CS204: HW#9 20160051 OhJun Kwon

1. The overall participants that are going to win the prize will be 3 people out of 100 participants. Therefore, the probability that a participant of the contest wins one of prizes will be $\underline{3/100}$.

2-(a) we can think the problem opposite way. The probability of rolling no sixes when a die is rolled four times is $(\frac{5}{6})^4$. Therefore, the probability of the complement event is $\underline{1-(\frac{5}{6})^4}$. $\therefore 1-(\frac{5}{6})^4 = \frac{671}{1296} = 0.518$

(b) The probability that a double six comes up when a pair of dices is (1/36). Then, the probability tha no double six comes up when a pair of dices 24 times is $(35/36)^{24}$. Therefore, the probability we want to calculate is the complement event of the event that we have calculated previously. $\therefore 1-(\frac{35}{36})^{24} = 0.491$.

; It is less than 1/2

(c) Yes. $0.518 > 0.491$.

3-(a) First, we can think the total permutation of a,a,2,3, which is $4!/2!$. Then, we can put 1 and 4 in the position of a's in order of 1,4, because the statement says "1 precedes 4". For example, 3a2a makes the sequence 3124. $\therefore \underline{12}$

(b) we can think this problem in same way as the 3-(a). If we put 4 and 1 consecutively in the a's positions, we can get sequences that satisfies the given statement. $\therefore \underline{12}$

(c) we can think this problem as picking 3's position in _4_a_a_, and put 1 and 2 in a's position randomly. Therefore, there is three possible 3's seats, and two ways to put 1 and 2. $\therefore 3\times 2 = \underline{6}$

(d) we can think this problem as putting 4 in front of the random parmutations of $\{1,2,3\}$ $\therefore 3! = \underline{6}$

(e) We can think this problem as putting 4 and 3 consecutively in a's postitions, and 2 and 1 consecutively in b's postitions in the permutations of aabb For examp →

(cont.) if we get abba out of aabb, it means 4213.

$\therefore \frac{4!}{2!2!} = \underline{6}$

4-(a) $(1/2)^5 = 1/32 = 0.03125$

(does not have a boy = all girls)

(b) $(\underbrace{1-0.1}_{\text{prob. of girl}})^5 = 0.9^5 = 0.59049$

(c) $\underbrace{\prod_{i=1}^{5}}_{\text{in 5 children}} \underbrace{\left(1-\underbrace{\left(0.51-\frac{i}{100}\right)}_{\text{prob. of boy}}\right)}_{\text{prob. of girl}} = (1-0.5)(1-0.49)(1-0.48)(1-0.47)(1-0.46)$

$= 0.5 \times 0.51 \times 0.52 \times 0.53 \times 0.54$

$= 0.03795012$

5. E[(score of T/F questions)]

= 50 × (score of a T/F question)

× (prob. of answering a T/F question correctly)

$= 50 \times 2 \times 0.9 = 90$.

E[(score of multiple-choice questions)]

= 25 × (score of a m-c ques.)

× (prob. of answering correctly)

$= 25 \times 4 \times 0.8 = 80$

$\therefore$ E[(total score)] $= 90 + 80 = \underline{170}$

6. $E[X] = 1 \times 6/36 + 2 \times 6/36 + 3 \times 6/36 + 4 \times 6/36 + 5 \times 6/36 + 6 \times 6/36$

$= \underline{7/2}$

(1 1)	(2 1)	(3 1)	(4 1)	(5 1)	(6 1)
(1 2)	(2 2)	(3 2)	(4 2)	(5 2)	(6 2)
(1 3)	(2 3)	(3 3)	(4 3)	(5 3)	(6 3)
(1 4)	(2 4)	(3 4)	(4 4)	(5 4)	(6 4)
(1 5)	(2 5)	(3 5)	(4 5)	(5 5)	(6 5)
(1 6)	(2 6)	(3 6)	(4 6)	(5 6)	(6 6)

Y

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

XY

2	6	12	20	30	42
3	8	15	24	35	48
4	10	18	28	40	54
5	12	21	32	45	60
6	14	24	36	50	66
7	16	27	40	55	72

Y	2	3	4	5	6	7	8	9	10	11	12
36p(Y)	1	2	3	4	5	6	5	4	3	2	1

XY	2	3	4	5	6	7	8	10	12	14	15	16
36p(XY)	1	1	1	1	2	1	1	1	2	1	1	1

XY	18	21	24	28	30	32	35	36	40
36p(XY)	1	1	1	1	1	1	1	1	2

XY	42	45	48	50	54	55	60	66	72
36p(XY)	1	1	1	1	1	1	1	1	1

(cont) $E[Y] = \sum y\, p(Y=y) = 7$

$E[XY] = \sum k\, p(XY=k) = 916/36 = 229/9$

$E[X]E[Y] = \frac{7}{2} \times 7 = \frac{49}{2} \neq \frac{229}{9} = E[XY]$

7. $X = \#$ of boys

prob. of given event $= P(X=2 \mid \underbrace{(X=1)\cup(X=2)}_{\text{at least one boy}})$

$$= \frac{P(X=2 \cap (X=1 \cup X=2))}{P((X=1)\cup(X=2))} = \frac{P(X=2)}{P(X=1 \cup X=2)}$$

$P(X=2)$: (b,b)

$P(X=1 \cup X=2)$: (b,b) (b,g) (g,b)

$$= \frac{1/4}{3/4} = \frac{1}{3}$$

8. X: answered the question correctly

Y: knew the answer

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X|Y)P(Y) + P(X|Y^c)P(Y^c)}$$

$$= \frac{1\cdot p}{1\cdot p + (1/m)\cdot(1-p)} = \frac{mp}{mp+1-p}$$

9. X: test result is positive

Y: a person has the disease

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X|Y)P(Y) + P(X|Y^c)P(Y^c)}$$

$$= \frac{0.95\cdot 0.05}{0.95\cdot 0.05 + 0.01\cdot 0.95}$$

$$= \frac{0.0475}{0.0475 + 0.0095}$$

$$= \frac{0.0475}{0.057} = \frac{5}{6}$$ approx. 83.3%

10.

HHHH	THTT
HHHT	$\boxed{\text{TTHH}}$
HHTH	TTHT
$\boxed{\text{HHTT}}$	TTTH
HTHH	TTTT
$\boxed{\text{HTHT}}$	
$\boxed{\text{HTTH}}$	
HTTT	
THHH	
$\boxed{\text{THHT}}$	
$\boxed{\text{THTH}}$	

$\therefore \frac{6}{16} = \frac{3}{8}$

(cf) $\frac{\overbrace{{}_4C_2}^{\text{choosing two seats for H or T}}}{\underbrace{{}_2\Pi_4}_{\text{choosing 4 from \{H,T\}}}} = \frac{6}{2^4} = \frac{3}{8}$