Four teams A, B, C, and D compete in a tournament. Teams A and B have the same chance of winning the tournament. Team C is twice as likely to win the tournament as team D. The probability that either team A or team C wins the tournament is 0.6. Find the probabilities of each team winning the tournament. State your answer as an integer between 0 and 9, making sure the answer is correctly rounded off.

$P(A) = 0.\boxed{2}$, $P(B) = 0.\boxed{2}$, $P(C) = 0.\boxed{4}$, $P(D) = 0.\boxed{2}$

Tournament Probabilities Problem Solution and Wolfram Alpha Documentation

Topics Covered in this Exercise:

* **Probability Basics**
* **Mutually Exclusive Events**
* **System of Linear Equations**
* **Algebraic Manipulation**

Problem Statement: Four teams A, B, C, and D compete in a tournament. * Teams A and B have the same chance of winning the tournament. * Team C is twice as likely to win the tournament as team D. * The probability that either team A or team C wins the tournament is 0.6.

Find the probabilities of each team winning the tournament.

Step 1: Define Variables

Let $P(A)$, $P(B)$, $P(C)$, and $P(D)$ be the probabilities that teams A, B, C, and D win the tournament, respectively.

Step 2: Formulate Equations based on the given information.

From the problem statement, we can write the following equations:

1. **Teams A and B have the same chance of winning**:

$$P(A) = P(B) \quad (Equation\ 1)$$

2. **Team C is twice as likely to win as team D**:

$$P(C) = 2 \times P(D) \quad (Equation\ 2)$$

3. **The probability that either team A or team C wins is 0.6**: Since A and C are mutually exclusive events (only one team can win), we can write:

$$P(A \text{ or } C) = P(A) + P(C) = 0.6 \quad (Equation\ 3)$$

4. **The sum of probabilities of all possible outcomes must be 1**:

$$P(A) + P(B) + P(C) + P(D) = 1 \quad (Equation\ 4)$$

Step 3: Solve the system of equations.

We have a system of 4 equations with 4 unknowns. Let's use substitution to solve it.

Substitute Equation 1 ($P(B) = P(A)$) into Equation 4:

$$P(A) + P(A) + P(C) + P(D) = 1$$

$$2P(A) + P(C) + P(D) = 1 \quad (Equation\ 5)$$

From Equation 3, we can express $P(C)$ in terms of $P(A)$:

$$P(C) = 0.6 - P(A) \quad (Equation\ 6)$$

Substitute Equation 6 into Equation 2:

$$0.6 - P(A) = 2 \times P(D)$$

$$P(D) = \frac{0.6 - P(A)}{2} \quad (Equation\ 7)$$

Now, substitute Equation 6 and Equation 7 into Equation 5:

$$2P(A) + (0.6 - P(A)) + \left(\frac{0.6 - P(A)}{2}\right) = 1$$

To eliminate the fraction, multiply the entire equation by 2:

$$2 \times (2P(A)) + 2 \times (0.6 - P(A)) + 2 \times \left(\frac{0.6 - P(A)}{2}\right) = 2 \times 1$$

$$4P(A) + 1.2 - 2P(A) + 0.6 - P(A) = 2$$

Combine like terms ($P(A)$ terms and constant terms):

$$(4P(A) - 2P(A) - P(A)) + (1.2 + 0.6) = 2$$

$$P(A) + 1.8 = 2$$

$$P(A) = 2 - 1.8$$

$$P(A) = 0.2$$

Step 4: Calculate the probabilities for each team.

Now that we have $P(A)$, we can find the other probabilities:

- **For P(A)**: $P(A) = 0.2$

- **For P(B)** (using Equation 1): $P(B) = P(A) = 0.2$
- **For P(C)** (using Equation 6): $P(C) = 0.6 - P(A) = 0.6 - 0.2 = 0.4$
- **For P(D)** (using Equation 2 or Equation 7): Using Equation 2: $P(C) = 2 \times P(D) \Longrightarrow 0.4 = 2 \times P(D) \Longrightarrow P(D) = \frac{0.4}{2} = 0.2$ (Alternatively, using Equation 7: $P(D) = \frac{0.6-P(A)}{2} = \frac{0.6-0.2}{2} = \frac{0.4}{2} = 0.2$)

Step 5: Verify the results.

Check if the sum of probabilities is 1: $P(A) + P(B) + P(C) + P(D) = 0.2 + 0.2 + 0.4 + 0.2 = 1.0$ (Correct)

Check if $P(A) + P(C) = 0.6$: $0.2 + 0.4 = 0.6$ (Correct)

The probabilities are: $P(A) = 0.2$ $P(B) = 0.2$ $P(C) = 0.4$ $P(D) = 0.2$

All answers are correctly rounded off to one decimal place, as implied by the input format (e.g., 0.2).

Wolfram Alpha Documentation

You can use Wolfram Alpha to solve systems of equations or verify individual steps.

- **Solving the system directly (conceptual input)**: `solve {Pa = Pb, Pc = 2*Pd, Pa + Pc = 0.6, Pa + Pb + Pc + Pd = 1}` (This would give you the values for Pa, Pb, Pc, Pd directly).
- **Verifying individual calculations (examples)**:
    - `0.6 - 0.2` (Result: 0.4)
    - `0.4 / 2` (Result: 0.2)
    - `0.2 + 0.2 + 0.4 + 0.2` (Result: 1.0)