Cross-Impact Analysis of Order Flow Imbalance in Equity Market Dynamics

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Abstract

This document presents a short study on cross-impact effects of Order Flow Imbalance (OFI) on short-term price changes across five equities (AAPL, AMGN, TSLA, JPM, XOM). We compute multi-level OFI, reduce it with PCA, and measure the contemporaneous and predictive impact of each stock's OFI on every other stock's intraday returns. We then extend the analysis with sector-level insights and demonstrate advanced predictive models (Random Forest and LSTM) to investigate whether cross-asset OFI signals can improve next-minute return forecasts.

1 Introduction

Cross-impact in equity markets refers to the notion that trading activity in one asset can influence the price dynamics of other, potentially related assets. Past works such as [**?**] and [**?**] showed that while a stock's own order flow significantly drives its price changes (self-impact), there can exist non-negligible *cross*-impact channels, especially among stocks in similar sectors or with fundamental linkages.

In this project, we explore *Order Flow Imbalance* (OFI) as a feature to measure net buying or selling pressure at multiple depths in the limit order book (LOB). After summing or integrating multi-level OFI via Principal Component Analysis (PCA), we empirically test:

- Whether and how each stock's OFI influences its own short-term returns (self-impact),
- Whether cross-asset OFI from other stocks explains or predicts returns (cross-impact),
- Whether more advanced methods (Random Forest, LSTM) can leverage cross-asset OFI to improve next-minute return forecasts.

2 Methodology

2.1 Data Sources & Preprocessing

We pull 1 week of MBP-10 (Market-By-Price, 10 levels) data from the Nasdaq TotalView-ITCH feed for five stocks: **AAPL**, **AMGN**, **TSLA**, **JPM**, and **XOM**, representing Tech, Healthcare, Consumer, Financial, and Energy sectors respectively. Each event update includes bid/ask size and price up to 10 levels. We then:

1. Convert timestamps and sort by event time.
2. Optionally filter to regular trading hours (e.g. 9:30–16:00 ET).
3. For each minute, sum changes in bid and ask sizes at each LOB level, and record the last mid-price of that minute to compute 1-min *log-returns*.

2.2 Multi-Level OFI Computation

At each LOB level ℓ, we track changes in the best bid size $\Delta\text{bid\_sz}_\ell$ and best ask size $\Delta\text{ask\_sz}_\ell$. Then the *Order Flow Imbalance* ($\texttt{OFI}_\ell$) at level ℓ over a given minute is:

$$\text{OFI}_\ell(t) \;=\; [\,\Delta\text{bid\_sz}_\ell(t)\,] \;-\; [\,\Delta\text{ask\_sz}_\ell(t)\,].$$

We do this for $\ell = 0, \ldots, 4$ (the top 5 levels). Summing the incremental changes within each 1-minute bin yields minute-level time-series of OFI across levels.

2.3 Principal Component Analysis (PCA)

To avoid overfitting from having too many separate level-based OFI features, we apply PCA to the vector $(\text{OFI}_0, \ldots, \text{OFI}_4)$. Let $\mathbf{w}$ be the first principal component. We define:

$$\text{OFI\_PCA}(t) \;=\; \mathbf{w} \,\cdot\, \big(\text{OFI}_0(t), \ldots, \text{OFI}_4(t)\big),$$

where $\text{OFI\_PCA}(t)$ is our single integrated OFI measure that explains the majority of the variance in multi-level OFI.

2.4 Cross-Impact Models

We consider two sets of regressions:

Contemporaneous Model.

$$r_i(t) \;=\; \alpha_i \;+\; \sum_j \beta_{i,j}\, \text{OFI\_PCA}_j(t) \;+\; \epsilon_i(t),$$

where $r_i(t)$ is the 1-minute log-return of stock i at minute t, and $\text{OFI\_PCA}_j(t)$ the integrated OFI of stock j. Coefficient $\beta_{i,j}$ measures how much OFI from *stock* j correlates with *stock* i's returns, *within the same minute.*

Lag-1 (Predictive) Model.

$$r_i(t) \;=\; \alpha_i \;+\; \sum_j \beta_{i,j}\, \text{OFI\_PCA}_j(t-1) \;+\; \epsilon_i(t).$$

Here, we shift the OFI by 1 minute to see if a lagged cross-asset OFI has predictive value for next-minute returns.

3 Empirical Results and Visualizations

3.1 OFI vs. Returns

In Figure 1, we illustrate `AAPL`'s OFI_PCA (top) vs. its 1-minute log-return (bottom). Large positive or negative swings in *OFI_PCA* sometimes coincide with the sign or amplitude of short-term price movements, but the relationship appears noisy. Other stocks (AMGN, TSLA, JPM, XOM) show similar patterns.

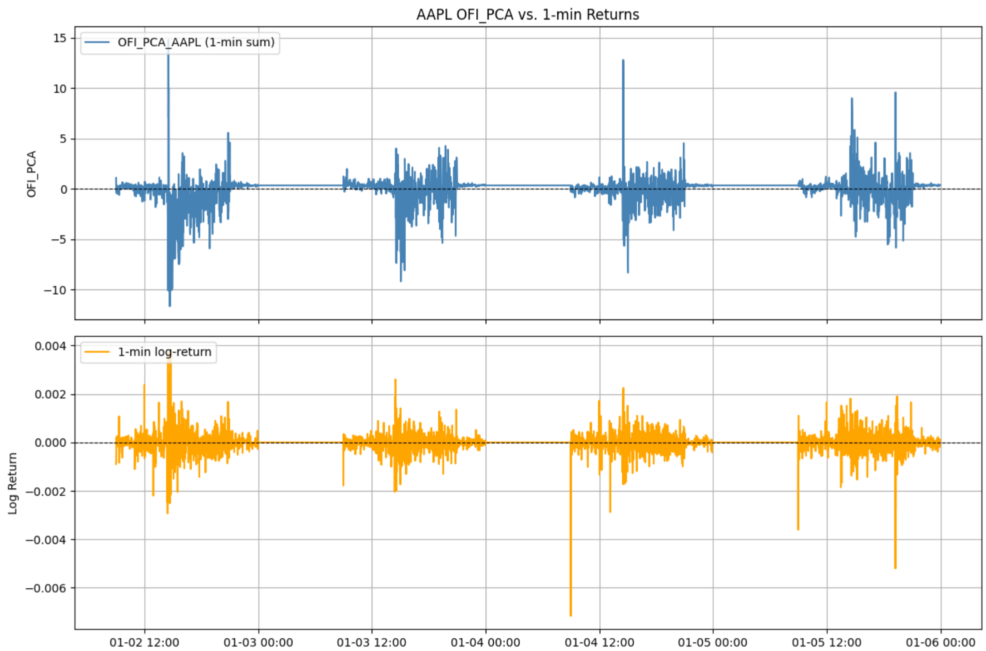

Figure 1: AAPL OFI_PCA vs. 1-min log-return over one week (minute-level).

3.2 Contemporaneous and Lag-1 Cross-Impact

We run the linear regressions for each stock's returns vs. *all* five `OFI_PCA` signals, generating coefficient matrices shown in Figures 2 (contemporaneous) and 3 (lag-1). The color scale indicates each $\beta_{i,j}$, typically on the order of 10^{-5}. While each stock's *self-impact* (diagonal) is generally the largest in magnitude, cross coefficients are non-trivial, though smaller. R^2 values remain very low (under 1%), suggesting these linear models explain only a fraction of return variance.

