CS204: Homework #1

Due on September 20, 2016 at 11:59pm

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Problem 1

(a) $q \wedge p \to \neg r$
(b) If the car won't start, then neither there is water in the cylinders nor the head gasket is blown.

Problem 2

(a) $\neg r \to \neg(p \vee q)$
(b) If you are in Kwangju, then you are in South Korea but not in Seoul.

Problem 3

(i) $a \iff \top, b \iff \top$

$$\begin{aligned}(a \vee b) \wedge (\neg(a \wedge b)) &\iff (\top \vee \top) \wedge \neg(\top \wedge \top)\\ &\iff \top \wedge \neg\top\\ &\iff \top \wedge \bot\\ &\iff \bot\end{aligned}$$

$$\begin{aligned}(a \leftrightarrow \neg b) &\iff (\top \leftrightarrow \neg\top)\\ &\iff (\top \leftrightarrow \bot)\\ &\iff \bot\end{aligned}$$

We can calculate all of the answers in this way and make a table 1.

a	b	$(a \vee b) \wedge (\neg(a \wedge b))$	$a \leftrightarrow \neg b$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

Table 1: truth table

If you look at the third and fourth column of the table 1, then you can tell that the result of the two expressions are same. Therefore, you can say that $(a \vee b) \wedge (\neg(a \wedge b))$ and $a \leftrightarrow \neg b$ are logically equivalent. □

Problem 4

A sufficient condition for the given statement: x is a multiple of four.

Solution

If $x = 4k$ (k is an integer), then $x/2 = 2k$, which means that $x/2$ is an even integer.

Problem 5

The result of $\neg(p \wedge q)$ can be deduced easily using the already-known value $p \wedge q$ which is shown in fourth column in the table 2.

Problem 5 continued on next page. . .

p	q	$p \uparrow q$	$p \wedge q$	$\neg(p \wedge q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

Table 2: truth table

The results of $p \uparrow q$ and $\neg(p \wedge q)$ are exactly same. Therefore, we can conclude that $p \uparrow q$ is logically equivalent to $\neg(p \wedge q)$. □

Problem 6

Part A

p	q	$(p \uparrow q) \uparrow (p \uparrow q)$	$p \wedge q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Table 3: truth table

The third and the fourth column on table 3 is identical, which means $(p \uparrow q) \uparrow (p \uparrow q)$ and $p \wedge q$ are logically equivalent. □

Part B

p	q	$p \uparrow p$	$q \uparrow q$	$(p \uparrow p) \uparrow (q \uparrow q)$	$p \vee q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Table 4: truth table

The sixth and the seventh column on table 4 is identical, which means $(p \uparrow p) \uparrow (q \uparrow q)$ and $p \vee q$ are logically equivalent. □

Part C

p	q	$p \uparrow (q \uparrow q)$	$p \rightarrow q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Table 5: truth table

Problem 6 continued on next page...

The sixth and the seventh column on table 4 is identical, which means $(p \uparrow p) \uparrow (q \uparrow q)$ and $p \vee q$ are logically equivalent. $\square$

Problem 7

Statement	Reasons
1. $p \wedge (q \vee r)$	given
2. $\neg(p \wedge q)$	given
3. $\neg p \vee \neg q$	De Morgan's law
4. $\neg q \vee \neg p$	commutativity
5. $q \to \neg p$	implication
6. p	simplification, 1
7. $\neg(\neg p)$	double negation
8. $\neg q$	*modus tollens*, 6, 5
9. $(q \vee r) \wedge p$	commutativity, 1
10. $q \vee r$	simplification
11. $r \vee q$	commutativity
12. $\neg(\neg r) \vee q$	double negation
13. $\neg r \to q$	implication
14. $\neg(\neg r)$	*modus tollens*, 8, 13
15. r	double negation
16. $p \wedge r$	conjunction, 6, 15

Table 6: proof sequence

Problem 8

Statement	Reasons
1. p	given
2. $p \to r$	given
3. $q \to \neg r$	given
4. $\neg q \vee \neg r$	implication
5. $\neg r \vee \neg q$	commutativity
6. $r \to \neg q$	implication
7. r	*modus ponens*, 1, 2
8. $\neg q$	*modus ponens*, 7, 6

Table 7: proof sequence

Problem 9

Is $a \to \neg a$ a contradiction?

Solution

No, it is not. If we have $\bot$ for a, then the given statement is $\bot \to \top$, which is true (not a contradiction). $\square$