Let X and Y denote two independent stochastic variables. Assume the PMF of X is

$$f_X(x) = \begin{cases} 0.2 & \text{if } x = 0 \\ 0.3 & \text{if } x = 1 \\ 0.5 & \text{if } x = 2 \\ 0 & \text{else} \end{cases}$$

and that the PMF of Y is

$$f_Y(y) = \begin{cases} 0.3 & \text{if } y = 0 \\ 0.4 & \text{if } y = 1 \\ 0.3 & \text{if } y = 2 \\ 0 & \text{else} \end{cases}$$

a. Find the following values. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X > 0) = 0.$ ☐

$\text{Var}(X) = 0.$ ☐

b. Find the following probabilities. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X < 2, Y > 1) = 0.$ ☐

$P(\{X < 2\} \cup \{Y < 2\}) = 0.$ ☐

More

c. Find the value below. State your answer as an integer between 0 and 99 so that the answer is given with two decimal precision. Please note that a negative sign has been pre-printed.

$Cov(2X - 5Y, 7X + 4Y + 1) = -$ ☐$.46$

PDF -> (E2022) -> Part 1 – 1.pdf

two independent stochastic variables
var
probability

Let $X \sim \text{Exponential}(3)$ and set $Y = e^{2X}$.

a. Determine the expected value of Y. State your answer as a positve integer.

$E[Y] = \square$

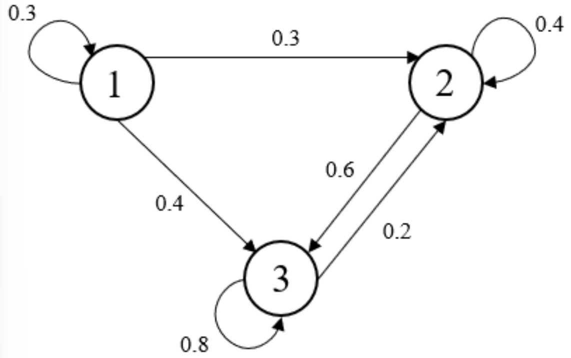
b. Find the PDF of Y. State your inputs as positve integers such that all values are stated as irreducible fractions.

$$f_Y(y) = \begin{cases} \frac{\square}{\square} y^{-\frac{\square}{\square}} & y > 1 \\ 0 & else \end{cases}$$

exponential pdf

PDF -> (E2022) -> Part 1 – 3.pdf

Let $\{X_n : n = 0, 1, \ldots\}$ denote a Markov Chain with states {1, 2, 3} and with the following state transition diagram:



Find the following probability. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X_5 = 3 \mid X_3 = 1, X_2 = 2) = 0.\square$

PDF -> (E2022) -> Part 1 – 4.pdf

markov chain probability

Let $X_1, \ldots, X_{1000}$ denote a sample with $X_i \sim Bernoulli(p)$ for all $i = 1, \ldots, 1000$. Assume we have observed the outcomes $x_1, \ldots, x_{1000} \in \{0, 1\}$ from the sample. You are informed that the average of all the x_i's is $\overline{x} = 0.54$ and that the sample variance is $s^2 = 0.45$. We are interested in determining whether or not the outcomes are evenly distributed in the sample space of the stated Bernoulli distribution.

a. Determine which of the below would be an appropriate alternative hypothesis for this test.

A $H_1: \ p \neq \frac{1}{2}$

B $H_1: \ \overline{x} \neq \frac{1}{2}$

C $H_1: \ \mu = 0.50$

D $H_1: \ \mu \neq 0.50$

E $H_1: \ p = \frac{1}{2}$

b. Set up a 90% confidence interval for p. Select the correct interval from the choices.

A [0.1753; 0.3526]

B [0.4556; 0.5523]

C [0.5102; 0.5872]

D [0.2670; 0.4231]

c. Assume we want to test the hypothesis mentioned above with $\alpha = 0.01$. Determine all the values below, and determine the correct decision based on the data. Select the value closest to your result.

Test Statistic:

The critical value:

The p-value:

Based on this we should ______ the null hypothesis.

Bernoulli alternative hypothesis confidence interval test statistic

critical value p-value p value null hypothesis

PDF -> (E2022) -> Part 1 – 5.pdf

Assume that 0.01% of the population has COVID-19 and that 20000 randomly chosen people are at a large gathering. What is the probability that at least 5 people at the gathering have COVID-19. Please state your answer as a **decimal value** correctly rounded to four decimal precision (e.g. 0.9876). Remember to use '.' as the decimal separator.

☐

PDF -> (E2022) -> Part 1 – 6.pdf probability at least randomly

Consider the following statistics collected from a sample of size 25: The sample mean is 310 and the sample standard deviation is 6. A calculated confidence interval for the mean is [306.6551 ; 313.3449]. Which confidence level was chosen? Assume distribution to be normal. Select the value below that is closest to the level.

A	90%
B	94%
C	95%
D	96%
E	97.5%

PDF -> (E2022) -> Part 1 – 8.pdf

sample sample size sample mean sample of size sample standard deviation
confidence interval confidence

Two Premier League teams, A and B, are to play a match. We know that the number of goals scored by Team A is modeled by a Poisson process $N_1(t)$ with rate $\lambda_1 = 0.02$ goals per minute, and the number of goals scored by Team B is modeled by a Poisson process $N_2(t)$with rate $\lambda_2 = 0.03$ goals per minute. The two processes are assumed to be independent. Let $N(t)$ be the total number of goals in the game up to and including time t. Assume the game lasts for 90 minutes with no overtime. State all your answers as decimal values correctly rounded off to two decimal precision for all problems below. Remember to use '.' as the decimal separator.

a. Find the probability that no goals are scored, i.e., the game ends with a 0-0 draw.

b. Find the probability that at least two goals are scored in the game. *Hint*: You can treat the number of goals scored by any of the two teams as a new Poisson process with rate $\lambda_1 + \lambda_2$ goals per minute.

c. Find the probability of the final score being:

Team A: 1, Team B: 2.

PDF -> (E2022) -> Part 1 – 10.pdf poisson process independent

Let $X_1, X_2, \ldots, X_n$ denote a random sample from the following normal distribution: $N(\mu, 9)$ for $n \in \mathbb{N}$, and let $\bar{X} = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$ denote the associated sample mean. How large must n be so that $\bar{X}$ is no more than 0.7 from μ with 95% confidence? State your answer as an integer between 0 and 99.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

The sample size must be at least $\boxed{71}$ in order for the sample mean to be no more than 0.7 from μ with 95% confidence.

(E2023).pdf - page 4 to 6 random sample normal distribution sample size

Let X be a discrete stochastic variable with the following probability mass function:

$$p_X(x) = \begin{cases} 1/3 & \text{for } x \in \{-1,0,1\} \\ 0 & \text{else.} \end{cases}$$

For all questions in this assignment, state your inputs as integers between 0 and 99 such that all answers are given as either integers or irreducible fractions

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. Find the expected value and variance of X.

