The Kalman filter and the square root Kalman filter

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Introduction

A linear Gaussian state space model is defined by the following two equations.

$$x_t = Fx_{t-1} + Eu_t + w_t$$

and

$$y_t = Hx_t + v_t,$$

where x_t is the $(k \times 1)$ state vector , y_t is the $(l \times 1)$ observation vector, u_t is the $(n \times 1)$ exogenous vector, v_t is the $(k \times 1)$ state noise, $w(t)$ is the $(l \times 1)$ observation noise, t is a time index. The Kalman Filter is used to estimate the state vector x_t given the observations y_t.

The Kalman filter is implemented using the following recursion:

1. Prediction step:
    1. $x_{t|t-1} = Fx_{t-1|t-1} + Eu_t$
    2. $P_{t|t-1} = FP_{t-1|t-1}F^t + V$
2. Innovation step:
    1. $e_t = y_t - Hx_{t|t-1}$
    2. $s_t = HP_{t-1|t-1}H^t + W$
3. Update step:
    1. $K_t = P_{t|t-1}H^t s_t^{-1}$
    2. $x_{t|t} = x_{t|t-1} + K_t e_t$