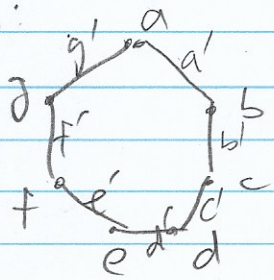
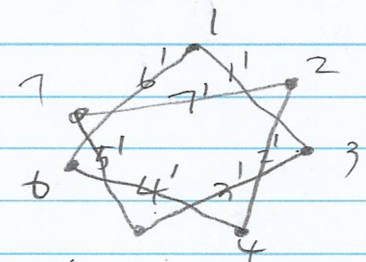
1.

G1 n_1	G1 e	G1 n_2	$\alpha(n_1)$	$\alpha(n_2)$	Edge of G2 joining $\alpha(n_1)$ and $\alpha(n_2)$	$\beta(e)$	Edge of G2 joining $\alpha(n_1)$ and $\alpha(n_2)$ = $\beta(e)$?
x_1	a_1	x_2	y_4	y_2	b_3	b_3	Yes
x_2	a_2	x_3	y_2	y_1	b_1	b_1	Yes
x_2	a_3	x_4	y_2	y_3	b_2	b_2	yes
x_1	a_4	x_5	y_4	y_5	b_5	b_5	yes
x_4	a_5	x_5	y_3	y_5	b_6	b_6	yes
x_3	a_6	x_5	y_1	y_5	b_4	b_4	yes.

2. G1:



G2:



$\alpha(a) = 1$ $\beta(a') = 1'$
$\alpha(b) = 3$ $\beta(b') = 3'$
$\alpha(c) = 5$ $\beta(c') = 5'$
$\alpha(d) = 7$ $\beta(d') = 7'$
$\alpha(e) = 2$ $\beta(e') = 2'$
$\alpha(f) = 4$ $\beta(f') = 4'$
$\alpha(g) = 6$ $\beta(g') = 6'$

$\therefore \underline{G1 \cong G2}$

G1 n_1	G1 e	G1 n_2	$\alpha(n_1)$	$\alpha(n_2)$	Edge of G2 joining $\alpha(n_1)$ and $\alpha(n_2)$	$\beta(e)$	Edge of G2 joining $\alpha(n_1)$ and $\alpha(n_2)$ = $\beta(e)$?
a	a'	b	1	3	1'	1'	Yes
b	b'	c	3	5	3'	3'	yes
c	c'	d	5	7	5'	5'	yes
d	d'	e	7	2	7'	7'	yes
e	e'	f	2	4	2'	2'	yes
f	f'	g	4	6	4'	4'	yes
g	g'	a	6	1	6'	6'	yes

3. (a) Yes. If we make an undirected graph which the vertices represent the teams and the edges represent the played games, the games can be represented with a graph with 11 vertices and $11 \times 6 / 2 = 33$ edges, and this graph can be drawn.
$\therefore$ 33 games will be played.

continued on next page →

3(cont.) (b) No. If we try to make a graph with conditions that I have stated on prob 3 (a), the graph's edge should be $11 \times 5/2 = 55/2$, which cannot be happened. In terms of the Euler's theorem, the sum of the degrees of the vertices should be twice the number of edges, i.e., the sum of the games that have been played by each teams should be even, which is not the case in the problem.

4. Mathematical induction

$T(n)$: a tree with n vertices.

$E(T)$ T: tree
$:= \#$ of edges in T

i) $n=1$

trivial. ✓

ii) $n=k$

$E(T(k)) = k-1$: induction hypothesis.

iii) $n=k+1$

$T(k+1)$

We can pick a terminal node (= a vertex with the degree 1) since there is no simple circuits in the tree. If we remove the edge that is connected to the terminal node, we can get a $T(k)$ and $T(1)$. The # of edges of each tree are $k-1$ and 0. If we put the edge we have deleted back, the total number of edge is $(k-1)+0+1 = k$

5. Mathematical induction on p

i) $p=0$

$L=(a_1) \rightarrow \text{Sum}(L)=a_1$ by part B ✓

ii) $p=n$ Induction hypothesis; $\text{Sum}(L)=a_1+a_2+\cdots+a_{2^n}$

iii) $p=n+1$

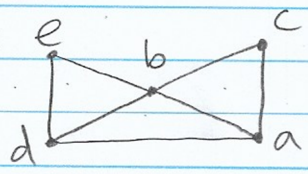
$L=((a_1,a_2,\cdots,a_{2^n}),(a_{2^n+1},a_{2^n+2},\cdots,a_{2^{n+1}}))$

$\text{Sum}(L)=\text{Sum}((a_1,a_2,\cdots,a_{2^n}))+\text{Sum}((a_{2^n+1},\cdots,a_{2^{n+1}}))$ by part R

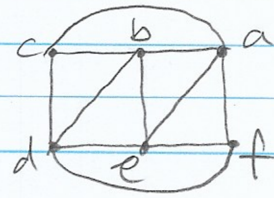
$=(a_1+a_2+\cdots+a_{2^n})+(a_{2^n+1}+\cdots+a_{2^{n+1}})$ by I.H

$=a_1+a_2+\cdots+a_{2^{n+1}}$ ✓

6\. (a) G1:



G2:



(b) faces: 4

e,b,d
b,c,a
d,b,a
e,b,c,a,d

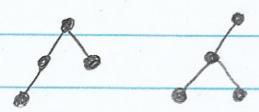
7

c,b,d
d,b,e
b,a,e
a,e,f
c,b,a
d,e,f
c,a,f,d

7\. 9 vertices
edges : $(2+2+2+3+3+3+4+4+5)/2 = 14$ (Euler's thm on degrees)
$\therefore$ faces $= e-v+2 = 14-9+2 = 7$

8\. (a)

(b)



9\. 1) T is a full binary tree w/ i internal vertices
$\longrightarrow$ T has $i+1$ terminal vertices, $2i+1$ total vertices.

B: 0 internal vertex
$\therefore$ 1 terminal vertices, 1 total vertices.

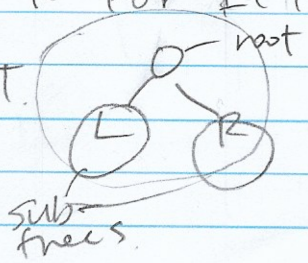
I(T) : # of internal vertices of tree T

Strong mathematical induction
; for $i \le k$ internal vertex $\rightarrow$ $i+1$ terminal vertices, $2i+1$ total

T(T) : terminal

R: For $I(T) = k+1$

Tot(T) : total



; L and R ; not empty ($\because$ full binary tree)

$\underset{k+1}{I(T)} = I(L) + I(R)$

$\therefore k = I(L) + I(R)$

$\Rightarrow \begin{cases} T(L) = I(L)+1 \\ Tot(L) = 2I(L)+1 \quad (\because I.H) \\ T(R) = I(R)+1 \\ Tot(R) = 2I(R)+1 \end{cases}$ $\left(\begin{matrix} I(L) \le k \\ I(R) \le k \end{matrix}\right)$

$\Rightarrow T(T) = T(L) + T(R) = I(L) + I(R) + 2 = k+2 = \underline{(k+1)+1}$

$Tot(T) = Tot(L) + Tot(R) + \underset{root}{1} = 2(I(L)+I(R)) + 3 = 2k+3 = \underline{2(k+1)+1}$