Discrete Random Variable Probability Calculations and Wolfram Alpha Documentation (Corrected)

This document provides a detailed solution to calculating probabilities for a discrete random variable with a given Probability Mass Function (PMF), including explanations of the steps and how to use Wolfram Alpha (or similar symbolic computation tools) to verify or perform the calculations.

Topics Covered in this Exercise:

* **Discrete Random Variables**
* **Probability Mass Function (PMF)**
* **Range of a Random Variable**
* **Calculating Probabilities for Discrete Events**
* **Conditional Probability for Discrete Events**

Problem Statement: Let X be a discrete random variable with the following PMF:

$$P_X(x) = \begin{cases} \frac{1}{2} & \text{for } x = 0 \\ \frac{1}{3} & \text{for } x = 1 \\ \frac{1}{6} & \text{for } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the following. State all inputs as integers between 0 and 99 and state all sets $\{x_1, x_2, \dots, x_n\}$ such that $x_1 < x_2 < \cdots < x_n$. Also, all resulting fractions must be irreducible.

Part a: Find the Range of X, R_X.

The range of a discrete random variable R_X is the set of all possible values that X can take, for which $P_X(x) > 0$. From the given PMF: $P_X(0) = \frac{1}{2} > 0$ $P_X(1) = \frac{1}{3} > 0$ $P_X(2) = \frac{1}{6} > 0$ For all other values of x, $P_X(x) = 0$.

So, the range of X is the set of values $\{0,1,2\}$, ordered from smallest to largest.

$$R_X = \{0,1,2\}$$

Part b: Find $P(X \geq 1.5)$

For a discrete random variable, $P(X \geq 1.5)$ means the sum of probabilities for all values of X in its range that are greater than or equal to 1.5. From $R_X = \{0,1,2\}$, the only value greater than or equal to 1.5 is $x = 2$.

$$P(X \geq 1.5) = P_X(2)$$

$$P(X \geq 1.5) = \frac{1}{6}$$

The fraction $\frac{1}{6}$ is irreducible.

- **Wolfram Alpha Input**: `probability X >= 1.5 where P(X=0)=1/2, P(X=1)=1/3, P(X=2)=1/6` (Result: 1/6)

Part c: Find $P(0 < X < 2)$

For a discrete random variable, $P(0 < X < 2)$ means the sum of probabilities for all values of X in its range that are strictly greater than 0 and strictly less than 2. From $R_X = \{0,1,2\}$, the only value strictly between 0 and 2 is $x = 1$.

$$P(0 < X < 2) = P_X(1)$$

$$P(0 < X < 2) = \frac{1}{3}$$

The fraction $\frac{1}{3}$ is irreducible.

- **Wolfram Alpha Input**: `probability 0 < X < 2 where P(X=0)=1/2, P(X=1)=1/3, P(X=2)=1/6` (Result: 1/3)

Part d: Find $P(X = 0 \mid X < 2)$

This is a conditional probability problem. The formula for conditional probability is:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

In this case, let A be the event $X = 0$ and B be the event $X < 2$.

Step 1: Find $P(A \cap B)$

The event $A \cap B$ means $(X = 0)$ AND $(X < 2)$. The intersection of these two conditions is simply $X = 0$. So, $P(A \cap B) = P(X = 0)$. From the PMF, $P(X = 0) = \frac{1}{2}$.

- **Wolfram Alpha Input**: `P(X=0)` for the given PMF (Result: 1/2)

Step 2: Find $P(B)$

The event B is $X < 2$. This means the sum of probabilities for values of X in its range that are strictly less than 2. From $R_X = \{0,1,2\}$, the values less than 2 are $x = 0$ and $x = 1$.

$$P(X < 2) = P_X(0) + P_X(1)$$

$$P(X < 2) = \frac{1}{2} + \frac{1}{3}$$

To add these fractions, find a common denominator, which is 6:

$$P(X < 2) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

- **Wolfram Alpha Input**: `probability X < 2 where P(X=0)=1/2, P(X=1)=1/3, P(X=2)=1/6` (Result: 5/6)

Step 3: Calculate $P(X = 0 \mid X < 2)$

$$P(X = 0 \mid X < 2) = \frac{P(X = 0)}{P(X < 2)} = \frac{\frac{1}{2}}{\frac{5}{6}}$$

$$= \frac{1}{2} \times \frac{6}{5} = \frac{6}{10} = \frac{3}{5}$$

The fraction $\frac{3}{5}$ is irreducible.

- **Wolfram Alpha Input**: `conditional probability X=0 given X < 2 where P(X=0)=1/2, P(X=1)=1/3, P(X=2)=1/6` (Result: 3/5)