

Answer 1.a

The sample mean and variance for each sample size using both methods is mentioned in the below screenshot. Computational time is also mentioned along with the proportion of values rejected in the Marsaglia and Bray method.

As we can see that the sample mean and sample variance are very close to their theoretical values. The error decreases substantially when the sample size is increased, i.e., more random numbers are generated.

The proportion of value rejected in the Marsaglia and Bray method is also compared with $1 - \pi/4$.

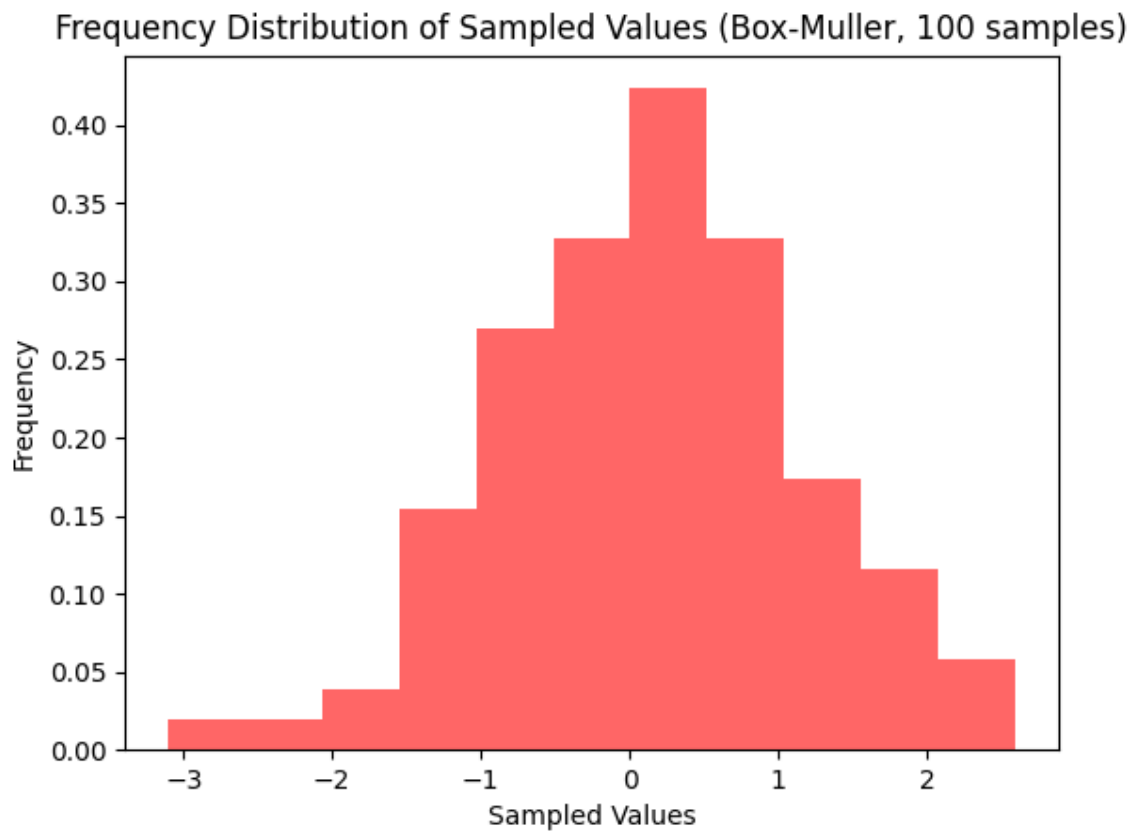
```
PS E:\SEM 5\MA323 MonteCarlo\Lab04> python -u "e:\SEM 5\MA323 MonteCarlo\Lab04\q1.py"
box_muller for n = 100
computational time = 0.0ms
mean = 0.14582374209896723
variance = 1.0706463796397083

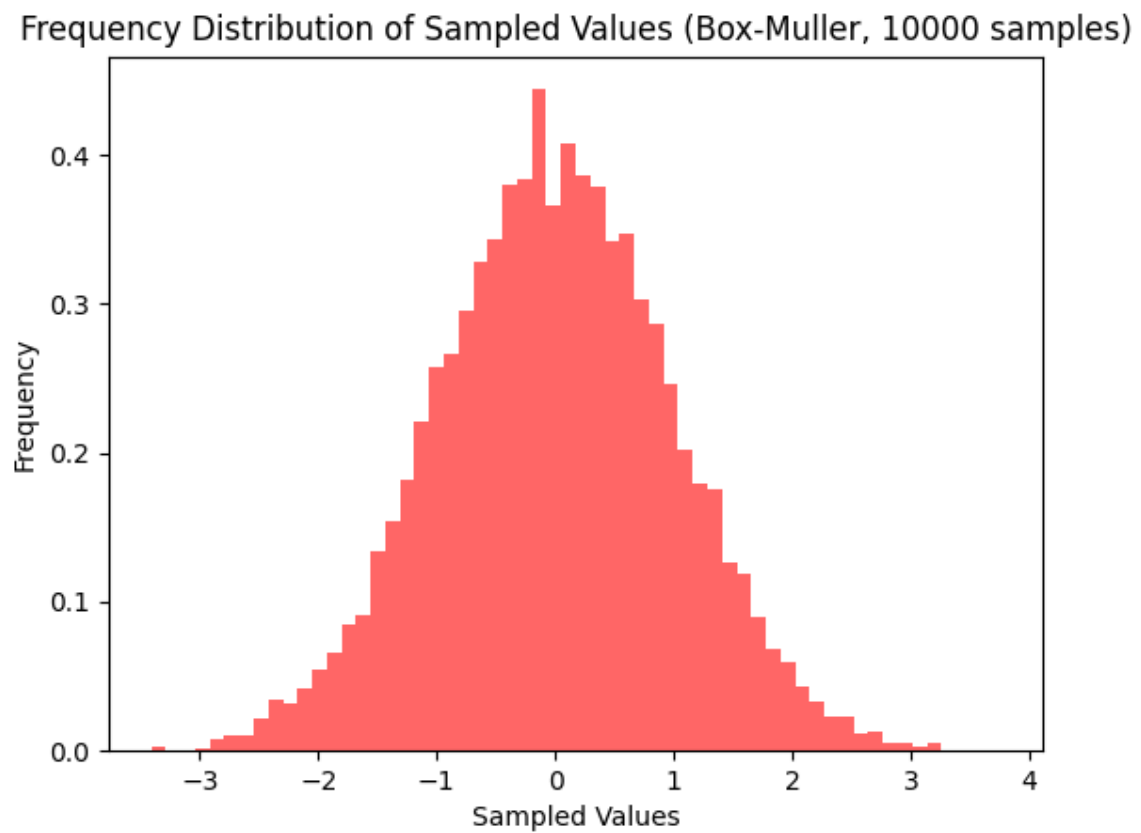
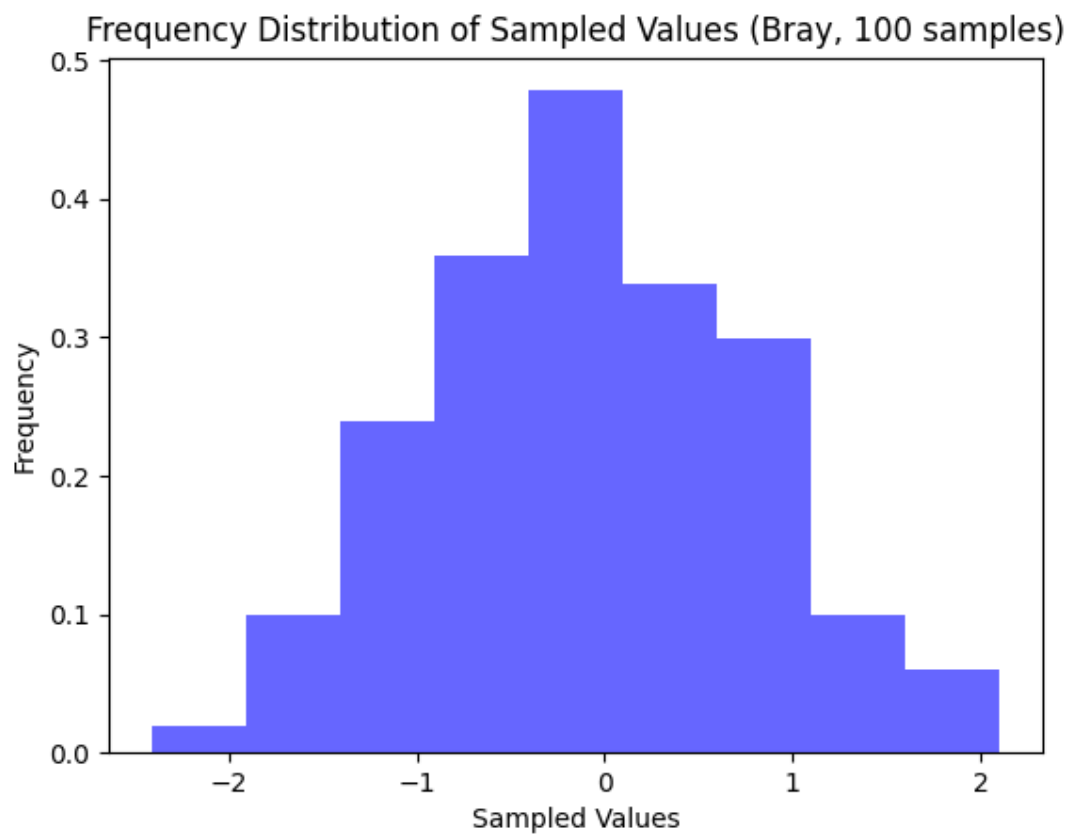
bray for n = 100
computational time = 0.0ms
Proportion rejected = 0.2366412213740458
1- $\pi/4$  = 0.21460183660255172
