Let X and Y denote two independent stochastic variables. Assume the PMF of X is

$$f_X(x) = \begin{cases} 0.2 & \text{if } x = 0 \\ 0.3 & \text{if } x = 1 \\ 0.5 & \text{if } x = 2 \\ 0 & \text{else} \end{cases}$$

and that the PMF of Y is

$$f_Y(y) = \begin{cases} 0.3 & \text{if } y = 0 \\ 0.4 & \text{if } y = 1 \\ 0.3 & \text{if } y = 2 \\ 0 & \text{else} \end{cases}$$

a. Find the following values. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X > 0) = 0.$☐

$\text{Var}(X) = 0.$☐

b. Find the following probabilities. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X < 2, Y > 1) = 0.$☐

$P(\{X < 2\} \cup \{Y < 2\}) = 0.$☐

More

c. Find the value below. State your answer as an integer between 0 and 99 so that the answer is given with two decimal precision. Please note that a negative sign has been pre-printed.

$Cov(2X - 5Y, 7X + 4Y + 1) = -$☐$.46$

PDF -> (E2022) -> Part 1 – 1.pdf

Let $X \sim \text{Exponential}(3)$ and set $Y = e^{2X}$.

a. Determine the expected value of Y. State your answer as a positve integer.

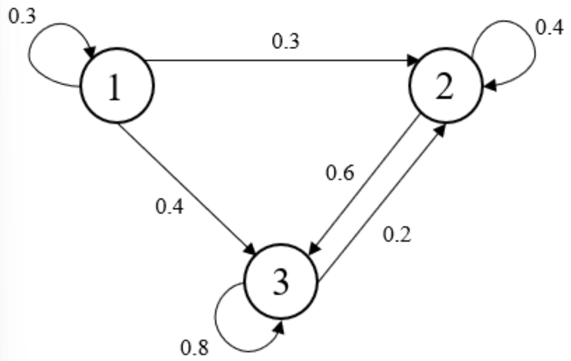
$E[Y] = \square$

b. Find the PDF of Y. State your inputs as positve integers such that all values are stated as irreducible fractions.

$$f_Y(y) = \begin{cases} \frac{\square}{\square} y^{-\frac{\square}{\square}} & y > 1 \\ 0 & else \end{cases}$$

PDF -> (E2022) -> Part 1 – 3.pdf

Let $\{X_n : n = 0, 1, \ldots\}$ denote a Markov Chain with states {1, 2, 3} and with the following state transition diagram:



Find the following probability. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X_5 = 3 \mid X_3 = 1, X_2 = 2) = 0.\square$

PDF -> (E2022) -> Part 1 – 4.pdf

Let $X_1, \ldots, X_{1000}$ denote a sample with $X_i \sim Bernoulli(p)$ for all $i = 1, \ldots, 1000$. Assume we have observed the outcomes $x_1, \ldots, x_{1000} \in \{0, 1\}$ from the sample. You are informed that the average of all the x_i's is $\overline{x} = 0.54$ and that the sample variance is $s^2 = 0.45$. We are interested in determining whether or not the outcomes are evenly distributed in the sample space of the stated Bernoulli distribution.

a. Determine which of the below would be an appropriate alternative hypothesis for this test.

A	$H_1: \ p \neq \frac{1}{2}$
B	$H_1: \ \overline{x} \neq \frac{1}{2}$
C	$H_1: \ \mu = 0.50$
D	$H_1: \ \mu \neq 0.50$
E	$H_1: \ p = \frac{1}{2}$

b. Set up a 90% confidence interval for p. Select the correct interval from the choices.

A	[0.1753; 0.3526]
B	[0.4556; 0.5523]
C	[0.5102; 0.5872]
D	[0.2670; 0.4231]

c. Assume we want to test the hypothesis mentioned above with $\alpha = 0.01$. Determine all the values below, and determine the correct decision based on the data. Select the value closest to your result.

Test Statistic:

The critical value:

The p-value:

Based on this we should ______ the null hypothesis.

PDF -> (E2022) -> Part 1 – 5.pdf

Assume that 0.01% of the population has COVID-19 and that 20000 randomly chosen people are at a large gathering. What is the probability that at least 5 people at the gathering have COVID-19. Please state your answer as a **decimal value** correctly rounded to four decimal precision (e.g. 0.9876). Remember to use '.' as the decimal separator.

