# Recap

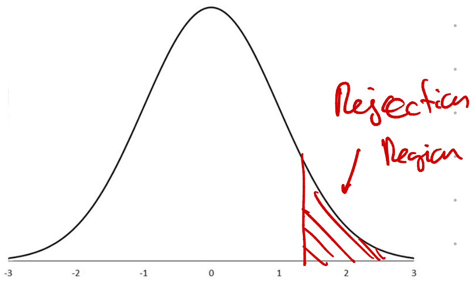
## Example: $\bar{x} = 998$, $n = 30$, $\sigma = 5.5$ (normal)


Two: Tailed:

$H_0: \mu = 1000$

$H_1: \mu \neq 1000$

$Z_{1-\alpha/2} = 1.96 = Z_{crit}$

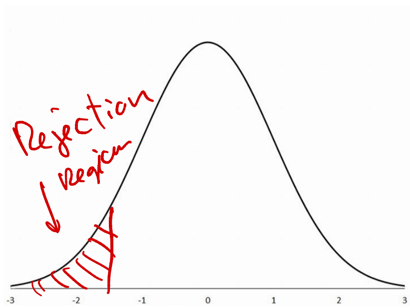



Right tailed:

$H_0: \mu \leq 1000$

$H_1: \mu > 1000$

$Z_{1-\alpha} = 1.64$




Left-tailed:

$H_0: \mu \geq 1000$

$H_1: \mu < 1000$

$Z_{1-\alpha} = -1.64$

Based on $\alpha$

## Test statistic:

$$Z_{test} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{998 - 1000}{5.5/\sqrt{30}} = -1.99$$

Rejection Criteria:

$|Z_{crit}| < |Z_{test}|$ Reject

else fail to reject ($H_0$ assumed to be true)

## P-Value:

The probability of obtaining an observation as extreme or more than the computed estimate ($\bar{x}$), <u>given</u> that $H_0$ is true.

P-value is a probability:

$$\text{p-value} = 2 \cdot (1 - P(Z \leq |Z_{test}|)) \quad (\text{two-tails})$$

only if two tail.

$$= 2 \cdot (1 - P(Z < 1.99)) = \underline{\underline{0.047}}$$

<u>Rejection Criteria:</u>

Reject if p-value $\leq \alpha$

<u>Types of tests:</u>

1) <u>Independent</u> samples: Test difference in two groups.

2) Paired test : Test compares <u>same</u> group at different times

3) one sample test : test one sample against hypotesised value

4) Two variables Test independence beteen two Categorical variable.

What can we test:

<u>one sample:</u>

→ mean
→ proportion
→ Variance / St.dev.

Two samples:

compare means of distinct groups

— " — same group

compare two proportions

compare two var/st.dev.

Two variables:

Compare independence between two categorial variables.

Two independent samples:

H1 : Females are more intelligent than non-females

H2 : Non-females are more intelligent than females

H3 : Non females and females differ in intelligence

$H1_0: \mu_f \leq \mu_m$ $H2_0: \mu_f \geq \mu_m$ $H3_0: \mu_f = \mu_m$

$H1_1\ \mu_f > \mu_m$ $H2_1: \mu_f < \mu_m$ $H3_1: \mu_f \neq \mu_m$

Paired-Test:

| | $W_1$ | $W_2$ | diff |
|---|---|---|---|
| Alice | 78 | 76 | 2 |
| Bob | 72 | 72 | 0 |
| Parth | 79 | 70 | 9 |
| Eve | 84 | 85 | -1 |

$H_0: \mu_D = 0$ weight difference

$H_0: \mu_D \geq 0$ weight gain

$H_0: \mu_D \leq 0$ weight loss

$\bar{D}$ difference in samples

$\Delta_0$ is hypothesised difference

# Test for Independence:

$P(A \cap B) = P(A) \cdot P(B)$

Discrete pmf

| | $y=0$ | $y=1$ | $y=2$ | $f(x)$ |
|---|---|---|---|---|
| $X=0$ | $a_{11}$ | $a_{21}$ | | $f_x(0)$ |
| $x=1$ | $a_{12}$ | $a_{22}$ | | $f_x(1)$ |
| $X=2$ | | | $a_{33}$ | $f_x(2)$ |
| $f(y)$ | $f_y(0)$ | $f_y(1)$ | $f_y(2)$ | |

observed values.

Based on marginals we can computed expected values ($a_{11}$ given $H_0$ is true)

$$\left.\begin{aligned} a_{11} &= f_x(0) \cdot f_y(0) \\ a_{21} &= f_x(0) \cdot f_y(1) \\ &\vdots \\ a_{33} &= f_x(2) \cdot f_y(2) \end{aligned}\right\}$$

$H_0$: X and Y are independent

$H_1$: X and Y are dependent.

$$\chi_0^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Observed

Expected