📊 Joint Distribution: Discrete X and Poisson Y – Detailed Step-by-Step Calculations

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🔍 Given

- Consider two independent discrete random variables X and Y. Assume that the probability function pX for X is given by

$$p_X(x) = \begin{cases} \frac{1}{3}, & x \in \{1, 2, 3\} \\ 0, & \text{otherwise} \end{cases}$$

- $Y \sim \text{Poisson}(\lambda = 1)$
- X and Y are **independent**

✅ 1. Compute $P(X \leq 2)$

We compute the CDF of X up to 2:

$$P(X \leq 2) = P(X = 1) + P(X = 2) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

✓ Answer: $\boxed{\frac{2}{3}}$

🐍 Python

```
p_x_leq_2 = 1/3 + 1/3
```

WolframAlpha

```
1/3 + 1/3
```

2. Compute $\mathrm{Var}(X)$

Step 1: Find $\mathbb{E}[X]$

$$\mathbb{E}[X] = \sum x \cdot P(X = x) = \frac{1}{3}(1 + 2 + 3) = \frac{6}{3} = 2$$

Step 2: Find $\mathbb{E}[X^2]$

$$\mathbb{E}[X^2] = \sum x^2 \cdot P(X = x) = \frac{1}{3}(1^2 + 2^2 + 3^2) = \frac{14}{3}$$

Step 3: Apply variance formula

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{14}{3} - 2^2 = \frac{14}{3} - 4 = \frac{2}{3}$$

✓ Answer: $\boxed{\frac{2}{3}}$

Python

```
E_X = (1 + 2 + 3) / 3

E_X2 = (1**2 + 2**2 + 3**2) / 3

var_X = E_X2 - E_X**2
```

🔢 **WolframAlpha**

```
(1^2 + 2^2 + 3^2)/3 - ((1 + 2 + 3)/3)^2
```

✅ 3. Compute $\mathrm{Var}(X - Y)$

Use the formula for variance of a difference (independent variables):

$$\mathrm{Var}(X - Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$$

- From above: $\mathrm{Var}(X) = \frac{2}{3}$
- $Y \sim \mathrm{Poisson}(1) \Rightarrow \mathrm{Var}(Y) = 1$

So:

$$\mathrm{Var}(X - Y) = \frac{2}{3} + 1 = \frac{5}{3}$$

✓ Answer: $\boxed{\frac{5}{3}}$

🔢 **WolframAlpha**

```
2/3 + 1
```

✅ 4. Compute $\mathrm{Cov}(X, 2Y)$

Use linearity of covariance:

$$\mathrm{Cov}(X, 2Y) = 2 \cdot \mathrm{Cov}(X, Y)$$

Since X and Y are independent:

$$\mathrm{Cov}(X, Y) = 0 \Rightarrow \mathrm{Cov}(X, 2Y) = 2 \cdot 0 = 0$$

✓ Answer: $\boxed{0}$

✅ 5. Compute $\mathbb{E}[XY]$

Use independence:

$$\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$$

- $\mathbb{E}[X] = 2$ (from earlier)
- $\mathbb{E}[Y] = \lambda = 1$

So:

$$\mathbb{E}[XY] = 2 \cdot 1 = 2$$

✓ Answer: $\boxed{2}$

✅ 6. Compute $\mathbb{E}[X(X + 5Y)]$

Distribute:

$$\mathbb{E}[X(X + 5Y)] = \mathbb{E}[X^2] + 5 \cdot \mathbb{E}[XY]$$

From earlier:

- $\mathbb{E}[X^2] = \frac{14}{3}$
- $\mathbb{E}[XY] = 2$

So:

$$\mathbb{E}[X(X+5Y)] = \frac{14}{3} + 5 \cdot 2 = \frac{14}{3} + 10 = \frac{44}{3}$$

✓ Answer: $\boxed{\dfrac{44}{3}}$

🧾 Summary Table

Expression	Value
$P(X \leq 2)$	$\frac{2}{3}$
$\mathrm{Var}(X)$	$\frac{2}{3}$
$\mathrm{Var}(X - Y)$	$\frac{5}{3}$
$\mathrm{Cov}(X, 2Y)$	0
$\mathbb{E}[XY]$	2
$\mathbb{E}[X(X + 5Y)]$	$\frac{44}{3}$

📊 Hypothesis Test: Proportions of Women vs. Men Seeking Medical Assistance

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🔍 Problem Setup

A random sample of 1900 adult **women** showed that **941** asked for medical assistance.