$E(X) = \square$

discrete stochastic variable probability mass function expected value variance

Cl

$Var(X) = \frac{\square}{\square}$

Let Y denote a stochastic variable that is independent of X and has the same PMF as X, i.e.

$$p_Y(y) = \begin{cases} 1/3 & \text{for } y \in \{-1,0,1\} \\ 0 & \text{else.} \end{cases}$$

b. Find the values below.

$Var(X - Y) = \frac{4}{3}$

PMF stochastic variable independent

cov

$Cov(X, Y + X) = \frac{\square}{\square}$

c. Find the values below

expected value

$E(|X - 1|) = \square$

$E(|X - 1| \cdot |Y - 1|) = \square$

d. Find the probabilities below.

$P(X \neq 0) = \frac{\square}{\square}$

(E2023).pdf - page 8 to 13

Let X and Y be two independent stochastic variables such that $X \sim \text{Binomial}(10, 0.5)$ and $Y \sim \text{Geometric}(1/5)$. Find the below probabilities. State your answer as an integer between 0 and 99 such that you supply four decimal precision.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

$P(X > 5 \cup Y \leq 3)$ =0.6 81 ✓ 0

$P(X > 5 \mid X + Y = 3)$ =0.000 0 ✓

(E2023).pdf - page 14 to 17

two independent stochastic variables
binomial
geometric
probability

Let X and Y be two jointly continuous random variables with joint PDF:

$$f_{XY}(x,y) = \begin{cases} \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6} & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $E\left[X^2 \mid Y = 1\right]$. State your input as a positive integer such that the answer is stated as an irreducible fraction.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

$$\frac{\boxed{21}}{50}$$

(E2023).pdf - page 18 to 21

two jointly continuous random variables
expected value joint

joint PDF

One way to design a spam filter is to look at the words in an email. In particular, some words are more frequent in spam emails. Suppose that we have the following information:

- 60% of emails are spam;
- 2% of spam emails contain the word "discount";
- 0.002% of non-spam emails contain the word "discount".

Suppose that an email is checked and found to contain the word "discount". Find the probabilities below. State all inputs as integers between 10 and 99 such that the answer is given as a decimal value with four decimal precision.

Documentation: You should supply the calculations leading up to the final result.

The probability that an email contains the word "discount": 0.0 12 ✓ 0

The probability that an email is spam given that it contains the word "discount": 0.9 99 ✓ 3

(E2023).pdf - page 22 to 25

probability

The number of cars, X, passing the Storebælt Bridge over a certain time period, can reasonably be assumed to follow a Poisson distribution:

$$X \sim \text{poisson}(\lambda t)$$

Over a time period of t = 3 hours, x = 1140 cars pass the bridge. Based on this information, the estimate for the time parameter is:

$$\hat{\lambda} = \frac{x}{t} = \frac{1140}{3} = 380 \text{ cars per hour}$$

Documentation: In (a) no documentation is required. In (b) and (c) you are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. An employee at the Storebælt Bridge claims that, on an annual basis and for a similar time period, an average of 400 cars pass per hour. Do the measured data (x = 1140 cars in t = 3 hours) agree with the employee's claim? To answer this question correctly, you need to conduct a hypothesis test. Identify the correct null hypothesis.

- [ ] $H_0 : \lambda_0 \neq 380$
- [ ] $H_0 : \lambda_0 \neq 1200$
- [ ] $H_0 : \lambda_0 \geq 1140$
- [ ] $H_0 : \lambda_0 \neq 400$
- [ ] $H_0 : \lambda_0 = 1140$
- [x] $H_0 : \lambda_0 = 400$ ✔
- [ ] $H_0 : \lambda_0 = 380$
- [ ] $H_0 : \lambda_0 \geq 380$
- [ ] $H_0 : \lambda_0 \neq 1140$
- [ ] $H_0 : \lambda_0 = 1200$

b. Determine the test statistic and the p-value for the test. State your inputs as integers between 10 and 99 so that the answer is displayed with four decimal precision.

Hint: Remember that the variance of a Poisson random variable is the same as the parameter.

Test statistic: -1.7 32 ✔ 1

P-value: 0.0 83 ✔ 3

poisson distribution   estimate time parameter   hypothesis test   hypothesis poisson   Check answers

c. Assuming $\lambda = 400$ cars per hour, as stated by the employee, determine the probability that 380 or fewer cars pass over the bridge in one hour.

$0.1\boxed{64}9$ ✔

(E2023).pdf - page 26 to 33

Let P and Q be two independent $N(0, 1)$ random variables and

independent   N(0,1)   cov

$$R = 7 - P + P^2$$
$$S = 4 - PQ.$$

Find $\text{Cov}(R, S)$. State your answer as an integer between 0 and 99.

Documentation: You must supply your calculations leading up to the result.

$$Cov(R,S) = \boxed{0}$$

(E2023).pdf - page 34 to 38

Let (X,Y) be a two-dimensional discrete stochastic vector with probability function $p_{X,Y}$ given by:

$$p_{X,Y}(x,y) = \begin{cases} \frac{1}{4}\cdot(1-p)p^y & \text{if } x\in\{1,2,3,4\} \text{ and } y\in\{0,1,2,\dots\} \\ 0 & \text{otherwise.} \end{cases}$$

two dimensional discrete stochastic vector

Here, $p\in]0,1[$.

two-dimensional discrete stochastic vector

For all questions in this assignment, state your inputs as integers between 0 and 99 such that all answers are given as either integers or irreducible fractions.

Documentation: You are expected to demonstrate how you obtained the results.

a. Find the marginal probability distributions of X and Y.

marginal probability distribution

$$p_X(x) = \begin{cases} \frac{1}{\boxed{4}} & \text{if } x\in\{1,2,3,4\} \\ 0 & \text{else} \end{cases}$$

$$p_Y(y) = \begin{cases} (\boxed{1}-p)p^y & \text{if } y\in\{0,1,2,..\} \\ 0 & \text{else} \end{cases}$$

In the rest of the assignment, feel free to use the fact that $p=\frac{1}{2}$

probability

b. Find the values below.

$$P(X\le 2) = \frac{\boxed{1}}{\boxed{2}}$$

$$P(Y\le 1) = \frac{\boxed{3}}{\boxed{4}}$$

$$P(\{X\le 2\}\cap\{Y\le 1\}) = \frac{\boxed{3}}{8}$$

$$P(\{X\le 2\}\cup\{Y\le 1\}) = \frac{7}{\boxed{8}}$$

c. Find the expected value and variance of X:

expected value variance

$$EX = \frac{\boxed{5}}{2}$$

$$\operatorname{Var} X = \frac{\boxed{5}}{4}$$

d. Find the values below. Feel free to use the fact that $EY=1$ and $\operatorname{Var} Y = 2$.

$$\operatorname{Var}(X-Y) = \frac{\boxed{}}{4}$$

$$\operatorname{Cov}(X, X-Y) = \frac{\boxed{}}{4}$$

$$E(Y\cdot|X-3|) = \boxed{}$$

var

cov

(E2024) Part1 - 1.pdf

Two competing coffee shops, Café A and Café B, are located on the same street. The number of customers arriving at each café follows independent Poisson processes:

- Café A receives customers at a rate of λ_1 = 1.5 customers per hour.
- Café B receives customers at a rate of λ_2 = 2 customers per hour.

independent poisson process
expected time
probability

The two cafés open at 8 AM and close at 5 PM, providing a 9-hour operational day.

