Condicional prabability

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(B|A\cup C) = \frac{P(B\cap(A\cup C))}{P(A\cup C)}$$

$$P(B|A,C) = \frac{P(B\cap A\cap C)}{P(A\cap C)}$$

3) Since $P(0 \leq X \leq 10) = 1$

$P(a \leq X \leq b) = (b-a) \times c$

$(10-0) \times c$

$10c = 1$

$c = \frac{1}{10}$

$$P(a \leq X \leq b) = \frac{b-a}{10}$$

1. $P(2 \leq X \leq 5) = \frac{(5-2)}{10} = \frac{3}{10}$

2. $P(X \leq 2 \mid X \leq 5) = \dfrac{P((X\leq 2) \cap (X \leq 5))}{P(X\leq 5)}$

$= \dfrac{P((X\leq 2) \text{ and } (X\leq 5)}{P(X\leq 5)}$

$= \dfrac{P(X\leq 2)}{(X\leq 5)}$

$= \dfrac{P(0\leq X\leq 2)}{P(0\leq X\leq 5)}$

$= \dfrac{\frac{2-0}{10}}{\frac{5-0}{10}}$

$= \dfrac{\frac{2}{10}}{\frac{5}{10}}$

$= \frac{2}{5}$

3. $P((3\leq X\leq 8) \mid (X\geq 4)) = \dfrac{P((3\leq X\leq 8) \cap (X\geq 4)}{P(X\geq 4)}$

$= \dfrac{P(4\leq X\leq 8)}{P(4\leq X\leq 10)}$

$$= \frac{\frac{8-4}{10}}{\frac{10-4}{10}}$$

$$= \frac{\frac{4}{10}}{\frac{6}{10}}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

4) 1. R_X = Possible Range for X

$R_X = \{0, 1, 2\}$

2. $P(X \geq 1.5) = P(X = 2)$

$$= P_X(2) = \frac{1}{6}$$

4. $P(X=0 \mid X<2) = \dfrac{P(X=0, X<2)}{P(X<2)}$

$$= \frac{P(X=0)}{P(X<2)}$$

$$= \frac{P_X(0)}{P_X(0) + P_X(1)}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}}$$

$$= \frac{\frac{1}{2}}{\frac{3}{6} + \frac{2}{6}}$$

$$= \frac{\frac{1}{2}}{\frac{5}{6}}$$

$$= \frac{\frac{3}{6}}{\frac{5}{6}}$$

$$= \frac{3}{5}$$

5) $X =$ n° cars being repaired

- $X \leq 3$
- $P_X(2) = P_X(1)$
- $P_X(0) = P_X(3)$
- $P_X(1 \text{ or } 2) = \frac{1}{2} P_X(0 \text{ or } 3)$
- $0 \leq X \leq 3$

$P_X(1) = P_X(2) = a$

$P_X(0) = P_X(3) = 2a$

$2a + a + a + 2a = 1$

$a = \frac{1}{6}$

$P_X(0) = \frac{2}{6}$

$P_X(1) = \frac{1}{6}$

6) 2. $P(X \leq 2 \text{ and } Y \leq 2) = P(X \leq 2) \cdot P(Y \leq 2)$

$= \left(\frac{1}{4} + \frac{1}{8}\right) \times \left(\frac{1}{6} + \frac{1}{6}\right)$

$= \frac{3}{8} \times \frac{2}{6}$

$= \frac{6}{48}$

$= \frac{1}{8}$

3. $P(x>2 \text{ OR } y>2) = P(x>2) + P(y>2) - P(x>2 \text{ and } y>2)$

$$= \left(\frac{1}{8}+\frac{1}{2}\right) + \left(\frac{1}{3}+\frac{1}{3}\right) - \left(P(x>2) \times P(y>2)\right.$$

$$= \frac{5}{8} + \frac{2}{3} - \left(\frac{5}{8} \times \frac{2}{3}\right)$$

$$= \frac{5}{8} + \frac{2}{3} - \frac{10}{24}$$

$$= \frac{5}{8} + \frac{2}{3} - \frac{5}{12}$$

$15 + 16 - 10$

$$= \frac{21}{24} = \frac{7}{8}$$

4. $P(x>2 \mid y>2) = P(x>2)$

$$= \frac{5}{8}$$

5. $P(x<y) = \sum_{k=1}^{4} P(x<y \mid y=k) \times P(y=k)$

$$= P(x<1 \mid y=1) \times P(y=1) + P(x<2 \mid y=2) \times P(y=2)$$
$$+ P(x<3 \mid y=3) \times P(y=3) + P(x<4 \mid y=4) \times P(y=4)$$