Comparing with 1- $\pi/4$  => 1.1026989569167183
mean = -0.07645551647266523
variance = 0.7771862423492388

box_muller for n = 10000
computational time = 0.004604409816885628ms
mean = -0.0011908136142448882
variance = 0.9803704493511021

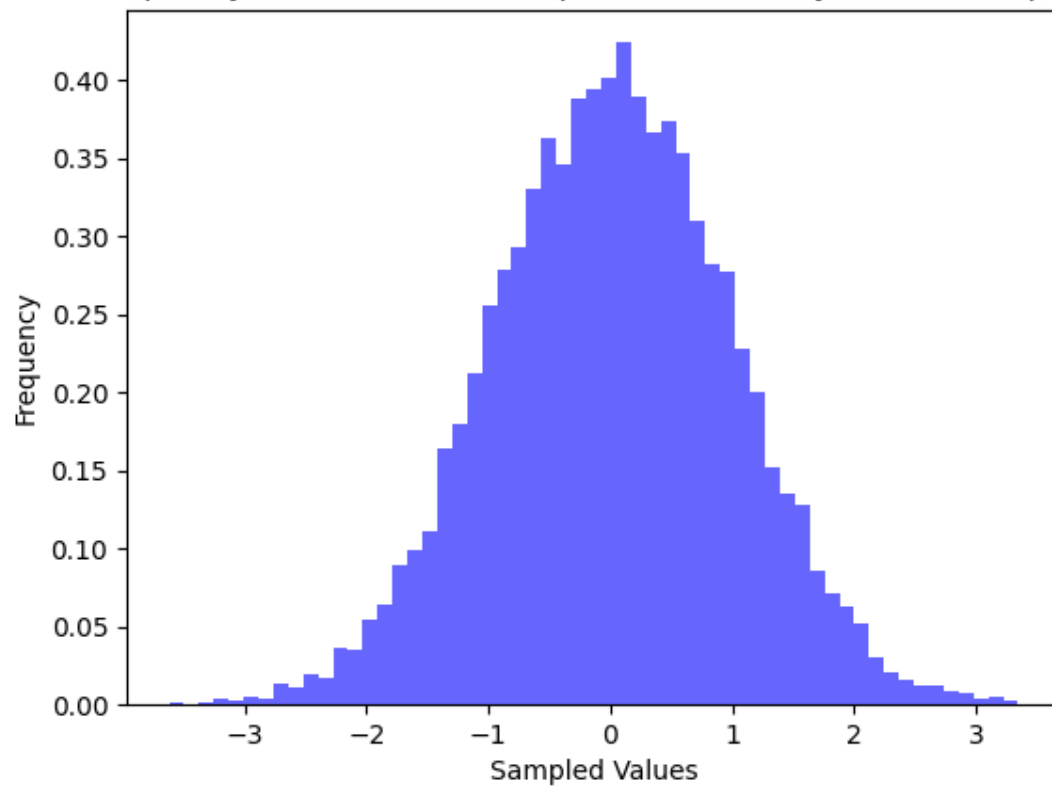
bray for n = 10000
computational time = 0.006414424840782113ms
Proportion rejected = 0.21457744266415332
1- $\pi/4$  = 0.21460183660255172
Comparing with 1- $\pi/4$  => 0.9998863293120666
mean = 0.01493015768657928
variance = 0.9626210558191095
```

