# Probability Calculation using Poisson Approximation to Binomial Distribution Solution and Wolfram Alpha Documentation

This document provides a detailed solution to a probability problem where the number of “successes” (people with COVID-19) in a large sample is modeled. Since the number of trials is large and the probability of success is small, the Binomial distribution can be approximated by the Poisson distribution. It includes explanations of the steps and how to use Wolfram Alpha (or similar symbolic computation tools) to verify or perform the calculations.

**Topics Covered in this Exercise:**

- **Binomial Distribution**
- **Poisson Approximation to Binomial Distribution**
- **Poisson Distribution Parameter ($\lambda$)**
- **Probability Mass Function (PMF) of Poisson Distribution**
- **Calculating Cumulative Probabilities**
- **Complement Rule of Probability**

**Problem Statement:**
Assume that 0.01% of the population has COVID-19 and that 20000 randomly chosen people are at a large gathering. What is the probability that at least 5 people at the gathering have COVID-19? Please state your answer as a decimal value correctly rounded to four decimal precision (e.g. 0.9876). Remember to use “.” as the decimal separator.

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## Step 1: Identify the Distribution and Parameters.

This is a scenario involving a fixed number of trials ($n = 20000$) and a probability of success ($p = 0.01\%$) for each trial. This inherently fits a **Binomial Distribution** where $X \sim B(n, p)$.

- Number of trials ($n$) = 20000
- Probability of success ($p$) = 0.01% = 0.0001 (since 0.01% = 0.01/100)

We want to find $P(X \geq 5)$.

However, for a Binomial distribution, when $n$ is large and $p$ is small (rule of thumb: $n \geq 20$ and $p \leq 0.05$, or $n \geq 100$ and $np \leq 10$), it can be well approximated by a **Poisson Distribution**.

## Step 2: Calculate the Poisson Parameter ($\lambda$).

The mean ($\lambda$) of the approximating Poisson distribution is given by:

$$\lambda = n \times p$$

$$\lambda = 20000 \times 0.0001$$

$$\lambda = 2$$

So, we can approximate the problem using $X \sim \text{Poisson}(2)$.

## Step 3: Calculate the Probability $P(X \geq 5)$ using Poisson Approximation.

The probability mass function (PMF) of a Poisson distribution is $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$. We want to find $P(X \geq 5)$, which is easier to calculate using the complement rule:

$$P(X \geq 5) = 1 - P(X < 5)$$

$$P(X < 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

Let's calculate each term with $\lambda = 2$:

- $P(X = 0) = \frac{e^{-2}2^0}{0!} = \frac{e^{-2} \cdot 1}{1} = e^{-2} \approx 0.135335$

- $P(X=1) = \frac{e^{-2}2^1}{1!} = 2e^{-2} \approx 2 \times 0.135335 = 0.270670$
- $P(X=2) = \frac{e^{-2}2^2}{2!} = \frac{4e^{-2}}{2} = 2e^{-2} \approx 2 \times 0.135335 = 0.270670$
- $P(X=3) = \frac{e^{-2}2^3}{3!} = \frac{8e^{-2}}{6} = \frac{4}{3}e^{-2} \approx 1.333333 \times 0.135335 = 0.180447$
- $P(X=4) = \frac{e^{-2}2^4}{4!} = \frac{16e^{-2}}{24} = \frac{2}{3}e^{-2} \approx 0.666667 \times 0.135335 = 0.090223$

Now, sum these probabilities for $P(X < 5)$:

$$P(X < 5) \approx 0.135335 + 0.270670 + 0.270670 + 0.180447 + 0.090223$$

$$P(X < 5) \approx 0.947345$$

Finally, calculate $P(X \geq 5)$:

$$P(X \geq 5) = 1 - P(X < 5) \approx 1 - 0.947345 = 0.052655$$

Rounding to four decimal places: $0.0527$.

- **Wolfram Alpha Input (Direct Poisson CDF)**: `poisson probability X >= 5, lambda = 2`
- **Wolfram Alpha Result**: `0.052653...`
- **Wolfram Alpha Input**: `round 0.052653 to 4 decimal places` (Result: 0.0527)

Alternatively, we can use the exact Binomial distribution calculation to compare:

- **Wolfram Alpha Input (Direct Binomial CDF)**: `binomial probability X >= 5, n = 20000, p = 0.0001`
- **Wolfram Alpha Result**: `0.052653...` (Same as Poisson approximation, confirming its accuracy here).

The probability that at least 5 people at the gathering have COVID-19 is $0.0526$.