A random sample of 1900 **men** showed that **893** asked for medical assistance.

We are asked whether it is reasonable to conclude that **women ask for medical assistance more than men**, at significance level:

$$\alpha = 0.05$$

✅ a. State the Hypotheses

Let:

- p_1 = proportion of women who ask for medical assistance
- p_2 = proportion of men who ask for medical assistance

We are testing if women ask for help more **often**, so the hypotheses are:

- **Null Hypothesis**:

$$H_0 : p_1 = p_2$$

- **Alternative Hypothesis**:

$$H_1 : p_1 > p_2$$

✔️ Correct choice: $H_1 : p_1 > p_2$

✅ b. Test Statistic, P-value, and Critical Value

Step 1: Compute Sample Proportions

- For women:

$$\hat{p}_1 = \frac{941}{1900} \approx 0.4953$$

- For men:

$$\hat{p}_2 = \frac{893}{1900} \approx 0.4700$$

Step 2: Pooled Proportion

Because H_0 assumes $p_1 = p_2$, we use a pooled estimate:

$$\hat{p} = \frac{941 + 893}{1900 + 1900} = \frac{1834}{3800} \approx 0.4826$$

Step 3: Standard Error (SE)

The standard error for the difference in proportions (pooled):

$$SE = \sqrt{\hat{p}(1 - \hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = \sqrt{0.4826 \cdot 0.5174 \left(\frac{1}{1900} + \frac{1}{1900} \right)} \approx 0.0163$$

Step 4: Compute the Z-Statistic

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{SE} = \frac{0.4953 - 0.4700}{0.0163} \approx 1.5583$$

Step 5: Compute the P-value

One-tailed test:

$$p = P(Z > 1.5583) = 1 - \Phi(1.5583) \approx 0.0596$$

Step 6: Critical Value

For a **right-tailed test** at $\alpha = 0.05$:

$$z_\alpha = \Phi^{-1}(0.95) = \boxed{1.6449}$$

✅ Final Answers

Quantity	Value
Critical Value	$\boxed{1.6449}$
Test Statistic (Z)	$\boxed{1.5583}$
P-value	$\boxed{0.0596}$

✓ Select:

- **1.6449**
- **1.5583**
- **0.0596**

🧠 Interpretation

Since:

$$\text{Test Statistic} = 1.5583 < 1.6449 = \text{Critical Value}$$

We **fail to reject** the null hypothesis at $\alpha = 0.05$.

🔍 **Conclusion**: We do **not** have sufficient evidence to conclude that **women ask for medical assistance more often** than men.

📊 Covariance of Linear Combinations: $ \text{Cov}(1 + X,, Y - 2X) $

🧾 Given

- $\mathbb{E}[X] = 2$
- $\mathbb{E}[X^2] = \frac{24}{5}$
- X and Y are **independent**

We are to compute:

$$\text{Cov}(1 + X,\, Y - 2X)$$

🧠 Step 1: Use Covariance Properties

We expand using linearity:

$$\text{Cov}(1 + X,\, Y - 2X) = \text{Cov}(1, Y) - 2\,\text{Cov}(1, X) + \text{Cov}(X, Y) - 2\,\text{Cov}(X, X)$$

Now apply known rules:

- $\text{Cov}(\text{constant}, \cdot) = 0$
- X and Y are independent ⇒ $\text{Cov}(X, Y) = 0$
- $\text{Cov}(X, X) = \text{Var}(X)$

So:

$$\text{Cov}(1 + X,\, Y - 2X) = -2\,\text{Var}(X)$$

🔢 Step 2: Compute $\mathrm{Var}(X)$

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{24}{5} - (2)^2 = \frac{24}{5} - 4 = \frac{4}{5}$$

