

# Homework ada9

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## homework1

ガウスカーネルモデルに対するラプラス正則化最小二乗分類を実装せよ

**answer**

In [2]:

```
%matplotlib inline

import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg, random
from sklearn.base import BaseEstimator
from sklearn import datasets, metrics
```

In [5]:

```

class LRRidge(BaseEstimator):
    def __init__(self,alpha=1.,beta=1.,gamma=10.,k=10):
        self.alpha=alpha
        self.beta=beta
        self.gamma=gamma
        self.k=k

    def fit(self,Xl,y,Xu):
        self.X=np.r_[Xl,Xu]
        #self.X=self.X[random.permutation(len(self.X))]
        self.X2=np.c_[np.sum(self.X**2,1)]
        Xl2=np.c_[np.sum(Xl**2,1)]
        Xu2=np.c_[np.sum(Xu**2,1)]
        Phil=np.exp(-self.gamma*(Xl2+self.X2.T-2*Xl.dot(self.X.T)))
        Phiu=np.exp(-self.gamma*(Xu2+self.X2.T-2*Xu.dot(self.X.T)))

        Phiu2=np.c_[np.sum(Phiu**2,1)]
        d=Phiu2+Phiu2.T-2*Phiu.dot(Phiu.T)
        p=np.c_[np.sort(d,axis=1)[:self.k+1]]
        W=d<=p
        W=(W+W.T)!=0
        D=np.diag(np.sum(W,axis=1))
        L=D-W

        n_features=Phil.shape[1]
        self.theta=linalg.pinv(Phil.T.dot(Phil)+self.alpha*np.identity(n_features)+self.beta*Phiu.T.dot(L)).do

    return self

    def predict(self,X):
        X2=np.c_[np.sum(X**2,1)]
        Phi=np.exp(-self.gamma*(X2+self.X2.T-2*X.dot(self.X.T)))
        return (Phi.dot(self.theta)>=0)*2-1

```

In [ ]:

```

N = 500 #サンプル数
X,y =datasets.make_moons(n_samples = N, noise = .05)
y = y*2 - 1 #値を0or1ではなく、-1or1にしている。

```

In [50]:

```

# demo
random.seed(1)
n=500
X,y=datasets.make_circles(n_samples=n,factor=.5,noise=.05)
#X,y=datasets.make_moons(n_samples=n,noise=.05)
y=y*2-1

nl=2 # number of labeled samples
idx=random.permutation(len(X))
il=idx[:nl]; iu=idx[nl:];
Xl=X[il]; Xu=X[iu];
yl=y[il]; yu=y[iu];

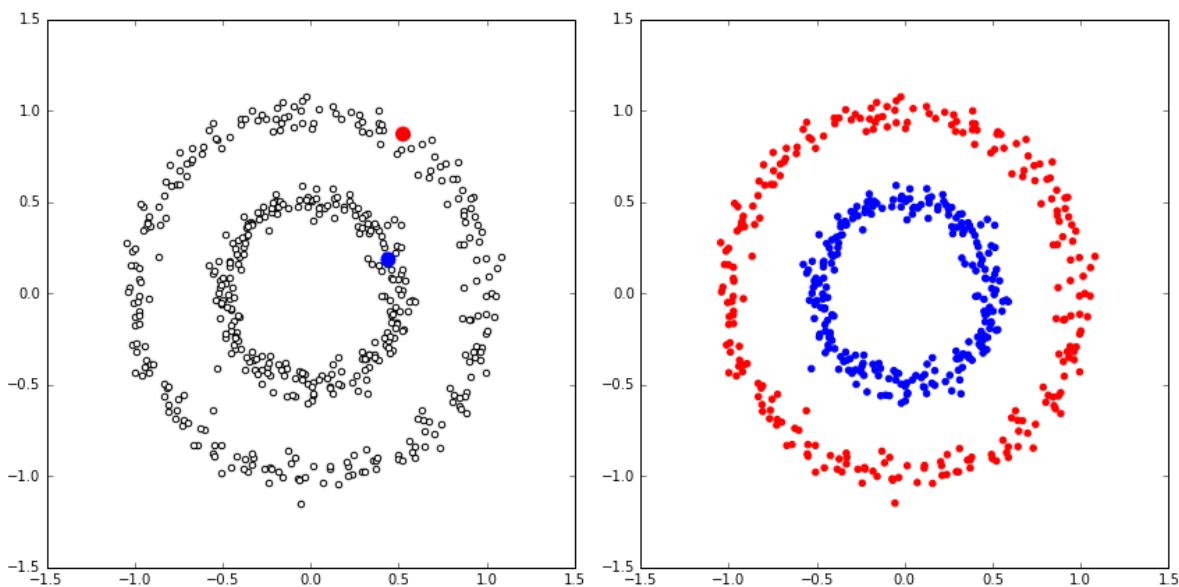
clf=LRRidge().fit(Xl,yl,Xu)
ypred=clf.predict(X)
yupred=clf.predict(Xu)

print("Accuracy (LRRidge):",metrics.accuracy_score(yu,yupred))

colors=np.array(["r","b"])
plt.figure(figsize=(12,6))
plt.subplot(121)
plt.scatter(Xu[:,0],Xu[:,1],c="w",s=20)
plt.scatter(Xl[:,0],Xl[:,1],color=colors[((1+yl)/2).astype(np.int)],s=100)
plt.subplot(122)
plt.scatter(X[:,0],X[:,1],color=colors[((1+ypred)/2).astype(np.int)])
plt.tight_layout()
plt.show()