☐

PDF -> (E2022) -> Part 1 – 6.pdf

Consider the following statistics collected from a sample of size 25: The sample mean is 310 and the sample standard deviation is 6. A calculated confidence interval for the mean is [306.6551 ; 313.3449]. Which confidence level was chosen? Assume distribution to be normal. Select the value below that is closest to the level.

A	90%
B	94%
C	95%
D	96%
E	97.5%

PDF -> (E2022) -> Part 1 – 8.pdf

Two Premier League teams, A and B, are to play a match. We know that the number of goals scored by Team A is modeled by a Poisson process $N_1(t)$ with rate $\lambda_1 = 0.02$ goals per minute, and the number of goals scored by Team B is modeled by a Poisson process $N_2(t)$with rate $\lambda_2 = 0.03$ goals per minute. The two processes are assumed to be independent. Let $N(t)$ be the total number of goals in the game up to and including time t. Assume the game lasts for 90 minutes with no overtime. State all your answers as decimal values correctly rounded off to two decimal precision for all problems below. Remember to use '.' as the decimal separator.

a. Find the probability that no goals are scored, i.e., the game ends with a 0-0 draw.

b. Find the probability that at least two goals are scored in the game. *Hint*: You can treat the number of goals scored by any of the two teams as a new Poisson process with rate $\lambda_1 + \lambda_2$ goals per minute.

c. Find the probability of the final score being:

Team A: 1,   Team B: 2.

PDF -> (E2022) -> Part 1 – 10.pdf

Let $X_1, X_2, \ldots, X_n$ denote a random sample from the following normal distribution: $N(\mu, 9)$ for $n \in \mathbb{N}$, and let $\bar{X} = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$ denote the associated sample mean. How large must n be so that $\bar{X}$ is no more than 0.7 from μ with 95% confidence? State your answer as an integer between 0 and 99.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

The sample size must be at least $\boxed{71}$ in order for the sample mean to be no more than 0.7 from μ with 95% confidence.

(E2023).pdf - page 4 to 6

Let X be a discrete stochastic variable with the following probability mass function:

$$p_X(x) = \begin{cases} 1/3 & \text{for } x \in \{-1, 0, 1\} \\ 0 & \text{else.} \end{cases}$$

For all questions in this assignment, state your inputs as integers between 0 and 99 such that all answers are given as either integers or irreducible fractions

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. Find the expected value and variance of X.

$E(X) = \square$

Ch

$Var(X) = \frac{\square}{\square}$

Let Y denote a stochastic variable that is independent of X and has the same PMF as X, i.e.

$$p_Y(y) = \begin{cases} 1/3 & \text{for } y \in \{-1, 0, 1\} \\ 0 & \text{else.} \end{cases}$$

b. Find the values below.

$Var(X - Y) = \frac{4}{3}$

$Cov(X, Y + X) = \frac{\square}{\square}$

c. Find the values below

$E(|X - 1|) = \square$

$E(|X - 1| \cdot |Y - 1|) = \square$

d. Find the probabilities below.

$P(X \neq 0) = \frac{\square}{\square}$

(E2023).pdf - page 8 to 13

Let X and Y be two independent stochastic variables such that $X \sim \text{Binomial}(10, 0.5)$ and $Y \sim \text{Geometric}(1/5)$. Find the below probabilities. State your answer as an integer between 0 and 99 such that you supply four decimal precision.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

$P(X > 5 \cup Y \leq 3)$ =0.6 81 ✔ 0

$P(X > 5 \mid X + Y = 3)$ =0.000 0 ✔

(E2023).pdf - page 14 to 17

Let X and Y be two jointly continuous random variables with joint PDF:

$$f_{XY}(x,y) = \begin{cases} \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6} & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $E\left[X^2 \mid Y = 1\right]$. State your input as a positive integer such that the answer is stated as an irreducible fraction.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

$$\frac{\boxed{21}}{50}$$

(E2023).pdf - page 18 to 21

One way to design a spam filter is to look at the words in an email. In particular, some words are more frequent in spam emails. Suppose that we have the following information:

- 60% of emails are spam;
- 2% of spam emails contain the word "discount";
- 0.002% of non-spam emails contain the word "discount".

Suppose that an email is checked and found to contain the word "discount". Find the probabilities below. State all inputs as integers between 10 and 99 such that the answer is given as a decimal value with four decimal precision.

Documentation: You should supply the calculations leading up to the final result.