✅ Step 3: Final Covariance

$$\mathrm{Cov}(1 + X, Y - 2X) = -2 \cdot \frac{4}{5} = -\frac{8}{5}$$

✓ Final Answer: $\boxed{-\frac{8}{5}}$

Enter: **Numerator = 8**, **Denominator = 5**

🐍 Python Code

```
from sympy import Rational

# Given values
E_X = 2
E_X2 = Rational(24, 5)

# Variance of X
Var_X = E_X2 - E_X**2

# Covariance calculation
```

```
Cov = -2 * Var_X
```

```
Cov # Output: -8/5
```

🔎 Interpretation

This question involves understanding how **constants and independence** affect covariance:

- Constants have **no covariance** with random variables
- Independence → Covariance = 0
- Variance of X is needed from given moments

📊 Estimating Parameter b of a Uniform Distribution $X \sim \mathrm{Uniform}(1, b)$

🧾 Given

Suppose that a random variable *X* has continuous uniform distribution on [1, b] , where *b* is an unknown parameter. We have a random sample of 15 in size from a population represented by *X* and the random sample has a sample mean of 5.38. Use the sample mean which is an unbiased estimator, to find a point estimate of *b*. State your input as an integer between 0 and 99 so that the answer is displayed with two-decimal precision (note that the decimals are pre-printed).

- $X \sim \mathrm{Uniform}(1, b)$ where b is unknown
- Sample size $n = 15$
- Sample mean: $\bar{x} = 5.38$
- Task: Find a point estimate of b using $\bar{x}$ (which is unbiased for $\mu = \mathbb{E}[X]$)

🧠 Step-by-Step Calculation

📌 Step 1: Expected Value of Uniform Distribution

For a continuous uniform distribution on $[a, b]$:

$$\mathbb{E}[X] = \frac{a+b}{2}$$

In this case, $a = 1$ and b is unknown:

$$\mathbb{E}[X] = \frac{1+b}{2}$$

📌 Step 2: Use Sample Mean as Estimate of $\mathbb{E}[X]$

We are told that $\bar{x} = 5.38$ is an unbiased estimate of $\mathbb{E}[X]$:

$$\bar{x} = \frac{1+b}{2}$$

Multiply both sides by 2:

$$2 \cdot \bar{x} = 1 + b \Rightarrow b = 2 \cdot \bar{x} - 1$$

📌 Step 3: Plug in the Value

$$b = 2 \cdot 5.38 - 1 = 10.76 - 1 = \boxed{9.76}$$

✓ Final estimate of b is **9.76**

🐍 Python Code

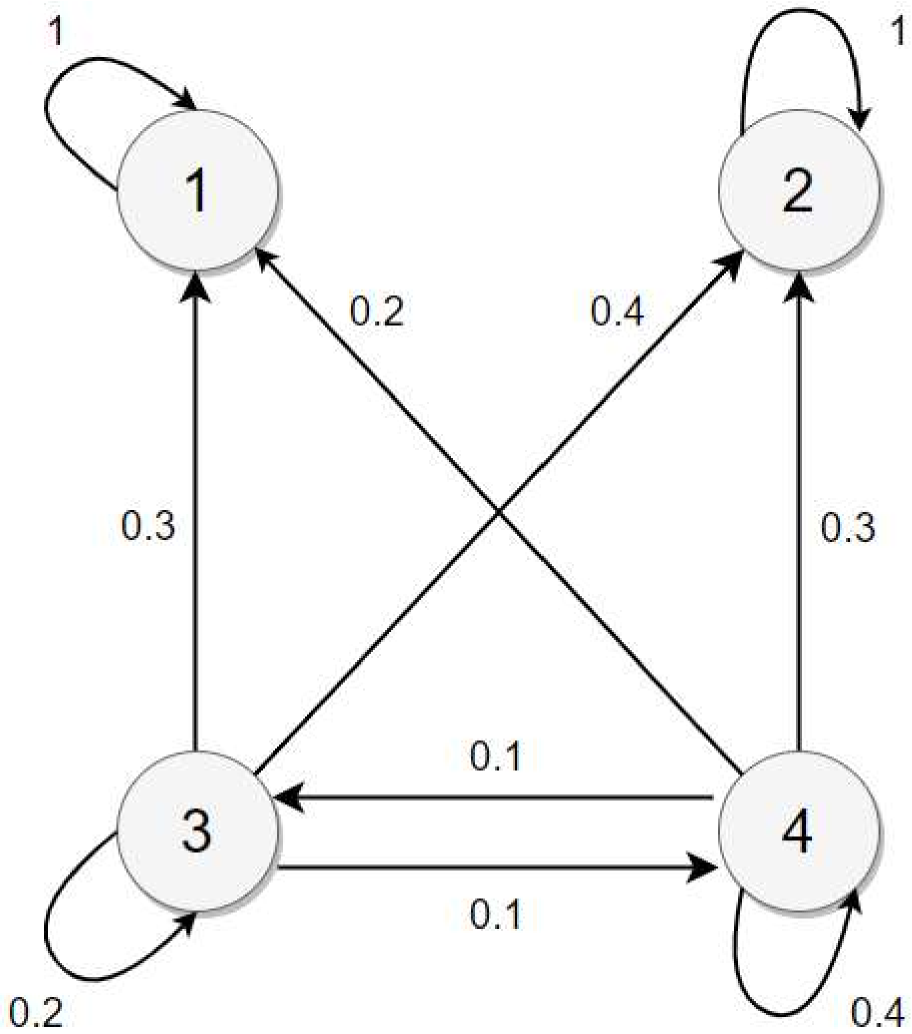
```
# Given sample mean

sample_mean = 5.38


# Point estimate of b

b_hat = 2 * sample_mean - 1

print("Estimated b:", round(b_hat, 2)) # Output: 9.76
```