Answer 1.b



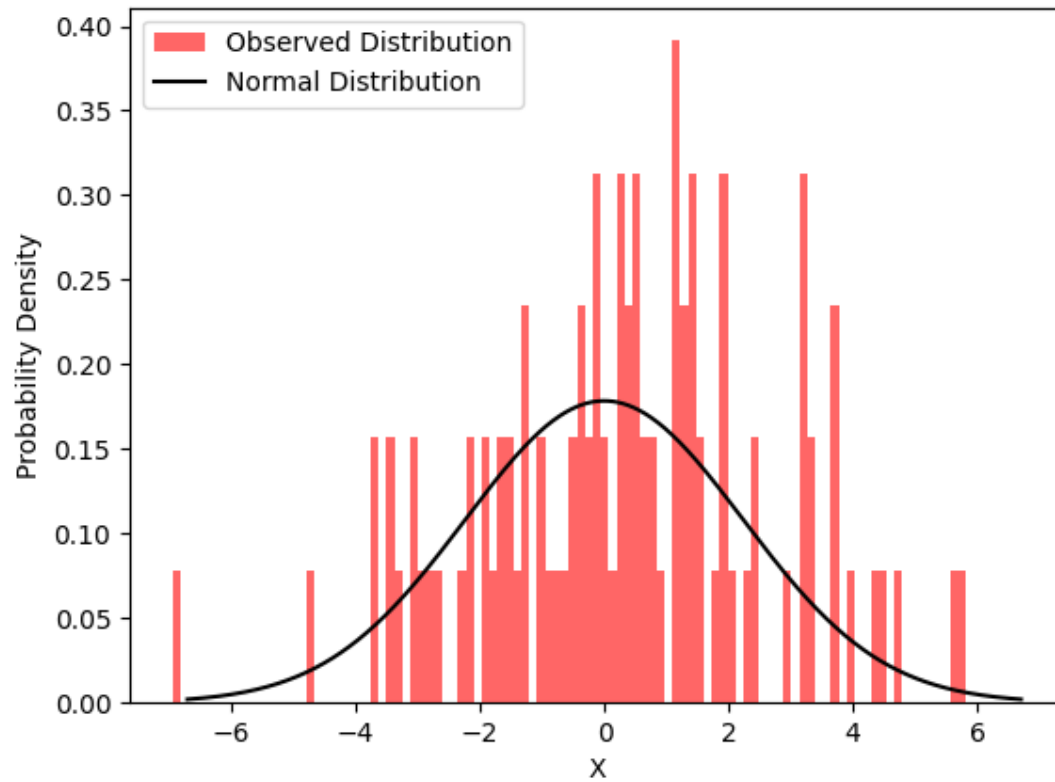


Frequency Distribution of Sampled Values (Bray, 10000 samples)

