Probability of Bolt Selection from Different Suppliers Solution and Wolfram Alpha Documentation

This document provides a detailed solution to a probability problem involving selecting bolts from a bin containing bolts from two different suppliers, without replacement. It includes explanations of the steps and how to use Wolfram Alpha (or similar symbolic computation tools) to verify or perform the calculations.

Topics Covered in this Exercise:

* **Combinations (Hypergeometric Distribution context)**
* **Probability without Replacement**
* **Conditional Probability (Implicitly)**
* **Calculating Probabilities of Specific Events**

Problem Statement: Each main bearing cap in an engine contains four bolts. The bolts are selected at random, without replacement, from a parts bin that contains 30 bolts from one supplier and 70 bolts from another supplier.

Part a: What is the probability that a main bearing cap contains all bolts from the same supplier?

This means either all four bolts are from supplier 1 OR all four bolts are from supplier 2. These are mutually exclusive events.

Total number of bolts in the bin: Total bolts = 30 (Supplier 1) + 70 (Supplier 2) = 100 bolts.

Number of bolts to be selected: 4 bolts.

Step 1: Probability that all 4 bolts are from Supplier 1.

- Number of ways to choose 4 bolts from Supplier 1: $\binom{30}{4}$
- Total number of ways to choose 4 bolts from 100: $\binom{100}{4}$

The probability is:

$$P(\text{all 4 from S1}) = \frac{\binom{30}{4}}{\binom{100}{4}}$$

Calculate the combinations: $\binom{30}{4} = \frac{30\times29\times28\times27}{4\times3\times2\times1} = 27{,}405$ $\binom{100}{4} = \frac{100\times99\times98\times97}{4\times3\times2\times1} = 3{,}921{,}225$

$$P(\text{all 4 from S1}) = \frac{27{,}405}{3{,}921{,}225} \approx 0.0069887$$

- **Wolfram Alpha Input**: `30 choose 4` (Result: 27405)
- **Wolfram Alpha Input**: `100 choose 4` (Result: 3921225)
- **Wolfram Alpha Input**: `27405 / 3921225` (Result: approximately 0.0069887)

Step 2: Probability that all 4 bolts are from Supplier 2.

- Number of ways to choose 4 bolts from Supplier 2: $\binom{70}{4}$
- Total number of ways to choose 4 bolts from 100: $\binom{100}{4}$ (same as above)

The probability is:

$$P(\text{all 4 from S2}) = \frac{\binom{70}{4}}{\binom{100}{4}}$$

Calculate the combination: $\binom{70}{4} = \frac{70\times69\times68\times67}{4\times3\times2\times1} = 916{,}895$

$$P(\text{all 4 from S2}) = \frac{916{,}895}{3{,}921{,}225} \approx 0.233816$$

- **Wolfram Alpha Input**: `70 choose 4` (Result: 916895)
- **Wolfram Alpha Input**: `916895 / 3921225` (Result: approximately 0.233816)

Step 3: Sum the probabilities for "all from same supplier".

Since these are mutually exclusive events:

$$P(\text{all from same supplier}) = P(\text{all 4 from S1}) + P(\text{all 4 from S2})$$

$$P(\text{all from same supplier}) = \frac{27{,}405}{3{,}921{,}225} + \frac{916{,}895}{3{,}921{,}225}$$

$$P(\text{all from same supplier}) = \frac{27{,}405 + 916{,}895}{3{,}921{,}225} = \frac{944{,}300}{3{,}921{,}225}$$

As a decimal, rounded to two decimal places: $\frac{944{,}300}{3{,}921{,}225} \approx 0.240801\ldots \approx 0.24$

- **Wolfram Alpha Input**: `(30 choose 4 + 70 choose 4) / (100 choose 4)` (Result: 0.240801...)
- **Wolfram Alpha Input**: `round 0.240801 to 2 decimal places` (Result: 0.24)

Part b: What is the probability that exactly three bolts are from the same supplier?

This means either three bolts are from supplier 1 and one from supplier 2 OR three bolts are from supplier 2 and one from supplier 1. These are mutually exclusive events.

Step 1: Probability that 3 bolts are from S1 and 1 bolt is from S2.

- Number of ways to choose 3 bolts from Supplier 1: $\binom{30}{3}$
- Number of ways to choose 1 bolt from Supplier 2: $\binom{70}{1}$
- Total number of ways to choose 4 bolts from 100: $\binom{100}{4}$ (same as above)

The number of ways to get this combination is $\binom{30}{3} \times \binom{70}{1}$.

$\binom{30}{3} = \frac{30 \times 29 \times 28}{3 \times 2 \times 1} = 4{,}060$

$\binom{70}{1} = 70$

Number of ways $= 4{,}060 \times 70 = 284{,}200$

Probability:

$$P(\text{3 S1, 1 S2}) = \frac{\binom{30}{3} \times \binom{70}{1}}{\binom{100}{4}} = \frac{284{,}200}{3{,}921{,}225} \approx 0.072478$$

- **Wolfram Alpha Input**: `(30 choose 3) * (70 choose 1)` (Result: 284200)
- **Wolfram Alpha Input**: `284200 / 3921225` (Result: approximately 0.072478)

Step 2: Probability that 3 bolts are from S2 and 1 bolt is from S1.

- Number of ways to choose 3 bolts from Supplier 2: $\binom{70}{3}$
- Number of ways to choose 1 bolt from Supplier 1: $\binom{30}{1}$

The number of ways to get this combination is $\binom{70}{3} \times \binom{30}{1}$.

$\binom{70}{3} = \frac{70 \times 69 \times 68}{3 \times 2 \times 1} = 54{,}740$

$\binom{30}{1} = 30$

Number of ways $= 54{,}740 \times 30 = 1{,}642{,}200$

Probability:

$$P(\text{3 S2, 1 S1}) = \frac{\binom{70}{3} \times \binom{30}{1}}{\binom{100}{4}} = \frac{1{,}642{,}200}{3{,}921{,}225} \approx 0.418797$$

- **Wolfram Alpha Input**: `(70 choose 3) * (30 choose 1)` (Result: 1642200)

- **Wolfram Alpha Input**: `1642200 / 3921225` (Result: approximately 0.418797)

Step 3: Sum the probabilities for "exactly three from the same supplier".

Since these are mutually exclusive events:

$$P(\text{exactly 3 from same supplier}) = P(3\text{ S1, 1 S2}) + P(3\text{ S2, 1 S1})$$

$$P(\text{exactly 3 from same supplier}) = \frac{284{,}200}{3{,}921{,}225} + \frac{1{,}642{,}200}{3{,}921{,}225}$$

$$P(\text{exactly 3 from same supplier}) = \frac{284{,}200 + 1{,}642{,}200}{3{,}921{,}225} = \frac{1{,}926{,}400}{3{,}921{,}225}$$

As a decimal, rounded to two decimal places: $\frac{1{,}926{,}400}{3{,}921{,}225} \approx 0.49126\ldots \approx 0.49$

- **Wolfram Alpha Input**: `((30 choose 3)*(70 choose 1) + (70 choose 3)*(30 choose 1)) / (100 choose 4)` (Result: 0.49126…)
- **Wolfram Alpha Input**: `round 0.49126 to 2 decimal places` (Result: 0.49)