A **real** number X is selected uniformly at random in the continuous interval [0,10]. (For example, X could be 3.87.)

Find the following probabilities. State all your inputs as integers between 0 and 99 such tthat your answers are irreducible fractions.

$P(2 \leq X \leq 5) =$

$\frac{3}{10}$

Check answer

$P(X \leq 2 \mid X \leq 5) =$

$\frac{2}{5}$

Check answer

$P(3 \leq X \leq 8 \mid X \geq 4)$

$\frac{2}{3}$

Uniform Distribution Probability Calculations and Wolfram Alpha Documentation

This document provides a detailed solution to calculating probabilities for a uniformly distributed continuous random variable, including explanations of the steps and how to use Wolfram Alpha (or similar symbolic computation tools) to verify or perform the calculations.

Topics Covered in this Exercise:

* **Uniform Continuous Distribution**
* **Probability for Continuous Random Variables**
* **Conditional Probability**

Problem Statement: A real number X is selected uniformly at random in the continuous interval $[0,10]$. Find the following probabilities. State all your inputs as integers between 0 and 99 such that your answers are irreducible fractions.

A continuous random variable X is uniformly distributed over an interval $[a, b]$ if its probability density function (PDF) is given by:

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

For this problem, the interval is $[0,10]$, so $a = 0$ and $b = 10$. Thus, the PDF is $f(x) = \frac{1}{10-0} = \frac{1}{10}$ for $0 \leq x \leq 10$, and 0 otherwise.

The probability of X falling within a subinterval $[c, d]$ (where $a \leq c \leq d \leq b$) is given by:

$$P(c \leq X \leq d) = \int_c^d f(x)\, dx = \int_c^d \frac{1}{b-a}\, dx = \frac{d-c}{b-a}$$

For this problem, $b - a = 10 - 0 = 10$. So, $P(c \leq X \leq d) = \frac{d-c}{10}$.

Part a: Find $P(2 < X < 5)$

Using the formula for probability in a uniform distribution: Here, $c = 2$ and $d = 5$.

$$P(2 < X < 5) = \frac{5-2}{10} = \frac{3}{10}$$

The fraction $\frac{3}{10}$ is irreducible.

- **Wolfram Alpha Input**: `probability X > 2 and X < 5 if X is uniformly distributed from 0 to 10` (Result: 3/10)

Part b: Find $P(X \leq 2 \mid X \leq 5)$

This is a conditional probability problem. The formula for conditional probability is:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

In this case, let A be the event $X \leq 2$ and B be the event $X \leq 5$.

Step 1: Find $P(A \cap B)$

The event $A \cap B$ means $X \leq 2$ AND $X \leq 5$. The intersection of these two conditions is simply $X \leq 2$. So, $P(A \cap B) = P(X \leq 2)$. Using the uniform distribution probability formula for $P(X \leq 2)$: Here, $c = 0$ and $d = 2$.

$$P(X \leq 2) = \frac{2-0}{10} = \frac{2}{10}$$

- **Wolfram Alpha Input**: `probability X <= 2 if X is uniformly distributed from 0 to 10` (Result: 1/5, which is 2/10)

Step 2: Find $P(B)$

The event B is $X \leq 5$. Using the uniform distribution probability formula for $P(X \leq 5)$: Here, $c = 0$ and $d = 5$.

$$P(X \leq 5) = \frac{5 - 0}{10} = \frac{5}{10}$$

- **Wolfram Alpha Input**: `probability X <= 5 if X is uniformly distributed from 0 to 10` (Result: 1/2, which is 5/10)

Step 3: Calculate $P(X \leq 2 \mid X \leq 5)$

$$P(X \leq 2 \mid X \leq 5) = \frac{P(X \leq 2)}{P(X \leq 5)} = \frac{\frac{2}{10}}{\frac{5}{10}}$$

$$= \frac{2}{10} \times \frac{10}{5} = \frac{2}{5}$$

The fraction $\frac{2}{5}$ is irreducible.

- **Wolfram Alpha Input**: `conditional probability X <= 2 given X <= 5 if X is uniformly distributed from 0 to 10` (Result: 2/5)

Part c: Find $P(3 \leq X \leq 8 \mid X \geq 4)$

This is another conditional probability problem. Let A be the event $3 \leq X \leq 8$ and B be the event $X \geq 4$.

Step 1: Find $P(A \cap B)$

The event $A \cap B$ means $(3 \leq X \leq 8)$ AND $(X \geq 4)$. The intersection of these two conditions is $4 \leq X \leq 8$. So, $P(A \cap B) = P(4 \leq X \leq 8)$. Using the uniform distribution probability formula for $P(4 \leq X \leq 8)$: Here, $c = 4$ and $d = 8$.

$$P(4 \leq X \leq 8) = \frac{8 - 4}{10} = \frac{4}{10}$$

- **Wolfram Alpha Input**: `probability X >= 4 and X <= 8 if X is uniformly distributed from 0 to 10` (Result: 2/5, which is 4/10)

Step 2: Find $P(B)$

The event B is $X \geq 4$. Given the interval is $[0,10]$, $X \geq 4$ means $4 \leq X \leq 10$. Using the uniform distribution probability formula for $P(4 \leq X \leq 10)$: Here, $c = 4$ and $d = 10$.

$$P(X \geq 4) = \frac{10 - 4}{10} = \frac{6}{10}$$

- **Wolfram Alpha Input**: `probability X >= 4 if X is uniformly distributed from 0 to 10` (Result: 3/5, which is 6/10)

Step 3: Calculate $P(3 \leq X \leq 8 \mid X \geq 4)$

$$P(3 \leq X \leq 8 \mid X \geq 4) = \frac{P(4 \leq X \leq 8)}{P(X \geq 4)} = \frac{\frac{4}{10}}{\frac{6}{10}}$$

$$= \frac{4}{10} \times \frac{10}{6} = \frac{4}{6} = \frac{2}{3}$$

The fraction $\frac{2}{3}$ is irreducible.

- **Wolfram Alpha Input**: `conditional probability 3 <= X <= 8 given X >= 4 if X is uniformly distributed from 0 to 10` (Result: 2/3)