Documentation: You are expected to demonstrate how you obtained the results.

a. Determine the probability that Café B serves the first customer of the day, and state your answer as a four precision decimal value. Remember to use dot (" . ") as decimal separator.

The probability that Café B serves the first customer of the day is ☐

b. Calculate the expected time until the first customer arrives at each café on a given day. State your answer as a positive integer that denotes minutes.

The expected time until the first customer for Café A is ☐ minutes.

The expected time until the first customer for Café B is ☐ minutes.

c. Determine the probability that neither café has any customers in the first hour of operation. State your answer as a four precision decimal value. Remember to use dot (" . ") as decimal separator.

$P(\text{No customers at A and B}) =$ ☐

d. Estimate the total number of customers that each café will serve in the first three hours. State your answer as a decimal value with one decimal precision. Remember to use dot (" . ") as decimal separator.

Together, Café A and Café B are expected to serve a total of ☐ customers in the first three hours.

(E2024) Part1 - 2.pdf

Three companies, A, B, and C, are competing in a market. The likelihood of customers shifting their preferences between these companies over a year is modeled by a Markov chain with three states representing each company. The transition probabilities are given by matrix P:

$$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

markov chain transition probabilities transition probability

State all answers as irreducible fractions.

Documentation: You are expected to demonstrate how you obtained the results.

a. Determine the probability that a customer who initially prefers Company B will prefer Company C after one year.

$\frac{\square}{\square}$

b. If a customer is initially with Company C, calculate the probability that they will be with Company A after two years.

$\frac{\square}{\square}$

c. Given the mean return times to state j:

- $r_1 = \frac{35}{12}$
- $r_2 = \frac{35}{9}$
- $r_3 = \frac{5}{2}$

mean return times

Determine the values in the vector given by

vector

$\pi_j = \lim_{n \to \infty} P\left(X_n = j \mid X_1 = i\right)$

$\pi_j = \left[\frac{\square}{\square} \quad \frac{\square}{\square} \quad \frac{\square}{\square} \right]$

d. In the long run, which company will have the largest market share?

- ○ Company A
- ○ Company B
- ○ Company C

(E2024) Part1 - 3.pdf

Please identify the correct distribution for each scenario. Note: All distributions mentioned relate to exactly one scenario. You must identify the distributions of all six scenarios correctly to obtain points. Please note that the Geometric and Negative Binomial distributions are treated as **distinct** distributions in this exercise acknowledging that the Geometric is simply a special case of the Negative Binomial.

Documentation: No documentation needed for this assignment.

	Binomial	Geometric	Negative Binomial	Hypergeometric	Poisson	Exponential
A cybersecurity firm employs an intrusion detection system that flags activities as malicious or benign. Given the probability of correctly identifying an activity, the firm wants to evaluate the system's accuracy over 100 distinct activities.	○	○	○	○	○	○
A network administrator monitors traffic to a server which on average receives 300 requests per minute. The distribution of requests over any given minute is	○	○	○	○	○	○

(E2024) Part1 - 4.pdf

distribution

Thirty software engineering students completed a series of courses taught by Professor Reginald Kooks. The effectiveness of these courses is being evaluated by comparing students' smartness scores, measured on a scale of 0 to 100, before and after the courses. Here are the summary statistics of the test:

- Number of students (n): 30
- The mean score before the course: 70
- The mean score after the courses: 76
- Standard deviation of scores before the courses (S_1): 10
- Standard deviation of scores after the courses (S_2): 15

We will assume minimal covariance and estimate the standard deviation of the differences (S_D) as:

$$S_D = \sqrt{S_1^2 + S_2^2}$$

standard deviation mean score covariance

We want to know whether following the courses have had a positive effect on the smartness score of the students (i.e. that score has increased), i.e. we want a significant test result.

Documentation: You only need to provide documentation of how you obtained your answer in question (c). You will need to use one or more probability distribution tables to solve some of the tasks in this assignment.

a. First, please identify the correct hypothesis associated with this test. Note μ_B andμ_A refer to the mean before and after, respectively, and μ_D to the Mean of the Differences.

hypothesis mean of the differences mean before mean after

A	$H_0 : \mu_D = 0 H_1 : \mu_D \neq 0$
B	$H_0 : \mu_B > \mu_A H_1 : \mu_B < \mu_A$
C	$H_0 : \mu_D = 10 H_1 : \mu_D \neq 10$

b. Find the critical value of the test, and state your answer as a decimal value with three decimal precision.

critical value

The critical value: [ ]

c. Find the test statistic. State you answer as a decimal value with three decimal precision.

test statistic

Test statistic: [ ]

d. Is there sufficient evidence to support the claim that the courses have effected and increase in students' smartness score?

- ○ Yes
- ○ No

evidence

(E2024) Part1 - 5.pdf

A software development team is working on implementing a series of new features into an existing application. The probability of successfully integrating each feature without bugs on the first try is initially estimated based on past performance. The team plans to implement 10 new features in the upcoming release. Let X denote the number of features successfully integrated without bugs out of 10 attempts. Historically, the probability of any single feature being successfully integrated without needing any bug fixes is 70%.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

State all answers as decimal values with 4 decimal precision. Remember to use dot (" . ") as decimal separator, except in the last exercise (e) where you must enter a positive integer.

a. Calculate the probability that exactly 7 out of the 10 features are successfully integrated without bugs.

probability

b. Find the probability that at least 8 features are integrated successfully.

Based on the success of the initial feature integration, the software team decides to utilize the observed success rate to inform the deployment of a new module or system upgrade. The team estimates that the success rate from above will directly impact the reliability of the system in a real-world environment, specifically in terms of uptime. Let *Y* represent the number of days in a given month (30 days) that the system achieves 99% or greater uptime. The probability of achieving 99% uptime any given day is assumed to be equivalent to achieving at least 8 successful integrations from (b), which represents a higher confidence in system stability due to previous successes.

c. Calculate the expected number of days the system achieves 99% or greater uptime in a 30-day month.

expected

d. Determine the probability that the system achieves 99% in more than 15 days in a 30-day month.

The software development team wants to ensure that when they deploy a small number of critical updates, at least one is successful. Based on historical data, the success rate of any single update being successfully deployed without any issues is 70% as mentioned above.

e. Determine the minimum number of critical updates n that the team should deploy to be 99% confident that at least one update is successfully deployed without issues. Since n must be an integer, round up to the nearest whole number.

minimum

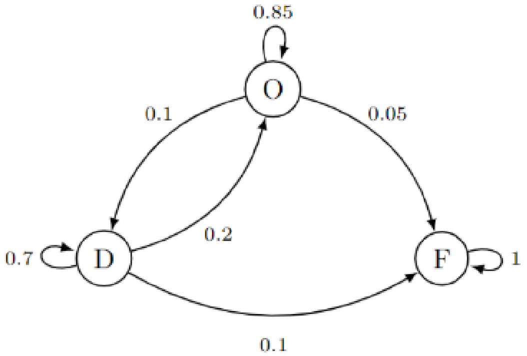
$n =$

(E2024) Part1 - 6.pdf

Suppose a server used for a large-scale web application can be in three states:

- **Operational (O)**: The server is fully functional.
- **Degraded (D)**: The server is running but with reduced capacity or speed.
- **Failure (F)**: The server is down and not serving any requests.

