

# Intro

1. Game Playing
2. Search
3. Simulated Annealing
4. Constraint Satisfaction
5. Probability
6. Bayes Nets
7. Machine Learning
8. Pattern Recognition through Time
9. Logic and Planning
10. Planning under Uncertainty

## Game Playing

1. adversarial search
2. minimax algorithm
3. alpha-beta pruning
4. evaluation functions
5. isolation game player

## Isolation game

isolate the other user on a checker board

**building a game tree:**

1. shows all the moves possible in the game: [example](#) (we're o)
2. first moves matters! (e.g. when o goes to top middle, it leads to many losses)
3. how to teach our computer player o to make better first move? --**minimax algorithm**

## Minimax Algorithm

mark game tree with up/ down arrows: up (max level) -- we try to maximize our score; down (min level) -- opponent tries to minimize our score

top of minimax tree is always a max level

## Propagating values up the tree

start from bottom nodes, if it's max level, take max of child nodes, vice versa. work our way up  
[reading]: 5.1-5.2

## Maximum num of nodes

$$25 * 24 * \dots * 1$$

better estimate:

$$25 * 24 * 12^4 * 11 * 12!$$

explanation:

1. middle of board has 16 moves but most cells have 12 or less moves (call it **branching factor**)
2. open moves:  $25 * 24$
3. last move when game ends 1, the one before 2, ... maximum choice before ending 12
4. moves before getting to 12 choices =  $25 - 2 - 12 = 11$
5. there are 11 moves with avg 12 choices after opening & before closing

[quiz] how many # nodes minimax algorithm needs to visit? (given b: branch factor, d: depth of tree)

$$\text{ans: } b^d$$

avg branch factor: 8 (from simulation) --  $8^{25}$  nodes

## Depth limited search

if we can't search endgame, how deep can we search?

assume we can search  $10^9$  nodes every second,

$$10^9 * 2 \text{ in 2 sec}$$

$$8^x = 2 * 10^8$$

$$\begin{aligned}
&\text{Depth Limited Search} \\
&10^9 \text{ nodes} \times 2 \text{ sec} = 2 \times 10^9 \text{ nodes} \\
&8^x < 2 \times 10^9 \\
&\log_8 8^x < \log_8 2 \times 10^9 \\
&x < \frac{\log_{10} 2 \times 10^9}{\log_{10} 8} \\
&x < 10.3
\end{aligned}$$

## Evaluation function

"maximize # moves player has left"

- when we search to leave nodes, we get good answer; but if we only search x levels deep (e.g. 2), we may get wrong answer

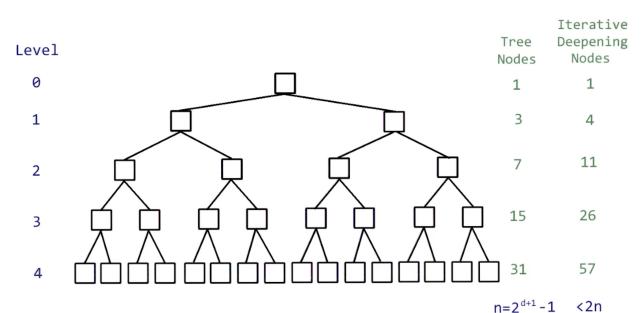
## Quiescent Search

calculate recommended move with depth 1, 2, ... until it doesn't change much --> reached enough depth;

often gives us better results @ beginning or ending of the game

## Iterative Deepening

at each level, compare & save result in case we run out of time. if time is up, return last saved result  
-- it's not inefficient because in game tree search, computation time is dominated by last level searched



when branch == 2, iterative deepening node < 2n, when branch >2, it's lower

## Bayes Nets

Replace denominator with constant

$$P(A|B) = P(B|A) * P(A) / P(B) = a * P(B|A) * P(A)$$

$$P(A'|B) = P(B|A') * P(A') / P(B) = a * P(B|A') * P(A')$$

$$A = \text{NOT } A' \rightarrow P(A|B) + P(A'|B) = 1$$

example: two test cancel

### Conditional Independence

$$P(+/+ | C) = P(+/C) * P(+/C)$$

conditional: only true if C is given

independent: two diagnoses don't rely on each other

foundation of bayes nets!