The probability that an email contains the word "discount": 0.0 12 ✔ 0

The probability that an email is spam given that it contains the word "discount": 0.9 99 ✔ 3

(E2023).pdf - page 22 to 25

The number of cars, X, passing the Storebælt Bridge over a certain time period, can reasonably be assumed to follow a Poisson distribution:

$$X \sim \text{poisson}(\lambda t)$$

Over a time period of t = 3 hours, x = 1140 cars pass the bridge. Based on this information, the estimate for the time parameter is:

$$\hat{\lambda} = \frac{x}{t} = \frac{1140}{3} = 380 \text{ cars per hour}$$

Documentation: In (a) no documentation is required. In (b) and (c) you are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. An employee at the Storebælt Bridge claims that, on an annual basis and for a similar time period, an average of 400 cars pass per hour. Do the measured data (x = 1140 cars in t = 3 hours) agree with the employee's claim? To answer this question correctly, you need to conduct a hypothesis test. Identify the correct null hypothesis.

- [ ] $H_0: \lambda_0 \neq 380$
- [ ] $H_0: \lambda_0 \neq 1200$
- [ ] $H_0: \lambda_0 \geq 1140$
- [ ] $H_0: \lambda_0 \neq 400$
- [ ] $H_0: \lambda_0 = 1140$
- [x] $H_0: \lambda_0 = 400$ ✓
- [ ] $H_0: \lambda_0 = 380$
- [ ] $H_0: \lambda_0 \geq 380$
- [ ] $H_0: \lambda_0 \neq 1140$
- [ ] $H_0: \lambda_0 = 1200$

b. Determine the test statistic and the p-value for the test. State your inputs as integers between 10 and 99 so that the answer is displayed with four decimal precision.

Hint: Remember that the variance of a Poisson random variable is the same as the parameter.

Test statistic: -1.7 32 ✓ 1

P-value: 0.0 83 ✓ 3

Check answers

c. Assuming $\lambda = 400$ cars per hour, as stated by the employee, determine the probability that 380 or fewer cars pass over the bridge in one hour.

0.1 64 9 ✓

(E2023).pdf - page 26 to 33

Let P and Q be two independent $N(0,1)$ random variables and

$$R = 7 - P + P^2$$
$$S = 4 - PQ.$$

Find $\text{Cov}(R, S)$. State your answer as an integer between 0 and 99.

Documentation: You must supply your calculations leading up to the result.

$Cov(R,S) = \boxed{0}$

(E2023).pdf - page 34 to 38

Let (X,Y) be a two-dimensional discrete stochastic vector with probability function $p_{X,Y}$ given by:

$$p_{X,Y}(x,y) = \begin{cases} \frac{1}{4} \cdot (1-p)p^y & \text{if } x \in \{1,2,3,4\} \text{ and } y \in \{0,1,2,\ldots\} \\ 0 & \text{otherwise.} \end{cases}$$

Here, $p \in]0,1[$.

For all questions in this assignment, state your inputs as integers between 0 and 99 such that all answers are given as either integers or irreducible fractions.

Documentation: You are expected to demonstrate how you obtained the results.

a. Find the marginal probability distributions of X and Y.

$$p_X(x) = \begin{cases} \frac{1}{\boxed{4}} & \text{if } x \in \{1,2,3,4\} \\ 0 & \text{else} \end{cases}$$

$$p_Y(y) = \begin{cases} (\boxed{1} - p)p^y & \text{if } y \in \{0,1,2,..\} \\ 0 & \text{else} \end{cases}$$

In the rest of the assignment, feel free to use the fact that $p = \frac{1}{2}$

b. Find the values below.

$P(X \leq 2) = \frac{\boxed{1}}{\boxed{2}}$

$P(Y \leq 1) = \frac{\boxed{3}}{\boxed{4}}$

$P(\{X \leq 2\} \cap \{Y \leq 1\}) = \frac{\boxed{3}}{8}$

$P(\{X \leq 2\} \cup \{Y \leq 1\}) = \frac{7}{\boxed{8}}$

c. Find the expected value and variance of X:

$EX = \frac{\boxed{5}}{2}$

$\text{Var}\, X = \frac{\boxed{5}}{4}$

d. Find the values below. Feel free to use the fact that $EY = 1$ and $\text{Var}\, Y = 2$.

$\text{Var}(X - Y) = \frac{\boxed{}}{4}$

$\text{Cov}(X, X - Y) = \frac{\boxed{}}{4}$

$E(Y \cdot |X - 3|) = \boxed{}$

(E2024) Part1 - 1.pdf

Two competing coffee shops, Café A and Café B, are located on the same street. The number of customers arriving at each café follows independent Poisson processes:

- Café A receives customers at a rate of λ_1 = 1.5 customers per hour.
- Café B receives customers at a rate of λ_2 = 2 customers per hour.