```

Accuracy (LRRidge): 1.0



## homework2

$$D_E(p_{\text{test}}, q_\pi) = 2E_{x' \sim p_{\text{test}} | x \sim q_\pi} \|x' - x\| - E_{x' \sim q_\pi | x \sim p_{\text{test}}} \|x' - x\| - E_{x' \sim q_\pi | x \sim q_\pi} \|x' - x\|$$

$$q_\pi(x) = \pi p_{\text{train}}(x|y = +1) + (1 - \pi) p_{\text{train}}(x|y = -1)$$

エネルギー距離が  $\pi$  の関数として次のように表現できることを示せ

$$J(\pi) = (2A_{+1,-1} - A_{+1,+1} - A_{-1,-1}) \pi^2 - 2(A_{+1,-1} - A_{-1,-1} - b_{+1} + b_{-1}) \pi + \text{Const}$$

**proof):**

第1項について

$$\begin{aligned} 2E_{x' \sim p_{\text{test}} | x \sim q_\pi} \|x' - x\| &= 2 \{ \pi E_{x' \sim p_{\text{test}} | x \sim p_{\text{train}}(x_{i+1})} \|x' - x\| + (1 - \pi) E_{x' \sim p_{\text{test}} | x \sim p_{\text{train}}(x_{i-1})} \|x' - x\| \} \\ &= 2(b_{+1} - b_{-1}) \pi + \text{Const} \end{aligned}$$

第2項について,  $\pi$  の関数ではないので const

第3項について

$$\begin{aligned} -E_{x', \tilde{x} \sim q_\pi} \|x' - x\| &= - \{ \pi^2 E_{x \sim p_{\text{train}}(x_{i+1}), \tilde{x} \sim p_{\text{train}}(x_{i+1})} \|x' - x\| + 2\pi(1 - \pi) E_{x \sim p_{\text{train}}(x_{i+1}), \tilde{x} \sim p_{\text{train}}(x_{i-1})} \|x' - x\| \\ &\quad + (1 - \pi)^2 E_{x \sim p_{\text{train}}(x_{i-1}), \tilde{x} \sim p_{\text{train}}(x_{i-1})} \|x' - x\| \} \\ &= (2A_{+1,-1} - A_{+1,+1} - A_{-1,-1}) \pi^2 - 2(A_{+1,-1} - A_{-1,-1}) \pi + \text{Const} \end{aligned}$$

以上より,

$$J(\pi) = (2A_{+1,-1} - A_{+1,+1} - A_{-1,-1}) \pi^2 - 2(A_{+1,-1} - A_{-1,-1} - b_{+1} + b_{-1}) \pi + \text{Const}$$

【Q.E.D】

## homework3

線形モデル  $f_\theta(x) = \theta^T x + \theta_0$  に対して, クラス比重み付き最小二乗法を実装せよ

**answer**

In [11]:

```
#dataset
from numpy import random as rnd
train_x = np.r_[rnd.normal(-2,1,(90,1)),rnd.normal(2,1,(10,1))]
train_x = np.c_[train_x,2*rnd.normal(0,1,(100,1))]
train_x = np.c_[train_x,np.ones((100,1))] #畳み込み
train_y = np.r_[np.ones((90,1)),1*np.ones((10,1))]
test_x = np.r_[rnd.normal(-2,1,(10,1)),rnd.normal(2,1,(90,1))]
test_x = np.c_[test_x,2*rnd.normal(0,1,(100,1))]
test_x = np.c_[test_x,np.ones((100,1))]
```

