# Power-Recycled Fabry-Perot Michelson Sensing Matrix

# Anarytical Calculation v.s. Finesse

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2020 July 31st

# Power Recycling Cavity Parameters

| Parameters | Design Values | Simulation |
|---|---|---|
| Power Recycling Gain | $G = \frac{t_R^2}{(1 - r_R r_{com})^2} = 32.9$<br>rcom: complex reflectivity of the FPMI part | 33.9 |
| PRC Length | 66.591 | |
| PRC FSR | 2.25 MHz | |
| PRC Finesse for carrier | 55 | |
| PRC cutoff frequency | 20 kHz | |

# PRFPMI Sensing Matrix (Analytical Calculation)

| | Φ- アーム差動 DARM | Φ- マイケルソン MICH | Φ+ アーム同相 CARM | Φp PRC長 PRCL |
|---|---|---|---|---|
| AS Q1 | $g_0 g_1 \lvert r'_{\rm reso}\rvert r_{\rm anti} \sin\alpha$ | $g_0 g_1 r_{\rm reso} r_{\rm anti} \sin\alpha$ | 0 | 0 |
| REFL I1 | 0 | 0 | $-g_0^2 \lvert r'_{\rm reso}\rvert r_{\rm rec1} + g_1^2 r_{\rm rec0} \lvert r'_{\rm anti}\rvert \cos\alpha$ | $-g_0^2 r_{\rm reso} r_{\rm rec1} - g_1^2 r_{\rm rec0} r_{\rm anti} \cos\alpha$ |
| REFL Q1 | $g_1^2 r_{\rm rec0} \lvert r'_{\rm anti}\rvert \sin\alpha$ | $-g_1^2 r_{\rm rec0} r_{\rm anti} \sin\alpha$ | 0 | 0 |
| POP I1 | 0 | 0 | $g_0 g_1 \frac{r_{\rm P}^2}{t_{\rm R}} (g_0 \lvert r'_{\rm reso}\rvert r_{\rm anti} + g_1 r_{\rm reso} \lvert r'_{\rm anti}\rvert) \cos\alpha$ | $g_0 g_1 \frac{r_{\rm P}^2}{t_{\rm R}} (g_0 - g_1) r_{\rm reso} r_{\rm anti} \cos\alpha$ |
| POP Q1 | | | | |

$$g_0 = \frac{t_{\rm R}}{1 - r_{\rm R} t_{\rm P}^2 r_{\rm reso}}$$

$$g_1 = \frac{t_{\rm R}}{1 + r_{\rm R} t_{\rm P}^2 r_{\rm anti} \cos\alpha}$$

$$r_{\rm rec0} = -r_{\rm R} + \frac{t_{\rm R}^2 t_{\rm P}^2 r_{\rm reso}}{1 - r_{\rm R} t_{\rm P}^2 r_{\rm reso}}$$

$$r_{\rm rec1} = -r_{\rm R} - \frac{t_{\rm R}^2 t_{\rm P}^2 r_{\rm anti} \cos\alpha}{1 + r_{\rm R} t_{\rm P}^2 r_{\rm anti} \cos\alpha}$$

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# PRFPMI Sensing Matrix (Analytical Calculation for f1)

| | Φ- アーム差動 DARM | Φ- マイケルソン MICH | Φ+ アーム同相 CARM | Φp PRC長 PRCL |
|---|---|---|---|---|
| AS Q1 | -2670 | -2.67 | 0 | 0 |
| REFL I1 | 0 | 0 | 30200 | 30.3 |
| REFL Q1 | 0.0010 | 0.203 | 0 | 0 |
| POP I1 | 0 | 0 | -10 | -0.0093 |
| POP Q1 | | | | |