The two cafés open at 8 AM and close at 5 PM, providing a 9-hour operational day.

Documentation: You are expected to demonstrate how you obtained the results.

a. Determine the probability that Café B serves the first customer of the day, and state your answer as a four precision decimal value. Remember to use dot (" . ") as decimal separator.

The probability that Café B serves the first customer of the day is ☐

b. Calculate the expected time until the first customer arrives at each café on a given day. State your answer as a positive integer that denotes minutes.

The expected time until the first customer for Café A is ☐ minutes.

The expected time until the first customer for Café B is ☐ minutes.

c. Determine the probability that neither café has any customers in the first hour of operation. State your answer as a four precision decimal value. Remember to use dot (" . ") as decimal separator.

$P(\text{No customers at A and B}) =$ ☐

d. Estimate the total number of customers that each café will serve in the first three hours. State your answer as a decimal value with one decimal precision. Remember to use dot (" . ") as decimal separator.

Together, Café A and Café B are expected to serve a total of ☐ customers in the first three hours.

(E2024) Part1 - 2.pdf

Three companies, A, B, and C, are competing in a market. The likelihood of customers shifting their preferences between these companies over a year is modeled by a Markov chain with three states representing each company. The transition probabilities are given by matrix P:

$$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

State all answers as irreducible fractions.

Documentation: You are expected to demonstrate how you obtained the results.

a. Determine the probability that a customer who initially prefers Company B will prefer Company C after one year.

$\frac{\square}{\square}$

b. If a customer is initially with Company C, calculate the probability that they will be with Company A after two years.

$\frac{\square}{\square}$

c. Given the mean return times to state j:

- $r_1 = \frac{35}{12}$
- $r_2 = \frac{35}{9}$
- $r_3 = \frac{5}{2}$

Determine the values in the vector given by

$\pi_j = \lim_{n \to \infty} P\left(X_n = j \mid X_1 = i\right)$

$\pi_j = \left[\frac{\square}{\square} \quad \frac{\square}{\square} \quad \frac{\square}{\square} \right]$

d. In the long run, which company will have the largest market share?

- ○ Company A
- ○ Company B
- ○ Company C

(E2024) Part1 - 3.pdf

Please identify the correct distribution for each scenario. Note: All distributions mentioned relate to exactly one scenario. You must identify the distributions of all six scenarios correctly to obtain points. Please note that the Geometric and Negative Binomial distributions are treated as **distinct** distributions in this exercise acknowledging that the Geometric is simply a special case of the Negative Binomial.

Documentation: No documentation needed for this assignment.

	Binomial	Geometric	Negative Binomial	Hypergeometric	Poisson	Exponential
A cybersecurity firm employs an intrusion detection system that flags activities as malicious or benign. Given the probability of correctly identifying an activity, the firm wants to evaluate the system's accuracy over 100 distinct activities.	○	○	○	○	○	○
A network administrator monitors traffic to a server which on average receives 300 requests per minute. The distribution of requests over any given minute is	○	○	○	○	○	○

(E2024) Part1 - 4.pdf

Thirty software engineering students completed a series of courses taught by Professor Reginald Kooks. The effectiveness of these courses is being evaluated by comparing students' smartness scores, measured on a scale of 0 to 100, before and after the courses. Here are the summary statistics of the test:

- Number of students (n): 30
- The mean score before the course: 70
- The mean score after the courses: 76
- Standard deviation of scores before the courses (S_1): 10
- Standard deviation of scores after the courses (S_2): 15

We will assume minimal covariance and estimate the standard deviation of the differences (S_D) as:

$$S_D = \sqrt{S_1^2 + S_2^2}$$

We want to know whether following the courses have had a positive effect on the smartness score of the students (i.e. that score has increased), i.e. we want a significant test result.

Documentation: You only need to provide documentation of how you obtained your answer in question (c). You will need to use one or more probability distribution tables to solve some of the tasks in this assignment.

a. First, please identify the correct hypothesis associated with this test. Note μ_B andμ_A refer to the mean before and after, respectively, and μ_D to the Mean of the Differences.