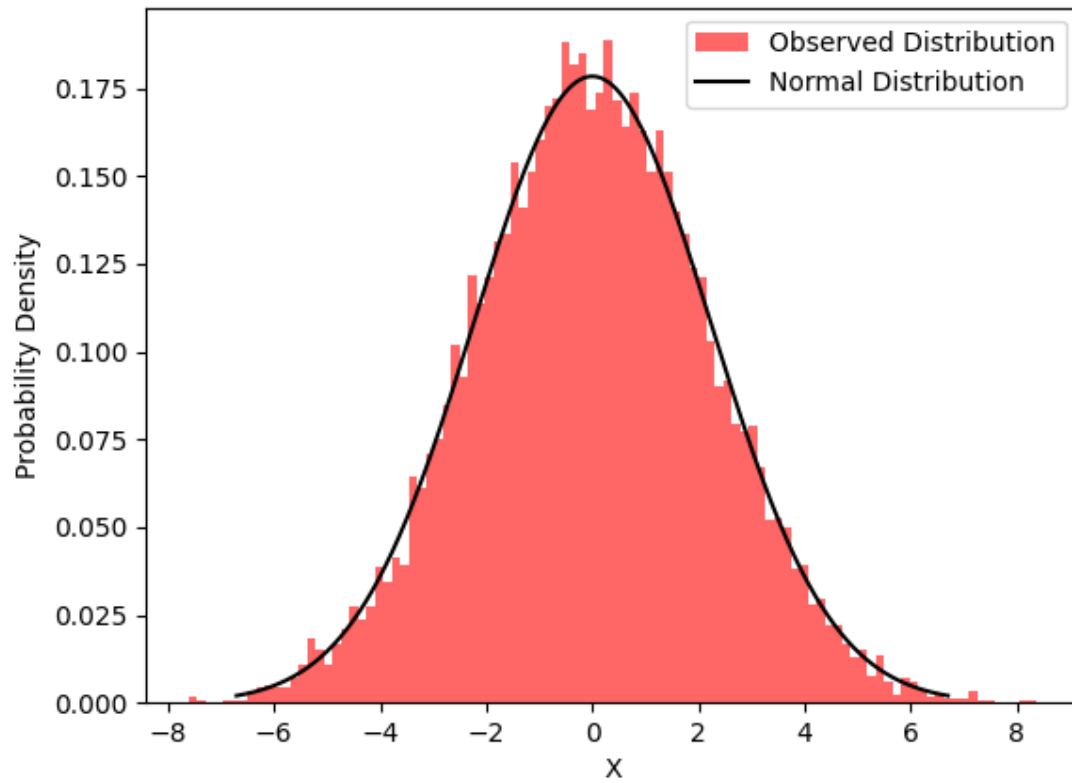


Answer 1.c

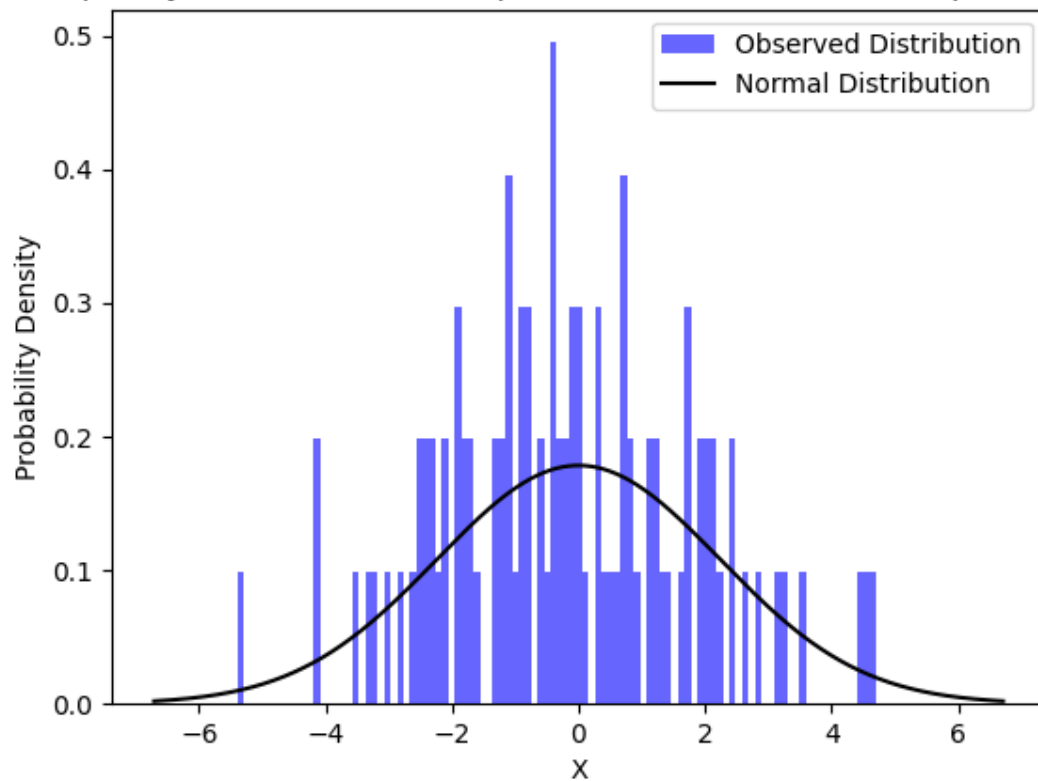
Frequency Distribution of Sampled Values ($N(0,5)$, 100 samples) (Box-Muller



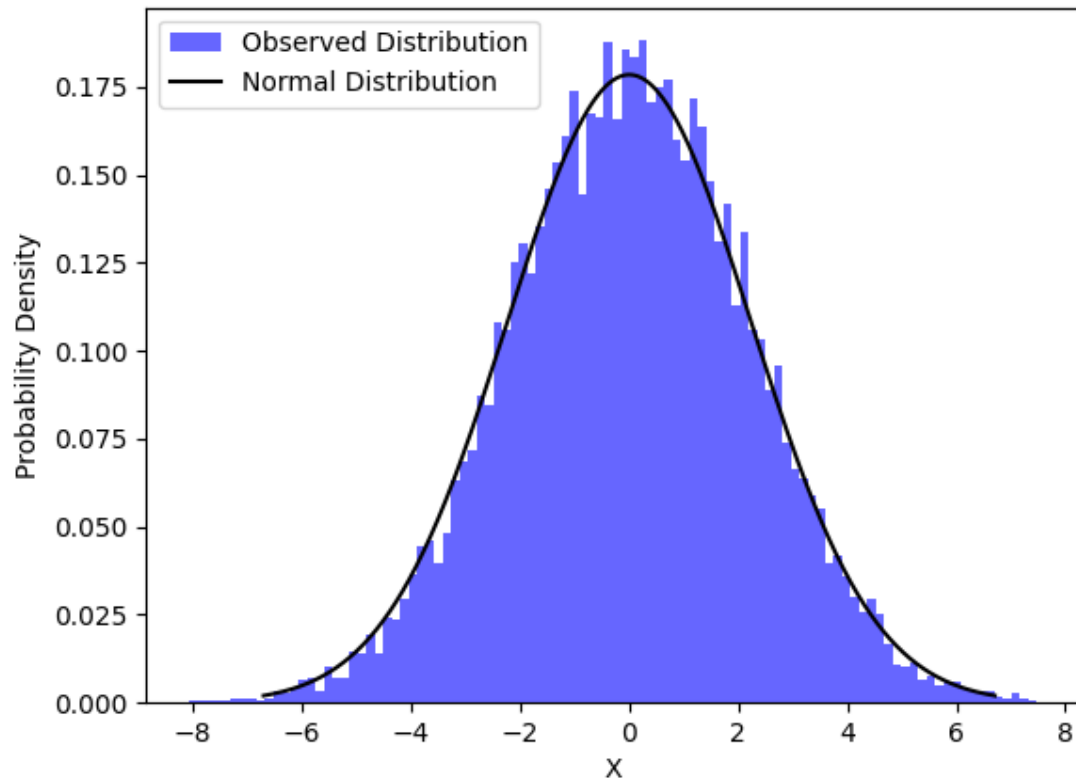
Frequency Distribution of Sampled Values ($N(0,5)$, 10000 samples) (Box-Muller)



