MAT237 Examples

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1. Let $S := [0, 100] \subset \mathbb{R}$, and let $f(x) = x^2$. Show f is uniformly continuous on S.
2. Let $S := [0, \infty) \subset \mathbb{R}$, and let $f(x) = x^2$. Show f is not uniformly continuous on S.
3. Prove that the equation

$$x^2 + y^2 = e^{(z-\frac{1}{2})^2 \cos(e^{-\sin(y/(x+2))})}$$

has a solution in the ball $B(2, \mathbf{0}) \subset \mathbb{R}^3$.
4. **"Moving the curve"**
Let C_1 be the set

$$\{(x, y) : x^2 + y^2 = 1, x \geq 0\}$$

oriented counterclockwise, and compute

$$\int_{C_1} 3x^2 y e^{x^3}\, dx + e^{x^3}\, dy$$

5. Let S be the set $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, z \geq 0\}$. Compute the area of S, as well as the integrals

$$\iint_S z\, dA, \text{ and } \iint_S (0, 0, z) \cdot \mathbf{n}\, dA$$

where the $\mathbf{n}$ oriented upwards.
6. Let $S := \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ with the unit normal $\mathbf{n}$ pointing outward, and compute

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dA, \quad \text{for } \mathbf{F} = (x + y, x - y\sin(yz), z\sin(yz))$$

7. **"Moving the surface"**
Let $S := \{(x, y, z) : x^2 + y^2 \leq 1, z = 1 - x^2 - y^2\}$ with the unit normal

oriented upward, and compute

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dA \quad \text{for } \mathbf{F} = (e^{yz}, \frac{x^2}{1+z^2}, 2z)$$

8.

$$F(x,y,z) := xy + xz\ln(yz) = 1$$

Note that $(1,1,1)$ is a solution. Does the equation implicitly determine z as a function $f(x,y)$ for (x,y) near $(1,1)$, with $f(1,1)=1$? If so, find a formula for $\partial_x f(x,y)$, and evaluate it at $(x,y)=(1,1)$.

9. Consider the system of equations

$$\begin{aligned} F_1(x,y,u,v) &= xye^u + \sin(v-u) = 0 \\ F_2(x,y,u,v) &= (x+1)(y+2)(u+3)(v+4) - 24 = 0 \end{aligned}$$

Note that $(0,0,0,0)$ is a solution.
Does the equation implicitly determine (u,v) as a function $\mathbf{f}$ of (x,y) near $(0,0)$, with $\mathbf{f}(0,0)=(0,0)$? If so, find a formula for $\partial_x\mathbf{f}(x,y)$ at $(x,y)=(0,0)$.