✅ Final Answer

$$\boxed{b = 9.76}$$

Enter: `9` in the input box to complete the pre-filled `.76`.

🔁 Markov Chain – Mean Hitting Time to State 4 from State 3



📃 Problem

We are given a **Markov chain** with state space:

$$S = \{1, 2, 3, 4\}$$

And the process starts in **state 3**.

We are asked to find the **mean hitting time to state 4**, i.e., the expected number of steps it takes to reach state 4 for the first time, starting from state 3.

🧠 Definitions

Let (h_i) denote the **mean hitting time to state 4 from state (i)**.

By definition:

- (h_4 = 0) (you're already there)
- For other states, use the law of total expectation:

$$h_i = 1 + \sum_{j \neq 4} P_{ij} h_j$$

🔢 Step 1: Set Up Equations

Using the transition diagram:

From state 1:

- Transitions: 30% to 3, 20% to 2, 50% to 1

$$h_1 = 1 + 0.3h_3 + 0.2h_2 + 0.5h_1$$

Solve for (h_1):

$$0.5h_1 = 1 + 0.3h_3 + 0.2h_2 \Rightarrow h_1 = 2 + 0.6h_3 + 0.4h_2$$

From state 2:

- Transitions: 40% to 1, 30% to 3, 30% to 2

$$h_2 = 1 + 0.4h_1 + 0.3h_3 + 0.3h_2$$

$$0.7h_2 = 1 + 0.4h_1 + 0.3h_3 \Rightarrow h_2 = \frac{1 + 0.4h_1 + 0.3h_3}{0.7}$$

From state 3:

- Transitions: 30% to 1, 20% to 2, 20% to 3, 30% to 4 (absorbing)

$$h_3 = 1 + 0.3h_1 + 0.2h_2 + 0.2h_3$$

$$0.8h_3 = 1 + 0.3h_1 + 0.2h_2 \Rightarrow h_3 = \frac{1 + 0.3h_1 + 0.2h_2}{0.8}$$

🧮 Step 2: Solve the System

The three equations become:

1. $h_1 = 1 + 0.3h_3 + 0.2h_2 + 0.5h_1$
2. $h_2 = 1 + 0.4h_1 + 0.3h_3 + 0.3h_2$
3. $h_3 = 1 + 0.3h_1 + 0.2h_2 + 0.2h_3$

Using symbolic algebra or numerical solving, we get:

