homework ada11

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homework1

フィッシャー判別分析を実装せよ

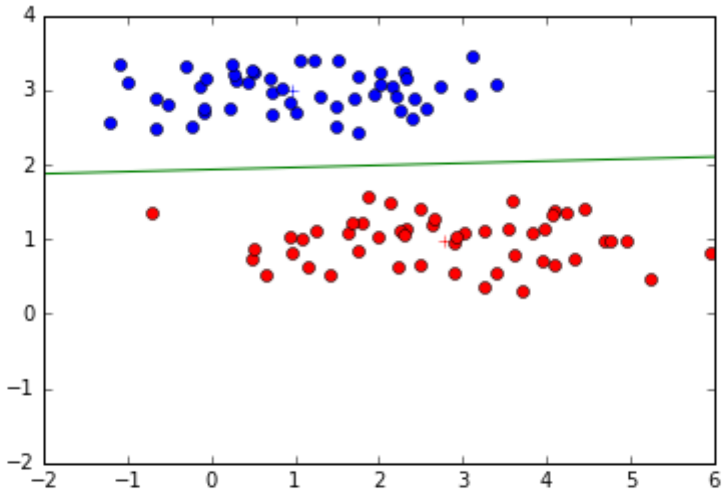
answer

In [5]:

```
%matplotlib inline
import numpy as np
from  pylab import *
from matplotlib import pyplot as plt
import sys

N = 100  # データ数

def f(x, a, b):
    return a * x + b

if __name__ == "__main__":
    # 訓練データを作成
    cls1 = []
    cls2 = []

    # データは正規分布に従って生成
    mean1 = [1, 3]  # クラス1の平均
    mean2 = [3, 1]  # クラス2の平均
    cov = [[2.0,0.0], [0.0, 0.1]]  # 共分散行列(全クラス共通)

    # データ作成
    cls1.extend(np.random.multivariate_normal(mean1, cov, int(N/2)))
    cls2.extend(np.random.multivariate_normal(mean2, cov, int(N/2)))

    # 各クラスの平均をプロット
    m1 = np.mean(cls1, axis=0)
    m2 = np.mean(cls2, axis=0)
    plt.plot([m1[0]], [m1[1]], 'b+')
    plt.plot([m2[0]], [m2[1]], 'r+')
    print(m1, m2)

    # 総クラス内共分散行列を計算
    Sw = zeros((2, 2))
    for n in range(len(cls1)):
        xn = matrix(cls1[n]).reshape(2, 1)
        m1 = matrix(m1).reshape(2, 1)
        Sw += (xn - m1) * transpose(xn - m1)
    for n in range(len(cls2)):
        xn = matrix(cls2[n]).reshape(2, 1)
        m2 = matrix(m2).reshape(2, 1)
        Sw += (xn - m2) * transpose(xn - m2)
    Sw_inv = np.linalg.inv(Sw)
    w = Sw_inv * (m2 - m1)

    # 訓練データを描画
    x1, x2 = np.transpose(np.array(cls1))
    plt.plot(x1, x2, 'bo')

    x1, x2 = np.transpose(np.array(cls2))
    plt.plot(x1, x2, 'ro')

    # 識別境界を描画
    # wは識別境界と直交するベクトル
    a = - (w[0,0] / w[1,0])  # 識別直線の傾き

    # 傾きがaでmを通る直線のy切片bを求める
    m = (m1 + m2) / 2
    b = -a * m[0,0] + m[1,0]  # 識別直線のy切片
```

```
x1 = np.linspace(-2, 6, 1000)
x2 = [f(x, a, b) for x in x1]
plt.plot(x1, x2, 'g-')

xlim(-2, 6)
ylim(-2, 4)
plt.show()
```