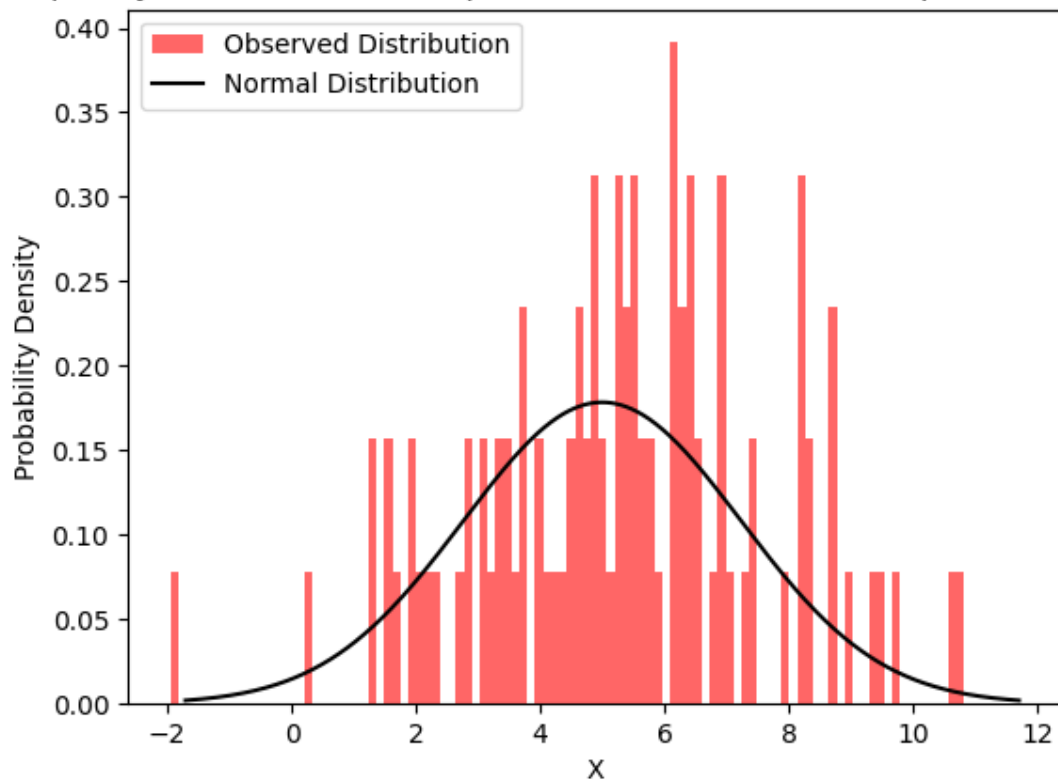
Frequency Distribution of Sampled Values ($N(0,5)$, 100 samples) (Bray)



