This problem involves working with a **discrete random variable** and a **transformation** of that variable. We are given the Probability Mass Function (PMF) of the original variable and need to find the PMF of the transformed variable. This falls under the topic of **functions of random variables**.

Problem Description:

Let Z be a discrete random variable with PMF $P_Z(k)$ defined as:

$$P_Z(k) = \begin{cases} 0.1 & \text{for } k = 0 \\ 0.3 & \text{for } k = 1 \\ 0.4 & \text{for } k = 2 \\ 0.2 & \text{for } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

We need to define $W = Z(Z-2)$ and find the PMF of W, $P_W(w)$. We must state the values of k (which are now values of w) as integers between 0 and 99 in ascending order, and the probabilities as decimal values with four decimal precision. A negative sign has been pre-printed for the first value of k. This suggests that one of the possible values for W might be negative.

Steps to Solve:

1. Determine the possible values of W by plugging in each possible value of Z:

The possible values for Z are $\{0, 1, 2, 3\}$.
Let's calculate $W = Z(Z-2)$ for each of these values:

- If $Z = 0$:
  $W = 0(0-2) = 0 \times (-2) = 0$
- If $Z = 1$:
  $W = 1(1-2) = 1 \times (-1) = -1$
- If $Z = 2$:
  $W = 2(2-2) = 2 \times 0 = 0$
- If $Z = 3$:
  $W = 3(3-2) = 3 \times 1 = 3$

So, the possible distinct values for W are $\{-1, 0, 3\}$.

2. Calculate the probability for each possible value of W:

- **For $W = -1$:**
  This occurs only when $Z = 1$.
  $P_W(-1) = P_Z(1) = 0.3$

- **For $W = 0$:**
  This occurs when $Z = 0$ **or** when $Z = 2$. Since these are mutually exclusive events, we sum their probabilities.
  $P_W(0) = P_Z(0) + P_Z(2) = 0.1 + 0.4 = 0.5$

- **For $W = 3$:**
  This occurs only when $Z = 3$.
  $P_W(3) = P_Z(3) = 0.2$

3. State the PMF of W in the required format:

The problem asks for the values of k (which are the values of W) in ascending order. So, we list them as $-1, 0, 3$. The probabilities should be to four decimal places.

$$P_W(w) = \begin{cases} 0.3000 & \text{for } w = -1 \\ 0.5000 & \text{for } w = 0 \\ 0.2000 & \text{for } w = 3 \\ 0 & \text{otherwise} \end{cases}$$

The provided solution template has the order `k=1`, `k=0`, `k=3` but the values must be in ascending order for k. The template also has `0.3` for `k=1` and `0.5` for `k=0`, which would correspond to our calculated probabilities for $W = -1$ and $W = 0$ if the values were indexed differently. However, the instruction is to state k values in ascending order. The negative sign pre-printed for the first value confirms our $W = -1$.

Let's match our results to the template's *implied* structure, assuming it means the first value in the list is the smallest W, the second is the next smallest, and so on.

The template suggests:

- Value of W for the first row: -1

- Probability for the first row: 0.3000

- Value of W for the second row: 0

- Probability for the second row: 0.5000
- Value of W for the third row: 3
- Probability for the third row: 0.2000

This matches our calculated values and the requirement for ascending order of k (which is w).

Topics Covered:

- **Discrete Random Variables:** Variables that can only take on a finite or countably infinite number of values.
- **Probability Mass Function (PMF):** A function that gives the probability that a discrete random variable is exactly equal to some value.
- **Functions of Random Variables:** How to derive the probability distribution of a new random variable that is a mathematical function of an existing random variable.
- **Mutually Exclusive Events:** Events that cannot occur at the same time. If a value of the new random variable can be produced by more than one value of the original variable, their probabilities are summed.

WolframAlpha/Computational Check:

While WolframAlpha cannot directly compute the PMF of a transformed random variable in this exact format, you can use it to verify the intermediate calculations or the logic.

- **Verifying values of W:**
  - `0*(0-2)` outputs `0`
  - `1*(1-2)` outputs `-1`
  - `2*(2-2)` outputs `0`
  - `3*(3-2)` outputs `3`
- **Verifying sums of probabilities:**
  - `0.1 + 0.3 + 0.4 + 0.2` outputs `1.0` (confirms original PMF is valid)
  - `0.3 + 0.5 + 0.2` outputs `1.0` (confirms new PMF is valid)

This problem primarily relies on careful calculation and mapping of the input variable’s probabilities to the output variable’s probabilities.