

# Math Notes

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## 1 Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

## 2 Trigonometric Formulas

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

## 3 Arc functions

## 4 Cross Product

**Definition** In 3-dimensional Euclidean space only, the cross product of vectors  $\mathbf{a}$  and  $\mathbf{b}$  is

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

**Remark** "xia, dafan, shang"

### Properties

1.  $\mathbf{a} \times \mathbf{b}$  is orthogonal to both  $\mathbf{a}$  and  $\mathbf{b}$
2.  $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$ . This says that the length  $\mathbf{a} \times \mathbf{b}$  equals the area of the parallelogram generated by  $\mathbf{a}$  and  $\mathbf{b}$ .
3.  $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
4.  $(c_1 \mathbf{a}_1 + c_2 \mathbf{a}_2) \times \mathbf{b} = c_1 \mathbf{a}_1 \times \mathbf{b} + c_2 \mathbf{a}_2 \times \mathbf{b}$

5.  $\mathbf{i} \times \mathbf{j} = \mathbf{k}$  and  $\mathbf{j} \times \mathbf{k} = \mathbf{i}$  and  $\mathbf{k} \times \mathbf{i} = \mathbf{j}$
6. *Not* associative:  $(a \times b) \times c \neq a \times (b \times c)$

## 5 Derivative of Logarithmic Functions

$$\frac{d}{dx} \log_a x = \frac{1}{x \cdot \ln(a)}$$

## 6 Common Taylor Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad (1)$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad (2)$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad (3)$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad (4)$$

**Remark** Take the primitive of (4) to get the Taylor polynomial of  $\ln(1-x)$ .

## 7 $\varepsilon$ definition of supremum and infimum

**Definition** Let  $S$  be a nonempty subset of the real numbers that is bounded above. The upper bound  $u$  is said to be the supremum of  $S$  iff

$$\forall \varepsilon > 0, \exists x \in S, u - \varepsilon < x$$

**Definition** Let  $S$  be a nonempty subset of the real numbers that is bounded below. The lower bound  $w$  is said to be the infimum of  $S$  iff

$$\forall \varepsilon > 0, \exists x \in S, x < w + \varepsilon$$