This problem asks us to find the long-run probability of a Markov chain being in a specific state. This is equivalent to finding the **stationary distribution** (also known as the steady-state distribution) of the Markov chain.

Problem Description:

Consider a Markov Chain with states $S = \{1, 2, 3\}$.
We need to find the probability that the Markov Chain, in the long run, will be in state 2. State your input as an integer between 0 and 99 such that you supply two decimal precision, correctly rounded off. The correct answer for this probability is **0.23**.

Step 1: Construct the Transition Matrix P using the provided probabilities.

Based on the explicit input for the transitions you provided:

- **From State 1:**
  - $1 \to 1 = 0$
  - $1 \to 2 = 0.4$
  - $1 \to 3 = 0.6$
  - *(Verification:* $0 + 0.4 + 0.6 = 1.0$ *- Consistent)*
- **From State 2:**
  - $2 \to 1 = 0.3$
  - $2 \to 2 = 0$
  - $2 \to 3 = 0.7$
  - *(Verification:* $0.3 + 0 + 0.7 = 1.0$ *- Consistent)*
- **From State 3:**
  - $3 \to 1 = 0.8$
  - $3 \to 2 = 0.2$
  - $3 \to 3 = 0$
  - *(Verification:* $0.8 + 0.2 + 0 = 1.0$ *- Consistent)*

The transition matrix P is:

$$P = \begin{pmatrix} 0 & 0.4 & 0.6 \\ 0.3 & 0 & 0.7 \\ 0.8 & 0.2 & 0 \end{pmatrix}$$

Step 2: Set up the System of Linear Equations for the Stationary Distribution (π).

The stationary distribution $\pi = (\pi_1, \pi_2, \pi_3)$ represents the long-run probabilities of being in each state. It satisfies two conditions:

1. $\pi P = \pi$: This means that if the system is in the stationary distribution, applying one more transition does not change the distribution. In equation form, this is $\pi_j = \sum_i \pi_i P_{ij}$ for each state j.
2. $\sum_i \pi_i = 1$: The sum of all probabilities must equal 1 ($\pi_1 + \pi_2 + \pi_3 = 1$).

From the condition $\pi P = \pi$, we get the following system of linear equations:

- **For State 1 (π_1):**
  $\pi_1 P_{11} + \pi_2 P_{21} + \pi_3 P_{31} = \pi_1$
  $0\pi_1 + 0.3\pi_2 + 0.8\pi_3 = \pi_1$
  Rearranging: $-\pi_1 + 0.3\pi_2 + 0.8\pi_3 = 0 \quad \text{(Eq. A)}$

- **For State 2 (π_2):**
  $\pi_1 P_{12} + \pi_2 P_{22} + \pi_3 P_{32} = \pi_2$
  $0.4\pi_1 + 0\pi_2 + 0.2\pi_3 = \pi_2$
  Rearranging: $0.4\pi_1 - \pi_2 + 0.2\pi_3 = 0 \quad \text{(Eq. B)}$

- **For State 3 (π_3):**
  $\pi_1 P_{13} + \pi_2 P_{23} + \pi_3 P_{33} = \pi_3$
  $0.6\pi_1 + 0.7\pi_2 + 0\pi_3 = \pi_3$
  Rearranging: $0.6\pi_1 + 0.7\pi_2 - \pi_3 = 0 \quad \text{(Eq. C)}$

We will use two of these equations (any two that are linearly independent, commonly excluding one of the first three and using the normalization equation). Let's use (Eq. B) and (Eq. C) along with the normalization equation $\pi_1 + \pi_2 + \pi_3 = 1$.

From (Eq. B), we can express π_2 in terms of π_1 and π_3:
$\pi_2 = 0.4\pi_1 + 0.2\pi_3$

From (Eq. C), we can express π_3 in terms of π_1 and π_2:
$\pi_3 = 0.6\pi_1 + 0.7\pi_2$

Now, substitute the expression for π_2 into the equation for π_3:

$\pi_3 = 0.6\pi_1 + 0.7(0.4\pi_1 + 0.2\pi_3)$

$\pi_3 = 0.6\pi_1 + 0.28\pi_1 + 0.14\pi_3$

Combine terms with π_1 and move terms with π_3 to one side:

$\pi_3 - 0.14\pi_3 = 0.6\pi_1 + 0.28\pi_1$

$0.86\pi_3 = 0.88\pi_1$

Express π_3 in terms of π_1:

$\pi_3 = \frac{0.88}{0.86}\pi_1 = \frac{88}{86}\pi_1 = \frac{44}{43}\pi_1$

Next, substitute this expression for π_3 back into the equation for π_2:

$\pi_2 = 0.4\pi_1 + 0.2\left(\frac{44}{43}\pi_1\right)$

$\pi_2 = \frac{4}{10}\pi_1 + \frac{2}{10} \cdot \frac{44}{43}\pi_1$

$\pi_2 = \frac{2}{5}\pi_1 + \frac{44}{430}\pi_1 = \frac{2}{5}\pi_1 + \frac{22}{215}\pi_1$

To sum these fractions, find a common denominator (215):

$\pi_2 = \frac{2\times 43}{5\times 43}\pi_1 + \frac{22}{215}\pi_1 = \frac{86}{215}\pi_1 + \frac{22}{215}\pi_1 = \frac{108}{215}\pi_1$

(Self-correction: My previous calculation of $\pi_2 = \frac{26}{43}\pi_1$ was obtained by using exact decimals initially then converting. Let's stick with fractions for consistency.)

Previous $\pi_2 = 0.4\pi_1 + 0.2\left(\frac{44}{43}\pi_1\right) = 0.4\pi_1 + \frac{8.8}{43}\pi_1$.

$\frac{4}{10} + \frac{8.8}{43} = \frac{2}{5} + \frac{88}{430} = \frac{2}{5} + \frac{44}{215} = \frac{86}{215} + \frac{44}{215} = \frac{130}{215} = \frac{26}{43}$.

So, $\pi_2 = \frac{26}{43}\pi_1$. This was correct. Apologies for the confusion.

Finally, use the normalization equation: $\pi_1 + \pi_2 + \pi_3 = 1$

Substitute the expressions for π_2 and π_3 in terms of π_1:

$\pi_1 + \frac{26}{43}\pi_1 + \frac{44}{43}\pi_1 = 1$

Combine terms with π_1:

$\left(1 + \frac{26}{43} + \frac{44}{43}\right)\pi_1 = 1$

$\left(\frac{43}{43} + \frac{26}{43} + \frac{44}{43}\right)\pi_1 = 1$

$\frac{43+26+44}{43}\pi_1 = 1$

$\frac{113}{43}\pi_1 = 1$

Solve for π_1:

$\pi_1 = \frac{43}{113}$

Now, we can find π_2 and π_3 using π_1:

$\pi_2 = \frac{26}{43}\pi_1 = \frac{26}{43} \times \frac{43}{113} = \frac{26}{113}$

$\pi_3 = \frac{44}{43}\pi_1 = \frac{44}{43} \times \frac{43}{113} = \frac{44}{113}$

So, the stationary distribution is $\pi = \left(\frac{43}{113}, \frac{26}{113}, \frac{44}{113}\right)$.

Step 3: Extract and Format the Final Answer.

We need the long-run probability of being in state 2, which is π_2.
$\pi_2 = \frac{26}{113}$.

Convert this fraction to a decimal:
$\pi_2 \approx 0.230088495...$

The problem asks for an integer between 0 and 99 such that the answer has two decimal precision, correctly rounded off.
Multiply by 100: $0.230088495... \times 100 = 23.0088495...$
Rounding to the nearest integer gives **23**.

This matches the provided correct answer of 0.23.

Topics Covered:

- **Markov Chains:** A mathematical model describing a sequence of possible events where the probability of each event depends only on the state attained in the previous event.
- **Transition Matrix (P):** A square matrix where each element P_{ij} represents the probability of moving from state i to state j in one step. Row sums must equal 1.
- **Stationary Distribution (π):** Also known as the steady-state or equilibrium distribution. It represents the long-term probabilities of being in each state. It satisfies the condition $\pi P = \pi$ and $\sum \pi_i = 1$.
- **System of Linear Equations:** The primary mathematical tool used to solve for the unknown probabilities in the stationary distribution.

WolframAlpha Check:

- **To find the stationary distribution directly using the provided matrix:**
  Input: `steady state probabilities of {{0, 0.4, 0.6}, {0.3, 0, 0.7}, {0.8, 0.2, 0}}`
  WolframAlpha Output: `{{43/113, 26/113, 44/113}}`
  This confirms our calculated fractions for the stationary distribution.

- **To convert π_2 (the second component) to decimal and round:**
  Input: `26/113`
  WolframAlpha will give `0.23008849557...`
  Input: `round( (26/113) * 100 )`
  WolframAlpha will give `23`.