Q: Does conditional independence indicate absolute independence?

(does  $P(A,B|C)=P(A|C)*P(B|C)$  mean  $P(A,B) = P(A)P(B)$ )?

A: NO.

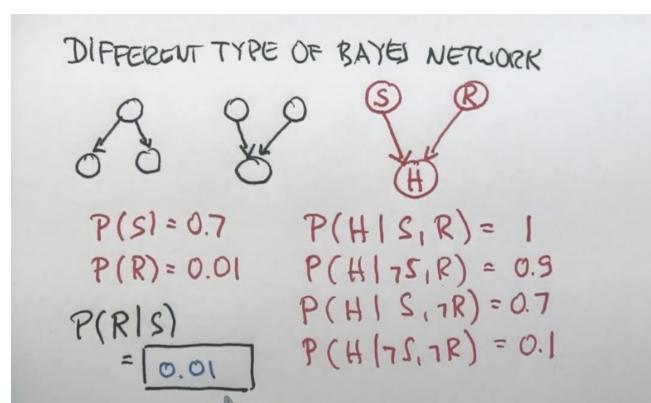
$P(+/+) = P(+/+, C)*P(C) + P(+/+, C')*P(C') \neq 0$ , hence not absolute independent

(note two +'s represent second & first diagnose)

Q: Does absolute independence indicate conditional independence?

A: NO.

### Confounding Cause



$P(R|S) = P(R)$  because 2 causes S,R are independent

### Explain away

observe: happy, also observe sunny  $\rightarrow$  P(raise) is lower

Q:

$P(S) = 0.7$	$P(H S, R) = 1$
$P(R) = 0.01$	$P(H _7S, R) = 0.9$
$P(R S)$	$P(H S, _7R) = 0.7$
$= 0.01$	$P(H _7S, _7R) = 0.1$

$P(R|S) = 0.01$   
 $P(R|_7S) = 0.01$   
 $P(R|S, R) = 1$   
 $P(H|S, R) = 1$   
 $P(H|_7S, R) = 0.9$   
 $P(H|S, _7R) = 0.7$   
 $P(H|_7S, _7R) = 0.1$

$P(R|H, S) = 0.0142$   
 $= \frac{P(H|R, S) \cdot P(R|S)}{P(H|R, S) \cdot P(R) + P(H|_7R, S) \cdot P(_7R)}$   
 $= \frac{1 \cdot 0.01}{0.01 + 0.7 \cdot 0.99} = 0.0142$

without knowing the weather (Sunny), chance of raise is higher given i'm happy

$$\begin{array}{ll}
 P(S) = 0.7 & P(H|S, R) = 1 \\
 P(R) = 0.01 & P(H|_7S, R) = 0.9 \\
 P(R|S) & P(H|S, _7R) = 0.7 \\
 & P(H|_7S, _7R) = 0.1 \\
 P(R|H, S) & = 0.01 \\
 P(R|H) & = 0.0142 \\
 \frac{P(H|R)P(R)}{P(H)} & P(H) = P(H|S, R)P(S, R) \\
 & + P(H|_7S, R)P(_7S, R) \\
 & + P(H|S, _7R)P(S, _7R) \\
 & + P(H|_7S, _7R)P(_7S, _7R) \\
 & = 0.57 \cdot 0.01 \\
 & = 0.5245
 \end{array}$$

$$\begin{array}{ll}
 P(S) = 0.7 & P(H|S, R) = 1 \\
 P(R) = 0.01 & P(H|_7S, R) = 0.9 \\
 P(R|S) & P(H|S, _7R) = 0.7 \\
 & P(H|_7S, _7R) = 0.1 \\
 P(R|H, S) & = 0.0142 \\
 P(R|H, _7S) & = 0.0083 \\
 \frac{P(H|R)P(R)}{P(H)} & P(H) = P(H|S, R)P(S, R) \\
 & + P(H|_7S, R)P(_7S, R) \\
 & + P(H|S, _7R)P(S, _7R) \\
 & + P(H|_7S, _7R)P(_7S, _7R) \\
 & = 0.9 \cdot 0.01 \\
 & = 0.0833
 \end{array}$$