$$g_0 = \frac{t_R}{1 - r_R t_P^2 r_{reso}}$$

$$g_1 = \frac{t_R}{1 + r_R t_P^2 r_{anti} \cos\alpha}$$

g0=5.8 r_reso = 0.998 r_anti ~ -1 r_rec0 ~ 0.89 cos α ~ 0.38

g1=0.50 r_reso’ = -996i r_anti’ ~ 0.001i r_rec1 ~ -0.89 sin α ~ 0.92

# PRFPMI Sensing Matrix (demodulated at 16.88MHz)

Demodulation phase
AS: Q = 213
REFL: I = 0, Q = 90
POP: I = 119 , Q=209

| | Φ- アーム差動 DARM | Φ- マイケルソン MICH | Φ+ アーム同相 CARM | Φp PRC長 PRCL |
|---|---|---|---|---|
| AS Q1 | -13 | -0.013 | 0 | 0 |
| REFL I1 | -145 | -0.14 | -290 | 0.3 |
| REFL Q1 | -0.0028 | 0.0010 | 0 | 0 |
| POP I1 | -0.05 | -4.5e-5 | -0.1 | 9e-5 |
| POP Q1 | 0 | -1e-5 | 0 | 0 |

Finesse diff gives wrong answer

Sign doesn't agree

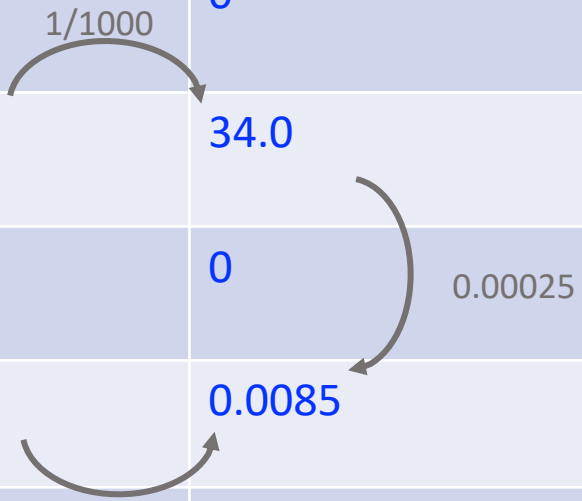
Sign doesn't agree

Error signals are 0 and correct
but diff results are wrong

Factor of 2 larger than the analyic formulae

# PRFPMI Sensing Matrix (Analytical Calculation for f2)

| | Φ- アーム差動 DARM | Φ- マイケルソン MICH | Φ+ アーム同相 CARM | Φp PRC長 PRCL |
|---|---|---|---|---|
| AS Q2 | -0.045 | -4.5e-5 | 0 | 0 |
| REFL I2 | 0 | 0 | 33900 | 34.0 |
| REFL Q2 | 1.1e-9 | 1.1e-6 | 0 | 0 |
| POP I2 | 0 | 0 | 8.7 | 0.0085 |
| POP Q2 | | | | |




g0=5.8
g1=0.16

r_reso = 0.998
r_reso' = -996i

r_anti ~ -1
r_anti' ~ 0.0001i

r_rec0 ~ 0.89
r_rec1 ~ -1

cos α ~ -1
sin α ~ 4.8e-5 ~ 0

# PRFPMI Sensing Matrix (demodulated at 45MHz)

Demodulation phase
AS: Q = 178
REFL: I = 0, Q = 90
POP: I = 78, Q= 168

| | Φ- アーム差動 DARM | Φ- マイケルソン MICH | Φ+ アーム同相 CARM | Φp PRC長 PRCL |
|---|---|---|---|---|
| AS Q2 | 0.008 | 8.1e-6 | 0 | 0 |
| REFL I2 | 160 | 0.00055 | 318 | -0.0012 |
| REFL Q2 | 0 | -2.1e-5 | 0 | 0 |
| POP I2 | 1.6 | -7e-5 | 3.2 | 0.00014 |
| POP Q2 | 0 | 1.5e-7 | 0 | 0 |

