Math Notes

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1 Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

2 Trigonometric Formulas

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$

$$\cos^2 a = \frac{1+\cos 2a}{2}$$

$$\sin^2 a = \frac{1-\cos 2a}{2}$$

3 Arc functions

Name	Usual notation	Definition	Domain	Range
arcsine	$y = \arcsin(x)$	$x = \sin(y)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
arccosine	$y = \arccos(x)$	$x = \cos(y)$	$[-1, 1]$	$[0, \pi]$
arctangent	$y = \arctan(x)$	$x = \tan(y)$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

4 Cross Product

Definition In 3-dimensional Euclidean space only, the cross product of vectors **a** and **b** is

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Remark "xia, dafan, shang"

As a Determinant

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

Properties

1. $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$
2. $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| sin\theta$. This says that the length $\mathbf{a} \times \mathbf{b}$ equals the area of the parallelogram generated by $\mathbf{a}$ and $\mathbf{b}$.
3. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
4. $(c_1\mathbf{a}_1 + c_2\mathbf{a}_2) \times \mathbf{b} = c_1\mathbf{a}_1 \times \mathbf{b} + c_2\mathbf{a}_2 \times \mathbf{b}$
5. $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ and $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ and $\mathbf{k} \times \mathbf{i} = \mathbf{j}$
6. Not associative: $(a \times b) \times c \neq a \times (b \times c)$

5 Derivative of Logarithmic Functions

$$\frac{d}{dx} \log_a x = \frac{1}{x \cdot ln(a)}$$

6 Common Taylor Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{1}$$
$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \tag{2}$$
$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \tag{3}$$
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \tag{4}$$

Remark Take the primitive of (4) to get the Taylor polynomial of $\ln(1-x)$.

7 ε definition of supremum and infimum

Definition Let S be a nonempty subset of the real numbers that is bounded above. The upper bound u is said to be the supremum of S iff

$$\forall \varepsilon > 0, \exists x \in S, u - \varepsilon < x$$

Definition Let S be a nonempty subset of the real numbers that is bounded below. The lower bound w is said to be the infimum of S iff

$$\forall \varepsilon > 0, \exists x \in S, x < w + \varepsilon$$

8 Odd-Even Decomposition

For any continuous function f, f can be decomposed into the sum of one even function and one odd function:

$$f(x) = \frac{1}{2}(f(x) + f(-x)) + \frac{1}{2}(f(x) - f(-x))$$

We can verify that the first part is even, and the second part is odd.