CS204 - HW#4

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1. (a) $A \cup B = \{2,3,4,5,6\}$
   $(A \cup B)' = \{1,7,8,9\}$
   (b) $(A \cap B) = \{3,4\}$
   $(A \cap B) \times A = \{(3,2),(3,3),(3,4),(4,2),(4,3),(4,4)\}$
   (c) $B \setminus A = \{5,6\}$
   $P(B \setminus A) = \{\emptyset, \{5\}, \{6\}, \{5,6\}\}$

2. (a) $|F \cap M'| > |S \cap C|$
   (b) All freshmen that are math major are CS majors.

3. A, B are disjoint $\iff A \cap B = \emptyset$
   $\iff |A \cap B| = 0$ ($\because$ def of $\emptyset$)
   ($|A \cup B| = |A| + |B| - |A \cap B|$; inclusion-exclusion principle)
   $\iff |A \cup B| = |A| + |B|$ ■

4. For example, for $X = \{1,2,3\}$ in $X \times X$ $(1,2)$ and $(2,1)$ are different elements but in $P2 = \{S \in P(X) \mid |S| = 2\}$ $\{1,2\}$ and $\{2,1\}$ are considered as a same elements, because they are the sets and sets do not have an order among the elements.
   Therefore, obviously, $|P1| \geq |P2|$

5. In order to be f to be total function, all of the domain of function f, which is P, should be defined, i.e. should have a value $f(p)$ for all $p \in P$.

6. (a) No.
   counter example) $\{1,2,5\}, \{1,5\} \in P(S)^*$
   $m(\{1,2,5\}) = 5 = m(\{1,5\})$
   (b) Yes. $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\} \in P(S)^*$
   $m(\{0\}) = 0$
   $m(\{1\}) = 1$
   $m(\{2\}) = 2$
   $m(\{3\}) = 3$
   $m(\{4\}) = 4$
   $m(\{5\}) = 5$
   → it maps all of the values from $S = \{0,1,2,3,4,5\}$

7. 1R1
   2R2 $\{1\}, \{2,3\}$
   3R3
   2R3
   3R2

8. 3R2, 2R4 are true while 3R4 is not. This makes the relation fails on transitivity which is one of the properties of equivalence relations.

9. i) reflexivity X

$\forall A \in P(X),\ A \cap A = A \neq \emptyset \Leftrightarrow A \not R A$

If we have a nonempty set for X, there will be exist a set whose size is nonzero and is subset of X. In that case, $A \cap A$ is not an empty set.

ii) Symmetry O

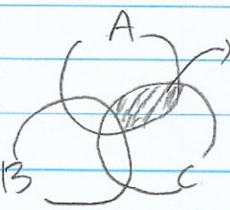
$\forall A, B \in P(X),\ ARB \overset{def}{\Leftrightarrow} A \cap B = \emptyset \overset{commutativity}{\Leftrightarrow} B \cap A = \emptyset \overset{def}{\Leftrightarrow} BRA$

iii) transitivity X

$\forall A, B, C \in P(X),\ ARB, BRC \not\Rightarrow ARC$

$\Updownarrow$ $\qquad\qquad\qquad$ $\Updownarrow$

$A \cap B = \emptyset,\ B \cap C = \emptyset \qquad A \cap C = \emptyset$



if we have any elements here, the above statement ($ARB, BRC \not\Rightarrow ARC$) is true

example). $X = \{1,2,3,4,5\}$

$A = \{1,5\} \in P(X) \qquad A \cap B = \emptyset$

$B = \{2\} \in P(X) \qquad B \cap C = \emptyset$

$C = \{3,5\} \in P(X) \qquad A \cap C = \{5\} \neq \emptyset$

10. $\{\emptyset\}$

$\{\{1\},\{2\},\{3\}\}$

$\{\{1,2\},\{1,3\},\{2,3\}\}$

$\{\{1,2,3\}\}$

11. (a) i) reflexivity

$\forall x \in \mathbb{Z},\ xRx \Leftrightarrow x + x = 2x$: even $\checkmark$

ii) symmetry

$\forall x, y \in \mathbb{Z},\ xRy \Leftrightarrow x + y = k$; k is even

$\Leftrightarrow y + x = k$; k is even

$\Leftrightarrow yRx$ $\checkmark$

iii) transitivity $\qquad (k, l \in \mathbb{Z})$

$\forall x, y, z \in \mathbb{Z},\ xRy, yRz \Leftrightarrow x + y = 2k,\ y + z = 2l$

$\Leftrightarrow x + 2y + z = 2(k + l)$

$\Leftrightarrow x + z = 2(k + l - y) \quad (k + l - y \in \mathbb{Z})$

$\Leftrightarrow xRz$ $\checkmark$ $\square$

$\therefore$ R is an equivalent relation.

11.(cont) (b) even + even = even
odd + odd = even
even + odd = odd
odd + even = odd

→ so the equivalence classes should be consist of only evens or only odds.

$\left(\begin{array}{l}\{2n \mid n \in \mathbb{Z}\} \\ \{2n+1 \mid n \in \mathbb{Z}\}\end{array}\right)$