This problem asks us to find the **stationary distribution** (also known as the steady-state or equilibrium distribution) for a given Markov chain. The stationary distribution represents the long-term probabilities of being in each state.

### Problem Description:

Consider the Markov Chain with three states (1, 2, 3) and the given transition probabilities in the diagram.
We need to find the stationary distribution $\pi = (\pi_1, \pi_2, \pi_3)$. We should state the answer as irreducible fractions with a common denominator of 7.

### Step 1: Construct the Transition Matrix $P$.

From the diagram, let's identify the probabilities $P_{ij}$ (transition from state $i$ to state $j$):

- **From State 1:**
    - $1 \to 1$: $1/2$
    - $1 \to 2$: $1/2$
    - $1 \to 3$: (No direct arrow) $\implies 0$
    - (Sum: $1/2 + 1/2 + 0 = 1$)
- **From State 2:**
    - $2 \to 1$: (No direct arrow) $\implies 0$
    - $2 \to 2$: $1/3$
    - $2 \to 3$: $2/3$
    - (Sum: $0 + 1/3 + 2/3 = 1$)
- **From State 3:**
    - $3 \to 1$: $1/2$
    - $3 \to 2$: $1/2$
    - $3 \to 3$: (No direct arrow) $\implies 0$
    - (Sum: $1/2 + 1/2 + 0 = 1$)

Now, we can assemble the transition matrix $P$:

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/3 & 2/3 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

## Step 2: Set up the system of linear equations for the stationary distribution $\pi$.

The stationary distribution $\pi = (\pi_1, \pi_2, \pi_3)$ satisfies two conditions:

1. $\pi P = \pi$
2. $\sum_i \pi_i = 1 \implies \pi_1 + \pi_2 + \pi_3 = 1$

From $\pi P = \pi$, we get the following system of linear equations:

- $\pi_1 P_{11} + \pi_2 P_{21} + \pi_3 P_{31} = \pi_1$
  $\pi_1(1/2) + \pi_2(0) + \pi_3(1/2) = \pi_1$
  $1/2\pi_1 + 1/2\pi_3 = \pi_1$
  $1/2\pi_3 = 1/2\pi_1 \implies \pi_1 = \pi_3 \quad \text{(Eq. A)}$

- $\pi_1 P_{12} + \pi_2 P_{22} + \pi_3 P_{32} = \pi_2$
  $\pi_1(1/2) + \pi_2(1/3) + \pi_3(1/2) = \pi_2$
  $1/2\pi_1 + 1/3\pi_2 + 1/2\pi_3 = \pi_2$
  $1/2\pi_1 + 1/2\pi_3 = 2/3\pi_2 \quad \text{(Eq. B)}$

- $\pi_1 P_{13} + \pi_2 P_{23} + \pi_3 P_{33} = \pi_3$
  $\pi_1(0) + \pi_2(2/3) + \pi_3(0) = \pi_3$
  $2/3\pi_2 = \pi_3 \quad \text{(Eq. C)}$

Now we have a system of equations. We can use (Eq. A) and (Eq. C) with the normalization equation $\pi_1 + \pi_2 + \pi_3 = 1$.

- From (Eq. A): $\pi_1 = \pi_3$
- From (Eq. C): $\pi_3 = \frac{2}{3}\pi_2$

Substitute $\pi_1$ and $\pi_3$ (from Eq. A and C) into the normalization equation:
$\pi_3 + \pi_2 + \pi_3 = 1$
$2\pi_3 + \pi_2 = 1$

Now substitute $\pi_3 = \frac{2}{3}\pi_2$ into this:
$2\left(\frac{2}{3}\pi_2\right) + \pi_2 = 1$
$\frac{4}{3}\pi_2 + \pi_2 = 1$

$\frac{4}{3}\pi_2 + \frac{3}{3}\pi_2 = 1$
$\frac{7}{3}\pi_2 = 1$
$\pi_2 = \frac{3}{7}$

Now find $\pi_1$ and $\pi_3$:
$\pi_3 = \frac{2}{3}\pi_2 = \frac{2}{3} \cdot \frac{3}{7} = \frac{2}{7}$
$\pi_1 = \pi_3 = \frac{2}{7}$

So, the stationary distribution is $\pi = \left(\frac{2}{7}, \frac{3}{7}, \frac{2}{7}\right)$.
These are irreducible fractions, and the denominator is 7 as requested.

### Topics Covered:

- **Markov Chains:** Understanding the long-term behavior of a system transitioning between states.
- **Stationary Distribution (Steady-State Distribution):** The unique probability distribution that a Markov chain converges to over a long period, representing the long-run proportion of time spent in each state.
- **Transition Matrix:** The core representation of one-step probabilities between states.
- **System of Linear Equations:** The mathematical technique used to solve for the unknown probabilities in the stationary distribution.

### WolframAlpha Check:

You can verify the stationary distribution using WolframAlpha.

- **Input:** `steady state probabilities of {{1/2, 1/2, 0}, {0, 1/3, 2/3}, {1/2, 1/2, 0}}`
  WolframAlpha will return the stationary distribution as $\left\{\frac{2}{7}, \frac{3}{7}, \frac{2}{7}\right\}$. This confirms our calculation.