Homework ada7

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homework1

$$(\hat{\mu}, \hat{\mathbf{\Sigma}}) = arg \min_{\mu, \mathbf{\Sigma}} J(\mu, \mathbf{\Sigma})$$

$$J(\mu, \mathbf{\Sigma}) = (\max(0, 1 - \mu^T \phi(\mathbf{x}) y))^2 + \phi(\mathbf{x})^T \mathbf{\Sigma} \phi(\mathbf{x}) + \gamma \left\{ log \frac{det(\tilde{\Sigma})}{det(\Sigma)} + tr(\tilde{\mathbf{\Sigma}}^{-1} \mathbf{\Sigma}) + (\mu - \tilde{\mu})^T \tilde{\Sigma}^{-1} (\mu - \tilde{\mu}) - d$$

2乗ヒンジ損失に対する適応正則化分類のμの解は，次式で表せることを示せ

$$\hat{\mu} = \tilde{\mu} + \frac{y \max(0, 1 - \tilde{\mu}^T \phi(\mathbf{x}) y)}{\phi(\mathbf{x})^T \tilde{\mathbf{\Sigma}} \phi(\mathbf{x}) + \gamma} \tilde{\mathbf{\Sigma}} \phi(\mathbf{x})$$

proof:

$\mu^T \phi(\mathbf{x}) = \phi(\mathbf{x})^Y \mu$より，

$$J(\mu, \mathbf{\Sigma}) = (\max(0, 1 - \phi(\mathbf{x})^T \mu y))^2 + \phi(\mathbf{x})^T \mathbf{\Sigma} \phi(\mathbf{x}) + \gamma \left\{ log \frac{det(\tilde{\Sigma})}{det(\Sigma)} + tr(\tilde{\mathbf{\Sigma}}^{-1} \mathbf{\Sigma}) + (\tilde{\Sigma}^{-1})^T (\mu - \tilde{\mu})^2 - d \right\}$$

$\max(0, 1 - \tilde{\mu}^T \phi(\mathbf{x}) y) = (1 - \tilde{\mu}^T \phi(\mathbf{x}) y)$とすると，

$$\frac{\partial J(\mu, \sigma)}{\partial \mu} = 0$$

及び逆行列の公式より，

$$\hat{\mu} = \tilde{\mu} + \frac{y(1 - \tilde{\mu}^T \phi(\mathbf{x}) y)}{\phi(\mathbf{x})^T \tilde{\mathbf{\Sigma}} \phi(\mathbf{x}) + \gamma} \tilde{\mathbf{\Sigma}} \phi(\mathbf{x})$$

また， $\max(0, 1 - \tilde{\mu}^T \phi(\mathbf{x}) y) = 0$の時，$\hat{\mu} = \tilde{\mu}$なので，

$$\hat{\mu} = \tilde{\mu} + \frac{y \max(0, 1 - \tilde{\mu}^T \phi(\mathbf{x}) y)}{\phi(\mathbf{x})^T \tilde{\mathbf{\Sigma}} \phi(\mathbf{x}) + \gamma} \tilde{\mathbf{\Sigma}} \phi(\mathbf{x})$$

【Q.E.D】

homework2

二乗ヒンジ損失に基づく適応正則化分類を線形モデル

$$f_\theta(\mathbf{x}) = f_\theta(x^{(1)}, x^{(2)}) = (x^{(1)} x^{(2)} 1) \theta$$

に対して実装せよ

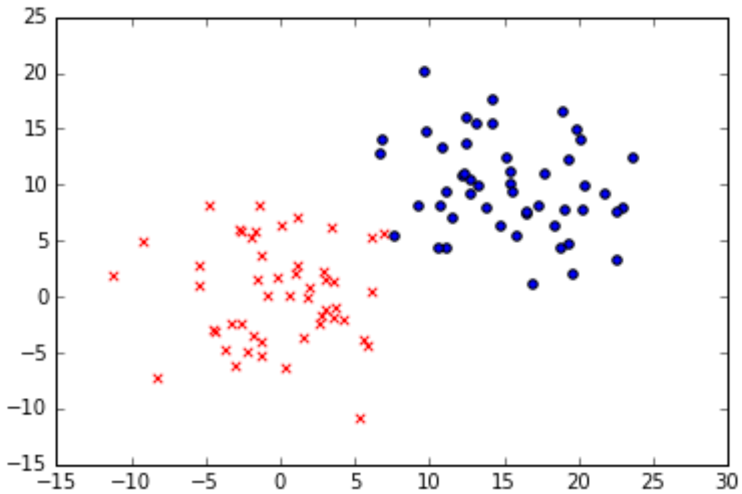
参考: [https://openbook4.me/projects/233/sections/1477 (https://openbook4.me/projects/233/sections/1477)](https://openbook4.me/projects/233/sections/1477)

