QE II – Major exam syllabus

Koji Sunami

Date of Exam: Jan 19 2023

Our purpose in this Qual is to exam the definitions and properties of Toric/log geometry, A/B-model, which are required materials for the research of Mirror Symmetry.

1: Toric Geometry

1. Construction of Normal Toric Variety.
   Define what is an abstract variety. Describe what is a normal fan and a rational polyhedral cone, and construct a projective space $\mathbb{P}^n$ and Hirzebruch surface $\mathscr{H}_n$. Also construct a weighted projective space $\mathbb{P}(1,1,2)$, and explain why it is compact but not smooth. Discuss a complete fan.

2. Weil/Cartier/Canonical Divisor.
   Divisors define the line bundles, in particular canonical divisor. discuss (very) ampleness of a polygon and compared with the ordinary definition of (very) ampleness. Define reflexive polytope and Gorenstein Fano variety.

3. resolution of singularity
   Define toric morphism and the refinement of a normal fan, and explain correspondence to blow up. Define the resolution of singularity. introduce projective resolution, crepant resolution, log resolution. Toric construction of Calabi-Yau manifold.

4. Mumford Degeneration.
   Explain what Mumford degeneration is.

2: Log Geometry

1. Log scheme
   Defining log scheme and its basic notions: morphism of log schemes, charts, coherence, smoothness, log derivation, differentials, deformation, and compactification. Explain toric variety is a log scheme. Describe toric blow up by the language of log geometry.

2. Twisted De Rham complex
   Define log de Rham complex over the partial compactification $\overline{X}$, and explain the formula of the hypercohomologies.

3: A/B-model

1. WDVV equation.
   Construct the moduli space $\overline{M}_{g,n}(X,\beta)$, and define Gromov-Witten invariant and construct WDVV equation. What is the quantum potential Φ and the quantum cohomology ring $H^*(X,\mathbb{C}[[y_0,...,y_m]])$.

2. Variation of Hodge Structure
   For A-model, define semi-infinite variation of Hodge structure parametrized by a space M, and explain the case of the quantum cohomology. Explain how variation of Hodge structure induces Frobenius manifold on the quantum cohomology. For B-model, define Landau-Ginzburg model, and define hypercohomology and twisted de Rham complex.

3. Mirror Symmetry
   Explain Barannikov period map which claims Mirror symmetry of two variations of Hodge structure of A-model and B-model. What is the mirror symmetry of $\mathbb{P}^n$.

References

[1] David A. Cox, Little, Schenck (2010) *Toric Variety.*

[2] Gross (2010) *Tropical Geometry and Mirror Symmetry.*

[3] Hartshorne (1977) *Algebraic Geometry.*

[4] Kashiwara, Shapiro (2006) *Categories and Sheaves.*

[5] Ogus (2006) *Lectures on Logarithmic Algebraic Geometry*

[6] Diane Maclagan, Bernd Sturmfels (2009) *Introduction to Tropical Geometry*