$P(R|H, S) <> P(R|H, S')$ : when we know one cause is False & the output, the other cause prob significantly increases

**Proof of independence doesn't imply conditional independence**

BAYES NETWORKS

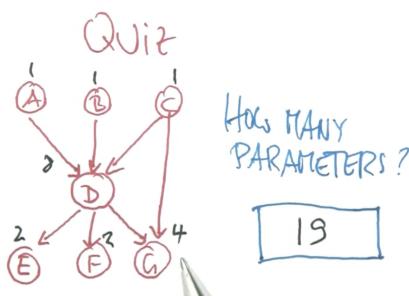
$$\begin{aligned}
 P(R|H, S) &= 0.0142 \neq P(R|H) \\
 P(R|S) &= 0.01 = P(R) \\
 P(R|H, _7S) &= 0.0833 \\
 RLS &\quad \cancel{R|S \neq H}
 \end{aligned}$$

## General Bayes Nets

enumerate all combo:  $2^N - 1$   
bayes nets representation: 10

advantage: shrinks probability values required significantly

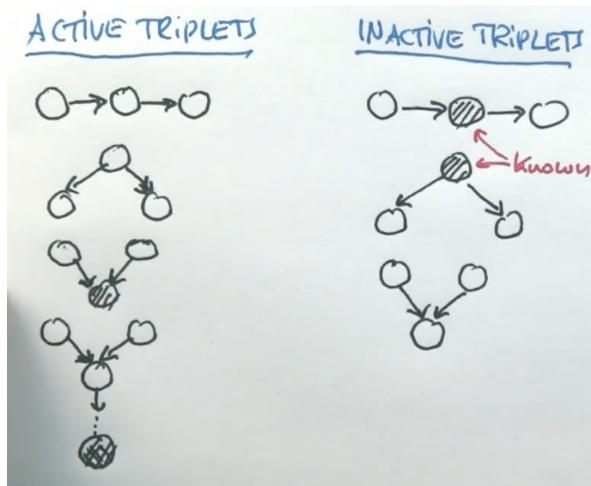
Quizes: count params



BAYES NETWORKS

$$\begin{aligned}
 P(A), P(B) \\
 P(C|A, B) \\
 P(D|C) P(E|C) \\
 P(A, B, C, D, E) = \\
 \underbrace{P(A) \cdot P(B)}_1 \cdot \underbrace{P(C|A, B)}_4 \cdot \underbrace{P(D|C)}_2 \cdot \underbrace{P(E|C)}_2
 \end{aligned}$$

## D-separation



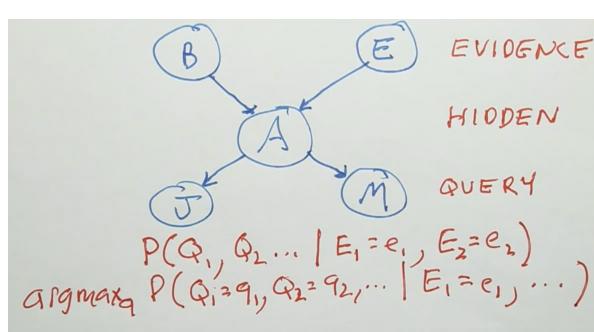
streamline: dependent

cut middle node in streamline: independent

cut parent node: children become independent

cut children node: parents become dependent

## Probabilistic Inference



evidence: probabilities we know

query: probabilities we are interested to find out

hidden: everything else

\* bayes networks can go both ways (B,E can be query too)