In [12]:

```
w = np.ones((train_y.shape))  
w[train_y==1] = 1/9  
w[train_y==-1] = 9  
W = np.diag(w[:,0])
```

In [13]:

```
tmp = np.linalg.inv(np.dot(np.dot(train_x.T,W),train_x))  
theta = np.dot(tmp,np.dot(np.dot(train_x.T,W),train_y))  
tmp = np.linalg.inv(np.dot(train_x.T,train_x))  
theta1 = np.dot(tmp,np.dot(train_x.T,train_y))
```

In [14]:

```

colors = np.array(['r','b'])
X = np.linspace(-5,5,1000)[:,:np.newaxis]

fig = plt.figure(figsize=(14,6))
plt.suptitle(u'class weighted Least-square method',fontsize=20,y=0.975)

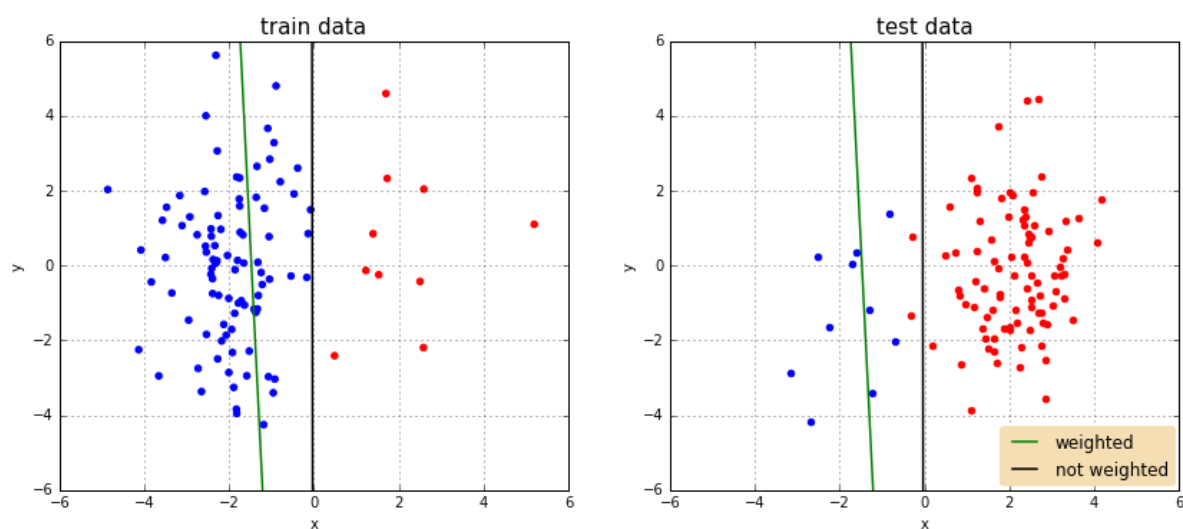
fig.add_subplot(1,2,1)
plt.title(u'train data',fontsize=15)
plt.scatter(train_x[:,0],train_x[:,1],color=colors[(train_y[:,0]+1).astype(int)])
plt.plot(X,theta[0]/theta[1]*X+theta[2]/theta[1],'g',linewidth=1.5)
plt.plot(X,theta1[0]/theta1[1]*X+theta[2]/theta[1],'black',linewidth=1.5)
plt.ylim(-6,6)
plt.xlim(-6,6)
plt.xlabel('x')
plt.ylabel('y')
plt.grid()

fig.add_subplot(1,2,2)
plt.title(u'test data',fontsize=15)
plt.scatter(test_x[:10,0],test_x[:10,1],color='b')
plt.scatter(test_x[10:,0],test_x[10:,1],color='r')
plt.plot(X,theta[0]/theta[1]*X+theta[2]/theta[1],'g',linewidth=1.5)
plt.plot(X,theta1[0]/theta1[1]*X+theta[2]/theta[1],'black',linewidth=1.5)
plt.ylim(-6,6)
plt.xlim(-6,6)
plt.xlabel('x')
plt.ylabel('y')
plt.grid()
legend = plt.legend([u'weighted',u'not weighted'],frameon=1,fancybox=True,\
                    bbox_to_anchor=(1.01, -0.01),loc='lower right')
frame = legend.get_frame()
frame.set_facecolor('wheat')
frame.set_edgecolor('None')

plt.subplots_adjust(top=0.85)
fig.savefig('ana9-2.png')

```

class weighted Least-square method



In [15]:

### fin