Monte Carlo Simulation - MA 323

Lab 02 - Report - Lakshya Kohli - 210123077

Answer 1:

A.The average number of iterations is exactly the bounding constant c. The reason for this is that the probability of acceptance is 1/c.

Also, the number of iterations needed to successfully generate X is a random variable having geometric distribution with expectation c as proved below:

No. of Iterations (N)

$\downarrow$ geometric distribution

probability of success

$$= \quad P(N=n) = (1-p)^{n-1}\, p \quad n \geq 1$$

also,

$$p = P\left(U \leq \frac{f(Y)}{cg(Y)}\right) \Rightarrow \frac{1}{c}$$

$\downarrow$ probability of acceptance

So, $E(N) = \frac{1}{p}$

$$\boxed{E(N) = c}$$

B, C, D, E

c_values = [max(f(x) in 0<x<1, 5, 10]

The sample mean and expectation of the PDF f is mentioned along with the approximate and exact value of P(0.25<=X<=0.75)

The mean of the number of iterations is also mentioned along with the average number of iterations as founded in part a.

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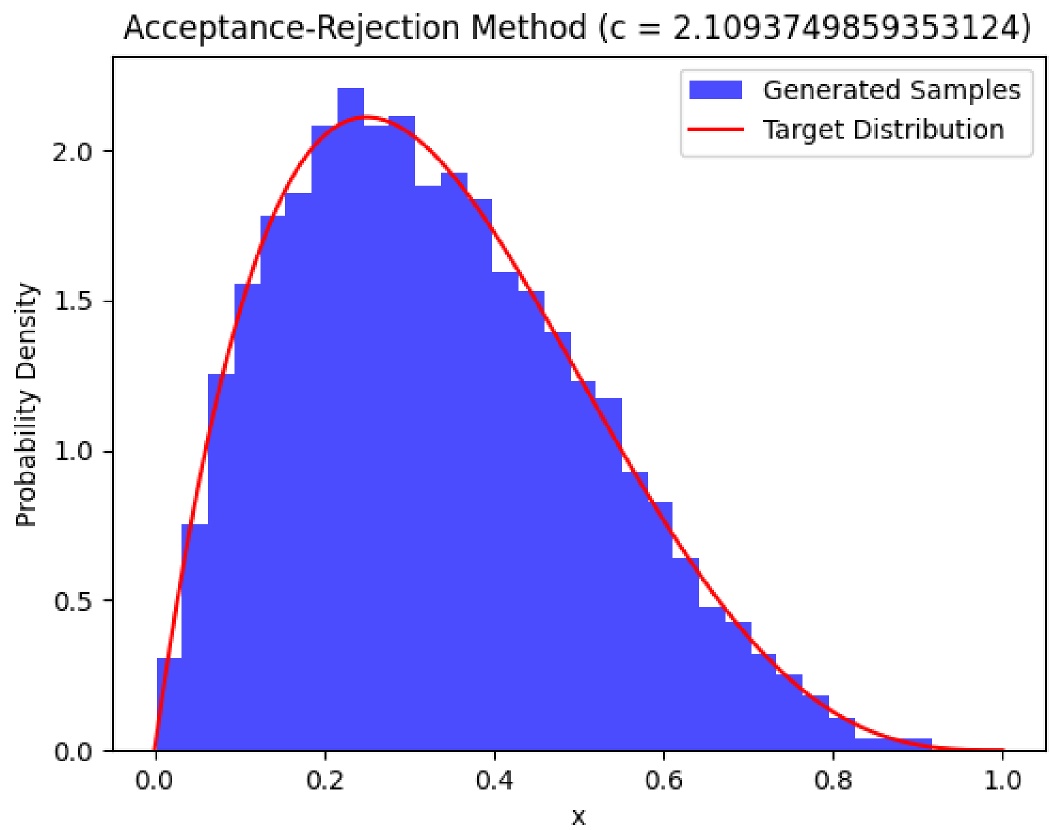
PS E:\SEM 5\MA323 MonteCarlo\Lab03> python -u "e:\SEM 5\MA323 MonteCarlo\Lab03\q1.py"
for c = 2.1093749859353124
(b) Sample Mean: 0.33459508380747716
Actual Mean: 0.3333333333333333
(c) Approximate value of P(0.25<=X<=0.75) = 0.6186
Exact value of P(0.25<=X<=0.75) = 0.6171875
(d) Average of count of number of iterations 2.1082
Average number of iterations (part a) 2.1093749859353124

