

# Homework ada10

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## homework1

次式を証明せよ

$$\sum_{i,i'=1}^n W_{i,i'} \|\mathbf{T}\mathbf{x}_i - \mathbf{T}\mathbf{x}_{i'}\|^2 = 2\text{tr}(\mathbf{T}\mathbf{X}\mathbf{L}\mathbf{X}^T\mathbf{T}^T)$$

$$\begin{aligned} \sum_{i,i'=1}^n W_{i,i'} \|\mathbf{T}\mathbf{x}_i - \mathbf{T}\mathbf{x}_{i'}\|^2 &= 2 \sum_{i=1}^n \mathbf{x}_i^T \mathbf{T}^T \mathbf{W}_{i,i'} \mathbf{T}\mathbf{x}_i - 2 \sum_{i'=1}^n \mathbf{x}_{i'}^T \mathbf{T}^T \mathbf{W}_{i,i'} \mathbf{T}\mathbf{x}_{i'} \\ &= 2 \sum_{i=1}^n \text{tr}(\mathbf{T}\mathbf{x}_i \mathbf{W}_{i,i'} \mathbf{x}_i^T \mathbf{T}^T) - 2 \sum_{i'=1}^n \text{tr}(\mathbf{T}\mathbf{x}_{i'} \mathbf{W}_{i,i'} \mathbf{x}_{i'}^T \mathbf{T}^T) \\ &= 2\text{tr}(\mathbf{T}\mathbf{X}(\mathbf{D} - \mathbf{W})\mathbf{X}^T\mathbf{T}^T) \\ &= 2\text{tr}(\mathbf{T}\mathbf{X}\mathbf{L}\mathbf{X}^T\mathbf{T}^T) \end{aligned}$$

【Q.E.D】

## homework2

適当な類似度行列に対して局所性保存射影を実装せよ

```
In [1]: %matplotlib inline
import numpy as np
from numpy import random as rnd
from scipy import linalg
from sklearn.decomposition import PCA
from matplotlib import pyplot as plt
from matplotlib.font_manager import FontProperties as fp
from matplotlib.ticker import *
```

```

In [2]: class LPP():
        def __init__(self,h=0.1):
            self.h = h

        def __get_rbf_matrix_(self,x,c,h=1.0):
            __Hx_ = np.tile(np.diag(np.dot(x,x.T)),(c.shape[0],1))
            __Hc_ = np.tile(np.diag(np.dot(c,c.T)),(x.shape[0],1))
            __G_ = np.dot(c,x.T)
            __K_ = np.exp(-(__Hx_-2*__G_+__Hc_.T)/(2*self.h**2))
            return __K_

        def fit(self,x):
            __W_ = self.__get_rbf_matrix_(x,x)
            __D_ = np.diag(np.sum(__W_,axis=1))
            __L_ = __D_ - __W_

            __A_ = np.dot(np.dot(x.T,__L_),x)
            __B_ = np.dot(np.dot(x.T,__D_),x)

            __B_inv_sqrt_ = linalg.inv(linalg.sqrtm(__B_))
            __C_ = np.dot(np.dot(__B_inv_sqrt_,__A_),__B_inv_sqrt_)

            #scipyでは固有値が昇順に並ぶ
            self._lambda, self.xi = linalg.eigh(__C_)

            return self

        def first_basis(self,X):
            __i_ = len(self.xi) - 1
            return self.xi[__i_][1] / self.xi[__i_][0] * X

        def second_basis(self,X):
            return self.xi[0][1] / self.xi[0][0] * X

```

```

In [3]: #dataset
n = 100
train_x1 = np.c_[2*rnd.normal(0,1,n), rnd.normal(0,1,n)]
train_y = np.sign(rnd.uniform(-1,1,n)).astype(int)
train_x2 = np.c_[2*rnd.normal(0,1,n), train_y+rnd.normal(0,1,n)/3]

```

```
In [4]: #LPPの実行
clf1 = LPP(h=0.1)
clf2 = LPP(h=0.1)
clf1.fit(train_x1)
clf2.fit(train_x2)

#比較のため、主成分分析の実行
pca1 = PCA()
pca2 = PCA()
pca1.fit(train_x1)
pca2.fit(train_x2)
```

```
Out[4]: PCA(copy=True, iterated_power='auto', n_components=None, random_state=None,
      svd_solver='auto', tol=0.0, whiten=False)
```

```

In [5]: def f(x, xi):
        i = len(xi) - 1
        y = xi[0][1] / xi[0][0] * x
        return y

        colors = np.array(['r', '', 'b'])
        X = np.linspace(-6, 6, 1000)

        #グラフ描
        fig = plt.figure(figsize=(12, 5))

        plt.suptitle("comparison between LPP and PCA", fontsize=20)

        fig.add_subplot(1, 2, 1)
        plt.title("1 class", fontsize=15)
        plt.plot(X, clf1.second_basis(X), color='r', label='LPP')
        plt.plot(X, f(X, pca1.components_), color='g', label='PCA')
        plt.scatter(train_x1[:, 0], train_x1[:, 1], color='b')
        plt.xlim(-5, 5)
        plt.ylim(-5, 5)
        plt.xlabel('$x^{\{1\}}$', size=15)
        plt.ylabel('$x^{\{2\}}$', size=15)
        legend = plt.legend(frameon=1, fancybox=True, \
                             bbox_to_anchor=(1.0, -0.01), loc='lower right')
        frame = legend.get_frame()
        frame.set_facecolor('wheat')
        frame.set_edgecolor('None')

        fig.add_subplot(1, 2, 2)
        plt.title('2 class', fontsize=15)
        plt.plot(X, clf2.first_basis(X), color='r', label='LPP')
        plt.plot(X, f(X, pca2.components_), color='g', label='PCA')
        plt.scatter(train_x2[:, 0], train_x2[:, 1], color='b')
        plt.xlim(-5, 5)
        plt.ylim(-5, 5)
        plt.xlabel('$x^{\{1\}}$', size=15)
        plt.ylabel('$x^{\{2\}}$', size=15)

        legend = plt.legend(frameon=1, fancybox=True, \
                             bbox_to_anchor=(1.0, -0.01), loc='lower right')

```

```
frame = legend.get_frame()
frame.set_facecolor('wheat')
frame.set_edgecolor('None')
```

```
plt.subplots_adjust(top=0.85)
fig.savefig('ana.png')
fig.show()
```

/Users/koichiro/.pyenv/versions/anaconda3-2.4.1/lib/python3.5/site-packages/matplotlib/figure.py:397: UserWarning: matplotlib is currently using a non-GUI backend, so cannot show the figure  
"matplotlib is currently using a non-GUI backend, "

## comparison between LPP and PCA