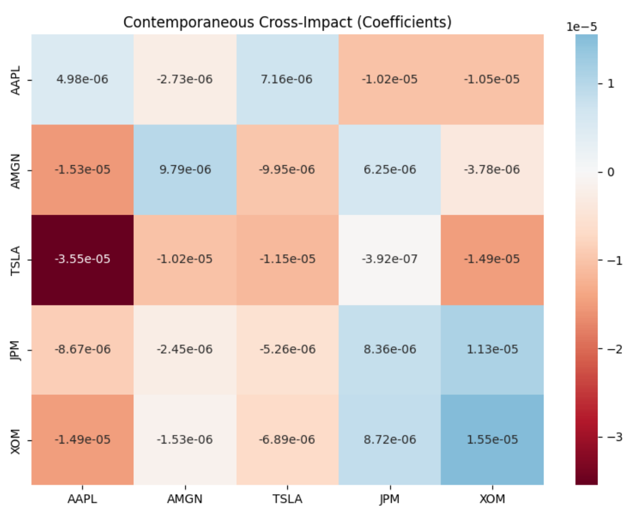

Figure 2: Contemporaneous cross-impact coefficients ($\beta_{i,j}$).

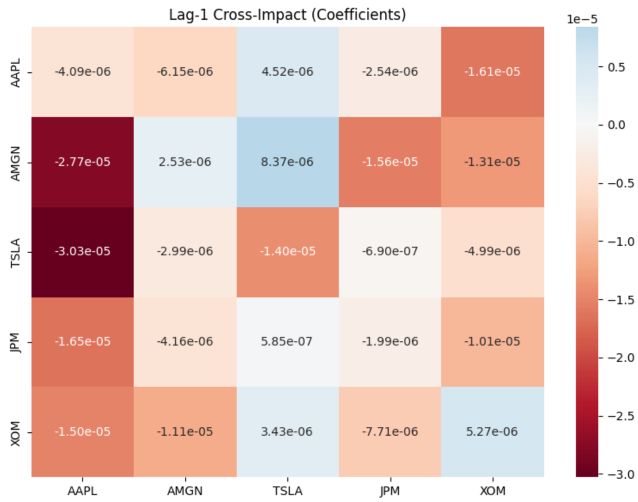

Figure 3: 1-min-lag cross-impact coefficients ($\beta_{i,j}$, for $r_i(t)$ regressed on $\text{OFI\_PCA}_j(t-1)$).

Across both models, we see that Tesla's OFI often has a slightly larger magnitude in cross-terms compared with other stocks; one might speculate TSLA order flow influences broader market sentiment or is more *attention-grabbing*. However, we still observe negative or inconclusive R^2 for real-time forecasting tasks discussed below.

3.3 Sector-Level OFI Correlations

To explore how aggregated OFI might connect stocks within the same sector, we group each stock by sector, average their OFI_PCA, and compute a correlation matrix (Figure 4). Tech (AAPL) and Healthcare (AMGN) show negative correlation, while Consumer (TSLA) and Financial (JPM) exhibit moderate positive correlation (0.47). Although the sample is small (5 stocks, 1 week), this hints that order flow across certain sectors can align or diverge in predictable ways.

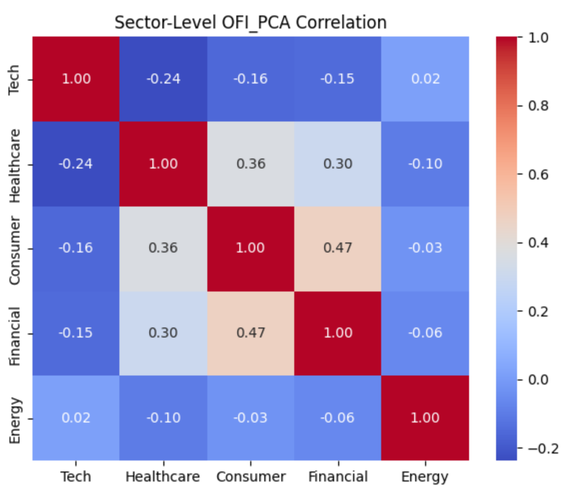

Figure 4: Sector-level OFI_PCA correlation for Tech, Healthcare, Consumer, Financial, Energy.

3.4 Pairwise Scatter: OFI vs. Returns

Figure 5 presents a scatter-matrix sampling 10% of minutes. Each row and column represent either an `OFI_PCA_Symbol` or `log_ret_Symbol`. We observe that most pairwise scatterplots appear as diffuse clouds, consistent with small coefficient estimates in the cross-impact regressions.

