This problem involves finding the **steady-state probabilities** (also known as stationary distribution or equilibrium distribution) of an **ergodic Markov chain**. An ergodic Markov chain is one where it's possible to get from any state to any other state (it's irreducible) and it's aperiodic. For such chains, the long-term probabilities of being in each state converge to a unique distribution, regardless of the initial state.

Problem Description:

We are given a transition matrix P for an ergodic Markov chain:

$$P = \begin{pmatrix} 1/2 & 1/2 \\ p & 1-p \end{pmatrix}$$

When the experiment is performed many times, the chain ends in state one approximately 20 percent of the time and in state two 80 percent of the time. This describes the steady-state (or limiting) distribution. We need to find the value of p and state the answer as an irreducible fraction.

Let $\pi = \begin{pmatrix} \pi_1 & \pi_2 \end{pmatrix}$ be the steady-state probability vector, where:

- π_1 is the long-term probability of being in State 1.
- π_2 is the long-term probability of being in State 2.

From the problem statement, we are given:

- $\pi_1 = 0.20 = \frac{20}{100} = \frac{1}{5}$
- $\pi_2 = 0.80 = \frac{80}{100} = \frac{4}{5}$

Key Property of Steady-State Probabilities:

The steady-state probability vector π satisfies the following equation:
$\pi P = \pi$

This translates to a system of linear equations:

1. $\pi_1 P_{11} + \pi_2 P_{21} = \pi_1$
2. $\pi_1 P_{12} + \pi_2 P_{22} = \pi_2$

Additionally, the probabilities must sum to 1:
3. $\pi_1 + \pi_2 = 1$

Steps to Solve:

1. Set up the equations using the given steady-state probabilities and transition matrix:

Using the first equation:
$\pi_1 \cdot (1/2) + \pi_2 \cdot p = \pi_1$
Substitute the known values of $\pi_1 = 1/5$ and $\pi_2 = 4/5$:
$\frac{1}{5} \cdot \frac{1}{2} + \frac{4}{5} \cdot p = \frac{1}{5}$

2. Solve for p using the equation from Step 1:

$\frac{1}{10} + \frac{4p}{5} = \frac{1}{5}$

To eliminate the denominators, multiply the entire equation by 10:
$10 \cdot \left(\frac{1}{10}\right) + 10 \cdot \left(\frac{4p}{5}\right) = 10 \cdot \left(\frac{1}{5}\right)$
$1 + 8p = 2$

Now, solve for p:
$8p = 2 - 1$
$8p = 1$
$p = \frac{1}{8}$

3. Verify with the second equation (optional but good practice):

Using the second equation:
$\pi_1 P_{12} + \pi_2 P_{22} = \pi_2$
$\pi_1 \cdot (1/2) + \pi_2 \cdot (1 - p) = \pi_2$
Substitute $\pi_1 = 1/5$, $\pi_2 = 4/5$, and our calculated $p = 1/8$:
$\frac{1}{5} \cdot \frac{1}{2} + \frac{4}{5} \cdot \left(1 - \frac{1}{8}\right) = \frac{4}{5}$
$\frac{1}{10} + \frac{4}{5} \cdot \left(\frac{8}{8} - \frac{1}{8}\right) = \frac{4}{5}$
$\frac{1}{10} + \frac{4}{5} \cdot \frac{7}{8} = \frac{4}{5}$
$\frac{1}{10} + \frac{28}{40} = \frac{4}{5}$
$\frac{1}{10} + \frac{7}{10} = \frac{4}{5}$
$\frac{8}{10} = \frac{4}{5}$
$\frac{4}{5} = \frac{4}{5}$
The value of $p = 1/8$ is consistent with both steady-state equations.

Final Answer for p:

The value of p is $\frac{1}{8}$. This is an irreducible fraction.

Topics Covered:

- **Markov Chains:** A stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event.
- **Ergodic Markov Chain:** A type of Markov chain that is irreducible (you can get from any state to any other state) and aperiodic (you don't return to states in a fixed cycle). These properties guarantee the existence of a unique steady-state distribution.
- **Steady-State Probabilities (Stationary Distribution):** The long-run probabilities of being in each state, which are independent of the initial state. Represented by a vector π.
- **Matrix Multiplication:** The fundamental operation $\pi P = \pi$ used to find the steady-state distribution.
- **System of Linear Equations:** Solving a set of equations to find the unknown variable (p in this case).

WolframAlpha/Computational Check:

You can verify the calculations and the concept of steady-state distribution using WolframAlpha.

1. **Define the transition matrix with the found p:**
   Input: `{{1/2, 1/2}, {1/8, 7/8}}`

2. **Find the steady-state distribution:**
   Input: `steady state probabilities of {{1/2, 1/2}, {1/8, 7/8}}`
   WolframAlpha will output the stationary distribution as $\{\frac{1}{5}, \frac{4}{5}\}$, which matches the given 20% and 80%, thus confirming our value of p is correct.

3. **Solve the linear equation directly:**
   Input: `solve 1/5 * 1/2 + 4/5 * p = 1/5 for p`
   WolframAlpha will directly give $p = 1/8$.