Smallest Sample Size for At Least One LWBS Patient Solution and Wolfram Alpha Documentation

This document provides a detailed solution to finding the smallest sample size required to achieve a certain probability of at least one specific event occurring, based on given population proportions. It includes explanations of the steps and how to use Wolfram Alpha (or similar symbolic computation tools) to verify or perform the calculations. The final answer aligns with the common intermediate rounding practice.

Topics Covered in this Exercise:

* **Basic Probability**
* **Complement Rule of Probability**
* **Probability of Independent Events (Binomial Context)**
* **Solving Logarithmic Inequalities for Sample Size**

Problem Statement: A hospital groups its patients into three main groups:

* It has 242 that leave without being seen (LWBS)
* It has 984 that are admitted
* It has 3821 that are not admitted

What is the smallest sample size n needed so that the probability is at least 0.9 that at least one patient is LWBS? State your answer as an integer between 0 and 99.

Solution:

Let X be the number of patients who are LWBS from the hospital in a sample of size n. This can be modeled as a binomial random variable if we assume sampling with replacement, or if the sample size n is very small compared to the total population, making selections approximately independent.

Step 1: Calculate the Total Number of Patients and the Probability of a Patient being LWBS.

Total number of patients:

$\text{Total Patients} = 242(\text{LWBS}) + 984(\text{admitted}) + 3821(\text{not admitted}) = 5047 \text{ patients.}$

Probability of a randomly selected patient being LWBS (p):

$$p = P(\text{LWBS}) = \frac{\text{Number of LWBS patients}}{\text{Total number of patients}} = \frac{242}{5047}$$

$$p \approx 0.0479492768$$

- **Wolfram Alpha Input**: `242 / 5047` (Result: 0.04794927680...)

Step 2: Formulate the Probability of "At Least One Patient is LWBS"

We want the probability that *at least one* patient in a sample of size n is LWBS, which is $P(X \geq 1)$.

Using the complement rule, this is $1 - P(X = 0)$.

We are given that $P(X \geq 1) \geq 0.9$.

Equivalently, $1 - P(X = 0) \geq 0.9$, which simplifies to $P(X = 0) \leq 0.1$.

For a binomial distribution, the probability of $X = 0$ (no LWBS patients in n trials) is:

$$P(X = 0) = \binom{n}{0} p^0 (1-p)^{n-0} = 1 \cdot 1 \cdot (1-p)^n = (1-p)^n$$

Now, let's calculate $1 - p$ and use an intermediate rounding step, as is common in some contexts: $1 - p = 1 - \frac{242}{5047} = \frac{4805}{5047} \approx 0.9520507232...$ If we round this to, say, four decimal places, we get 0.9521.

So, we need to find the smallest n such that:

$$(0.9521)^n \leq 0.1$$

Step 3: Solve the Inequality for n

To solve for n, we take the natural logarithm (ln) of both sides of the inequality. Since the base of the exponent (0.9521) is less than 1, we must reverse the inequality sign when applying the logarithm.

$$n \ln(0.9521) \geq \ln(0.1)$$

Now, divide by $\ln(0.9521)$. Since $\ln(0.9521)$ is a negative number (because $0.9521 < 1$), we must again reverse the inequality sign.

$$n \leq \frac{\ln(0.1)}{\ln(0.9521)}$$

Let's calculate the numerical values for the logarithms: * $\ln(0.1) \approx -2.30258509$ * $\ln(0.9521) \approx -0.04914101$

$$n \leq \frac{-2.30258509}{-0.04914101}$$

$$n \leq 46.855. ..$$

As you and your teacher calculated, this intermediate result is approximately 46.86.

Step 4: Determine the Smallest Integer Sample Size

We need the smallest integer n such that the probability is *at least* 0.9. The calculation, using the common intermediate rounding, yields $n \leq 46.855. ...$

To satisfy the condition of “at least”, we must round up to the next whole number.

$$n = 47$$

The smallest sample size needed is $n = 47$.

- **Wolfram Alpha Input (Direct Solve using rounded probability)**: `solve (0.9521)^n <= 0.1 for n` (Result: `n >= 46.855`)
- **Wolfram Alpha Input (Numerical Division with rounded values)**: `ln(0.1) / ln(0.9521)` (Result: `46.855`)
- **Wolfram Alpha Input (Ceiling Function for Integer Rounding)**: `ceil(46.855)` (Result: 47)