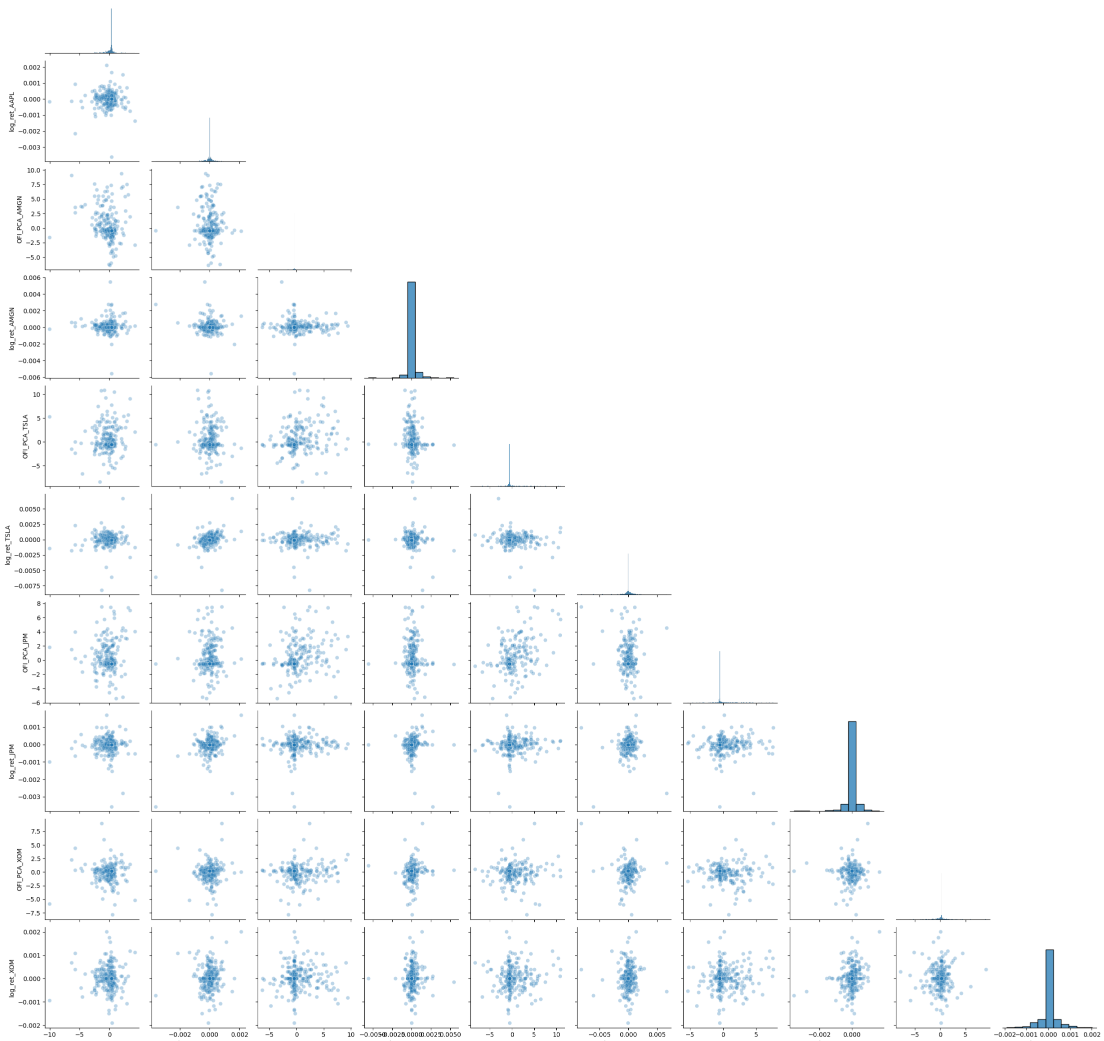

Figure 5: Scatter matrix of OFI_PCA and log-returns across the five stocks. Only mild relationships appear in the clouds.

