.11091120380106667
2.	100	0.4979223786754022	0.12587712356223194
3.	1000	0.488395462251308	0.12560387481263918
4.	10000	0.5052312727512848	0.12504274561358916
5.	100000	0.4999847890241483	0.1249264130104649

Observations:

1. The distribution function of X is identical to the CDF F(x) from which random variable X was generated. This is because F(x) is a continuous strictly increasing function and U is a uniform distribution function on [0, 1], so, $F^{-1}(U)$ will be a sample from F. This clearly shows the Inverse Transform Method and the corresponding theorem.

2. The distribution functions of X approaches the plot of F(x) as the sample size increases.

Answer 4:



Random Variable Range	Frequency
0-499	4969
500-999	5124
1000-1499	5088
1500-1999	4988
2000-2499	5028
2500-2999	4920
3000-3499	5004
3500-3999	4862
4000-4499	5032
4500-4999	5045
5000-5499	4987
5500-5999	5118
6000-6499	4887
6500-6999	4929
7000-7499	5028
7500-7999	4952
8000-8499	5103
8500-8999	4916
9000-9499	5037

9499-1000	4976

Observations:

1. From the above bar plot we can observe that the random variable X has almost uniform distribution (similar frequencies) over the values 1, 3, 5, 7, 9……..,999
2. This clearly shows the Inverse transform method for discrete random variable with finite support and the corresponding theorem.