# Math Notes

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## Contents

# 1   Hyperbolic Functions

$$sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$cosh(x) = \frac{e^x + e^{-x}}{2}$$

# 2   Trigonometric Formulas

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

# 3   Arc functions

| Name | Usual notation | Definition | Domain | Range |
|---|---|---|---|---|
| **arcsine** | $y = \arcsin(x)$ | $x = \sin(y)$ | $[-1, 1]$ | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ |
| **arccosine** | $y = \arccos(x)$ | $x = \cos(y)$ | $[-1, 1]$ | $[0, \pi]$ |
| **arctangent** | $y = \arctan(x)$ | $x = \tan(y)$ | $\mathbb{R}$ | $(-\frac{\pi}{2}, \frac{\pi}{2})$ |

# 4   Cross Product

**Definition**   In 3-dimensional Euclidean space only, the cross product of vectors **a** and **b** is

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

**Remark**   ”xia, dafan, shang”

**Properties**

1. $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$

2. $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|sin\theta$. This says that the length $\mathbf{a} \times \mathbf{b}$ equals the area of the parallelogram generated by $\mathbf{a}$ and $\mathbf{b}$.

3. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

4. $(c_1\mathbf{a}_1 + c_2\mathbf{a}_2) \times \mathbf{b} = c_1\mathbf{a}_1 \times \mathbf{b} + c_2\mathbf{a}_2 \times \mathbf{b}$

5. $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ and $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ and $\mathbf{k} \times \mathbf{i} = \mathbf{j}$

6. *Not* associative: $(a \times b) \times c \neq a \times (b \times c)$

# 5   Derivative of Logarithmic Functions

$$\frac{d}{dx} \log_a x = \frac{1}{x \cdot ln(a)}$$

# 6   Common Taylor Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{1}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \tag{2}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \tag{3}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \tag{4}$$

**Remark**   Take the primitive of (4) to get the Taylor polynomial of $\ln(1-x)$.

# 7   $\varepsilon$ definition of supremum and infimum

**Definition**   Let S be a nonempty subset of the real numbers that is bounded above. The upper bound $u$ is said to be the supremum of S iff

$$\forall \varepsilon > 0, \exists x \in S, u - \varepsilon < x$$

**Definition**    Let S be a nonempty subset of the real numbers that is bounded below. The lower bound $w$ is said to be the infimum of S iff

$$\forall \varepsilon > 0, \exists x \in S, x < w + \varepsilon$$

# 8   Odd-Even Decomposition

For any continuous function $f$, $f$ can be decomposed into the sum of one even function and one odd function:

$$f(x) = \frac{1}{2}(f(x) + f(-x)) + \frac{1}{2}(f(x) - f(-x))$$

We can verify that the first part is even, and the second part is odd.