

# The Kalman filter and the square root Kalman filter

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## Introduction

### The Kalman filter and the square root Kalman filter

#### A linear Gaussian state space model

A linear Gaussian state space model is defined by the following two equations.

$$x_t = Fx_{t-1} + Eu_t + w_t$$

and

$$y_t = Hx_t + v_t,$$

where  $x_t$  is the  $(k \times 1)$  state vector,  $y_t$  is the  $(l \times 1)$  observation vector,  $u_t$  is the  $(n \times 1)$  exogenous vector,  $v_t$  is the  $(k \times 1)$  state noise,  $w(t)$  is the  $(l \times 1)$  observation noise,  $t$  is a time index. The Kalman Filter is used to estimate the state vector  $x_t$  given the observations  $y_t$ .

The Kalman filter is implemented using the following recursion:

1. Prediction step:

$$1. x_{t|t-1} = Fx_{t-1|t-1} + Eu_t$$

$$2. P_{t|t-1} = FP_{t-1|t-1}F^t + V$$

2. Innovation step:

$$1. e_t = y_t - Hx_{t|t-1}$$

$$2. s_t = HP_{t-1|t-1}H^t + W$$

3. Update step:

1.  $K_t = P_{t|t-1} H^t s_t^{-1}$
2.  $x_{t|t} = x_{t|t-1} + K_t e_t$
3.  $P_{t|t} = (I - K_t H) P_{t|t-1}$

where  $x_{t|t}$  is the estimated state vector at,  $P_t$  is the estimated state covariance matrix, and  $K_t$  is the Kalman gain at time t. See Kitagawa, (2010) for more details.

### The square root Kalman filter

The square root Kalman filter is an alternative to the Kalman filter that is numerically more stable. The square root Kalman filter is implemented using the following recursion. Tracy (2022) proposes the following recursion for the square root Kalman filter only using QR decompositions (The QR Kalman filter).

The QR Kalman filter is implemented using the following recursion

1. Prediction step

1.  $x_{t|t-1} = F x_{t-1|t-1} + E u_t$
2.  $\Sigma_{t|t-1} = gr_r(\Sigma_{t-1|t-1} F^t, \Gamma_v)$

2. Innovation step:

1.  $e_t = y_t - H x_{t|t-1}$
2.  $G_t = gr_r(\Sigma_{t|t-1} H^t, \Gamma_w)$

3. Update step:

1.  $K_t = [G_t^{-1} (G_t^{-t} H) \Sigma_{t|t-1}^t (\Sigma_{t|t-1})^t]^t$
2.  $x_{t|t} = x_{t|t-1} + K_t e_t$
3.  $\Sigma_{t|t} = gr_r(\Sigma(t|t-1)(I - K(t)H)^t, \Gamma_w K(t)^t)$

### Running Code

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1 + 1
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[1] 2
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You can add options to executable code like this

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