The transitions between these states occur according to the probabilities reflecting the server's reliability and maintenance effectiveness:



markov chain

The server starts fully functional 95% of the time and the rest of the time the server starts out degraded, running with reduced capacity or speed.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code. No documentation needed for the final assignment (d).

State all answers as decimal values with 4 decimal precision. Remember to use dot (" . ") as decimal separator.

a. Given that the server starts fully operational, what is the probability that the server is in the Failure (F) state after two time steps?

b. Calculate the probability of transitioning from the Degraded (D) state to the Failure (F) state in exactly three transitions.

probability transition

c. Calculate the mean hitting time for the server to go from Operation (O) to Failure (F) (t_O) and the mean hitting time for the server to go from Degraded (D) to Failure (F) (t_D). Both values are positive integers.

mean time

$t_O =$

$t_D =$

d. What will happen with the system in the long run?

A	In the long run, the system will spend equal time in the Operational (O), Degraded (D), and Failure (F) states, with each state having a probability of 1/3.
B	In the long run, the Markov chain will oscillate between the Operational (O) and Degraded (D) states with equal probabilities, while the probability of being in the Failure (F) state will be 0.
C	In the long run, the Markov chain will reach a steady-state where the probability of being in the Operational (O) state is 0.5, the probability of being in the Degraded (D) state is 0.3, and the probability of being in the Failure (F) state is 0.2.
D	In the long run, the Markov chain will reach the absorbing state of Failure (F) with probability 1. This means that regardless of the initial state, the system will eventually fail and remain in the Failure state indefinitely. The probabilities of being in the Operational (O) and Degraded (D) states will both be 0 in the long run.

More

(E2024) Part1 - 7.pdf

Let Z be a discrete random variable with PMF

pmf W

$$P_Z(k) = \begin{cases} 0.1 & \text{for } k = 0 \\ 0.3 & \text{for } k = 1 \\ 0.4 & \text{for } k = 2 \\ 0.2 & \text{for } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

distribution probabilities

Define $W = Z(Z-2)$. Find the PMF of W. Please state the values of k as integers between 0 and 99 and in ascending order like in the formula above (i.e. $-k < k < k+1$). Please state the distribution probabilities as decimal values with four decimal precision. Note that in the first value for k, a negative sign has been pre-printed.

Documentation: Please supply the calculations leading to your answers.

$$P_W(k) = \begin{cases} \boxed{0.3} & for\ k = -\boxed{1} \\ \boxed{0.5} & for\ k = \boxed{0} \\ \boxed{0.2} & for\ k = \boxed{3} \\ 0 & otherwise \end{cases}$$

(ME2024) Part1 - 1.pdf

Math Problem

Let A and B be two independent $N(0,1)$ random variables, and

random variables two independent

N(0,1)

$$U = 5 + A - A^2 B,$$
$$V = 2 + B.$$

corr

Find $\mathbf{Corr}(U, V)$. Please enter two integers between 0 and 99 such that the answer displayed is an irreducible fraction. Also, note that a negative sign has been pre-printed. You can use the fact that $\mathbf{Var}(U) = 4$ and $\mathbf{Var}(V) = 1$

Documentation: Please supply the calculations leading to the answer.

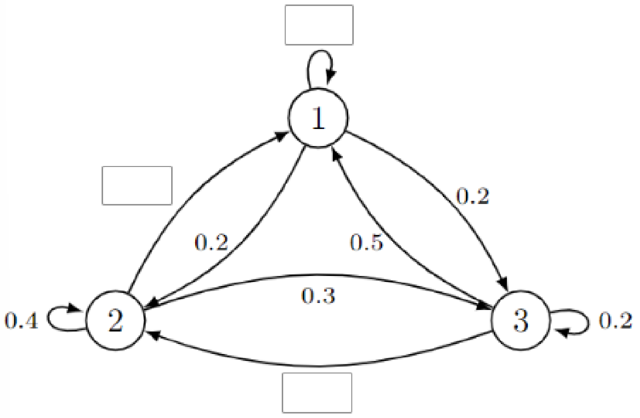
$$\mathrm{Corr}(U,V) = -\frac{\boxed{1}}{\boxed{2}}$$

(ME2024) Part1 - 2.pdf

Consider a Markov chain with three states $S = \{1, 2, 3\}$. markov chain transition

a. The state transition diagram of such a Markov chain is shown below. Fill out the missing transition probabilities. State your inputs as decimal values between 0.1 and 0.9 (both included) with one decimal precision and remember to use dot as the decimal separator.

Documentation: No documentation is needed for parts (a) and (b) but please supply all calculations leading to your answers in part (c).



b. Determine the state transition matrix of the above Markov chain, including the values found in (a). State your inputs as decimal values between 0.1 and 0.9 (both included) with one decimal precision and remember to use dot as the decimal separator.

$P =$

c. If we know $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{3}$, find $P(X_1 = 2, X_2 = 3, X_3 = 1)$. State your inputs as two integers between 0 and 99 such that the answer is given as an irreducible fraction.

(ME2024) Part1 - 3.pdf

The time it takes to complete a particular task is measured for employees in a company. A new software tool was introduced to help complete the task more efficiently. Completion times in minutes for a random sample of employees before and after using the new software were measured. Let μ_B denote the mean completion time before using the software and let μ_A denote the mean completion time after using the software. If the improvement in task completion time is more than 2 minutes, we would like to detect it. State the correct hypotheses for this test.

Documentation: No documentation is need for this assignment

H_0 : $\mu_B - \mu_A \leq 2$

H_1 : $\mu_B - \mu_A > 2$

$\mu_B - \mu_A \neq 0$ | $\mu_B - \mu_A \geq 2$ | $\mu_B - \mu_A \neq 2$ | $\mu_B - \mu_A > 0$ | $\mu_B - \mu_A < 0$ | $\mu_B - \mu_A = 0$

$\mu_B - \mu_A < 2$ | $\mu_B - \mu_A \geq 0$ | $\mu_B - \mu_A = 2$ | $\mu_B - \mu_A \leq 0$

(ME2024) Part1 - 4.pdf random sample mean time mean before mean after

Consider the following test in a regression problem:

test regression problem

$H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$

test statistic

The following has been computed:

significance

$\hat{\beta}_1 = 14.947 \quad n = 20,$
$S_{xx} = 0.68088, \quad \hat{\sigma}^2 = 1.18$

absolute critical value

Documentation: Please supply the calculations leading to the answers. Note, you will need to use a t-table for this assignment.

a. Calculate the test statistic for this test. State your answer as a decimal value with four decimal precision.

$T_0 = \boxed{11.5540}$

b. Assuming we want to do the test with a significance of 0.01. What will then be the absolute critical value for the test? State your answer as a decimal value with three decimal precision.