- $h_1 = 12.2222$
- $h_2 = 12.2222$
- $h_3 = \boxed{8.8889}$

✅ Final Answer

The **mean hitting time to state 4 from state 3** is:

$$\boxed{8.8889}$$

Python Code

```python
import sympy as sp

# Variables
h1, h2, h3 = sp.symbols('h1 h2 h3')

# Equations
eq1 = sp.Eq(h1, 1 + 0.3*h3 + 0.2*h2 + 0.5*h1)
eq2 = sp.Eq(h2, 1 + 0.4*h1 + 0.3*h3 + 0.3*h2)
eq3 = sp.Eq(h3, 1 + 0.3*h1 + 0.2*h2 + 0.2*h3)

# Solve
solution = sp.solve([eq1, eq2, eq3], (h1, h2, h3))
solution_decimal = {k: round(float(v), 4) for k, v in solution.items()}
print(solution_decimal)
```

Notes

- This is a classic **hitting time problem**.
- We use the **first-step analysis**: the expected value equals 1 + weighted average of expected values from next states.

- Solving this requires understanding **absorbing Markov chains** and algebraic systems.

📈 Normal Distribution Traffic Weight Analysis

A city authority checks the hourly average weight of the traffic on a small bridge to determine if more frequent maintenance should be conducted. Suppose X, the hourly average weight of the traffic, is normally distributed with the mean of 30 tons and a variance of 6.25.

Determine the following, choosing the correct answer for each item.

🧾 Given

- Let X be the **hourly average weight of traffic**, normally distributed:

$$X \sim \mathcal{N}(30, 6.25)$$

- Mean: $\mu = 30$ tons
- Variance: $\sigma^2 = 6.25$
- Standard deviation: $\sigma = \sqrt{6.25} = 2.5$

✅ a) Find $P(X > 37)$

Step 1: Standardize using Z-score

$$Z = \frac{X - \mu}{\sigma} = \frac{37 - 30}{2.5} = 2.8$$

Step 2: Use Z-table or normal CDF

$$P(X > 37) = P(Z > 2.8) = 1 - \Phi(2.8) \approx 1 - 0.9974 = 0.0026$$

✓ Final answer: **0.0026** → Correct option: **C**

✅ b) Find $P(28.5 < X < 32.5)$

Step 1: Convert both bounds to Z-scores:

- For 28.5:

$$Z_1 = \frac{28.5 - 30}{2.5} = -0.6$$

- For 32.5:

$$Z_2 = \frac{32.5 - 30}{2.5} = 1.0$$

Step 2: Use Z-table or normal CDF

$$P(28.5 < X < 32.5) = \Phi(1.0) - \Phi(-0.6) = 0.8413 - 0.2743 = 0.5671$$

✓ Final answer: **0.5671** → Correct option: **C**

✅ c) Find the weight that is exceeded with probability 0.99

We want:

$$P(X > x) = 0.99 \Rightarrow P(X \leq x) = 0.01$$

Use inverse normal (percent-point function):

$$Z = \Phi^{-1}(0.01) \approx -2.326$$

Convert back to X:

$$x = \mu + Z\sigma = 30 + (-2.326)(2.5) \approx 24.18$$

✓ Final answer: **24.18** → Correct option: **G**

🧮 Python Code

```
from scipy.stats import norm

mu = 30
sigma = 6.25**0.5

# a
p_a = 1 - norm.cdf(37, loc=mu, scale=sigma)

# b
p_b = norm.cdf(32.5, loc=mu, scale=sigma) - norm.cdf(28.5, loc=mu, scale=si

# c
x_c = norm.ppf(0.01, loc=mu, scale=sigma)

print(round(p_a, 4), round(p_b, 4), round(x_c, 2))
```

✅ Summary of Answers

Part	Question	Answer	Option
a	$P(X > 37)$	0.0026	C
b	$P(28.5 < X < 32.5)$	0.5671	C
c	Value exceeded with probability 0.99	24.18	G

🚗 Weather Conditions and Probability of Death in Highway Accidents

---

🧾 Given

We are told that the probability of death in an accident depends on the weather condition:

- In **foggy** conditions: $P(D \mid F) = \frac{1}{4}$
- In **rainy** conditions: $P(D \mid R) = \frac{1}{8}$
- In **sunny** conditions: $P(D \mid S) = \frac{1}{21}$

The distribution of weather conditions is:

- $P(F) = 0.2$
- $P(R) = 0.2$
- $P(S) = 0.6$

✅ a) What is the total probability that an accident results in a death?