4 Extended Discussion

4.1 Implications for Trading Strategies

Although the linear cross-impact models yield low explanatory power, this does not necessarily imply *no* cross-impact is present in real markets. Intraday microstructure can be heavily influenced by high-frequency participants, and any discernible signals can quickly be arbitraged. Moreover, certain fleeting patterns may appear or vanish at sub-minute intervals. Our 1-minute binning can obscure some microstructure-level dynamics.

For a high-frequency or execution-focused trader, even small cross coefficients could be relevant. For instance, a TSLA-specific OFI shock might *slightly* forecast downward pressure in AMGN on a short time scale if the two are strongly correlated in some factor dimension (index or momentum signals). However, in our week-long sample, these cross-asset effects remain subtle.

4.2 Interpreting Lagged Effects

The 1-minute-lag results show similarly modest or slightly higher R^2 than the contemporaneous model in some cases, which suggests a limited but non-zero capacity to forecast the next minute's returns from current minute's OFI. While the magnitude of cross-term coefficients is often overshadowed by the self-term (own OFI), the cross-terms occasionally match or exceed the self-term, especially for certain stocks like TSLA or XOM. This might imply that large order flow shocks in one asset could briefly transmit to others.

5 Bonus: Advanced Predictive Modeling

5.1 Random Forest Experiment

We train a Random Forest regressor to predict `AAPL`'s next-minute log-return based on the lagged OFI_PCA signals from *all five* stocks. In Figure 6, the R^2 is negative, meaning the model's predictions explain less variance than a naive constant predictor. Incorporating extra features like mid-price or partial sector dummies in Figure 7 likewise does not yield positive predictive power. This underscores how difficult it is to predict short-horizon returns even with cross-asset features.

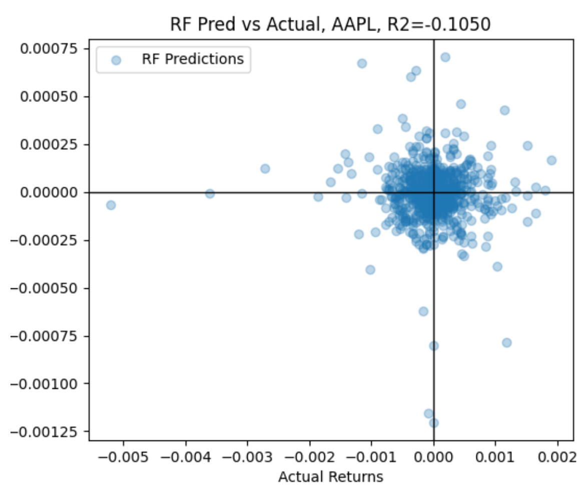

Figure 6: Random Forest predictions vs. actual 1-min returns for AAPL (no extra features). Negative R^2 indicates poor predictive fit.

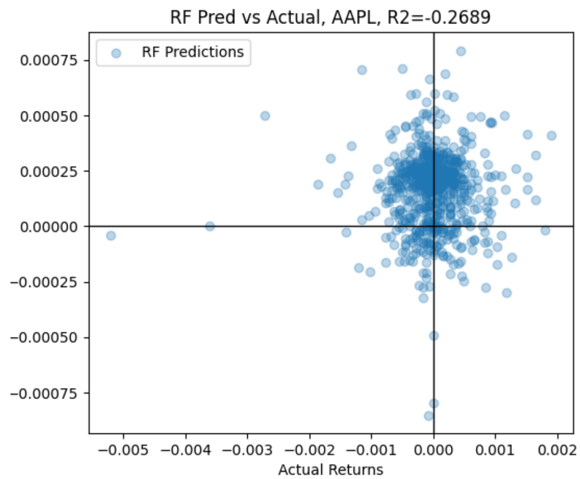

Figure 7: Random Forest with additional (extra) features for AAPL. Still negative R^2.