$T_{crit} = \boxed{2.878}$

confidence interval for the slope

c. Set up a 95% confidence interval for the slope. State the lower and upper bounds with four decimal precision.

lower and upper bounds

lower bounds

$\boxed{12.1825} \leq \beta_1 \leq \boxed{17.7115}$

(ME2024) Part1 - 5.pdf

Assume the following transition matrix P for an ergodic Markov chain:

ergodic markov chain

$$P = \begin{bmatrix} 1/2 & 1/2 \\ p & 1-p \end{bmatrix}$$

When the experiment is performed many times, the chain ends in state one approximately 20 percent of the time and in state two 80 percent of the time. Find a value for p and state your answer as an irreducible fraction.

$p = \frac{\square}{\square}$

(ME2024) Part1 - 6.pdf

Let A and B be two independent stochastic variables such that $A \sim Poisson(3)$ and $B \sim Geometric(1/2)$. poisson geometric

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. Find $P(A = 2 \text{ or } B = 2)$. State your input as a decimal value with four decimal precision. Remember to use dot as decimal separator.

$P(A = 2 \text{ or } B = 2) =$ ☐ two independent stochastic variables

b. Find $P(B = 3 \mid A + B = 4)$ and state your input as a decimal value with four decimal precision. Remember to use dot as decimal separator.

$P(B = 3 \mid A + B = 4) =$ ☐

c. Find the values below. In both cases, state your answer as a positive integer.

poisson geometric

poisson(3) geometric(1/2)

$E[A + B] =$ ☐

$E\left[(A + B)^2\right] =$ ☐ expected value

(ME2024) Part1 - 7.pdf

A Markov chain has states $0, 1, 2$ with transition probabilities

transition probabilities

$$P = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}.$$

markov chain

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code. Please state all answers as decimal values with four decimal precision.

mean return time

Find the mean return time to state 1. Round your answer to the nearest integer.

$r_1 =$ ☐

(ME2024) Part1 - 8.pdf

Consider two independent discrete random variables X and Y. Assume that the probability function p_X for X is given by

$$p_X(x) = \begin{cases} \frac{1}{3} & \text{if } x \in \{1,2,3\} \\ 0 & \text{otherwise,} \end{cases}$$

two independent discret random variables

poisson distribution

poisson-distributed

and that Y is Poisson-distributed with parameter $\lambda = 1$.

For all questions in this assignment, state your inputs as integers between 0 and 99 such that all answers are given as either integers or irreducible fractions.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

Find the values below.

$P(X \leq 2) = \frac{\square}{\square}$

var

cov

expected value

$Var(X) = \frac{\square}{\square}$

$Var(X - Y) = \frac{\square}{\square}$

$Cov(X, 2Y) = \square$

$E(XY) = \square$

$E(X(X + 5Y)) = \frac{\square}{3}$

(RE2023).pdf Page 1 to 5

A random sample of 1900 adult women indicated that 941 of them asked for medical assistance last time they felt sick. The same sized sample of men indicated that 893 of them asked for medical assistance.

Documentation: In (a) no documentation is required. In (b) you are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. Is it reasonable to conclude that women ask for medical assistance more often than men? Set up an appropriate hypothesis test to answer this question using $\alpha = 0.05$. Assuming p_1 refers to the rate of women asking for medical assistance and that p_2 refers to the rate of men asking for medical assistance, identify the correct alternative hypothesis of this test and check the answer.

- ○ $H_1: p_1 \neq p_2$
- ○ $H_1: p_1 < p_2$
- ○ $H_1: p_1 = p_2$
- ○ $H_1: p_1 + p_2 > 1$
- ○ $H_1: p_1 - p_2 > 1$
- ○ $H_1: p_1 \geq p_2$
- ○ $H_1: p_1 - p_2 \geq 0$
- ○ $H_1: p_1 > p_2$
- ○ $H_1: p_1 - p_2 \neq 0$
- ○ $H_1: p_1 \leq p_2$

hypothesis

critical value

test statistic

p value

p-value

b. Determine the critical value, the test statistic and the p-value for the test. Identify the three values below, i.e. you need to choose three of the choices listed below.

- ☐ 1.5583
- ☐ 0.1192
- ☐ 1.5287
- ☐ 2.3263
- ☐ 0.0596
- ☐ 0.0298
- ☐ 0.5
- ☐ 1.5722
- ☐ 1.2816
- ☐ 2.578
- ☐ 1.6449
- ☐ 1.96

(RE2023).pdf Page 6 to 9

Assuming X and Y are independent and given $E(X) = 2$ and $E(X^2) = \frac{24}{5}$, find the value below. E(X) E(x2)

State you inputs as two positive integers such that the answer displays an irreducible fraction (note a negative sign is pre-printed).

Documentation: You are expected to demonstrate how you obtained the result either by supplying manual calculations or Python code.

$Cov(1 + X, Y - 2X) = -\frac{\square}{\square}$ cov X Y

(RE2023).pdf Page 10 to 12

Suppose that a random variable X has continuous uniform distribution on [1, b] , where b is an unknown parameter. We have a random sample of 15 in size from a population represented by X and the random sample has a sample mean of 5.38. Use the sample mean which is an unbiased estimator, to find a point estimate of b. State your input as an integer between 0 and 99 so that the answer is displayed with two-decimal precision (note that the decimals are pre-printed). point estimate sample mean unbiased estimator

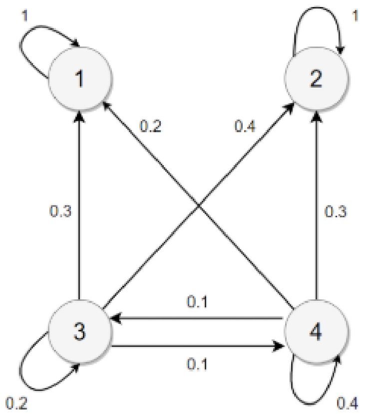
Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

$\boxed{9}.76$

random variable continuous uniform distribution [1,b] unknown parameter random sample

(RE2023).pdf Page 13 to 15

Consider a Markov chain $\{X_n : n = 0, 1, 2, \ldots\}$ with state space $S = \{1, 2, 3, 4\}$ and state transition diagram given by



markov chain

State your input as a **decimal** value so that you supply four decimal precision. Remember to use '.' (dot) as the decimal separator.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

Find the mean hitting time of state 4 given that the proces started in state 3.

mean hitting time

(RE2023).pdf Page 16 to 20

hourly average mean variance

A city authority checks the hourly average weight of the traffic on a small bridge to determine if more frequent maintenance should be conducted. Suppose X, the hourly average weight of the traffic, is normally distributed with the mean of 30 tons and a variance of 6.25.

Determine the following, choosing the correct answer for each item.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. $P(X > 37)$

A	0.0125
B	0.2302
C	0.0072
D	0.1418
E	0.1314

b. $P(28.5 < X < 32.5)$

A	0.2503
B	0.2007
C	0.5412
D	0.2764

c. Weight, which is exceeded with probability 0.99.

