Expectation, Variance, Covariance — Reference Guide

In statistics, “Cov” refers to covariance, “Var” refers to variance, and “E” (or sometimes “μ”) refers to the expected value or mean

General Definitions (Any Distribution)

Concept	Formula
Expectation	$E[X] = \sum x \cdot P(X = x)$ or $E[X] = \int x f(x)\, dx$
Variance	$\mathrm{Var}(X) = E[X^2] - (E[X])^2$
Covariance	$\mathrm{Cov}(X, Y) = E[XY] - E[X]E[Y]$
If Independent	$\mathrm{Cov}(X, Y) = 0$
Linear Scaling	$\mathrm{Var}(aX + b) = a^2 \cdot \mathrm{Var}(X)$
Covariance Scaling	$\mathrm{Cov}(aX, bY) = ab \cdot \mathrm{Cov}(X, Y)$

Discrete Uniform Distribution

Definition: X is uniformly distributed over $\{1, 2, 3, 4\}$

Formulas:

- $E[X] = \frac{a+b}{2}$
- $\mathrm{Var}(X) = \frac{(b-a+1)^2-1}{12}$

Example:

Let $X \sim \text{Uniform}(\{1,2,3,4\})$

- $E[X] = \frac{1+4}{2} = 2.5$
- $\text{Var}(X) = \frac{(4-1+1)^2-1}{12} = \frac{16-1}{12} = \frac{15}{12} = 1.25$

Binomial Distribution

Definition: $X \sim \text{Binomial}(n = 5, p = 0.4)$

Formulas:

- $E[X] = np$
- $\text{Var}(X) = np(1-p)$

Example:

- $E[X] = 5 \cdot 0.4 = 2$
- $\text{Var}(X) = 5 \cdot 0.4 \cdot 0.6 = 1.2$

Poisson Distribution

Definition: $X \sim \text{Poisson}(\lambda = 3)$

Formulas:

- $E[X] = \lambda$
- $\text{Var}(X) = \lambda$

Example:

- $E[X] = 3$
- $\text{Var}(X) = 3$

Uniform (Continuous)

Definition: $X \sim \text{Uniform}(a = 2, b = 6)$ -> [2,6]

Formulas:

- $E[X] = \frac{a+b}{2}$
- $\text{Var}(X) = \frac{(b-a)^2}{12}$

Example:

- $E[X] = \frac{2+6}{2} = 4$
- $\text{Var}(X) = \frac{(6-2)^2}{12} = \frac{16}{12} = \frac{4}{3}$

Normal Distribution

Definition: $X \sim N(\mu = 10, \sigma^2 = 4)$

Formulas:

- $E[X] = \mu$
- $\text{Var}(X) = \sigma^2$

Example:

- $E[X] = 10$
- $\text{Var}(X) = 4$

Covariance — Medium Example

Let:

- $X \sim \text{Bernoulli}(0.6)$
- $Z = 2X + 3X^2$

Find: $\text{Cov}(X, Z)$

Use:

- $E[X] = 0.6$, since $X \in \{0, 1\}$
- $X^2 = X$, so $Z = 5X$
- $E[Z] = 5E[X] = 3$
- $E[XZ] = E[5X^2] = 5E[X] = 3$

Then:

- $\text{Cov}(X, Z) = E[XZ] - E[X]E[Z] = 3 - (0.6)(3) = 3 - 1.8 = 1.2$

Answer: $\boxed{1.2}$

Summary of Key Properties

- $\text{Cov}(aX + b, cY + d) = ac \cdot \text{Cov}(X, Y)$
- If X, Y independent: $\text{Cov}(X, Y) = 0$
- $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab \cdot \text{Cov}(X, Y)$
- If X, Y independent: $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$