

CS204: Homework #6

Due on November 10, 2016 at 11:59pm

Prof. Sungwon Kang — Section A

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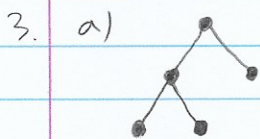
1. B. let Λ : empty string
 $\lambda \in X$

R1. If $\lambda \in X$, then $\lambda 1 \in X$

R2. If $\lambda, \gamma \in X$, then $\lambda 0 \gamma 0 \in X$

2. B. $0 \in X$

R. If $\lambda \in X$, then $\lambda + 2, \lambda - 2 \in X$



4. a) $f(0, n) = n + 1$

$$\begin{aligned} f(1, n) &= f(0, f(1, n-1)) = f(1, n-1) + 1 \\ &= f(1, 0) + n = f(0, 1) + n \\ &= n + 2 \end{aligned}$$

$$\begin{aligned} f(2, n) &= f(1, f(2, n-1)) = f(2, n-1) + 2 \\ &= f(2, 0) + 2n = f(1, 1) + 2n \\ &= 2n + 3 \end{aligned} \quad \dots \dots \dots (1)$$

$$f(3, n) = f(2, f(3, n-1)) = 2f(3, n-1) + 3$$

$$\begin{aligned} f(3, 4) &= 2f(3, 3) + 3 \\ &= 2(2f(3, 2) + 3) + 3 = 4f(3, 2) + 9 \\ &= 4(2f(3, 1) + 3) + 9 = 8f(3, 1) + 21 \\ &= 8(2f(3, 0) + 3) + 21 = 16f(3, 0) + 45 \\ &= 16f(2, 1) + 45 = 16 \cdot 5 + 45 \\ &= \underline{125} \end{aligned}$$

b) $f(3, n) = 2^{n+3} - 3$

pf) Base case $n=0$

$$f(3, 0) = f(2, 1) = 5$$

$$2^{0+3} - 3 = 8 - 3 = 5$$

Inductive case

I.H: $f(3, n) = 2^{n+3} - 3$ for $n=k$

For $n=k+1$:

$$\begin{aligned} f(3, k+1) &= f(2, f(3, k)) \\ &= f(2, 2^{k+3} - 3) \quad (\because \text{I.H.}) \\ &= 2(2^{k+3} - 3) + 3 \quad (\because (1)) \\ &= 2^{(k+1)+3} - 6 + 3 \\ &= 2^{(k+1)+3} - 3 \end{aligned}$$

5. Base case $m=n=1$

$$f(1,1)=5$$

$$2(1+1)+1=5 \quad \checkmark$$

Inductive case $\forall m, n$ s.t.

$$\text{I.H.}; f(m,n)=2(m+n)+1 \quad \text{for } (1,1) \leq (m,n) \leq (a,b)$$

$$\text{i) } (a+1, b)$$

$$\Rightarrow a > 1 \text{ or } (a=1 \text{ and } b > 1) (*)$$

$$\text{i-1) } b=1$$

$$\Rightarrow a > 1 (*)$$

$$f(a+1, b) = f(a, b) + 2 \quad (\because b=1 \text{ and } a > 1)$$

$$= 2(a+b)+1+2 \quad (\because \text{I.H.})$$

$$= 2(a+1+b)+1 \quad \checkmark$$

$$\text{i-2) } b > 1$$

$$f(a+1, b) = f(a+1, b-1) + 2 \cdot 1$$

$$= f(a+1, b-2) + 2 \cdot 2$$

$$\vdots$$

$$= f(a+1, 1) + 2(b-1)$$

$$= f(a, 1) + 2b$$

$$= 2(a+1)+1+2b \quad (\because \text{I.H. } (1,1) \leq (a,1) \leq (a,b))$$

$$= 2(a+1+b)+1 \quad \checkmark$$

$$\text{ii) } (a, b+1)$$

$$\text{ii-1) } b=0$$

$$\Rightarrow a > 1 (*)$$

$$f(a, b+1) = f(a-1, b+1) + 2 \cdot 1 \quad (\because b+1=1)$$

$$= 2(a-1+b+1)+1+2 \quad (\because \text{I.H. } (1,1) \leq (a-1, b+1) \leq (a,b))$$

$$= 2(a+(b+1))+1 \quad \checkmark$$

$$\text{ii-2) } b > 0$$

$$f(a, b+1) = f(a, b) + 2 \quad (\because b+1 > 1)$$

$$= 2(a+b)+1+2 \quad (\because \text{I.H.})$$

$$= 2(a+(b+1))+1 \quad \checkmark$$

$$\text{iii) } (a+1, b+1)$$

$$\text{iii-1) } b=0$$

$$\Rightarrow a > 1 (*)$$

$$f(a+1, b+1) = f(a, b+1) + 2 \quad (\because a+1 > 1 \text{ and } b+1=1)$$

$$= f(a-1, b+1) + 2 \cdot 2 \quad (\because a > 1 \text{ and } b+1=1)$$

$$= 2(a+b)+1+2 \cdot 2 \quad (\because \text{I.H. } (1,1) \leq (a-1, b+1) \leq (a,b))$$

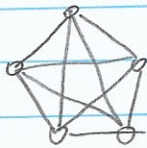
$$= 2(a+1+(b+1))+1 \quad \checkmark$$

 \Rightarrow cont.