A	15.46
B	23.68
C	24.18

(RE2023).pdf Page 21 to 24

Accidents on highways are one of the main causes of death or injury in developing countries and the weather conditions have an impact on the rates of death and injury. In foggy, rainy, and sunny conditions, 1/4, 1/8, and 1/21 of the accidents result in death, respectively. Sunny conditions occur 60% of the time, while rainy and foggy conditions each occur 20% of the time.

For both questions, identify the correct answer, here given with 4 decimal precision. probability weather accidents

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. What is the probability that an accident results in a death?

A	0.1103
B	0.0964
C	0.0987
D	0.1152

b. Given that an accident without deaths occurred, what is the conditional probability that it was foggy at the time? Remember to use non-rounded values in your calculations.

conditional probability

A	0.1721
B	0.1750
C	0.1603
D	0.1673
E	0.1587

(RE2023).pdf Page 25 to 28

A database contains 48 heart signal recordings with 22 from females patients and 26 from male patients. For a classification task, an analyst randomly selects 36 records for predictive model training and keeps the other 12 records for testing the model performance.

For all questions, identify the correct answer. expected number

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. What is the expected number of female patient recordings selected for training? hyper geometric distribution

A	16.5
B	17.5
C	19
D	18

b. What is the probability that at least three male patient recordings are selected for training? probability

A	0.95
B	1
C	0.94
D	0

c. What is the probability that the same number of male and female patients are used for testing the model performance?

A	0.5
B	0.22
C	0
D	0.25

(RE2023).pdf Page 29 to 33

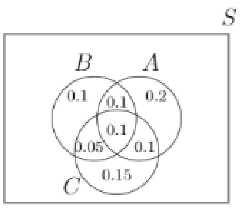
Four teams A, B, C, and D compete in a tournament. Teams A and B have the same chance of winning the tournament. Team C is twice as likely to win the tournament as team D. The probability that either team A or team C wins the tournament is 0.6. Find the probabilities of each team winning the tournament. State your answer as an integer between 0 and 9, making sure the answer is correctly rounded off.

$P(A) = 0.\boxed{2}$, $P(B) = 0.\boxed{2}$, $P(C) = 0.\boxed{4}$, $P(D) = 0.\boxed{2}$

football team probability

Exam 1-3 – question1.pdf

Let A, B, and C be three events with probabilities given:



venn diagram

probability

Find the following probabilities. State all inputs as integers between 0 and 99 such that your answers are irreducible fractions.

$P(A \mid B) =$

$$\frac{\boxed{4}}{\boxed{7}}$$

Check answer

$P(C \mid B) =$

$$\frac{\boxed{3}}{\boxed{7}}$$

$P(B \mid A \cup C) =$

$$\frac{\boxed{5}}{\boxed{14}}$$

Check answer

$P(B \mid A,\, C) =$

$$\frac{\boxed{1}}{\boxed{2}}$$

Exam 1-3 – question2.pdf

uniformly random
continuous interval

A **real** number X is selected uniformly at random in the continuous interval [0,10]. (For example, X could be 3.87.)

Find the following probabilities. State all your inputs as integers between 0 and 99 such tthat your answers are irreducible fractions.

$P(2 \leq X \leq 5) =$

$$\frac{\boxed{3}}{\boxed{10}}$$

Check answer

$P(X \leq 2 \mid X \leq 5) =$

$$\frac{\boxed{2}}{\boxed{5}}$$

Check answer

$P(3 \leq X \leq 8 \mid X \geq 4)$

$$\frac{\boxed{2}}{\boxed{3}}$$

Exam 1-3 – question3.pdf

Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} \frac{1}{2} & \text{for } x = 0 \\ \frac{1}{3} & \text{for } x = 1 \\ \frac{1}{6} & \text{for } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

discrete random variable PMF

Rx

Find the following. State all inputs as integers between 0 and 99 and state all sets $\{x_1, x_2, \ldots, x_n\}$ such that $x_1 < x_2 < \ldots < x_n$. Also, all resulting fractions must be irreducible.

$R_X = \{\boxed{0}, \boxed{1}, \boxed{2}\}$

Check answer

$P(X \geq 1.5) = \frac{\boxed{1}}{\boxed{6}}$

$P(0 < X < 2) = \frac{\boxed{1}}{\boxed{3}}$

Check answer

$P(X = 0 \mid X < 2) =$

$\frac{\boxed{3}}{\boxed{5}}$

Exam 1-3 – question4.pdf

Let X be the number of the cars being repaired at a repair shop. We have the following information:

- At any time, there are at most 3 cars being repaired.
- The probability of having 2 cars at the shop is the same as the probability of having one car.
- The probability of having no car at the shop is the same as the probability of having 3 cars.
- The probability of having 1 or 2 cars is half of the probability of having 0 or 3 cars.
- The shop can handle no more than 3 cars.

Find the following. State all inputs as integers between 0 and 99 and state all sets $\{x_1, x_2, \ldots, x_n\}$ such that $x_1 < x_2 < \ldots < x_n$. Also, all resulting fractions must be irreducible.

$R_X = \{\boxed{0}, \boxed{1}, \boxed{2}, \boxed{3}\}$

probability

Check answer

$P_X(0) = P_X(3) = \dfrac{\boxed{1}}{\boxed{3}}$

Check answer

$P_X(1) = P_X(2) = \dfrac{\boxed{1}}{\boxed{6}}$

Exam 1-3 – question5.pdf

Let X and Y be two independent discrete randoms variable with the following PMFs

$$P_X(k) = \begin{cases} \frac{1}{4} & \text{for } k = 1 \\ \frac{1}{8} & \text{for } k = 2 \text{ and } k = 3 \\ \frac{1}{2} & \text{for } k = 4 \\ 0 & \text{otherwise} \end{cases}$$

two independent discrete randoms variable pmf

and

$$P_Y(k) = \begin{cases} \frac{1}{6} & \text{for } k = 1 \text{ and } k = 2 \\ \frac{1}{3} & \text{for } k = 3 \\ \frac{1}{3} & \text{for } k = 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the following. State all inputs as integers between 0 and 99 and state all sets $\{x_1, x_2, \ldots, x_n\}$ such that $x_1 < x_2 < \ldots < x_n$. Also, all resulting fractions must be irreducible.

$R_X = R_Y = \{\boxed{1}, \boxed{2}, \boxed{3}, \boxed{4}\}$

Check answer

$P(X \leq 2 \text{ and } Y \leq 2) =$

$$\frac{\boxed{6}}{\boxed{48}}$$

$P(X > 2 \text{ or } Y > 2) =$

$$\frac{\boxed{7}}{\boxed{8}}$$

Check answer

$P(X > 2 \mid Y > 2) =$

$$\frac{\boxed{5}}{\boxed{8}}$$

Check answer

$P(X < Y) =$

$$\frac{\boxed{1}}{\boxed{3}}$$

Exam 1-3 – question6.pdf

Consider the following circuit.

The probability that each device functions correctly is $p_1 = 0.83$ and $p_2 = 0.75$. Assume that devices fail independently. Let X denote the number of defective devices. Find the mean of X. State your answer as an integer between 0 and 99, such that you supply two decimal precision correctly rounded off.

