Face Recognition Device: Geometric and Negative Binomial Distribution Solution and Wolfram Alpha Documentation

This document provides a detailed solution to a probability problem involving the performance of a face recognition device. It covers concepts related to geometric and negative binomial distributions, including the mean number of trials until failure and the probability of the first failure occurring at a specific trial. It includes explanations of the steps and how to use Wolfram Alpha (or similar symbolic computation tools) to verify or perform the calculations.

Topics Covered in this Exercise:

* **Geometric Distribution**
* **Negative Binomial Distribution**
* **Probability Mass Function (PMF)**
* **Expected Value (Mean) of Geometric Distribution**
* **Expected Value (Mean) of Negative Binomial Distribution**
* **Independent Trials**

Problem Statement: A research team has developed a face recognition device to match photos in a database. From laboratory tests, the recognition accuracy is 92% and trials are assumed to be independent.

Part a: If the research team continues to run laboratory tests, what is the mean number of trials until failure?

This question relates to Geometric **Distribution**. A Geometric distribution models the number of Bernoulli trials needed to get the first "success" (in this case, "failure" of the device).

Let p be the probability of "success" (which is recognition accuracy).

So $p = 0.92$. Let q be the probability of "failure" (which means the device does *not* recognize, i.e., it makes an error). So, $q = 1 - p = 1 - 0.92 = 0.08$.

The mean number of trials *until* the first failure (including the failure itself) for a geometric distribution with probability of success q is given by $E[X] = \frac{1}{q}$.

$$E[\text{trials until failure}] = \frac{1}{q} = \frac{1}{0.08}$$

$$E[\text{trials until failure}] = 12.5$$

The mean number of trials until failure is 12.5. This matches the provided answer.

- **Wolfram Alpha Input**: `mean of geometric distribution with success probability 0.08` (Result: 12.5)

Part b: What is the probability that the first failure occurs on the tenth trial?

This is also a question about the **Geometric Distribution**. The probability mass function (PMF) of a geometric distribution, $P(X = k)$, for the first success occurring on the k-th trial is given by:

$$P(X = k) = (1 - q)^{k-1}q$$

where q is the probability of success on a single trial (in this context, the probability of failure), and k is the number of trials.

Here, $q = 0.08$ (probability of failure). We want the first failure to occur on the tenth trial, so $k = 10$.

$$P(X = 10) = (1 - 0.08)^{10-1} \times 0.08$$

$$P(X = 10) = (0.92)^9 \times 0.08$$

Calculate $(0.92)^9$: $(0.92)^9 \approx 0.472166$

Now, multiply by 0.08: $P(X = 10) \approx 0.472166 \times 0.08 \approx 0.03777328$

Rounding to four decimal places: 0.0378.

The probability that the first failure occurs on the tenth trial is 0.0378. This matches the provided answer.

- **Wolfram Alpha Input**: `probability that the first success is on the 10th trial for geometric distribution with success probability 0.08` (Result: 0.0377733)
- **Wolfram Alpha Input**: `round 0.0377733 to 4 decimal places` (Result: 0.0378)

Part c: To improve the recognition algorithm, a chief engineer decides to collect 10 failures. How many trials are expected to be needed?

This question relates to the **Negative Binomial Distribution**. A Negative Binomial distribution models the number of Bernoulli trials (X) needed to get r "successes" (in this case, r "failures").

Let r be the desired number of failures = 10. Let q be the probability of a single trial being a "failure" (as defined in Part a), so $q = 0.08$.

The expected number of trials needed to get r successes (failures in this context) in a negative binomial distribution is given by:

$$E[X] = \frac{r}{q}$$

$$E[\text{trials for 10 failures}] = \frac{10}{0.08}$$

$$E[\text{trials for 10 failures}] = 125$$

The expected number of trials needed is 125. This matches the provided answer.

- **Wolfram Alpha Input**: `mean of negative binomial distribution with r=10 and p=0.08` (Result: 125) (Note: Wolfram Alpha's negative binomial parameters usually define p as the probability of success for each trial, and r as the number of successes desired. So we use $p = 0.08$ (failure probability) for our 'success' here.)