Regression Hypothesis Test

We are testing:

- $H_0 : \beta_1 = 0$
- $H_1 : \beta_1 \neq 0$

Given:

- $\hat{\beta}_1 = 14.947$
- $n = 20$
- $S_{xx} = 0.68088$
- $\sigma^2 = 1.18$

(a) Test Statistic T_0

The formula for the test statistic is:

$$T_0 = \frac{\hat{\beta}_1 - 0}{\sqrt{\frac{\sigma^2}{S_{xx}}}}$$

Plug in the values:

$$T_0 = \frac{14.947}{\sqrt{\frac{1.18}{0.68088}}} = \frac{14.947}{\sqrt{1.7333}} = \frac{14.947}{1.3159} \approx 11.3605$$

Answer:

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T₀ = 11.3605
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(b) Critical Value T_{crit}

Significance level: $\alpha = 0.01$
Degrees of freedom: $df = n - 2 = 20 - 2 = 18$

From the **t-distribution table**, for a **two-tailed test** at $\alpha = 0.01$:

$$T_{\text{crit}} = t_{0.005,18} \approx 2.878$$

Answer:

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T_crit = 2.878
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© 95% Confidence Interval for the Slope

The general formula is:

$$\hat{\beta}_1 \pm t_{\alpha/2,df} \cdot \sqrt{\frac{\sigma^2}{S_{xx}}}$$

We use:

- $t_{0.025,18} \approx 2.101$
- $\sqrt{\frac{1.18}{0.68088}} \approx 1.3159$

So:

$$\text{Margin of error} = 2.101 \cdot 1.3159 \approx 2.7645$$

Confidence interval:

$$14.947 \pm 2.7645 = [12.1825,\ 17.7115]$$

Answer:

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12.1825 ≤ β₁ ≤ 17.7115
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