CS 204	Discrete Mathematics	Fall 2016
	Homework 1 (70 pts)	
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1. (8 pts) Let the following statements be given.

   p = "There is water in the cylinders."

   q = "The head gasket is blown."

   r = "The car will start."

   (a) Translate the following statement into symbols of formal logic.

   If the head gasket is blown and there's water in the cylinders, then the car won't start.

   (b) Translate the following statement into everyday English.

   $r \rightarrow \neg(q \vee p)$

2. (8 pts) Let the following statements be given.

   p = "You are in Seoul."

   q = "You are in Kwangju."

   r = "You are in South Korea."

   (a) Translate the following statement into symbols of formal logic.

   If you are not in South Korea, then you are not in Seoul or Kwangju.

   (b) Translate the following statement into everyday English.

   $q \rightarrow (r \vee \neg p)$

3. (5 pts) Use truth tables to show that $(a \vee b) \wedge (\neg(a \wedge b))$ is logically equivalent to $a \leftrightarrow \neg b$.

4. (5 pts) Mathematicians say that "Statement P is a sufficient condition for statement Q" if $P \rightarrow Q$ is true. In other words, in order to know that Q to be true, it is necessary to know that P is true. Let x be an integer. Give a sufficient condition on x for x/2 to be an even integer.

5. (5 pts) The NAND connective $\uparrow$ is defined by the following truth table.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

Use truth tables to show that $p \uparrow q$ is logically equivalent to $\neg (p \wedge q)$. (This explains the name NAND: Not AND.)

6. (12 pts) The NAND connective is important because it is easy to build an electronic circuit that computes the NAND of two signals. Such a circuit is called a logic gate. Moreover, it is possible to build logic gates for the other logical connectives entirely out of NAND gates. Prove this fact by proving the following equivalences, using truth tables.

(a) $(p \uparrow q) \uparrow (p \uparrow q)$ is logically equivalent to $p \wedge q$.

(b) $(p \uparrow p) \uparrow (q \uparrow q)$ is logically equivalent to $p \vee q$

(c) $p \uparrow (q \uparrow q)$ is logically equivalent to $p \rightarrow q$.

7. (5 pts) Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statement	Reasons
1. $p \wedge (q \vee r)$	given
2. $\neg(p \wedge q)$	given
3. $\neg p \vee \neg q$	
4. $\neg q \vee \neg p$	
5. $q \to \neg p$	
6. p	
7. $\neg(\neg p)$	
8. $\neg q$	
9. $(q \vee r) \wedge p$	
10. $q \vee r$	
11. $r \vee q$	
12. $\neg(\neg r) \vee q$	
13. $\neg r \to q$	
14. $\neg(\neg r)$	
15. r	
16. $p \wedge r$	

8. (12 pts) Write a proof sequence for the following assertion. Justify each step.

$$\left.\begin{array}{l} p \\ p \to r \\ q \to \neg r \end{array}\right\} \Rightarrow \neg q$$

9. (10 pts) Is $a \to \neg a$ a contradiction? Why or why not?