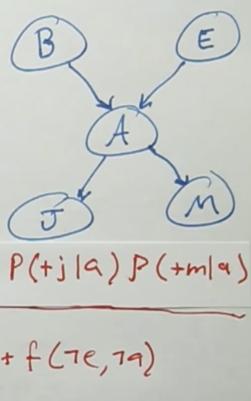
## Enumeration

go through all possible probabilities, add them up and come up with the query probability

### ENUMERATION

$$P(+b | +j, +m) = \\ P(+b, +j, +m) / P(+j, +m)$$

$$P(+b, +j, +m) = \\ \sum_e \sum_a P(+b, +j, +m, e, a) = \\ \sum_e \sum_a \underbrace{P(+b) P(e) P(a | +b, e) P(+j | a) P(+m | a)}_{f(e, a)} \\ = f(+e, +a) + f(+e, -a) + f(-e, +a) + f(-e, -a)$$



### Speed up Enumeration

techniques:

$$\sum_e \quad \sum_a \quad P(+b) \quad P(e) \quad P(a | +b, e) P(+j | a) P(+m | a)$$

### 1. pull out terms

#### PULLING OUT TERMS

$$P(+b) \quad \sum_e P(e) \quad \sum_a \quad \underbrace{P(a | +b, e) P(+j | a) P(+m | a)}$$

### 2. maximize independence

for streamline network:

node0->node1->node2->...->noden : O(N)

for complete network (every pair nodes are connected)

O(2^N)

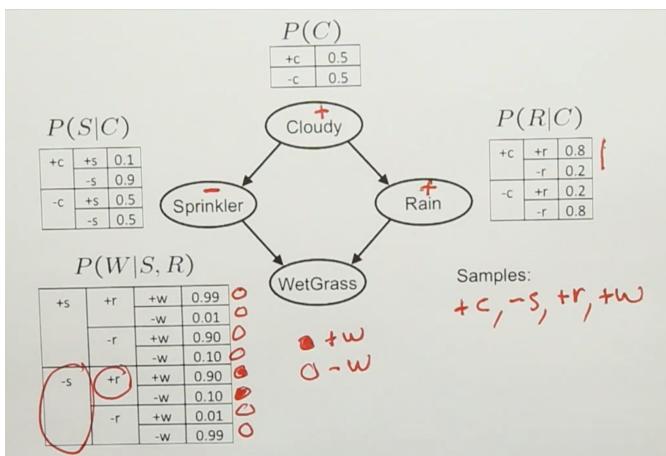
### Variable Elimination

enumeration repeats many nodes

elimination: combine some variables, enumerate smaller networks & work on combined nodes

1. combine nodes into small network
2. sumout & eliminate small network

## Approximate Inference (Sampling)



sample ancestor ->  
sample next level conditional to ancestor  
output ->  
carry on ->  
until end of the network  
-> repeat

**sampling method is consistent**  
with inf samples, approach to true joint prob

**rejection sampling**  
when compute conditional prob: remove  
samples that don't match with conditions

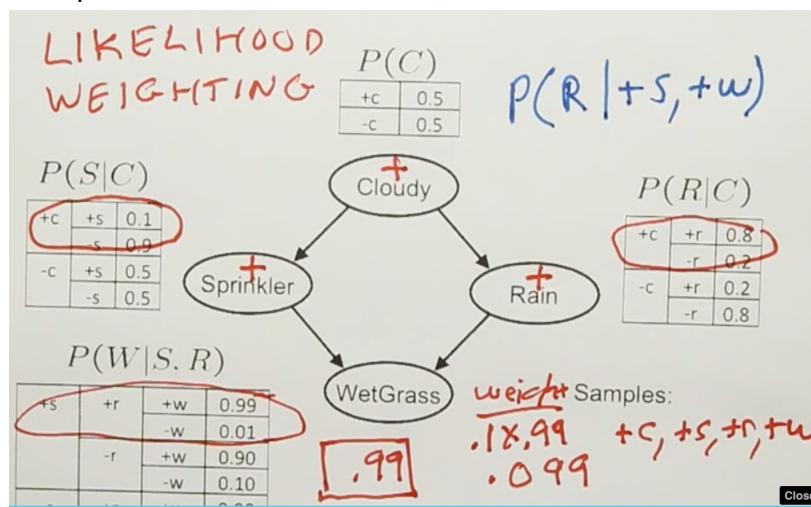
and only keep those that do. e.g. when calculate  $P(+W | -C)$ , the  $+c, -s, +r, +w$  sample is not useful -> remove