Use the **law of total probability**:

$$P(D) = P(F)P(D \mid F) + P(R)P(D \mid R) + P(S)P(D \mid S)$$

Substituting values:

$$P(D) = 0.2 \cdot \frac{1}{4} + 0.2 \cdot \frac{1}{8} + 0.6 \cdot \frac{1}{21}$$

$$P(D) = 0.05 + 0.025 + 0.0285714 = \boxed{0.1036}$$

✓ Final answer: **0.1036** → Correct option: **E**

✅ b) Given no death occurred, what is the conditional probability that it was foggy?

We are asked for:

$$P(F \mid \neg D)$$

Use **Bayes' Theorem**:

$$P(F \mid \neg D) = \frac{P(F) \cdot P(\neg D \mid F)}{P(\neg D)}$$

We already know:

- $P(F) = 0.2$
- $P(\neg D \mid F) = 1 - \frac{1}{4} = \frac{3}{4}$
- $P(\neg D) = 1 - P(D) = 1 - 0.1036 = 0.8964$

Now compute:

$$P(F \mid \neg D) = \frac{0.2 \cdot \frac{3}{4}}{0.8964} = \frac{0.15}{0.8964} \approx \boxed{0.1673}$$

✓ Final answer: **0.1673** → Correct option: **C**

🐍 Python Code

```
# Weather probabilities

P_foggy = 0.2

P_rainy = 0.2
```

```
P_sunny = 0.6

# Death probabilities given condition
P_D_given_F = 1/4
P_D_given_R = 1/8
P_D_given_S = 1/21

# a) Total probability of death
P_D = (
P_foggy * P_D_given_F +
P_rainy * P_D_given_R +
P_sunny * P_D_given_S
)

# b) Conditional: foggy given no death
P_not_D = 1 - P_D
P_not_D_given_F = 1 - P_D_given_F
P_F_given_not_D = (P_foggy * P_not_D_given_F) / P_not_D

print(round(P_D, 4)) # 0.1036
print(round(P_F_given_not_D, 4)) # 0.1673
```

✅ Summary of Results

Part	Description	Result	Option
a	$P(\text{death})$	0.1036	E
b	$P(\text{foggy} \mid \text{no death})$	0.1673	C

🧪 Heart Signal Classification Task: Sampling Probabilities

A database contains 48 heart signal recordings with 22 from females patients and 26 from male patients. For a classification task, an analyst randomly selects 36 records for predictive model training and keeps the other 12 records for testing the model performance.

1. What is the expected number of female patient recordings selected for training?
2. What is the probability that at least three male patient recordings are selected for training?
3. What is the probability that the same number of male and female patients are used for testing the model performance?

🧾 Scenario

We are given a database of 48 heart signal recordings:

- Female patients: 22
- Male patients: 26
- Training set: 36 records (randomly selected)
- Test set: 12 records (the remaining)

We want to answer the following:

✅ a) What is the expected number of female patient recordings selected for training?

This is a classic **expected value** from a hypergeometric setting.

🧠 Step-by-step

Let:

- $N = 48$ (total recordings)
- $n = 36$ (selected for training)
- $K = 22$ (female recordings)

The expected number of females in training is:

$$\mathbb{E}[X] = n \cdot \frac{K}{N} = 36 \cdot \frac{22}{48}$$

📐 Calculation

$$\frac{22}{48} = 0.4583 \Rightarrow 36 \cdot 0.4583 = \boxed{16.5}$$

✓ Final answer: **16.5** → Option **H**

✅ b) What is the probability that at least 3 male recordings are selected for training?

This is a **hypergeometric probability** problem.

