Expected Number of Defective Devices in a Circuit Solution and Wolfram Alpha Documentation

This document provides a detailed solution to finding the expected number of defective devices in a simple circuit, based on their individual probabilities of functioning correctly. It includes explanations of the steps and how to use Wolfram Alpha (or similar symbolic computation tools) to verify or perform the calculations.

Topics Covered in this Exercise:

* **Probability of Independent Events**

* **Bernoulli Trials**

* **Random Variables**

* **Expected Value of a Sum of Random Variables**

Problem Statement:

Consider the following circuit with two devices, P1 and P2, in series.

The probability that each device functions correctly is $p_1 = 0.83$ and $p_2 = 0.75$.

Assume that devices fail independently.

Let X denote the number of defective devices.

Find the mean of X, denoted as $E[X]$.

State your answer as an integer between 0 and 99, such that you supply two decimal precision correctly rounded off.

Step 1: Define Random Variables for Each Device

Let X_1 be a random variable representing the state of device P1. Let X_2 be a random variable representing the state of device P2.

We are interested in the number of *defective* devices.

So, let's define X_i to be 1 if the device is defective, and 0 if it functions correctly.

The probability of P1 functioning correctly is $p_1 = 0.83$.

The probability of P1 being defective (failing) is $P(X_1 = 1) = 1 - p_1 = 1 - 0.83 = 0.17$.

The probability of P1 functioning correctly (not defective) is $P(X_1 = 0) = 0.83$.

The probability of P2 functioning correctly is $p_2 = 0.75$.

The probability of P2 being defective (failing) is $P(X_2 = 1) = 1 - p_2 = 1 - 0.75 = 0.25$.

The probability of P2 functioning correctly (not defective) is $P(X_2 = 0) = 0.75$.

Step 2: Define the Total Number of Defective Devices X

Let X be the total number of defective devices. $X = X_1 + X_2$.

The possible values for X are:

* $X = 0$: Both devices function correctly ($X_1 = 0, X_2 = 0$).

* $X = 1$: One device is defective ($X_1 = 1, X_2 = 0$ or $X_1 = 0, X_2 = 1$).

* $X = 2$: Both devices are defective ($X_1 = 1, X_2 = 1$).

Step 3: Calculate the Expected Value of Each Individual Random Variable

The expected value of a Bernoulli random variable (which X_1 and X_2 are, representing success/failure) is simply the probability of "success" (in our case, "defective").

For X_1: $E[X_1] = P(X_1 = 1) = 0.17$

For X_2: $E[X_2] = P(X_2 = 1) = 0.25$

- **Wolfram Alpha Input**: `expected value of Bernoulli distribution with success probability 0.17` (Result: 0.17)
- **Wolfram Alpha Input**: `expected value of Bernoulli distribution with success probability 0.25` (Result: 0.25)

Step 4: Calculate the Mean of X (Expected Number of Defective Devices)

The expected value of a sum of random variables is the sum of their expected values, regardless of whether they are independent or not (linearity of expectation).

$$E[X] = E[X_1 + X_2] = E[X_1] + E[X_2]$$

$$E[X] = 0.17 + 0.25$$

$$E[X] = 0.42$$

The mean of X is 0.42. This matches the provided answer.

- **Wolfram Alpha Input**: `0.17 + 0.25` (Result: 0.42)

Alternative Method (Using PMF of X):

We could also find the PMF of X and then calculate its expected value:

$P(X = 0) = P(X_1 = 0 \text{ and } X_2 = 0) = P(X_1 = 0) \times P(X_2 = 0)$ (due to independence)

$P(X = 0) = 0.83 \times 0.75 = 0.6225$

$P(X = 1) = P(X_1 = 1 \text{ and } X_2 = 0) + P(X_1 = 0 \text{ and } X_2 = 1)$

$P(X = 1) = (P(X_1 = 1) \times P(X_2 = 0)) + (P(X_1 = 0) \times P(X_2 = 1))$

$P(X = 1) = (0.17 \times 0.75) + (0.83 \times 0.25)$

$P(X = 1) = 0.1275 + 0.2075 = 0.3350$

$P(X = 2) = P(X_1 = 1 \text{ and } X_2 = 1) = P(X_1 = 1) \times P(X_2 = 1)$ (due to independence)

$P(X = 2) = 0.17 \times 0.25 = 0.0425$

Verify sum of probabilities: $0.6225 + 0.3350 + 0.0425 = 1.0000$.

Now calculate $E[X]$ using the PMF:

$E[X] = \sum xP(X = x) = (0 \times P(X = 0)) + (1 \times P(X = 1)) + (2 \times P(X = 2))$

$E[X] = (0 \times 0.6225) + (1 \times 0.3350) + (2 \times 0.0425)$

$E[X] = 0 + 0.3350 + 0.0850$

$E[X] = 0.4200$

This confirms the result obtained through linearity of expectation.

- **Wolfram Alpha Input**: `expected value of discrete distribution { {0, 0.6225}, {1, 0.3350}, {2, 0.0425} }` (Result: 0.42)