... but this end up generating lots unuseful samples

### likelihood weighting

generate samples that we always can keep (e.g. C always negative, and generate S, R)  
BUT, resulting samples are **inconsistent**

to fix it, assign likelihood to each sample & weighing correctly. it will make sampling **consistent**  
example:



### **gibbs sampling**

- takes all evidence (not just upstream evidence)

- uses MCMC

- **consistent**

1. generate random sample

2. pick ONE non-evidence variable, resample it **based on** other variables

3. repeat

(note variables are dependent in this gibbs sampling, which is opposite of rejection sampling)

### **monty hall**

door 1: 1/3

door 2: 2/3

because we learned something new about second door, but nothing new about first door -- opening 1st door is never an option

## **Pattern Recognition Through Time**

### **Dynamic Time Warping (DTW)**

in time series analysis, DTW is an algorithm measuring similarity between two temporal sequences, which may vary in speed

### **Soe Chiba Bounds**

won't allow match over certain bounds

### **HMMs**

Markov Chain with self transition & out transition

expected time frame spent in a state =  $1/(1-p)$  where p=self-transition prob

=> dummy state (initial X0, p =0, timeframe spent on X0: 1)

Q: what property does delta\_y (observed sequence) help distinguish between two gestures ("we" & "i" in sign language)?

A: 1. probability distributions in middle states; 2. likely time spent in middle states

### **Viterbi Trellis**

$$P(\text{state } s \text{ at } t) = P(\text{transition}) * P(s)$$

Baum Welch  
(EM)

## Logic and Planning

### Propositional Logic

(alarm example) Burglary, Earthquake, Alarm, MaryCalling, JohnCalling  
use T/F

$(E \vee B) \Rightarrow A$  ( $E$  or  $B$  is T implies  $A$  is T)

$A \Rightarrow (J \wedge M)$  ( $A$  implies  $J$  and  $M$ )

$J \Leftrightarrow M$  (equivalent)

$J \Leftrightarrow \neg M$  (opposite)

p.s.  $\vee$ : or;  $\wedge$ : and;

### Truth Tables

		TRUTH TABLES					
$P$	$Q$	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$	
false	false	true	false	false	true	true	
false	true	true	false	true	true	false	
true	false	false	false	true	false	false	
true	true	false	true	true	true	true	

true  $\Rightarrow$  true; false  $\Rightarrow$  false: true

different from ordinary english!

### Terminology

valid: T in every model in every possible combination of symbols

satisfiable: T in some but not necessarily all

VALID	SATISFIABLE
$\neg \neg P$	$P \vee \neg P$
$P \wedge \neg P$	$P \wedge (\neg P \vee (P \Leftrightarrow Q))$
$P \vee Q \vee (P \Leftrightarrow Q)$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
$(P \Rightarrow Q) \vee (Q \Rightarrow P)$	$((Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)) \Rightarrow ((Food \wedge Drunks) \Rightarrow Party)$

### Propositional Logic Limitations

1. only T/F, no capability to handle uncertainty
2. can't tell objects or relationships b.w. objects
3. no shortcuts

## First Order Logic

### types:

	<u>world</u>	<u>beliefs</u>
first-order logic:	relationship, objects, functions	T/F/unknown
propositional logic:	facts	T/F/unknown
probability theory:	facts	[0, 1]

### representation:

atomic: problem solving; transition b.w. A&B

factored:

structured:

### models:

propositional: symbol {P: T, Q: F}

first-order:

constants:{A,B,C,D,1,2,3,CEE}

functions (mapping b.w. objects):

number of: {A->1, B->3, C->3, D->2}

relations:

above: {[A,B], [C,D]}

vowel: {[A]}

rainy: {} not rainy {}