-3.8e-6
0.12
4.4e-5

Finesse diff gives wrong answer

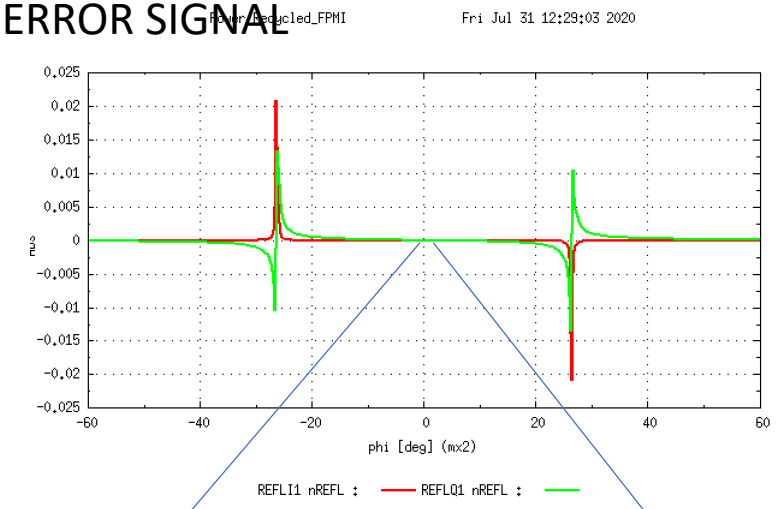
Sign doesn't agree

Error signals are 0 and correct
but diff results are wrong

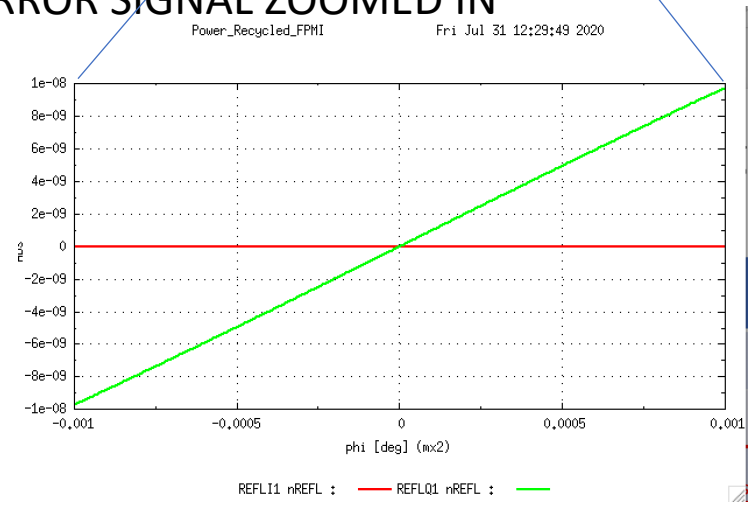
Totally disagreed?
Checked that the assumption r_SB ~= r_anti was not the problem

# DARM diff at REFL/POP something wrong

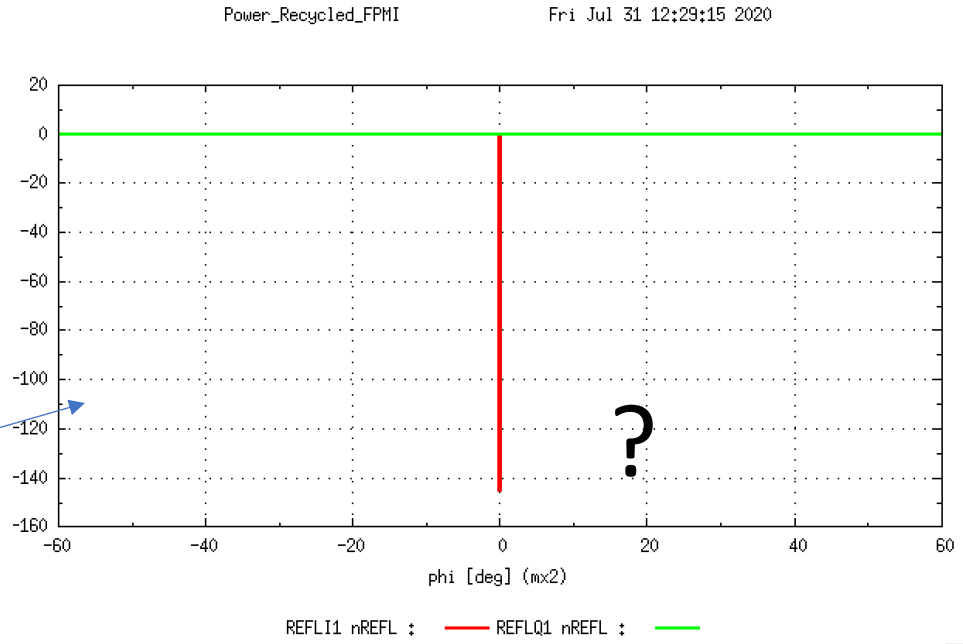
ERROR SIGNAL




ERROR SIGNAL ZOOMED IN




DIFF (SLOPE)




Changing deriv_h in kat.ini doesn't solve