

CS204: Homework #9

Due on December 15, 2016 at 11:59pm

Prof. Sungwon Kang — Section A

20160051 Ohjun Kwon

1. The overall participants that are going to win the prize will be 3 people out of 100 participants. Therefore, the probability that a participant of the contest wins one of prizes will be $\underline{3/100}$.

2-(a) we can think the problem opposite way. The probability of rolling no sixes when a die is rolled four times is $(\frac{5}{6})^4$. Therefore, the probability of the complement event is $\underline{1 - (\frac{5}{6})^4}$. $\therefore 1 - (\frac{5}{6})^4 = \frac{671}{1296} = 0.518$

(b) The probability that a double six comes up when a pair of dices is $(1/36)$. Then, the probability that no double six comes up when a pair of dices 24 times is $(35/36)^{24}$. Therefore, the probability we want to calculate is the complement event of the event that we have calculated previously. $\therefore 1 - (\frac{35}{36})^{24} = 0.491$.

i It is less than $1/2$.

(c) Yes. $0.518 > 0.491$.

3-(a) First, we can think the total permutation of a, a, 2, 3, which is $4!/2!$. Then, we can put 1 and 4 in the position of a's in order of 1, 4, because the statement says "1 precedes 4". For example, 3a2a makes the sequence 3124. $\therefore \underline{12}$

(b) we can think this problem in same way as the 3-(a). If we put 4 and 1 consecutively in the a's positions, we can get sequences that satisfies the given statement. $\therefore \underline{12}$

(c) we can think this problem as picking 3's position in -4-a-a-, and put 1 and 2 in a's position randomly. Therefore, there is three possible 3's seats, and two ways to put 1 and 2. $\therefore 3 \times 2 = \underline{6}$

(d) we can think this problem as putting 4 in front of the random permutations of {1, 2, 3}. $\therefore 3! = \underline{6}$

(e) we can think this problem as putting 4 and 3 consecutively in a's positions, and 2 and 1 consecutively in b's positions in the permutations of aabb. For example, $\underline{2}$ \rightarrow

(cont.) if we get abba out of aabb, it means 4213.

$$\therefore \frac{4!}{2!2!} = 6$$

4-(a) $(1/2)^5 = 1/32 = 0.03125$

(does not have a boy = all girls)

(b) $(1-0.1)^5 = 0.9^5 = 0.59049$

↑ prob. of girl

(c) $\prod_{i=1}^5 (1 - (0.51 - \frac{i}{100})) = (1-0.5)(1-0.49)(1-0.43)(1-0.47)(1-0.46)$

↑ prob. of boy

↑ prob. of girl

~ 5 children

$$= 0.5 \times 0.51 \times 0.52 \times 0.53 \times 0.54 = 0.03795012$$

5. $E[\text{Score of T/F questions}]$

$$= 50 \times (\text{Score of a T/F question})$$

\times (prob. of answering a T/F question correctly)

$$= 50 \times 2 \times 0.9 = 90$$

$E[\text{Score of multiple-choice questions}]$

$$= 25 \times (\text{Score of a m-c ques.})$$

\times (prob. of answering correctly)

$$= 25 \times 4 \times 0.8 = 80$$

$$\therefore E[\text{Total score}] = 90 + 80 = 170$$

6. $E[X] = 1 \times 6/36 + 2 \times 6/36 + 3 \times 6/36 + 4 \times 6/36 + 5 \times 6/36 + 6 \times 6/36$

$$= 7/2$$

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)	Y	2	3	4	5	6	7
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)		3	4	5	6	7	8
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)		4	5	6	7	8	9
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)		5	6	7	8	9	10
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)		6	7	8	9	10	11
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)		7	8	9	10	11	12

XY

							Y	2	3	4	5	6	7	8	9	10	11	12
2	6	12	20	30	42	36P(Y)	1	2	3	4	5	6	5	4	3	2	1	

3	8	15	24	35	48	XY	2	3	4	5	6	7	8	10	12	14	15	16
---	---	----	----	----	----	----	---	---	---	---	---	---	---	----	----	----	----	----

4	10	18	28	40	54	36P(XY)	1	1	1	2	1	1	1	2	1	1	1	
---	----	----	----	----	----	---------	---	---	---	---	---	---	---	---	---	---	---	--

5	12	21	32	45	60	XY	18	21	24	28	30	32	35	36	40			
---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	--	--	--

6	14	24	36	50	66	36P(XY)	1	1	1	1	1	1	1	1	1	2		
---	----	----	----	----	----	---------	---	---	---	---	---	---	---	---	---	---	--	--

7	16	27	40	55	72	XY	42	45	48	50	54	55	60	66	72			
---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	--	--	--

36P(XY)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
---------	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

(cont) $E[Y] = \sum y p(Y=y) = 7$

$$E[XY] = \sum k p(XY=k) = 916/36 = 229/9$$

$$E[X]E[Y] = \frac{7}{2} \times 7 = \frac{49}{2} \neq \frac{229}{9} = E[XY]$$

7. $X = \#$ of boys

prob. of given event = $P(X=2 | (X=1 \cup X=2))$ at least one boy.

$$= \frac{P(X=2 \cap (X=1 \cup X=2))}{P(X=1 \cup X=2)} = \frac{P(X=2)}{P(X=1 \cup X=2)}$$

$$= \frac{1/4}{3/4} = \left(\frac{1}{3}\right)$$

(b,b)
(b,g)
(g,b)

8. X : answered the question correctly

Y : knew the answer

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X|Y)P(Y) + P(X|Y^c)P(Y^c)}$$

$$= \frac{1 \cdot p}{1 \cdot p + (1/m) \cdot (1-p)} = \frac{mp}{mp + 1-p}$$

9. X : test result is positive

Y : a person has the disease

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X|Y)P(Y) + P(X|Y^c)P(Y^c)}$$

$$= \frac{0.95 \cdot 0.05}{0.95 \cdot 0.05 + 0.01 \cdot 0.95}$$

$$= \frac{0.0475}{0.0475 + 0.0095}$$

$$= \frac{0.0475}{0.057} = \left(\frac{5}{6}\right) \quad \text{approx. } 83.3\%$$

10.

HHHH	THTT
HHHT	TTTH
HHHT	TTHT
HTTT	TTTH
HTHH	TTTT
HTHT	
HTTH	
HTTT	
THTT	
THTH	
TTTH	

$$\frac{6}{16} = \left(\frac{3}{8}\right)$$

(f) $\frac{\binom{4}{2}}{2^4} = \frac{6}{16} = \left(\frac{3}{8}\right)$

choosing two seats for H or T

choosing 4 from {H, T}