A	$H_0 : \mu_D = 0 H_1 : \mu_D \neq 0$
B	$H_0 : \mu_B > \mu_A H_1 : \mu_B < \mu_A$
C	$H_0 : \mu_D = 10 H_1 : \mu_D \neq 10$

b. Find the critical value of the test, and state your answer as a decimal value with three decimal precision.

The critical value:

c. Find the test statistic. State you answer as a decimal value with three decimal precision.

Test statistic:

d. Is there sufficient evidence to support the claim that the courses have effected and increase in students' smartness score?

○ Yes

○ No

(E2024) Part1 - 5.pdf

A software development team is working on implementing a series of new features into an existing application. The probability of successfully integrating each feature without bugs on the first try is initially estimated based on past performance. The team plans to implement 10 new features in the upcoming release. Let X denote the number of features successfully integrated without bugs out of 10 attempts. Historically, the probability of any single feature being successfully integrated without needing any bug fixes is 70%.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

State all answers as decimal values with 4 decimal precision. Remember to use dot (" . ") as decimal separator, except in the last exercise (e) where you must enter a positive integer.

a. Calculate the probability that exactly 7 out of the 10 features are successfully integrated without bugs.

b. Find the probability that at least 8 features are integrated successfully.

Based on the success of the initial feature integration, the software team decides to utilize the observed success rate to inform the deployment of a new module or system upgrade. The team estimates that the success rate from above will directly impact the reliability of the system in a real-world environment, specifically in terms of uptime. Let Y represent the number of days in a given month (30 days) that the system achieves 99% or greater uptime. The probability of achieving 99% uptime any given day is assumed to be equivalent to achieving at least 8 successful integrations from (b), which represents a higher confidence in system stability due to previous successes.

c. Calculate the expected number of days the system achieves 99% or greater uptime in a 30-day month.

d. Determine the probability that the system achieves 99% in more than 15 days in a 30-day month.

The software development team wants to ensure that when they deploy a small number of critical updates, at least one is successful. Based on historical data, the success rate of any single update being successfully deployed without any issues is 70% as mentioned above.

e. Determine the minimum number of critical updates n that the team should deploy to be 99% confident that at least one update is successfully deployed without issues. Since n must be an integer, round up to the nearest whole number.

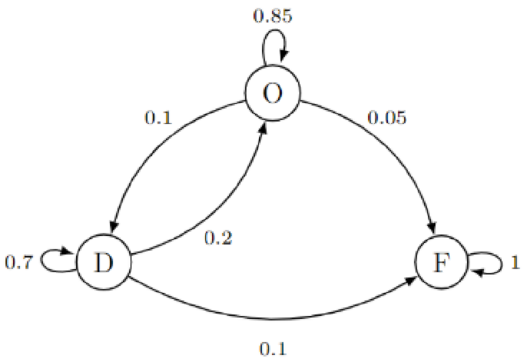
$n =$

(E2024) Part1 - 6.pdf

Suppose a server used for a large-scale web application can be in three states:

- **Operational (O)**: The server is fully functional.
- **Degraded (D)**: The server is running but with reduced capacity or speed.
- **Failure (F)**: The server is down and not serving any requests.

The transitions between these states occur according to the probabilities reflecting the server's reliability and maintenance effectiveness:



The server starts fully functional 95% of the time and the rest of the time the server starts out degraded, running with reduced capacity or speed.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code. No documentation needed for the final assignment (d).

State all answers as decimal values with 4 decimal precision. Remember to use dot (" . ") as decimal separator.

a. Given that the server starts fully operational, what is the probability that the server is in the Failure (F) state after two time steps?

b. Calculate the probability of transitioning from the Degraded (D) state to the Failure (F) state in exactly three transitions.

c. Calculate the mean hitting time for the server to go from Operation (O) to Failure (F) $(t_O)$and the mean hitting time for the server to go from Degraded (D) to Failure (F) (t_D). Both values are positive integers.

$t_O =$

$t_D =$

d. What will happen with the system in the long run?

A	In the long run, the system will spend equal time in the Operational (O), Degraded (D), and Failure (F) states, with each state having a probability of 1/3.
B	In the long run, the Markov chain will oscillate between the Operational (O) and Degraded (D) states with equal probabilities, while the probability of being in the Failure (F) state will be 0.
C	In the long run, the Markov chain will reach a steady-state where the probability of being in the Operational (O) state is 0.5, the probability of being in the Degraded (D) state is 0.3, and the probability of being in the Failure (F) state is 0.2.
D	In the long run, the Markov chain will reach the absorbing state of Failure (F) with probability 1. This means that regardless of the initial state, the system will eventually fail and remain in the Failure state indefinitely. The probabilities of being in the Operational (O) and Degraded (D) states will both be 0 in the long run.