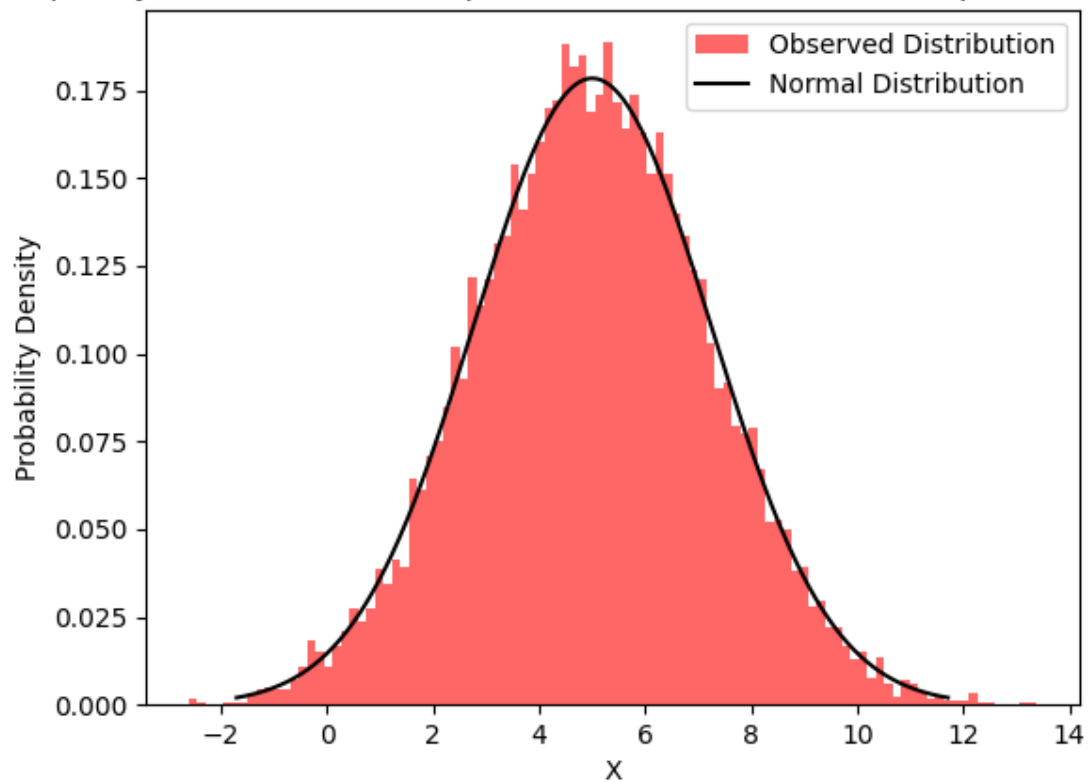
Frequency Distribution of Sampled Values ($N(0,5)$, 10000 samples) (Bray)



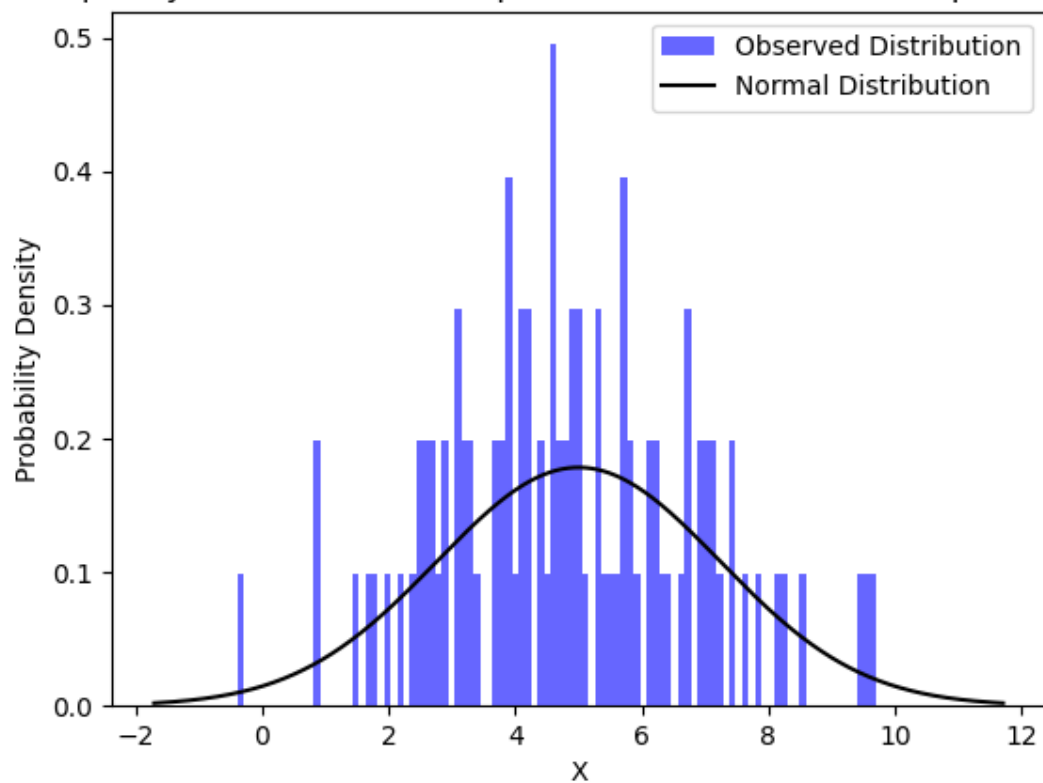
Frequency Distribution of Sampled Values ($N(5,5)$, 100 samples) (Box-Muller)



