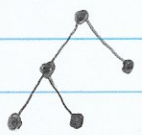
CS204: HW#6 20160051 Ohjun Kwon

1. B. let λ: empty string
   $\lambda \in X$
   R1. If $x \in X$, then $x1 \in X$
   R2. If $x, y \in X$, then $x0y0 \in X$

2. B. $0 \in X$
   R. If $x \in X$, then $x+2, x-2 \in X$

3. a)  b) 

4. a) $f(0,n) = n+1$

$f(1,n) = f(0, f(1,n-1)) = f(1,n-1)+1$
$= f(1,0)+n = f(0,1)+n$
$= n+2$

$f(2,n) = f(1, f(2,n-1)) = f(2,n-1)+2$
$= f(2,0)+2n = f(1,1)+2n$
$= 2n+3$ ..... (1)

$f(3,n) = f(2, f(3,n-1)) = 2f(3,n-1)+3$

$f(3,4) = 2f(3,3)+3$
$= 2(2f(3,2)+3)+3 = 4f(3,2)+9$
$= 4(2f(3,1)+3)+9 = 8f(3,1)+21$
$= 8(2f(3,0)+3)+21 = 16f(3,0)+45$
$= 16f(2,1)+45 = 16 \cdot 5+45$
$= \underline{125}$

b) $f(3,n) = 2^{n+3} - 3$

pf) Base case $n=0$
$f(3,0) = f(2,1) = 5$
$2^{0+3} - 3 = 8-3 = 5$

Inductive case
I.H: $f(3,n) = 2^{n+3} - 3$ for $n=k$
For $n=k+1$:

$f(3,k+1) = f(2, f(3,k))$
$= f(2, 2^{k+3}-3)$ ($\because$ I.H.)
$= 2(2^{k+3}-3)+3$ ($\because$ (1))
$= 2^{(k+1)+3} - 6+3$
$= 2^{(k+1)+3} - 3$

5. Base case $m=n=1$

$f(1,1)=5$

$2(1+1)+1=5$ ✓

Inductive case

I.H.; $f(m,n)=2(m+n)+1$ for $\forall m,n$ s.t. $(1,1)\le(m,n)\le(a,b)$

$\Rightarrow a>1$ or $(a=1$ and $b>1)$ (*)

i) $(a+1,b)$

i-1) $b=1$

$\Rightarrow a>1$ (*)

$f(a+1,b)=f(a,b)+2$ ($\because b=1$ and $a>1$)

$=2(a+b)+1+2$ ($\because$ I.H.)

$=2((a+1)+b)+1$ ✓

i-2) $b>1$

$f(a+1,b)=f(a+1,b-1)+2\cdot 1$

$=f(a+1,b-2)+2\cdot 2$

$\vdots$

$=f(a+1,1)+2(b-1)$

$=f(a,1)+2b$

$=2(a+1)+1+2b$ ($\because$ I.H. $(1,1)\le(a,1)\le(a,b)$)

$=2((a+1)+b)+1$ ✓

ii) $(a,b+1)$

ii-1) $b=0$

$\Rightarrow a>1$ (*)

$f(a,b+1)=f(a-1,b+1)+2\cdot 1$ ($\because b+1=1$)

$=2(a-1+b+1)+1+2$ ($\because$ I.H. $(1,1)\le(a-1,b+1)\le(a,b)$)

$=2(a+(b+1))+1$ ✓

ii-2) $b>0$

$f(a,b+1)=f(a,b)+2$ ($\because b+1>1$)

$=2(a+b)+1+2$ ($\because$ I.H.)

$=2(a+(b+1))+1$ ✓

iii) $(a+1,b+1)$

iii-1) $b=0$

$\Rightarrow a>1$ (*)

$f(a+1,b+1)=f(a,b+1)+2$ ($\because a+1>1$ and $b+1=1$)

$=f(a-1,b+1)+2\cdot 2$ ($\because a>1$ and $b+1=1$)

$=2(a+b)+1+2\cdot 2$ ($\because$ I.H. $(1,1)\le(a-1,b+1)\le(a,b)$)

$=2((a+1)+(b+1))+1$ ✓

$\Rightarrow$ Cont.