More

(E2024) Part1 - 7.pdf

Let Z be a discrete random variable with PMF

$$P_Z(k) = \begin{cases} 0.1 & \text{for } k = 0 \\ 0.3 & \text{for } k = 1 \\ 0.4 & \text{for } k = 2 \\ 0.2 & \text{for } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

Define $W = Z(Z-2)$. Find the PMF of W. Please state the values of k as integers between 0 and 99 and in ascending order like in the formula above (i.e. $-k \ < k < k+1$). Please state the distribution probabilities as decimal values with four decimal precision. Note that in the first value for k, a negative sign has been pre-printed.

Documentation: Please supply the calculations leading to your answers.

$$P_W(k) = \begin{cases} \boxed{0.3} & for\ k = -\boxed{1} \\ \boxed{0.5} & for\ k = \boxed{0} \\ \boxed{0.2} & for\ k = \boxed{3} \\ 0 & otherwise \end{cases}$$

(ME2024) Part1 - 1.pdf

Math Problem

Let A and B be two independent $N(0,1)$ random variables, and

$$U = 5 + A - A^2B,$$
$$V = 2 + B.$$

Find $\mathrm{Corr}(U,V)$. Please enter two integers between 0 and 99 such that the answer displayed is an irreducible fraction. Also, note that a negative sign has been pre-printed. You can use the fact that $\mathrm{Var}(U) = 4$ and $\mathrm{Var}(V) = 1$

Documentation: Please supply the calculations leading to the answer.

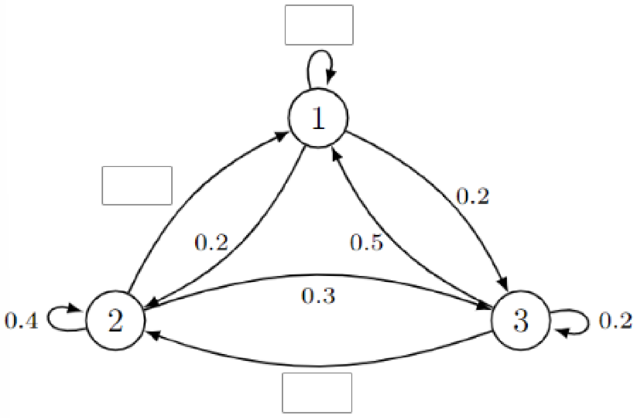
$$\mathrm{Corr}(U,V) = -\frac{\boxed{1}}{\boxed{2}}$$

(ME2024) Part1 - 2.pdf

Consider a Markov chain with three states $S = \{1, 2, 3\}$.

a. The state transition diagram of such a Markov chain is shown below. Fill out the missing transition probabilities. State your inputs as decimal values between 0.1 and 0.9 (both included) with one decimal precision and remember to use dot as the decimal separator.

Documentation: No documentation is needed for parts (a) and (b) but please supply all calculations leading to your answers in part (c).



b. Determine the state transition matrix of the above Markov chain, including the values found in (a). State your inputs as decimal values between 0.1 and 0.9 (both included) with one decimal precision and remember to use dot as the decimal separator.

$P =$

c. If we know $P(X_1 = 1) = P(X_2 = 2) = \frac{1}{3}$, find $P(X_1 = 2, X_2 = 3, X_3 = 1)$. State your inputs as two integers between 0 and 99 such that the answer is given as an irreducible fraction.

(ME2024) Part1 - 3.pdf

The time it takes to complete a particular task is measured for employees in a company. A new software tool was introduced to help complete the task more efficiently. Completion times in minutes for a random sample of employees before and after using the new software were measured. Let μ_B denote the mean completion time before using the software and let μ_A denote the mean completion time after using the software. If the improvement in task completion time is more than 2 minutes, we would like to detect it. State the correct hypotheses for this test.