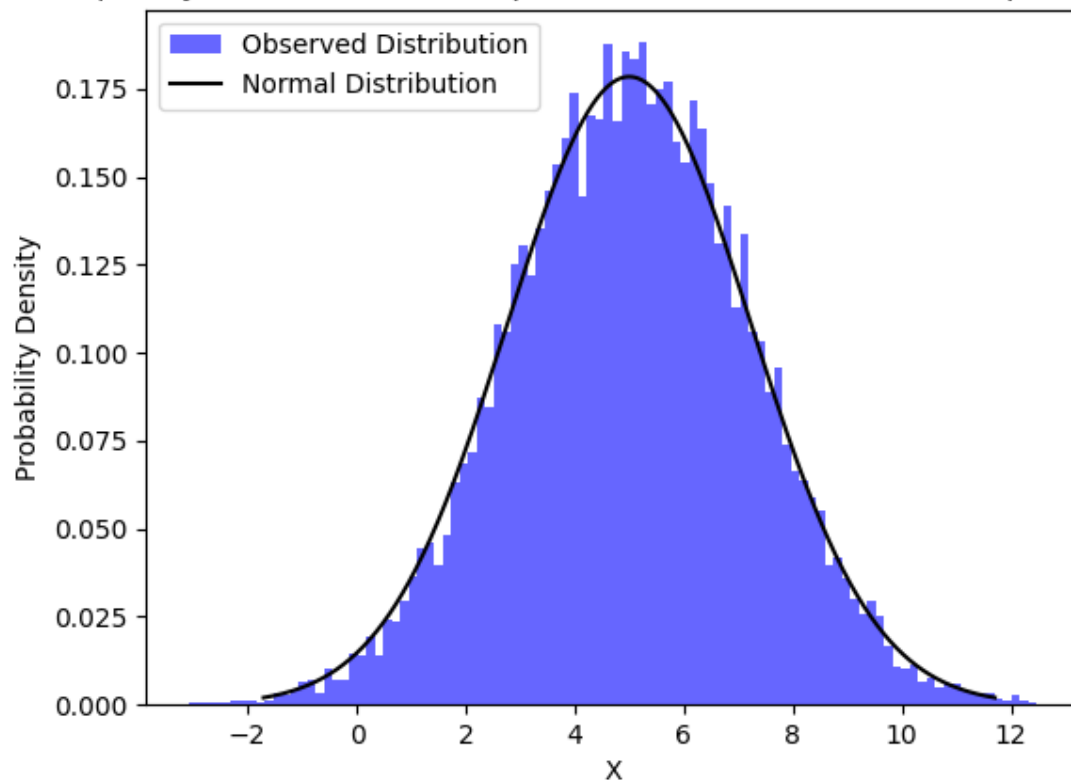
Frequency Distribution of Sampled Values ($N(5,5)$, 10000 samples) (Box-Muller)



Frequency Distribution of Sampled Values ($N(5,5)$, 100 samples) (Bray)



Frequency Distribution of Sampled Values ($N(5,5)$, 10000 samples) (Bray)



I have used the fact that if $Z \sim N(0, 1)$, then $\mu + \sigma Z \sim N(\mu, \sigma^2)$. Using this I've plotted the corresponding histograms for both $N(0,5)$ and $N(5,5)$ for both the n values using both the methods along with the actual normal distribution in the same plot.

Observations:

1. In both situations, the distribution produced by the sample of random variables gets closer to the theoretical distribution as the sample size rises. When the sample size is less at first, there is a significant deviation from the predicted curve.

The anomalies in the plots are more pronounced and stray somewhat from the normal distribution when the sample size is smaller (size = 100).

2. It is clearly seen that the mean and variance varies with the graph as Normal distribution is changed. The peak is attained at the mean, and the spread of the values depend on the variance. In both the cases of $N(0, 5)$ and $N(5, 5)$, the plots convey this information clearly.

The plot for $N(5, 5)$ is shifted to 5 units to the right of the $N(0, 5)$ and its variance is the same as that of $N(0, 5)$.

Answer 2:

The computational time is attached in the answer 1.

According to the computational time observed in the above values, the Box-Muller Method is faster than the Marsaglia and Bray Method for sample size = 10000 and the contrary is observed for sample size = 100.

The Marsaglia and Bray technique should be quicker than the Box-Muller method since we don't have to evaluate the "cos" and "sin" functions. However, in real-world situations, we might not always see this pattern. This is so that we may cycle through the Acceptance Rejection approach (used in the Marsaglia and Bray Method) and accept just the appropriate values while rejecting the undesirable ones. The Marsaglia approach becomes somewhat slower than the BoxMuller method as the sample size rises due to high overhead.

Answer 3:

Theoretically, the proportion of values that are rejected should be close to $1 - \pi/4$. This is so because it is a piece of unit area that was left over from a box. We are only selecting the random integers such that they are contained within a circle that is outlined within a square with a unit area.

As a consequence, the size of the remaining square, which represents the percentage of the values that were rejected, is $1 - \pi/4$.

When the sample size is big (10000), the divergence from the theoretical value is quite small, but when the sample size is small (100), there is a small variation from the observed value.