$$= 0 \times \frac{1}{6} + \frac{1}{4} \times \frac{1}{6} + \left(0 + \frac{1}{4} + \frac{1}{8}\right) \times \frac{1}{3} + \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{8}\right) \times \frac{1}{3}$$

$$= 0 + \frac{1}{24} + \frac{3}{24} + \frac{4}{24}$$

$$= \frac{8}{24} = \frac{1}{3}$$

7) $p_1 = 0{,}83$ $\quad$ p_1 failure $= 1 - 0{,}83 = 0{,}17$

$p_2 = 0{,}75$ $\quad$ p_2 failure $= 1 - 0{,}75 = 0{,}25$

$x_1 = 1$ if device p_1 fails, 0 otherwise

$x_2 = 1$ if device p_2 fails, 0 otherwise

$X = x_1 + x_2$

Linearity of Expectation

$$EX = EX_1 + EX_2 = P(p_1 \text{ fails}) + P(p_2 \text{ fails}) = 0{,}17 + 0{,}25 = 0{,}42$$

8) bearing cap $= 4$ bolts $\qquad$ 100 bolts $= 30$ Supply A $+ 70$ Supply B

a. $R_1 =$ Supply A $\qquad$ $R_2 =$ Supply B

$x_1 =$ bolts from S_1 $\qquad$ $x_2 =$ bolts from S_2

$$P(x_1 = 4 \cup x_2 = 4) = ?$$

hypergeometric Distribution

$$P(x_1 = 4 \cup x_2 = 4) = \frac{\binom{R_1}{x_1}\binom{N-R_1}{n-x_1}}{\binom{N}{n}} + \frac{\binom{R_2}{x_2}\binom{N-R_1}{n-x_1}}{\binom{N}{n}}$$

$$\frac{\binom{30}{4}\binom{100-30}{4-4}}{\binom{100}{4}} + \frac{\binom{70}{4}\binom{100-70}{4-4}}{\binom{100}{4}} \approx 0{,}24$$

$$P(x_1=3 \cup x_2=3) = \frac{\binom{30}{3}\binom{100-30}{4-3}}{\binom{100}{4}} + \frac{\binom{70}{3}\binom{100-70}{4-3}}{\binom{100}{4}}$$

$$\approx 0,49$$

9) $P(X=0) = \binom{n}{x} p^x (1-p)^{n-x}$

$X =$ n° patients LWBS, So $P(x \geq 1) > 0,9$

$1 - P(x=0)$ OR $P(x=0) \leq 0,1$

$p =$ Probability of success

$n =$ number of trials / experiments

$$p = \frac{242}{5047}$$

$$P(x=0) = \binom{n}{0}\left(\frac{242}{5047}\right)^0 \left(1-\frac{242}{5047}\right)^{n-0} \leq 0,1$$

$$\binom{n}{0} = 1 \times \left(\frac{242}{5047}\right)^0 = 1$$

$$\left(1-\frac{242}{5047}\right)^n \leq 0,1$$

$$n \ln\left(1-\frac{242}{5047}\right) \geq \ln(0.1)$$

$$n \geq \frac{\ln(0.1)}{\ln\left(1-\frac{242}{5047}\right)} \approx 46.86 \approx 47$$

10) a) $P(RA) = 0{,}92$ mean number of trials until failure

$q = 1 - P(RA) = 0{,}08$

$E(x) = \frac{1}{0{,}08} = 12{,}5$

mean geometric distribution $= \frac{1}{q}$

b) probability first failure occurs in k-th trial

$P(x = k) = p^{k-1} q$

$p(x = 10) = p^{9} q$

$P(x = 10) = 0{,}92^{9}\ 0{,}08 = 0{,}0378$

C) number of trials needed to collect k failures

$\frac{R}{q}$ R = number of failures

$E(y) = \frac{10}{0{,}08} = 125$

11) a) $P(x = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$ $\lambda = 10{,}5$

$$P(x < 15) = \sum_{x=1}^{14} \frac{e^{-10{,}5}\ 10{,}5^{x}}{x!} = 0{,}89$$

b) $\lambda = 10{,}5/6 = 1{,}75$

$$P(x=0) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-1{,}75}\,1{,}75^{0}}{0!} = 0{,}17$$

c) $\lambda = 5{,}25$

$$P(x=3) = \frac{e^{-5{,}25}\,5{,}25^{3}}{3!} = 0{,}13$$

12) a) PDF

$$\int_{10}^{15} \frac{(x-10)}{5h}\,dx + \int_{15}^{20} \frac{1}{h}\,dx + \int_{20}^{25} -\frac{(x-25)}{5h}\,dx = 1$$

