Homework 3, FRE-6971, due 4/19/2023, 3pm

Problem 1 (10 points)

Read Chapters 2-3 of the Fisher & Gilles paper.

1-Factor Affine model:

$\mathrm{dr}(t) = \{\mu - \kappa r(t)\}\mathrm{dt} + \sqrt{\gamma r(t) + \sigma}\ \mathrm{dW(t)}$

Prove that p(t, T)=$e^{A(t,T)-B(t,T)r(t)}$

where functions A & B satisfy the Ricatti equations:

$\frac{\mathrm{dA}}{\mathrm{dt}} - \mu B + \frac{1}{2}\sigma B^2 = 0$
$\frac{\mathrm{dB}}{\mathrm{dt}} - \kappa B - \frac{1}{2}\gamma B^2 + 1 = 0$

Problem 2 (40 points)

Assume $[r(0), \mu, \kappa, \gamma, \sigma]$=[0.04, 0.0025, 0.05, 0, 0.01] in the above model.

1. Implement the solver for the Ricatti equations and find the solution for a given set of parameters. You can use scipy.integrate.odeint, for instance.

2. EXTRA CREDIT: Find the analytical solution for the zero-coupon bond price, starting from the pricing formula for the zero-coupon bond we derived in class, and compare the result with the solution of the above system of ODEs.

Problem 3 (10 ponts):

Assume the dynamics are as follows:

$\mathrm{dr}(t) = \{\mu - \kappa r^2(t)\}\mathrm{dt} + \sqrt{\gamma r(t) + \sigma}\ \mathrm{dW(t)}$

1. Prove that this model is not from the Affine class

2. Is zero coupon bond price p(t,T) still a solution to a system of ODEs? Explain your answer in detail