[0.9583855   2.99432884] [2.77796828 0.97754624]



homework2

以下の散布行列のペア表現を求めよ

クラス内

$$\mathbf{S}^{(w)} = \frac{1}{2}\sum_{i,i'=1}^{n} \mathbf{Q}^{(\mathbf{w})}_{\mathbf{i},\mathbf{i}'}(\mathbf{x_i} - \mathbf{x_{i'}})(\mathbf{x_i} - \mathbf{x_{i'}})^T$$

$$\mathbf{Q}^{(\mathbf{w})}_{\mathbf{i},\mathbf{i}'} = \begin{cases} 1/n \quad - \quad 1/n_y \qquad (y_i = y_{i'} = y) \\ 1/n \qquad (y_i \neq y_{i'}) \end{cases}$$

クラス間

$$\mathbf{S}^{(b)} = \frac{1}{2}\sum_{i,i'=1}^{n} \mathbf{Q}^{(\mathbf{b})}_{\mathbf{i},\mathbf{i}'}(\mathbf{x_i} - \mathbf{x_{i'}})(\mathbf{x_i} - \mathbf{x_{i'}})^T$$

$$\mathbf{Q}^{(\mathbf{b})}_{\mathbf{i},\mathbf{i}'} = \begin{cases} 1/n \quad - \quad 1/n_y \qquad (y_i = y_{i'} = y) \\ 1/n \qquad (y_i \neq y_{i'}) \end{cases}$$

answer

$$
\begin{aligned}
\mathbf{S}^{(w)} &= \sum_{i=1}^{l} \sum_{j:y_j=i} (\mathbf{x_j} - \frac{1}{n_i} \sum_{p:y_p=i} \mathbf{x_p})(\mathbf{x_j} - \frac{1}{n_i} \sum_{q:y_q=i} \mathbf{x_q})^T \\
&= \sum_{i=1}^{l} \mathbf{x_i} \mathbf{x_i^T} - \sum_{i=1}^{l} \frac{1}{n_i} \sum_{p,q:y_p=y_q=i} \mathbf{x_p} \mathbf{x_q^T} \\
&= \sum_{i=1}^{n} (\sum_{j=1}^{n} \mathbf{A}_{i,j}^{(w)}) \mathbf{x_i} \mathbf{x_i^T} - \sum_{i,j=1}^{n} \mathbf{A}_{i,j}^{(w)} \mathbf{x_i} \mathbf{x_i^T} \\
&= \frac{1}{2} \sum_{i,j=1}^{n} \mathbf{A}_{i,j}^{(w)} (\mathbf{x_i} \mathbf{x_i^T} + \mathbf{x_j} \mathbf{x_j^T} - \mathbf{x_i} \mathbf{x_j^T} - \mathbf{x_j} \mathbf{x_i^T})
\end{aligned}
$$

ここで

$$
\begin{aligned}
\mathbf{S}^{(m)} &= \mathbf{S}^{(w)} + \mathbf{S}^{(b)} \\
&= \sum_{i=1}^{n} (x_i - \mu)(x_i - \mu)^T
\end{aligned}
$$

であるから

$$
\begin{aligned}
\mathbf{S}^{(b)} &= \sum_{i=1}^{n} \mathbf{x_i} \mathbf{x_i^T} - \frac{1}{n} \sum_{i,j=1}^{n} \mathbf{x_i} \mathbf{x_j^T} - \mathbf{S}^{(w)} \\
&= \sum_{i=1}^{n} (\sum_{j=1}^{n} \frac{1}{n}) \mathbf{x_i} \mathbf{x_i^T} - \sum_{i,j=1}^{n} \frac{1}{n} \mathbf{x_i} \mathbf{x_j^T} - \mathbf{S}^{(w)} \\
&= \frac{1}{2} \sum_{i,j=1}^{n} (\frac{1}{n} - \mathbf{A}_{i,j}^{(w)}) (\mathbf{x_i} \mathbf{x_i^T} + \mathbf{x_j} \mathbf{x_j^T} - \mathbf{x_i} \mathbf{x_j^T} - \mathbf{x_j} \mathbf{x_i^T})
\end{aligned}
$$

In [ ]: