CS204: Homework #2

Due on September 29, 2016 at 11:59pm

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Problem 1

(a) All penguins are dangerous.
(b) Some penguins are dangerous.
(c) There is no penguin that is dangerous.
(d) Some penguins are not dangerous.

Problem 2

(a) $(\forall x)(L(x) \to F(x))$
(b) $(\forall x)(L(x) \land F(x))$

Problem 3

(a) $(\exists x)P(x)$
(b) $(\forall x)(P(x) \to \neg Q(x))$
(c) $(\exists x)(\neg P(x) \land \neg Q(x))$

Problem 4

(a) $(\forall x)(\exists y)(N(x) \to P(x, y))$
(b)

$$\begin{aligned}
\neg\forall x\exists y(N(x) \to P(x, y)) &\Leftrightarrow \exists x\neg\exists y(N(x) \to P(x, y)) && (1)\\
&\Leftrightarrow \exists x\forall y\neg(N(x) \to P(x, y)) && (2)\\
&\Leftrightarrow \exists x\forall y\neg(\neg N(x) \lor P(x, y)) && (3)\\
&\Leftrightarrow \exists x\forall y(\neg\neg N(x) \land \neg P(x, y)) && (4)\\
&\Leftrightarrow \exists x\forall y(N(x) \land \neg P(x, y)) && (5)
\end{aligned}$$

(1): universal negation
(2): existential negation
(3): material implication
(4): De Morgan's law
(5): double negation

(c) $\exists x\forall y(N(x) \land \neg P(x, y))$
There is some integer x such that $x \neq 0$ and $xy \neq 1$ for all integers y.
(d) **(b)** is true.
Proof
case **(a)**: for all integers x, y that satisfies $xy = 1$ is uniquely exists as $y = 1/x$, which does not belong to the integer set. The statement satisfies When x is -1 or 1, but it does not cover all integers.
case **(b)**: if we select 2 as x, then it satisfies two statement, $x \neq 0$ and $xy \neq 1$, for any integer y.

Problem 5

(a) For all traders who work at the Tokyo Stock Exchange, there exists a trader that makes less money than him/her.
(b) There exists a trader A, that makes more money than any trader other than A.

Problem 5 continued on next page...

(c) **(a)** is an impossible statement, because there should be a minimum in a finite set(traders who work at the Tokyo Stock Exchange). Then, if we pick a minimum, then there will be no traders that satisfy the statement. Therefore, the minimum is the counter-example of the statement, and it proves that the **(a)** is a false statement. In the similar context, if we pick a trader who has a maximum income for A, there will be no trader who make more money other than A. It satisfies the statement **(b)**.

Problem 6

Domain: $\mathbb{R}$
$R(x)$: x is rational.

Given statement: $\forall x \forall y((R(x) \wedge \neg R(y)) \rightarrow \neg R(x+y))$
Negation:

$$\begin{aligned}
\neg \forall x \forall y((R(x) \wedge \neg R(y)) \rightarrow \neg R(x+y)) &\Leftrightarrow \exists x \neg \forall y((R(x) \wedge \neg R(y)) \rightarrow \neg R(x+y)) && (6)\\
&\Leftrightarrow \exists x \exists y \neg((R(x) \wedge \neg R(y)) \rightarrow \neg R(x+y)) && (7)\\
&\Leftrightarrow \exists x \exists y \neg(\neg(R(x) \wedge \neg R(y)) \vee \neg R(x+y)) && (8)\\
&\Leftrightarrow \exists x \exists y(\neg\neg(R(x) \wedge \neg R(y)) \wedge \neg\neg R(x+y)) && (9)\\
&\Leftrightarrow \exists x \exists y((R(x) \wedge \neg R(y)) \wedge R(x+y)) && (10)\\
&\Leftrightarrow \exists x \exists y(R(x) \wedge \neg R(y) \wedge R(x+y)) && (11)
\end{aligned}$$

(6): universal negation
(7): universal negation
(8): material implication
(9): De Morgan's law
(10): double negation
(11): associativity

Problem 7

(a)

$$\begin{aligned}
\neg(\exists x)(R(x) \wedge B(x)) &\Leftrightarrow (\forall x) \neg(R(x) \wedge B(x)) && (12)\\
&\Leftrightarrow (\forall x)(\neg R(x) \vee \neg B(x)) && (13)\\
&\Leftrightarrow (\forall x)(R(x) \rightarrow \neg B(x)) && (14)
\end{aligned}$$

(12): existential negation
(13): De Morgan's law
(14): material implication

(b) There is no triangle that is a right triangle and also has an obtuse angle.
(c) For all triangles x, if x is a right triangle, then x does not have an obtuse angle.

Problem 8

(a) I will give a example by using predicates from **Problem 7**.
Domain: triangles
Predicates:

Problem 8 continued on next page...

$R(x)=$ x is a right triangle.
$B(x)=$ x has an obtuse angle.

$(\exists x)(P(x) \wedge Q(x))$: There exists a triangle that is a right triangle and also has an obtuse angle.
$(\exists x)P(x) \wedge (\exists x)Q(x)$: There exists a triangle that is a right triangle, and there also exists a triangle that has an obtuse angle.

The first statement is false, because if a triangle has both right and obtuse angle, the sum of the interior angles exceeds 180°. However, as we know, the sum of the interior angles of a triangle should be 180°. For the second sentence, of course, there exists a right triangle, and a triangle that has an obtuse angle. Therefore, the second statement is true. The result of the statements are different, so they are not logically equivalent.

(b) The given two statements can be translated into ordinary English as below.
$(\exists x)(P(x) \vee Q(x))$: There exists a triangle that is a right triangle or has an obtuse angle.
$(\exists x)P(x) \vee (\exists x)Q(x)$: There exists a triangle that is a right triangle, or there exists a triangle that has an obtuse angle.

The first and second sentences can be satisfied either a right triangle or a triangle that has an obtuse angle.

Problem 9

(a) $\exists x T(x), \forall x(T(x) \to P(x)) \vdash \exists y(T(y) \wedge P(y))$

$$
\begin{array}{r|llll}
1 & \exists x T(x) & & & \\
2 & \forall x(T(x) \to P(x)) & & & \\
\hline
3 & a & \vert\ T(a) \to P(a) & & \forall\text{E}, 2 \\
4 & & \vert\ \quad \vert\ T(a) & & \\
5 & & \vert\ \quad \vert\ P(a) & & \to\text{E}, 3, 4 \\
6 & & \vert\ \quad \vert\ T(a) \wedge P(a) & & \wedge\text{I}, 4, 5 \\
7 & & \vert\ T(a) \wedge P(a) & & \exists\text{E}, 1, 4\text{–}6 \\
8 & \exists y(T(y) \wedge P(y)) & & & \exists\text{I}, 7
\end{array}
$$

(b) $\forall x(P(x) \wedge Q(x)) \vdash \forall x P(x) \wedge \forall y Q(y)$

$$
\begin{array}{r|lll}
1 & \forall x(P(x) \wedge Q(x)) & & \\
\hline
2 & t & \vert\ P(t) \wedge Q(t) & \forall\text{E}, 1 \\
3 & & \vert\ P(t) & \wedge\text{E}, 2 \\
4 & & \vert\ Q(t) & \wedge\text{E}, 2 \\
5 & \forall x P(x) & & \forall\text{I}, 3 \\
6 & \forall y Q(y) & & \forall\text{I}, 4 \\
7 & \forall x P(x) \wedge \forall y Q(y) & & \wedge\text{I}, 5, 6
\end{array}
$$