1. The problem is based on Project Evaluation and Review Technique (PERT), a project planning tool. Consider the software project described in following table. It has 10 tasks (activities), indexed by $j = 1, \ldots, 10$. The project is completed when all of the tasks are completed. A task can begin only after all of its predecessors have been completed. The project starts at time 0. Task j starts at time

j	Task	Predecessors	Mean time (in Days)
1	Planning	None	4
2	Database Design	1	4
3	Module Layout	1	2
4	Database Capture	2	5
5	Database Interface	2	2
6	Input Module	3	3
7	Output Module	3	2
8	GUI Structure	3	3
9	I/O Interface Implementation	5,6,7	2
10	Final Testing	4,8,9	2

S_j, takes time T_j and ends at time $E_j = S_j + T_j$. Any task j with no predecessors (here only task 1) starts at $S_j = 0$. The start time for a task with predecessors is the maximum of the ending times of its predecessors. For example, $S_4 = E_2$ and $S_9 = \max\{E_5, E_6, E_7\}$. The project as a whole ends at time E_{10}. Assume that T_j are independent exponentially distributed random variables with means (say θ_j) given in the final column of the table.

   (a) Write E_{10} in terms of T_j, $j = 1, 2, \ldots, 10$.

   (b) Find an approximate value of mean of E_{10} using a simple Monte Carlo. You may take $n = 10000$. Save the generated sample as it will be used.

   (c) Plot a histogram of generated values of E_{10}. Comment on the shape (mainly skewness) of the histogram.

   (d) Assume that there will be a severe penalty should the project miss a deadline in 70 days. Find an approximate value of the probability that the project miss the deadline using a simple Monte Carlo. You may use the same sample that you generated in (1b). Also calculate the standard deviation. Comment on the performance.

   (e) Find an approximate value of the probability that the project miss the deadline using importance sampling technique. To write q, take T_j are independent exponential with mean $\lambda_j = 4\theta_j$. Compute the standard deviation and effective sample size.

   (f) Find an approximate value of the probability that the project miss the deadline using importance sampling technique. Here to write q, take T_j are independent exponential with mean λ_j, where $\lambda_j = \kappa\theta_j$ for $j = 1, 2, 4, 10$ and $\lambda_j = \theta_j$ for $j \notin \{1, 2, 4, 10\}$. Take $\kappa = 3.0, 4.0, 5.0$. Compute the standard deviation and effective sample size for each values of κ.

   (g) Compare results that you obtained in 1e and 1f.

   (h) Obtain the confidence interval for the probability taking the value of κ that has minimum effective sample size among 3.0, 4.0, 5.0.

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