

To find the correlation coefficient $\text{Corr}(U, V)$, we use the formula:

$$\text{Corr}(U, V) = \frac{\text{Cov}(U, V)}{\sqrt{\text{Var}(U)\text{Var}(V)}}$$

We are given $\text{Var}(U) = 4$ and $\text{Var}(V) = 1$.

Thus, the denominator is $\sqrt{4 \times 1} = \sqrt{4} = 2$.

Next, we need to calculate $\text{Cov}(U, V) = E[UV] - E[U]E[V]$.

1. Calculate $E[V]$:

$$V = 2 + B$$

$$E[V] = E[2 + B] = E[2] + E[B]$$

Since $B \sim N(0, 1)$, $E[B] = 0$.

$$E[V] = 2 + 0 = 2$$

2. Calculate $E[U]$:

$$U = 5 + A - A^2B$$

$$E[U] = E[5 + A - A^2B] = E[5] + E[A] - E[A^2B]$$

Since $A \sim N(0, 1)$, $E[A] = 0$.

Since A and B are independent, A^2 and B are also independent. Therefore,

$$E[A^2B] = E[A^2]E[B].$$

For $A \sim N(0, 1)$, $E[A^2] = \text{Var}(A) + (E[A])^2 = 1 + 0^2 = 1$.

So, $E[A^2B] = (1)(0) = 0$.

$$E[U] = 5 + 0 - 0 = 5$$

3. Calculate $E[UV]$:

$$E[UV] = E[(5 + A - A^2B)(2 + B)]$$

Expand the product:

$$E[UV] = E[10 + 5B + 2A + AB - 2A^2B - A^2B^2]$$

Using linearity of expectation and independence properties:

$$E[UV] = E[10] + E[5B] + E[2A] + E[AB] - E[2A^2B] - E[A^2B^2]$$

- $E[10] = 10$
- $E[5B] = 5E[B] = 5(0) = 0$
- $E[2A] = 2E[A] = 2(0) = 0$
- $E[AB] = E[A]E[B] = (0)(0) = 0$ (since A and B are independent)
- $E[2A^2B] = 2E[A^2]E[B] = 2(1)(0) = 0$ (since A^2 and B are independent)

- $E[A^2 B^2] = E[A^2]E[B^2]$ (since A^2 and B^2 are independent)
 - $E[B^2] = \text{Var}(B) + (E[B])^2 = 1 + 0^2 = 1.$
 - So, $E[A^2 B^2] = (1)(1) = 1.$

Substitute these values:

$$E[UV] = 10 + 0 + 0 + 0 - 0 - 1 = 9$$

4. Calculate $\text{Cov}(U, V)$:

$$\text{Cov}(U, V) = E[UV] - E[U]E[V]$$

$$\text{Cov}(U, V) = 9 - (5)(2)$$

$$\text{Cov}(U, V) = 9 - 10 = -1$$

5. Calculate $\text{Corr}(U, V)$:

$$\text{Corr}(U, V) = \frac{-1}{\sqrt{4 \times 1}} = \frac{-1}{2}$$

The correlation coefficient $\text{Corr}(U, V)$ is $-\frac{1}{2}$.