Math Notes

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Contents

1 Hyperbolic Functions

$$sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$cosh(x) = \frac{e^x + e^{-x}}{2}$$

2 Trigonometric Formulas

$$cos(a+b) = cos(a)cos(b) - sin(a)sin(b)$$

$$sin(a+b) = cos(a)sin(b) + sin(a)cos(b)$$

3 Arc functions

4 Cross Product

Definition In 3-dimensional Euclidean space only, the cross product of vectors $\mathbf{a}$ and $\mathbf{b}$ is

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Remark "xia, dafan, shang"

Properties

1. $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$
2. $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|sin\theta$. This says that the length $\mathbf{a} \times \mathbf{b}$ equals the area of the parallelogram generated by $\mathbf{a}$ and $\mathbf{b}$.
3. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
4. $(c_1\mathbf{a}_1 + c_2\mathbf{a}_2) \times \mathbf{b} = c_1\mathbf{a}_1 \times \mathbf{b} + c_2\mathbf{a}_2 \times \mathbf{b}$
5. $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ and $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ and $\mathbf{k} \times \mathbf{i} = \mathbf{j}$
6. *Not* associative: $(a \times b) \times c \neq a \times (b \times c)$

5 Derivative of Logarithmic Functions

$$\frac{d}{dx}\log_a x = \frac{1}{x \cdot ln(a)}$$

6 Common Taylor Series

$$\begin{aligned}
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \\
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n
\end{aligned}$$