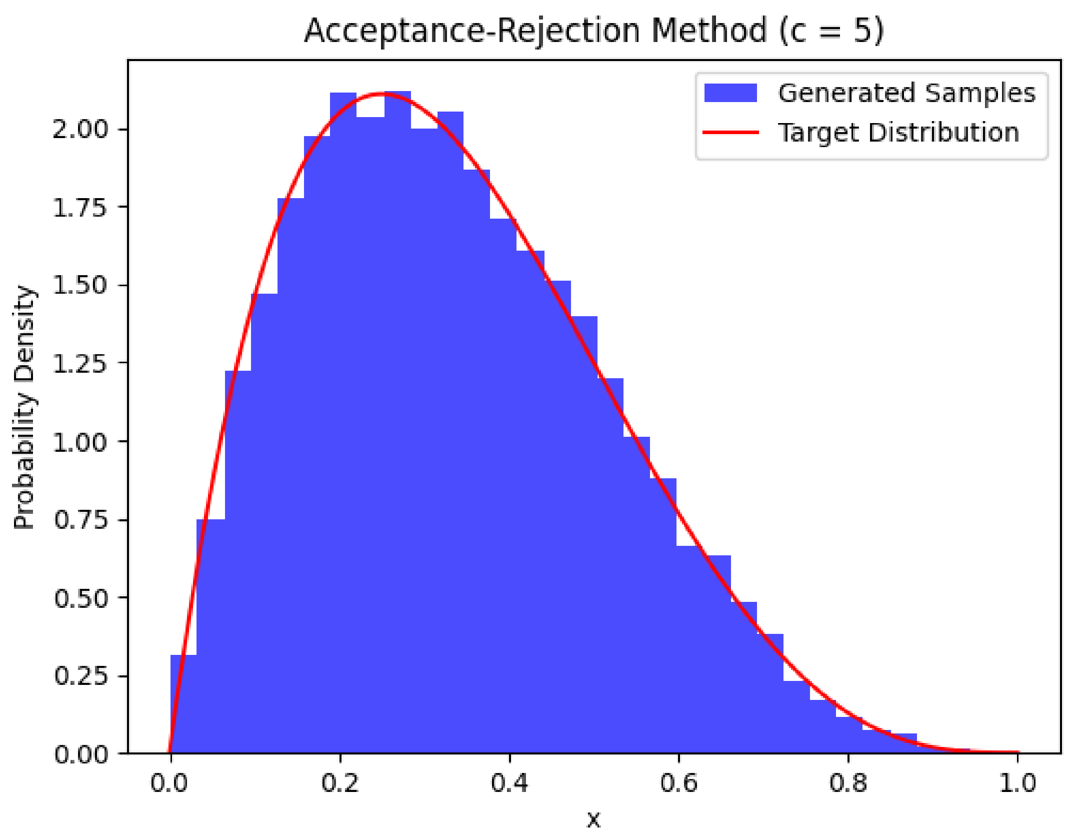
for c = 5
(b) Sample Mean: 0.33680119189016566
Actual Mean: 0.3333333333333333
(c) Approximate value of P(0.25<=X<=0.75) = 0.6251
Exact value of P(0.25<=X<=0.75) = 0.6171875
(d) Average of count of number of iterations 4.997
Average number of iterations (part a) 5

for c = 10
(b) Sample Mean: 0.33568466833757465
Actual Mean: 0.3333333333333333
(c) Approximate value of P(0.25<=X<=0.75) = 0.6235
Exact value of P(0.25<=X<=0.75) = 0.6171875
(d) Average of count of number of iterations 9.7997
Average number of iterations (part a) 10