Documentation: No documentation is need for this assignment

H_0 : $\mu_B - \mu_A \leq 2$

H_1 : $\mu_B - \mu_A > 2$

$\mu_B - \mu_A \neq 0$ | $\mu_B - \mu_A \geq 2$ | $\mu_B - \mu_A \neq 2$ | $\mu_B - \mu_A > 0$ | $\mu_B - \mu_A < 0$ | $\mu_B - \mu_A = 0$

$\mu_B - \mu_A < 2$ | $\mu_B - \mu_A \geq 0$ | $\mu_B - \mu_A = 2$ | $\mu_B - \mu_A \leq 0$

(ME2024) Part1 - 4.pdf

Consider the following test in a regression problem:

$H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$

The following has been computed:

$\hat{\beta}_1 = 14.947 \quad n = 20,$
$S_{xx} = 0.68088, \quad \hat{\sigma}^2 = 1.18$

Documentation: Please supply the calculations leading to the answers. Note, you will need to use a t-table for this assignment.

a. Calculate the test statistic for this test. State your answer as a decimal value with four decimal precision.

$T_0 = \boxed{11.5540}$

b. Assuming we want to do the test with a significance of 0.01. What will then be the absolute critical value for the test? State your answer as a decimal value with three decimal precision.

$T_{crit} = \boxed{2.878}$

c. Set up a 95% confidence interval for the slope. State the lower and upper bounds with four decimal precision.

$\boxed{12.1825} \leq \beta_1 \leq \boxed{17.7115}$

(ME2024) Part1 - 5.pdf

Assume the following transition matrix P for an ergodic Markov chain:

$$P = \begin{bmatrix} 1/2 & 1/2 \\ p & 1-p \end{bmatrix}$$

When the experiment is performed many times, the chain ends in state one approximately 20 percent of the time and in state two 80 percent of the time. Find a value for p and state your answer as an irreducible fraction.

$p = \frac{\square}{\square}$

(ME2024) Part1 - 6.pdf

Let A and B be two independent stochastic variables such that $A \sim Poisson(3)$ and $B \sim Geometric(1/2)$.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. Find $P(A = 2 \text{ or } B = 2)$. State your input as a decimal value with four decimal precision. Remember to use dot as decimal separator.

$P(A = 2 \text{ or } B = 2) = $ ☐

b. Find $P(B = 3 \mid A + B = 4)$ and state your input as a decimal value with four decimal precision. Remember to use dot as decimal separator.

$P(B = 3 \mid A + B = 4) = $ ☐

c. Find the values below. In both cases, state your answer as a positive integer.

$E[A + B] = $ ☐

$E\left[(A + B)^2\right] = $ ☐

(ME2024) Part1 - 7.pdf

A Markov chain has states $0, 1, 2$ with transition probabilities

$$P = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}.$$

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code. Please state all answers as decimal values with four decimal precision.

Find the mean return time to state 1. Round your answer to the nearest integer.

$r_1 = $ ☐

(ME2024) Part1 - 8.pdf

Consider two independent discrete random variables X and Y. Assume that the probability function p_X for X is given by

$$p_X(x) = \begin{cases} \frac{1}{3} & \text{if } x \in \{1, 2, 3\} \\ 0 & \text{otherwise,} \end{cases}$$

and that Y is Poisson-distributed with parameter $\lambda = 1$.

For all questions in this assignment, state your inputs as integers between 0 and 99 such that all answers are given as either integers or irreducible fractions.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

Find the values below.

$P(X \leq 2) = \frac{\square}{\square}$

$Var(X) = \frac{\square}{\square}$

$Var(X - Y) = \frac{\square}{\square}$

$Cov(X, 2Y) = \square$

$E(XY) = \square$

$E\big(X(X + 5Y)\big) = \frac{\square}{3}$

(RE2023).pdf Page 1 to 5

A random sample of 1900 adult women indicated that 941 of them asked for medical assistance last time they felt sick. The same sized sample of men indicated that 893 of them asked for medical assistance.

Documentation: In (a) no documentation is required. In (b) you are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. Is it reasonable to conclude that women ask for medical assistance more often than men? Set up an appropriate hypothesis test to answer this question using $\alpha = 0.05$. Assuming p_1 refers to the rate of women asking for medical assistance and that p_2 refers to the rate of men asking for medical assistance, identify the correct alternative hypothesis of this test and check the answer.

- ○ $H_1: \ p_1 \neq p_2$
- ○ $H_1: \ p_1 < p_2$
- ○ $H_1: \ p_1 = p_2$
- ○ $H_1: \ p_1 + p_2 > 1$
- ○ $H_1: \ p_1 - p_2 > 1$
- ○ $H_1: \ p_1 \geq p_2$
- ○ $H_1: \ p_1 - p_2 \ \geq 0$
- ○ $H_1: \ p_1 > p_2$
- ○ $H_1: \ p_1 - p_2 \neq 0$
- ○ $H_1: \ p_1 \leq p_2$

b. Determine the critical value, the test statistic and the p-value for the test. Identify the three values below, i.e. you need to choose three of the choices listed below.