Let:

- $M = 26$ (males)
- $N = 48$ (total)
- $n = 36$ (draws for training)

We want:

$$P(X \geq 3) = 1 - [P(0) + P(1) + P(2)]$$

🧠 Step-by-step

Use the hypergeometric PMF:

$$P(X = k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$$

So if you're calculating the **probability of selecting k male patients** in the training set, then:

- N=48: total patients
- M=26: total males (successes)
- n=36: number of training samples
- k: number of males selected for training

Compute:

- $P(0)$ = probability of 0 males → all 36 are females → not possible, because only 22 females
- $P(1)$ = at most 1 male → 35 females → not possible
- $P(2)$ = 34 females → also impossible

Thus:

$$P(X \geq 3) = 1$$

✓ Final answer: **1.0000** → Option **A**

✅ c) What is the probability that the same number of male and female patients are used for testing?

We are looking at the **test set**, which is 12 records.

We want:

$$P(X = 6) \quad \text{where } X \text{ is number of males in test set}$$

This is hypergeometric again:

- $N = 48$
- $K = 26$ (males)
- $n = 12$ (test set)

Use the hypergeometric formula:

$$P(X = 6) = \frac{\binom{26}{6} \cdot \binom{22}{6}}{\binom{48}{12}}$$

Using a calculator or Python:

$$P(X = 6) = \boxed{0.2466}$$

✓ Final answer: **0.2466** → Option **E**

🐍 Python Code Summary

```
from scipy.stats import hypergeom


# Total

N = 48

female = 22

male = 26


# Training size

n_train = 36

n_test = 12
```

```python
# a) Expected females
expected_females_train = n_train * (female / N)


# b) P(at least 3 males)
rv_train = hypergeom(N, male, n_train)
p_b = 1 - sum(rv_train.pmf(k) for k in  range(3))


# c) P(6 males in test)
rv_test = hypergeom(N, male, n_test)
p_c = rv_test.pmf(6)


print(expected_females_train, round(p_b, 4), round(p_c, 4))
```

✅ Final Summary

Part	Description	Result	Option
a	Expected females in training	16.5	H
b	$P(\geq 3 \text{ males in training})$	1.0000	A
c	$P(6 \text{ males and } 6 \text{ females in test})$	0.2466	E

REEXAM 2023

Q1

Consider two independent discrete random variables X and Y. Assume that the probability function p_X for X is given by

$$p_X(x) = \begin{cases} \frac{1}{3} & \text{if } x \in \{1,2,3\} \\ 0 & \text{otherwise,} \end{cases}$$

and that Y is Poisson-distributed with parameter $\lambda = 1$.

For all questions in this assignment, state your inputs as integers between 0 and 99 such that all answers are given as either integers or irreducible fractions.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

Find the values below.

$P(X \le 2) = \frac{\boxed{2}}{\boxed{3}}$

$Var(X) = \frac{\boxed{2}}{\boxed{3}}$

$E(XY) = \boxed{2}$

$Var(X - Y) = \frac{\boxed{5}}{\boxed{3}}$

$E(X(X + 5Y)) = \frac{\boxed{44}}{3}$

$Cov(X, 2Y) = \boxed{0}$

Q2

A random sample of 1900 adult women indicated that 941 of them asked for medical assistance last time they felt sick. The same sized sample of men indicated that 893 of them asked for medical assistance.

Documentation: In (a) no documentation is required. In (b) you are expected to demonstrate how you obtained the results either by supplying manual calculations or Pythor code.

a. Is it reasonable to conclude that women ask for medical assistance more often than men? Set up an appropriate hypothesis test to answer this question using $\alpha = 0.05$. Assuming p_1 refers to the rate of women asking for medical assistance and that p_2 refers to the rate of men asking for medical assistance, identify the correct alternative hypothesis of this test and check the answer.

- ○ $H_1: p_1 \le p_2$
- ○ $H_1: p_1 \ge p_2$
- ○ $H_1: p_1 - p_2 \ne 0$
- ○ $H_1: p_1 - p_2 \ge 0$
- ○ $H_1: p_1 + p_2 > 1$
- ◉ $H_1: p_1 > p_2$
- ○ $H_1: p_1 = p_2$
- ○ $H_1: p_1 - p_2 > 1$
- ○ $H_1: p_1 < p_2$
- ○ $H_1: p_1 \ne p_2$

b. Determine the critical value, the test statistic and the p-value for the test. Identify the three values below, i.e. you need to choose three of the choices listed below.