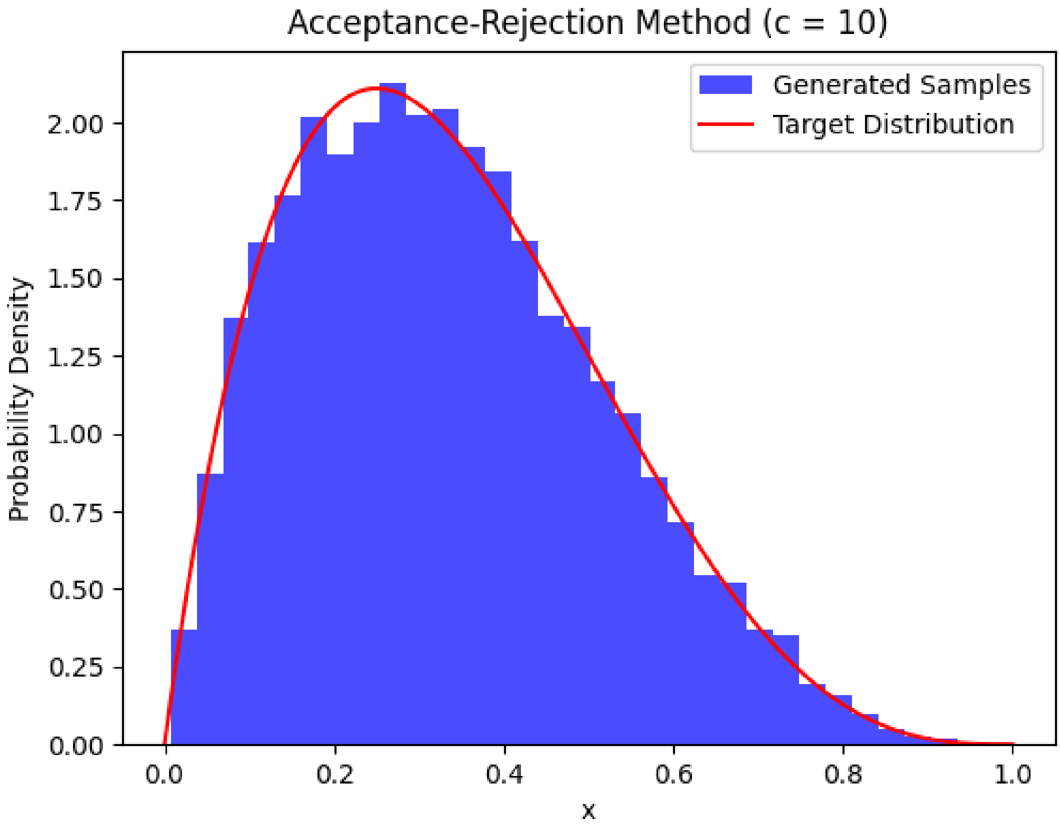
PS E:\SEM 5\MA323 MonteCarlo\Lab03>
```



Acceptance-Rejection Method (c = 2.1093749859353124)
Generated Samples
Target Distribution
Probability Density
x



Acceptance-Rejection Method (c = 5)
Generated Samples
Target Distribution
Probability Density
x



Acceptance-Rejection Method (c = 10)
Generated Samples
Target Distribution
Probability Density
x
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.0
0.2
0.4
0.6
0.8
1.0

Answer 2:

The rejection constant for alpha = 0.9 is 1.3840078872571917
The dominating PDF is g(x) = (x^(alpha-1))/A, where A = (1/alpha) + (1/e)