5.2 LSTM Neural Network

In a second demonstration, we feed OFI sequences (with a 5-timestep rolling window) into an LSTM to predict AAPL's next-minute returns. Figure 8 again shows a negative R^2. The training and validation losses in Figure 9 suggest the model can overfit to training data but does not generalize. The extremely short prediction horizon (1-minute) likely accentuates random noise and microstructure friction. Future improvements could involve multi-day or multi-week patterns, or additional macro/industry signals.

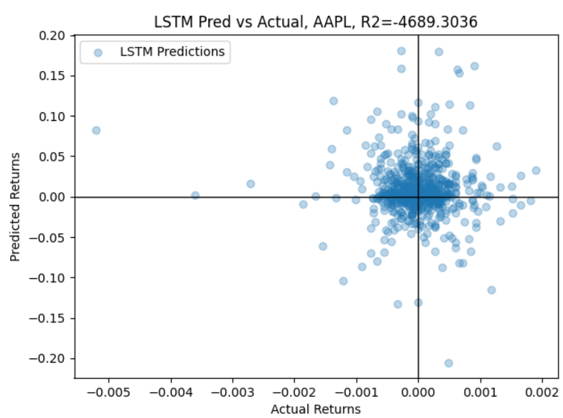

Figure 8: LSTM predictions vs. actual for AAPL's 1-min returns, with a 5-timestep input of lagged OFI_PCA. Very poor R^2 suggests high noise.

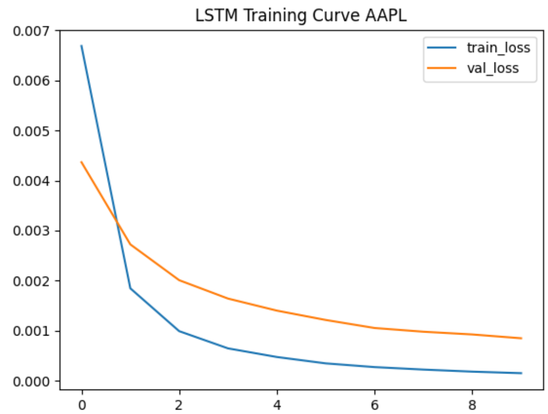

Figure 9: LSTM training curve for AAPL, showing steady training improvement but relatively high validation loss.

6 Conclusion & Next Steps

We have demonstrated a pipeline for:

- Pulling and cleaning tick-level LOB data,
- Computing multi-level OFI and merging them via PCA,
- Performing linear cross-impact regressions to assess contemporaneous vs. lagged impacts,
- Investigating sector correlations, and
- Testing advanced ML models (Random Forest, LSTM) in a short-horizon setting.

6.1 Summary of Findings

- **OFI Computation & PCA:** Summing Δbid_sz and Δask_sz at multiple LOB levels, then consolidating via PCA effectively yields a single factor (OFI_PCA) capturing the main imbalance dimension.
- **Linear Cross-Impact Models:** Both contemporaneous and 1-min-lag regressions show modest R^2 ($< 1\%$). However, cross-asset coefficients do appear at times, albeit smaller in magnitude than a stock's own OFI coefficient.
- **Sector-Level Correlation:** Aggregating OFI_PCA by sector indicates moderate correlation among some sector pairs (Consumer-Financial), and negative correlation among others (Tech-Healthcare).
- **Random Forest & LSTM:** Short-horizon return prediction from solely cross-asset OFI signals remains unproductive in this sample. Negative R^2 points to difficulty in extracting profitable signals at 1-minute intervals from these features alone.

6.2 Proposed Next Steps

Potential extensions include:

- Evaluating sub-minute or event-based horizons, where cross-impact might manifest more acutely among HFT participants.
- Integrating additional predictive covariates, such as macro news or correlated index-level order flows.
- Testing multi-week or multi-quarter data to capture patterns that transcend a short 1-week snapshot.
- Using more complex neural architectures or kernel-based cross-impact frameworks for non-linear, multi-dimensional interactions.

Overall, the results confirm that while *self-impact* dominates at high frequency, cross-asset OFI influences can exist. Yet exploiting them for short-horizon alpha may be challenging without a broader, multi-factor approach.

References

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