$EX = 0.\boxed{42}$

Exam 1-3 – question7.pdf independently mean of x p1 p2

Each main bearing cap in an engine contains four bolts. The bolts are selected at random, without replacement, from a parts bin that contains 30 bolts from one supplier and 70 bolts from another supplier. State your answers as integers between 0 and 99 such that you supply two decimal precision, correctly rounded off.

a. What is the probability that a main bearing cap contains all bolts from the same supplier?

$0.\boxed{24}$

probability

Check answer

b. What is the probability that exactly three bolts are from the same supplier?

$0.\boxed{49}$

Exam 1-3 – question8.pdf

A hospital groups its patients into three main groups:

- It has 242 that leave without beeing seen (LWBS)
- It has 984 that are admitted
- It has 3821 that are not admitted

What is the smallest sample size needed so that the probability is at least 0.9 that at least one patient is LWBS? State your answer as an integer between 0 and 99.

$n = \boxed{47}$

Exam 1-3 – question9.pdf sample size smallest sample size probability

A research team has developed a face recognition device to match photos in a database. From laboratory tests, the recognition accuracy is 92% and trials are assumed to be independent.

a. If the research team continues to run laboratory tests, what is the mean number of trials until failure? State your answer as an integer between 0 and 99.

$\boxed{12}.5$

trials until failure

Check answer

b. What is the probability that the first failure occurs on the tenth trial? State your answer as an integer between 100 and 999 such that you supply four decimal precision, correctly rounded off.

$0.0\boxed{378}$

first failure k trial

Check answer

c. To improve the recognition algorithm, a chief engineer decides to collect 10 failures. How many trials are expected to be needed? State your answer as an integer between 0 and 999.

$\boxed{125}$

k failure

Exam 1-3 – question10.pdf

First identify the correct distribution and then answer the questions.

Let X denote the number of patients arriving at a walk-in test center between 1:00 and 2:00 PM with a mean of 10.5. Then X follows a

Poisson distribution.

poisson distribution

Check answer

For the following questions, state your answer as an integer between 0 and 99 such that you supply two decimal precision, correctly rounded off.

a. What is the probability that fewer than 15 patients arrive at the clinic between 1:00 and 2:00 PM?

0.89

probability time

Check answer

b. What is the probability that no patients arrive between 1:00 and 1:10 pm?

0.17

Check answer

c. Suppose that 20 patients arrive between 1:00 and 1:30 PM. What is the probability that three more patients arrive between 1:30 and 2:00 PM?

0.13

Exam 1-3 – question11.pdf

Let a continuous random variable X denote the time spent on a cell phone (in hours) per month with the following probability density function where $h \neq 0$:

$$f(x) = \begin{cases} \frac{x-10}{5h} & 10 \leq x < 15 \\ \frac{1}{h} & 15 \leq x < 20 \\ -\frac{x-25}{5h} & 20 \leq x \leq 25 \\ 0 & \text{otherwise} \end{cases}$$

continuous random variable time

a. Find the value of h. State your answer as an integer between 0 and 99.

$h = \boxed{10}$

h

Check answer

b. Find $P(X < 17.5)$. State your answer as an integer between 0 and 99 such that you provide two decimal precision, correctly rounded off.

$0.\boxed{5}$

Check answer

c. Find $P(X < 22.0)$. State your answer as an integer between 0 and 99 such that you provide two decimal precision, correctly rounded off.

$0.\boxed{91}$

d. Find x such that $P(X < x) = 0.95$. State your answer as an integer between 0 and 99 such that your answer is rounded off correctly to the nearest integer.

$x = \boxed{23}$

expected value

Check answer

e. Find the expected value of X. State your answer as an integer between 0 and 99.

$EX = \boxed{17}.5$

Check answer

variance

e. Find the variance of X. State your answer as an integer between 0 and 99.

$Var(X) = \boxed{10}.42$

Exam 1-3 – question12.pdf

A credit card company monitors cardholder transaction habits to detect any unusual activity. Suppose that the dollar value of unusual activity for a customer in a month follows a normal distribution with mean \$250 and variance \$391. Answer each question below and state your inputs as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

a. What is the probability of \$250 to \$300 in unusual activity in a month? normal distribution mean variance

0. 4 9 4 3

Check answer

probability

b. What is the probability of more than \$300 in unusual activity in a month?

0. 0 0 5 7

Check answer

k independently

c. Suppose that 10 customer accounts independently follow the same normal distribution. What is the probability that at least one of these customers exceeds \$300 in unusual activity in a month?

0. 0 5 5 8

Exam 1-3 – question13.pdf

Web crawlers need to estimate the frequency of changes to Web sites to maintain a current index for Web searches. Assume that the changes to a Web site follow a Poisson process with a mean of 6 days. Let a random variable X denote the time (in days) until the next change.

a. What is the probability that the next change occurs in less than 4.5 days? State your answer as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

0. 5 2 7 6

poisson process mean

Check answer

b. What is the probability that the time until the next change is greater than 9.5 days? State your answer as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

0. 2 0 5 3

Check answer

c. What is the time of the next change that is exceeded with probability 90%? State your answer as an integer between 0 and 99 such that you supply 2 decimal precision, correctly rounded off.

0. 63 day

d. What is the probability that the next change occurs in less than 12.5 days, given that it has not yet occurred after 3.0 days? State your answer as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

0. 7 9 4 7

Exam 1-3 – question14.pdf

Two methods of measuring surface smoothness are used to evaluate a paper product. The measurements are recorded as deviations from the nominal surface smoothness in coded units. The joint probability distribution of the two measurements is a uniform distribution over the region $0 < x < 4,\ 0 < y,\ \text{and}\ x - 1 < y < x + 1$. That is, $f_{XY}(x, y) = c$ for X and Y in the region.

a. Determine the value for c such that $f_{XY}(x, y)$ is a joint probability density function. State your inputs as two integers between 0 and 99 such that your answer is an irreducible fraction.

$\frac{2}{15}$

Check answer

deviations

joint probability distribution

uniform distribution

fxy(x,y)

c

b. $P(X < 0.5, Y < 0.6) =$ 0.04

Check answer

c. $P(X < 0.5) =$ 0.083

Check answer

d. $E(X) =$ 2.111

Check answer

e. Find the marginal probability density function of X for $1 < x < 4$. State your inputs as two integers between 0 and 99 such that your answer is an irreducible fraction.

marginal probability density function

$\frac{4}{15}$

Check answer

exam4-7.pdf page 1-5

Suppose that a random variable X has continuous uniform distribution on [1, a] where a is an unknown parameter. We have a random sample of 15 in size from a population represented by X: 4.5, 1.3, 8.6, 6.4, 7.4, 4.3, 7.2, 1.6, 4.4, 2.0, 8.4, 6.5, 7.3, 3.4, 7.4. Find a point estimate, $\hat{a}$, of a. Use an unbiased estimator. State your answer as en integer between 0 and 99.