- ☐ 1.5583
- ☐ 0.1192
- ☐ 1.5287
- ☐ 2.3263
- ☐ 0.0596
- ☐ 0.0298
- ☐ 0.5
- ☐ 1.5722
- ☐ 1.2816
- ☐ 2.578
- ☐ 1.6449
- ☐ 1.96

(RE2023).pdf Page 6 to 9

Assuming X and Y are independent and given $E(X) = 2$ and $E(X^2) = \frac{24}{5}$, find the value below.

State you inputs as two positive integers such that the answer displays an irreducible fraction (note a negative sign is pre-printed).

Documentation: You are expected to demonstrate how you obtained the result either by supplying manual calculations or Python code.

$$Cov(1 + X, Y - 2X) = -\frac{\square}{\square}$$

(RE2023).pdf Page 10 to 12

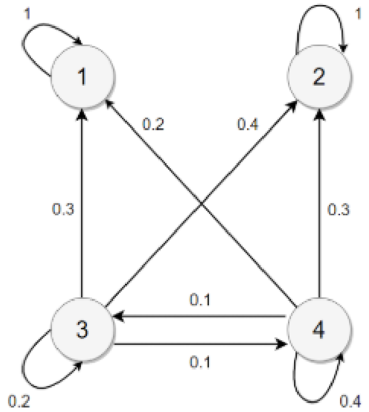
Suppose that a random variable *X* has continuous uniform distribution on [1, b] , where *b* is an unknown parameter. We have a random sample of 15 in size from a population represented by *X* and the random sample has a sample mean of 5.38. Use the sample mean which is an unbiased estimator, to find a point estimate of *b*. State your input as an integer between 0 and 99 so that the answer is displayed with two-decimal precision (note that the decimals are pre-printed).

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

$\boxed{9}.76$

(RE2023).pdf Page 13 to 15

Consider a Markov chain $\{X_n : n = 0, 1, 2, \ldots\}$ with state space $S = \{1, 2, 3, 4\}$ and state transition diagram given by



State your input as a **decimal** value so that you supply four decimal precision. Remember to use '.' (dot) as the decimal separator.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

Find the mean hitting time of state 4 given that the proces started in state 3.

(RE2023).pdf Page 16 to 20

A city authority checks the hourly average weight of the traffic on a small bridge to determine if more frequent maintenance should be conducted. Suppose X, the hourly average weight of the traffic, is normally distributed with the mean of 30 tons and a variance of 6.25.

Determine the following, choosing the correct answer for each item.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. $P(X > 37)$

A	0.0125
B	0.2302
C	0.0072
D	0.1418
E	0.1314

b. $P(28.5 < X < 32.5)$

A	0.2503
B	0.2007
C	0.5412
D	0.2764

c. Weight, which is exceeded with probability 0.99.

A	15.46
B	23.68
C	24.18

(RE2023).pdf Page 21 to 24

Accidents on highways are one of the main causes of death or injury in developing countries and the weather conditions have an impact on the rates of death and injury. In foggy, rainy, and sunny conditions, 1/4, 1/8, and 1/21 of the accidents result in death, respectively. Sunny conditions occur 60% of the time, while rainy and foggy conditions each occur 20% of the time.

For both questions, identify the correct answer, here given with 4 decimal precision.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. What is the probability that an accident results in a death?

A	0.1103
B	0.0964
C	0.0987
D	0.1152

b. Given that an accident without deaths occurred, what is the conditional probability that it was foggy at the time? Remember to use non-rounded values in your calculations.

A	0.1721
B	0.1750
C	0.1603
D	0.1673
E	0.1587

(RE2023).pdf Page 25 to 28

A database contains 48 heart signal recordings with 22 from females patients and 26 from male patients. For a classification task, an analyst randomly selects 36 records for predictive model training and keeps the other 12 records for testing the model performance.

For all questions, identify the correct answer.

Documentation: You are expected to demonstrate how you obtained the results either by supplying manual calculations or Python code.

a. What is the expected number of female patient recordings selected for training?

A	16.5
B	17.5
C	19
D	18

b. What is the probability that at least three male patient recordings are selected for training?

A	0.95
B	1
C	0.94
D	0

c. What is the probability that the same number of male and female patients are used for testing the model performance?

A	0.5
B	0.22
C	0
D	0.25

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