5. (cont.)

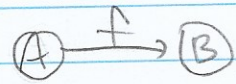
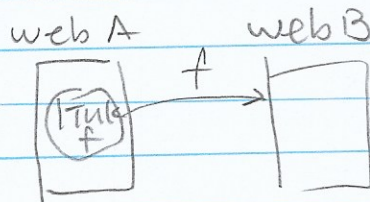
$$\begin{aligned}
 \text{iii-2) } b > 0 \quad (a > 1 \text{ or } (a=1 \text{ and } b > 1)) \\
 f(a+1, b+1) &= f(a, b+1) + 2 \quad (\because b+1 > 1) \\
 &= f(a-1, b+1) + 2 \cdot 2 \quad (\because b+1 > 1) \\
 &= 2(a+b) + 1 + 2 \cdot 2 \quad (\because \text{I.H. } (1,1) \leq (a-1, b+1) \leq (a,b)) \\
 &= 2((a+1) + (b+1)) + 1 \quad \checkmark \quad \blacksquare
 \end{aligned}$$

6.



10 edges.

7. a) Yes, it is a directed graph. If we click a link on the website, it connects to the other webpage. Therefore, we can think

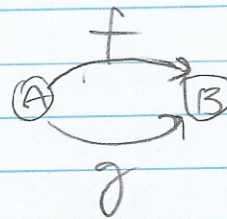
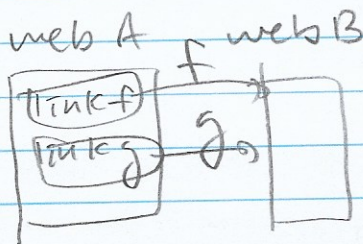


a link as an edge, which has a direction property.

b) No. If we make webpages which have no links on them, the graph has nodes which are not connected any pages.

c) No, we can make a disconnected graph, which is not a complete graph. (b)

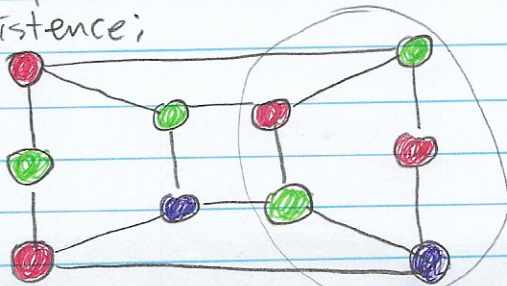
d) No, we can make a webpage that have multiple links that connect to the same page.



e) The outdegree of p means the number of links on page p .

f) The indegree of p represents the number of links that can be used to access page p .

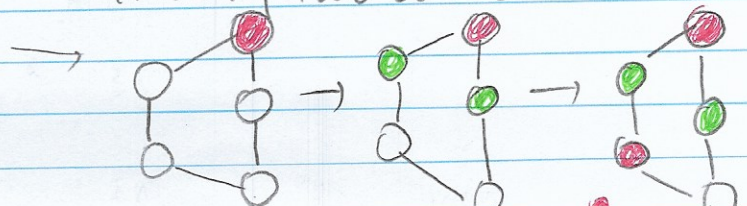
8. existence;



3 colors.

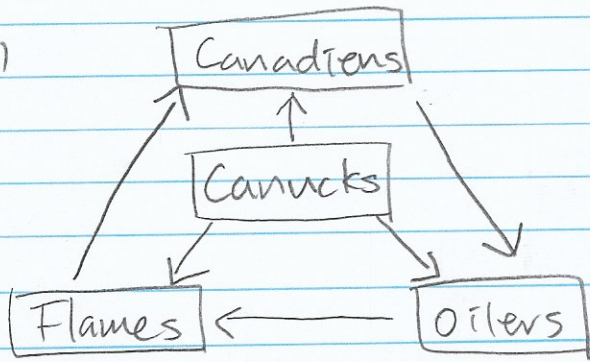
minimality;

Assume that we can color the graph with only two colors.



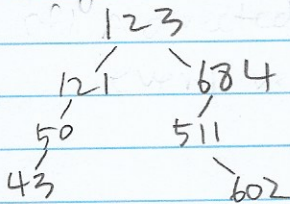
This should be other than Fail

9. a)

b) Canadiens \rightarrow Oilers \rightarrow Flames \rightarrow Canadiens

c) we cannot decide ranks within the circuit because there is always a team which have defeated them regardless of what team we picked.

10. a)



b) 3, For example, we have to visit 3 edges if we want to find 43 in the tree, which is the deepest (that has a maximum number of edges to reach there from the root) element in the tree.

c) 123, 50, 602, 43, 121, 511, 684