$\boxed{9}$.76

random variable continuous uniform distribution [1,a] unknown parameter

random sample point estimate a hat a^ ahat

Exam3-7.pdf page 13-14 unbiased estimator

Consider the one-sided confidence interval expressions for a mean of normal population with known variance. State your answer as an integer between 0 and 99 such that the confidence level is stated as a two decimal percentage.

one-sided confidence interval

What confidence level will cause $|z_\alpha|$ be equal to 1.93? A $\boxed{97}$.32% confidence level.

one sided confidence interval

What confidence level will cause $|z_\alpha|$ be equal to 1.18? A $\boxed{88}$.10% confidence level.

mean

za

What confidence level will cause $|z_\alpha|$ be equal to 1.45? A $\boxed{92}$.65% confidence level.

confidence level

Exam3-7.pdf page 14-15

Consider results of 20 randomly chosen people who have run a marathon. Their times, in minutes, are as follows: 137, 146, 150, 163, 166, 179, 186, 193, 196, 213, 218, 225, 236, 239, 254, 269, 271, 279, 282, 295.

Calculate a 90% upper confidence bound on the mean time of the race. Assume distribution to be normal. Use the ceiling function such that your answer is an integer between 0 and 999.

$\mu \leq \boxed{230}$

mean time normal distribution ceilling function

Check answer

Exam3-7.pdf page 16-18

Assume a random sample has been taken from a normal distribution. You are told that the standard error was 1.55, the standard deviation was 6.02, and the sum of all observations was 797.90. Based on this information find the values below.

Using the floor function, the sample size is n = $\boxed{15}$.

normal distribution

random sample

sum of all observations

standard error

The sample mean is $\hat{x} = \boxed{53}$.193. State your answer as an integer between 0 and 99.

standard deviation

sample mean

sample size

xhat x^ x hat

interval

Find a 95% CI on the population mean using the values found above. Select the answer that best encapsulates the interval.

population mean

A	[45 ; 62]
B	[46 ; 49]
C	[43 ; 57]
D	[50 ; 59]
E	[42; 58]
F	[43 ; 53]
G	[50 ; 57]

Exam3-7.pdf page 18-21

To estimate the portion of voters who plan to vote for Candidate A in an election, a random sample of size *n* from the voters is chosen. The sampling is done with replacement. Let θ be the portion of voters who plan to vote for Candidate A among all voters. How large does *n* need to be so that we can obtain a 90% confidence interval with 3 % margin of error? State your answer as an integer between 0 and 99.

$n \geq \boxed{752}$ confidence interval margin of error

random sample size n sampling replacement

Exam3-7.pdf page 21-23

A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed with known standard deviation of 0.3 volt, and the manufacturer wishes to test $H_0 : \mu_0 = 5$, using n = 10 units. The acceptance region for this test is $4.85 \leq \mu \leq 5.15$. Find the value of α. State your answer as an integer between 10 and 99 such that you supply two decimal precision, correctly rounded off.

$\alpha = 0.\boxed{11}$ standard deviation hypothesis a

Exam3-7.pdf page 23-25

The United States Golf Association tests golf balls to ensure that they conform to the rules of golf. Balls are tested for weight, diameter, roundness, and overall distance. The overall distance test is conducted by hitting balls with a driver swung by a mechanical device nicknamed "Iron Byron" after the legendary great Byron Nelson, who's swing the machine is said to emulate. From a sample of 100 distances (in meters) achieved by a particular brand of golf ball in the overall distance test, the following was obtained: $\bar{x} = 260.3$ and $s = 13.41$.

Can you support the claim, **using a t-test**, at a 0.05 level of significance that the mean distance achived by this particular golf ball exceeds 280 meters? First identify the correct hypotheses by dragging the correct hypothesis to the appropriate placeholder. Then state the test statistic and the critical value.

H_0 : $\mu_0 = 280$

H_1 : $\mu_0 > 280$

sample distance test xhat x hat x^ s t-test t test

hypothesis level significance

hypotheses mean distance

$\mu_0 \geq 280$ $\mu_0 < 280$ $\mu_0 \neq 280$ $\mu_0 \leq 280$

test statistic

critical value

fail to reject

Please state all values as integers between 0 and 99 such that you supply two decimal precision, correctly rounded off.

Test Statistic = -14.69

Critical Value = 1.66

Check answers

We Fail to reject

Exam3-7.pdf page 25-28

A recent study among 254 computer science graduates from Aarhus University was made in order to determine how successful the former students were in their current employment. 98 of these students had taken a course in linear algebra and of these 92 were classified as "successful" in their current employment. 136 of the students who had not taken a course in linear algebra were classified as "successful" in their current employment.

Is there evidence to support the claim that computer science graduates who had taken a linear algebra course were more successful in their current employment than those who had not taken such a course with a significance of 0.05? Let p_A denote the proportion of succesful students who had taken linear algebra and let p_{A^c} denote the proportion successful students who had not taken the linear algebra course. First identify the correct hypotheses by dragging the correct hypothesis to the appropriate placeholder. Then state the test statistic, the critical value, and the p-value.

H_0 : $p_A = p_{A^c}$

H_1 : $p_A > p_{A^c}$

evidence support claim

hypothesis

significance

test statistic
critical value
p value p-value

reject

$p_A < p_{A^c}$ | $p_A \neq p_{A^c}$ | $p_A \geq p_{A^c}$ | $p_A \leq p_{A^c}$

The test statistic is 1.71

Check answer

The critical value is 1.64

Check answer

The p-value is 0.0432

Check answer

Based on the above we Reject

Check answer

Exam3-7.pdf page 33-35

To estimate efficiency of a drug for weight loss, the clinical trial was performed. The results are presented in the table below (data also available in "WeightLoss.xlsx".

Patient number	Weight before trial, kg	Weight after trial, kg
1	85.2	83.5
2	79.6	78.1
3	75.8	73.2
4	76.2	74.0
5	91	90.2
6	89.8	87.0
7	82.0	79.9
8	81.7	78.5
9	67.3	64.0
10	68.4	65.1
11	70.0	67.8
12	74	70.0
13	66.8	64.6
14	60	58.6
15	94	92.9
16	88.2	88.0

hypothesis

evidence support claim

significance

mean weight

average weight

test statistic
critical value

two-sided confidence interval

Is there evidence to support the claim that the drug has had an effect (negative or positive) with a significance of 0.01? Let μ_B the mean weigth before the clinical trial and let μ_A denote the average weight after the trial. First identify the correct hypotheses by dragging the correct hypothesis to the appropriate placeholder. Then state the test statistic and the critical value.

H_0 : $\mu_B - \mu_D = 0$

H_1 : $\mu_B - \mu_D \neq 0$

The test statistic is 8.45 (absolute value)

Check answer

The critical value is 2.95 (absolute value)

Check answer

Construct a 99% two-sided confidence interval for the difference in weight.

A [1.21 ; 2.62]

B [2.00 ; 3.00]

More

Assume they now want to set up a test to support the claim that the drug has had a positive effect, i.e. has led to weight loss. How should the hypotheses of such a test look?

H_0 : $\mu_B - \mu_D = 0$

H_1 : $\mu_B - \mu_D > 0$

Exam3-7.pdf page 42-45