Lab 06 – Report – Lakshya Kohli – 210123077

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Answer 1.

M	I_M (estimated value of I)	95% Confidence Interval
100	1.9947105074120772	[1.90764626644459, 2.0817747483795648]
1000	1.9854670058109818	[1.9581113983963325, 2.012822613225631]
10000	2.0068336334099994	[1.998195161598582, 2.015472105221417]
100000	2.000019269038806	[1.99728645453245, 2.0027520835451615]

Observations:

1. The actual (exact) value of I is equal to 2 (calculated by integration shown below)

$$I = E[\exp(\sqrt{U})]$$
$$g(x) = \exp(\sqrt{x})$$
$$x \sim U(0,1) \quad \text{pdb} \quad ?$$
$$f(x) = 1 \quad 0 < x < 1$$

$$I = \int_0^1 g(t)\, f(t)\, dt$$
$$= \int_0^1 e^{\sqrt{t}}\, dt$$

Let $\sqrt{t} = p$
$$\frac{dt}{2\sqrt{t}} = dp$$
$$dt = 2p\, dp$$

$$I = \int_0^1 2p\, e^p\, dp$$
$$= 2[p e^p - \int e^p\, dp]_0^1$$
$$= 2[p e^p - e^p]_0^1$$
$$= 2$$

$$\boxed{I = 2}$$

2. The value of I_M converges to the actual value of I, that is, 2 as M increases.