$h = 10$

b) $$P(x<17{,}5) = \int_{10}^{15} \frac{x-10}{5\cdot 10}\,dx + \int_{15}^{17{,}5} \frac{1}{10}\,dx = 0{,}5$$

c) $$P(x<22) = \int_{10}^{15} \frac{x-10}{5\times 10}\,dx + \int_{15}^{20} \frac{1}{10}\,dx + \int_{20}^{22} -\frac{(x-25)}{5\times 10}\,dx$$

$= 0{,}91$

d) $P(X < x) = 0{,}95$

$$\int_{10}^{15} \frac{(x-10)}{5 \times 10} = 0{,}25 \qquad 0{,}25 + 0{,}5 = 0{,}75$$

$$\int_{15}^{20} \frac{1}{10}\, dx = 0{,}5$$

Therefore $P(X < x) = 0{,}95$ is between 20 and 25

$$\int_{20}^{23} -\frac{(x-25)}{5 \times 10}\, dx = 0{,}21 \qquad 0{,}75 + 0{,}21 = 0{,}96$$

So $X < 23$, because 23 is already 0,01 above

e) $E(X) = \int_{10}^{25} x f(x)\, dx$

$$E(X) = \int_{10}^{15} x \times \frac{x-10}{5 \times 10}\, dx + \int_{15}^{20} x \times \frac{1}{10}\, dx + \int_{20}^{25} x \times -\left(\frac{x-25}{5 \times 10}\right) dx$$

$E(X) = 17{,}5$

f) $V(X) = \int_{10}^{25} x^2 f(x)\,dx - (E(X))^2 = 10,42$

13) a) probability between 250 and 300

$P(250 < X < 300)$

$Z = \frac{X - \text{mean}}{\text{standard deviation}}$ → 250

$\sqrt{\text{variance}}$ → 391

$Z_2 = \frac{300 - 250}{\sqrt{391}}$

$Z_1 = \frac{250 - 250}{\sqrt{391}}$

$P(Z_1 < Z < Z_2) = \Phi(Z_2) - \Phi(Z_1) = 0,9943 - 0,5000$
$= 9,4943$

$\Phi(2,5286)$ using Z table $\Phi(2,53) = 9,9943$

$Z \leq$

$\Phi(0) = 0,5$

b) $P(X > 300) = 1 - \Phi\left(\frac{300-250}{\sqrt{391}}\right)$

$= 1 - 0{,}9943$

$= 0{,}0057$

c) $P(X > 300)^{10}$

$1 - \left(\Phi\left(\frac{300-250}{\sqrt{391}}\right)\right)^{10} = 0{,}0558$

14) a) Poisson process exponential distribution

$P(X \leq x) = 1 - e^{\lambda x}$

$P(X \leq 4{,}5) = 1 - e^{-\lambda \cdot 4{,}5}$

$\lambda = \frac{1}{6}$ = changes per day

$P(X \leq 4{,}5) = 1 - e^{-\frac{1}{6} \cdot 4{,}5} = 0{,}5277$

b) $P(X > 9{,}5) = e^{-\frac{1}{6} \cdot 9{,}5} = 0{,}2053$

c) $P(x > t) = 0{,}90$

$P(x > t) = e^{-\lambda t}$

$e^{-\frac{1}{6}t} = 0{,}90$

$\ln\left(e^{-\frac{1}{6}t}\right) = \ln(0{,}90)$

$-\frac{1}{6}t = \ln(0{,}90)$

$-\frac{1}{6}t = -0{,}105361$

$t = -6(-0{,}105361)$

$t = 0{,}63$

d) $P(x < 12{,}5 \mid x > 3)$

$P(x < 3 + 9{,}5 \mid x > 3) = P(x < 9{,}5)$

$P(x < 9{,}5) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{6} \cdot 9{,}5}$

$= 0{,}7947$