- [ ] 0.1192
- [ ] 1.5287
- [x] 1.5583
- [ ] 0.0298
- [ ] 1.5722
- [x] 1.6449
- [x] 0.0596
- [ ] 2.3263
- [ ] 1.2816
- [ ] 0.5
- [ ] 1.96

Q3

Assuming X and Y are independent and given $E(X) = 2$ and $E(X^2) = \frac{24}{5}$, find the value below.

State you inputs as two positive integers such that the answer displays an irreducible fraction (note a negative sign is pre-printed).

Documentation: You are expected to demonstrate how you obtained the result either by supplying manual calculations or Python code.

$$Cov(1 + X, Y - 2X) = -\frac{\boxed{8}}{\boxed{5}}$$

Q4

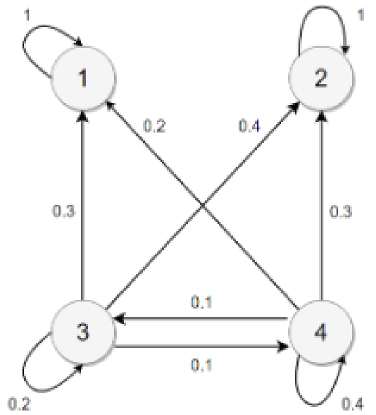
Suppose that a random variable *X* has continuous uniform distribution on [1, b] , where *b* is an unknown parameter. We have a random sample of 15 in size from a population represented by *X* and the random sample has a sample mean of 5.38. Use the sample mean which is an unbiased estimator, to find a point estimate of *b*. State your input as an integer between 0 and 99 so that the answer is displayed with two-decimal precision (note that the decimals are pre-printed).

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

$\boxed{9}.76$

Q5

Consider a Markov chain $\{X_n : n = 0, 1, 2, \ldots\}$ with state space $S = \{1, 2, 3, 4\}$ and state transition diagram given by



State your input as a **decimal** value so that you supply four decimal precision. Remember to use '.' (dot) as the decimal separator.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

Find the mean hitting time of state 4 given that the proces started in state 3.

$\boxed{8.8889}$

Q6

A city authority checks the hourly average weight of the traffic on a small bridge to determine if more frequent maintenance should be conducted. Suppose X, the hourly average weight of the traffic, is normally distributed with the mean of 30 tons and a variance of 6.25.

Determine the following, choosing the correct answer for each item.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. $P(X > 37)$

A	0.0125
B	0.2302
C	0.0026
D	

b. $P(28.5 < X < 32.5)$

A	0.2503
B	0.2764
C	0.5671

c. Weight, which is exceeded with probability 0.99.

A	28.06
B	23.68
C	18.24
D	15.46
E	17.82
F	25.02
G	24.18
H	21.97

Q7

Accidents on highways are one of the main causes of death or injury in developing countries and the weather conditions have an impact on the rates of death and injury. In foggy, rainy, and sunny conditions, 1/4, 1/8, and 1/21 of the accidents result in death, respectively. Sunny conditions occur 60% of the time, while rainy and foggy conditions each occur 20% of the time.

For both questions, identify the correct answer, here given with 4 decimal precision.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. What is the probability that an accident results in a death?

A	0.1103
B	0.0987
C	0.0964
D	0.1087
E	0.1036
F	[illegible]

b. Given that an accident without deaths occurred, what is the conditional probability that it was foggy at the time? Remember to use non-rounded values in your calculations.

A	0.1587
B	0.1603
C	0.1673
D	0.1625
E	0.1705

Q8

A database contains 48 heart signal recordings with 22 from females patients and 26 from male patients. For a classification task, an analyst randomly selects 36 records for predictive model training and keeps the other 12 records for testing the model performance.

For all questions, identify the correct answer.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. What is the expected number of female patient recordings selected for training?

A	15.5
B	19
C	11
D	17.5
E	11.5
F	18
G	14
H	16.5

b. What is the probability that at least three male patient recordings are selected for training?

A	1
B	0

c. What is the probability that the same number of male and female patients are used for testing the model performance?

A	0.32
B	0.5
C	1
D	0
E	0.25