5. (cont.) iii-2) $b>0$ $(a>1$ or $(a=1$ and $b>1))$

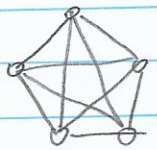
$f(a+1, b+1) = f(a, b+1) + 2$ $(\because b+1>1)$

$= f(a-1, b+1) + 2\cdot 2$ $(\because b+1>1)$

$= 2(a+b)+1+2\cdot 2$ $(\because$ I.H. $(1,1) \le (a-1, b+1) \le (a,b))$

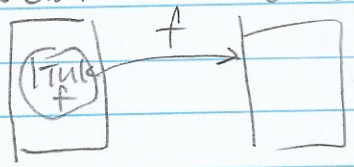
$= 2((a+1)+(b+1))+1$ ✓ ∎

6.



10 edges.

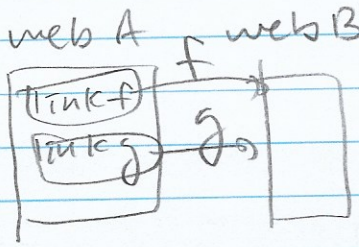
7. a) Yes, it is a directed graph. If we click a link on the website, it connects to the other webpage. Therefore, we can think a link as an edge which has a direction property.

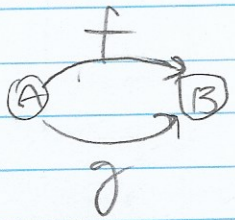


b) No. If we make webpages which have no links on them, the graph has nodes which are not connected any pages.

c) No, we can make a disconnected graph. which is not a complete graph. (b)

d) No, we can make a webpage that have multiple links that connect to the same page.

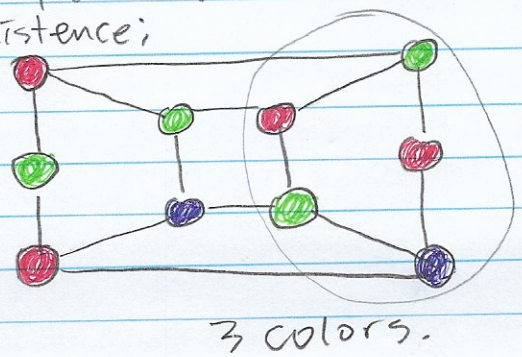




e) The outdegree of p means the number of links on page p.

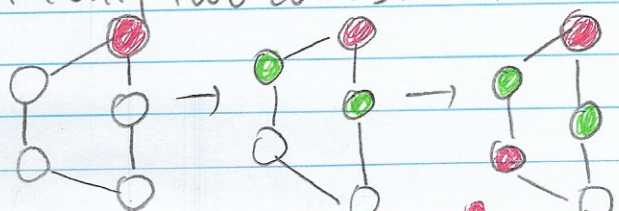
f) The indegree of p represents the number of links that can be used to access page p.

8. existence;

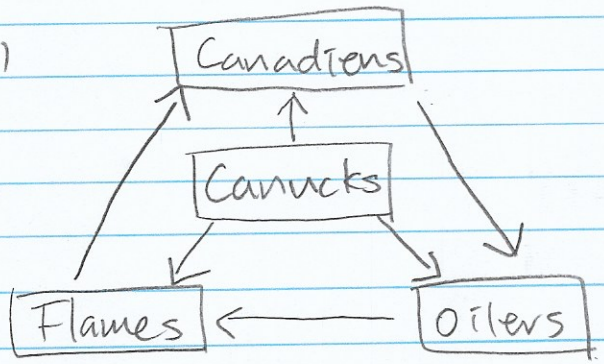


minimality;

Assume that we can color the graph with only two colors.



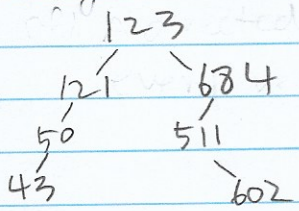
9. a)



b) Canadiens → Oilers → Flames → Canadiens

c) we cannot decide ranks within the circuit because there is always a team which have defeated them regardless of what team we picked.

10. a)



b) 3, For example, we have to visit 3 edges if we want to find 43 in the tree. which is the deepest (that has a maximum number of edges to reach there from the root) element in the tree.

c) 123, 50, 602, 43, 121, 511, 684