In [3]:

```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from pandas import DataFrame, Series
from numpy.random import rand, multivariate_normal
from sklearn.utils import shuffle
from sklearn.model_selection import train_test_split
from numpy import dot   #np.dotはめんどくさいのでdotでつかえるようにする
```

In [4]:

```
N1 = 50       # t=+1 のデータ数
Mu1 = [15,10]  # t=+1 の中心座標     Muはμ

N2 = 50       # t=-1 のデータ数
Mu2 = [0,0]    # t=-1 の中心座標

variance = 20

# データセット を用意
cov1 = np.array([[variance,0],[0,variance]])  #分散共分散行列
cov2 = np.array([[variance,0],[0,variance]])

df1 = DataFrame(multivariate_normal(Mu1,cov1,N1),columns=['x','y'])
df1['type'] = 1
df2 = DataFrame(multivariate_normal(Mu2,cov2,N2),columns=['x','y'])
df2['type'] = -1
df = pd.concat([df1,df2],ignore_index=True) #縦に結合

plt.scatter(df1["x"],df1["y"], c='blue', marker='o')
plt.scatter(df2["x"],df2["y"], c='red', marker='x')

plt.show()
```



In [6]:

```
df = shuffle(df).reset_index().drop("index", axis = 1)    #dfをシャッフル
t = df["type"]
data = df[["x", "y"]]
phi = data
phi["bias"] = 1
```

In [7]:

```
#初期値
theta = np.random.normal(loc=0, scale=0.1, size=3)
sigma = np.random.normal(loc=0, scale=0.1, size=(3,3))
```

In [12]:

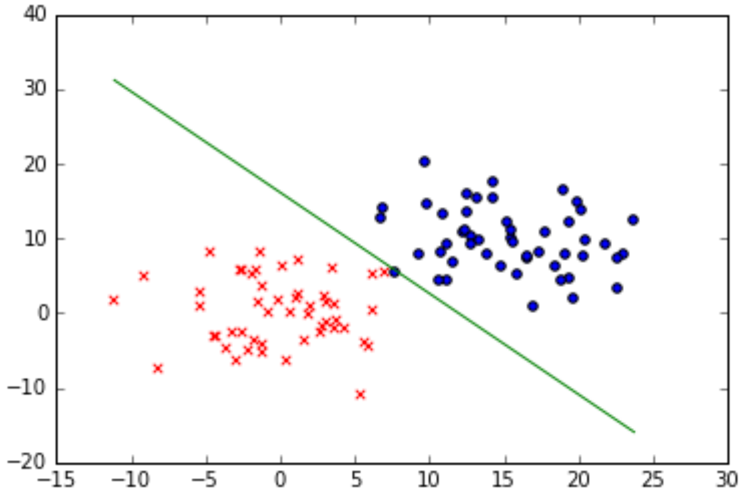
```
for i in range(100):    #データ点すべてを使う
    tmp = dot(phi.ix[i][:, np.newaxis], phi.ix[i][:, np.newaxis].T)
    gamma = 0.0001
    beta = dot(dot(phi.ix[i][:, np.newaxis].T,  sigma) , phi.ix[i][:, np.newaxis])  + gamma
    sigma = sigma - dot(dot(sigma, tmp), sigma) / beta    #シグマ更新
    theta = theta + t[i] * np.max(1 - dot(theta[:, np.newaxis].T, phi.ix[i]) * t[i], 0) * dot(sigma, phi.ix[i]) /
    theta = theta.flatten()
    # print(theta)
```

In [10]:

```
def f(x):    #モデル関数
    y = - theta[0] /theta[1] * x - theta[2] / theta[1]
    return y
```

In [11]:

```
plt.scatter(df1["x"],df1["y"], c='blue', marker='o')
plt.scatter(df2["x"],df2["y"], c='red', marker='x')
linex = np.arange(df["x"].min(),df["x"].max(),0.01)
liney = f(linex)                              #求めたfを使って直線をかく
plt.plot(linex, liney, color='green')
plt.show()
```



In [ ]:

```
```