I. TEORIJA RELATIVNOSTI

str. 5 / 2

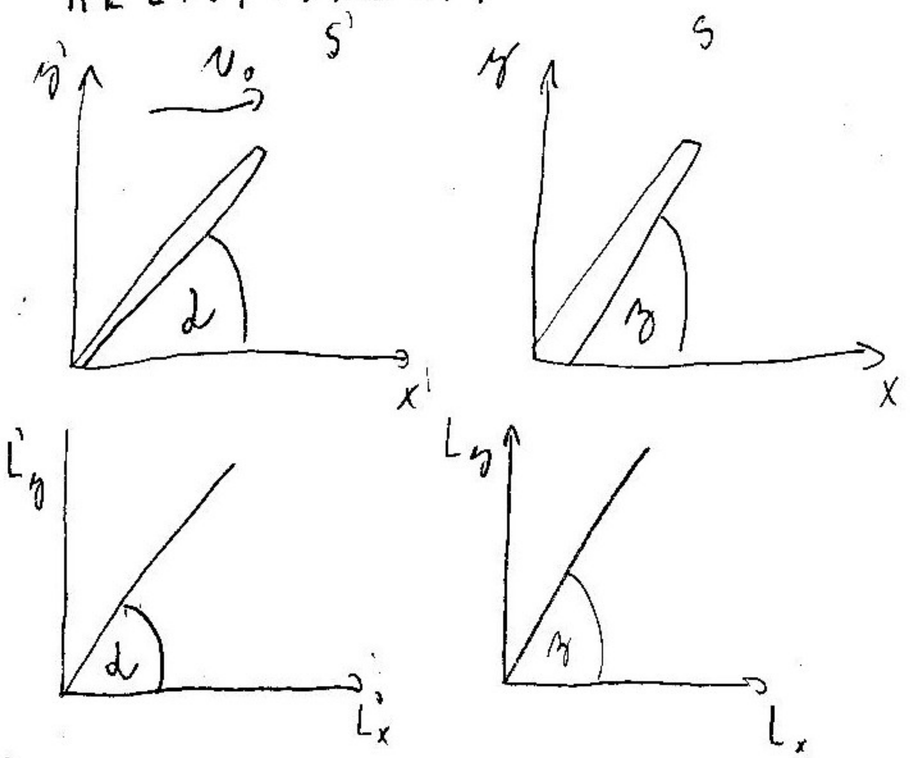
$L_0 = 1\,m$

$v_0 = 0,8\, c_0$

$\alpha = 30^\circ$

a) $L = ?$

b) $\beta = ?$



$\rightsquigarrow tg\,\beta = \dfrac{L_y}{L_x}$, $L_y = L_y'$

$L_x' = \dfrac{1}{\gamma} L_x =$

$\rightsquigarrow tg\,\alpha = \dfrac{L_y'}{L_x'} = \dfrac{L_y' \cdot \gamma}{L_x} = \dfrac{L_y \cdot \gamma}{L_x} = tg\,\beta \cdot \gamma \Rightarrow$

$\rightsquigarrow tg\,\beta = \dfrac{tg\,\alpha}{\gamma} = tg\,\alpha \sqrt{1-\beta^2} = tg\,\alpha \cdot 0,6 = tg\,30 \cdot 0,6 = 0,346$

$\beta = \dfrac{v}{c} = 0,8$

$\beta^2 = 0,64$

$\beta = \underline{\underline{19,1^\circ}}$ ✓

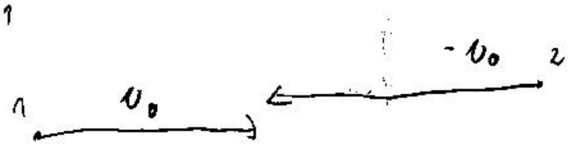
$\rightsquigarrow L_0' = \sqrt{L_x'^2 + L_y'^2} = \sqrt{\dfrac{L_x^2}{\gamma^2} + L_y^2} = \sqrt{L_0^2\left(\dfrac{\cos^2\beta}{\gamma^2} + \sin^2\beta\right)} = \underline{\underline{0,65\,m}}$ ✓

$L_x = L_0 \cos\beta$

$L_y = L_0 \sin\beta$

3) $v = 0{,}99\, v_0$

v_{21}



$$v_x' = \frac{v_x - v_0}{1 - \frac{v_0 \cdot v_x}{c^2}}$$

$$v_{21} = \frac{-v - v_0}{1 + \frac{v_0 \cdot v_x}{c^2}} = \frac{-2 v_0}{1 + \frac{v_0^2}{c^2}} = 0{,}99995\, c$$

$$v_{12} = \frac{2v}{1 + \frac{v_0^2}{c^2}} = \underline{\underline{0{,}99995\, c}}$$

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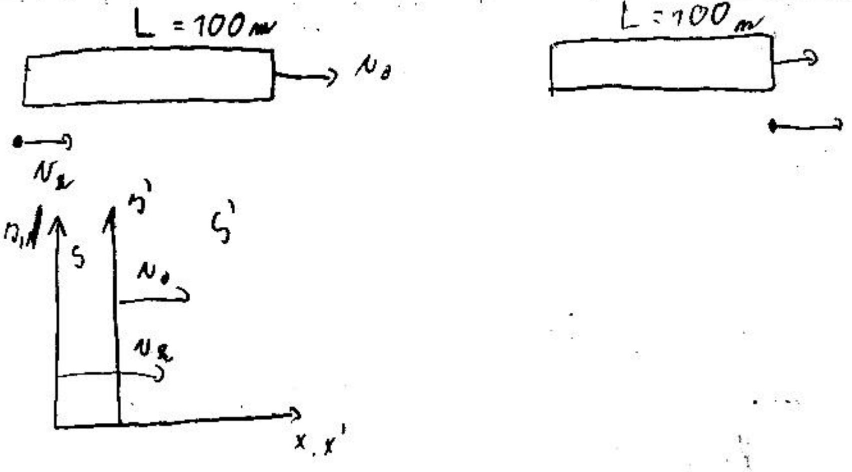
4) $L = 100\, m$

$v_0 = 0{,}5\, c_0$

$v_k = 0{,}9\, c_0$

$t' = ?$



1) kugla ustrijeljena $S(0,0)$, $S'(0,0)$

2) kugla doleti ludje $S(x,t)$, $S'(x', t')$

$x = v_k \cdot t$ $\qquad x' = L_0$

Uvrstimo Lorentzove transf.

$\to x' = \gamma(x - v_0 \cdot t)$ $\qquad \to t = \gamma\left(t' + \frac{v_0 \cdot x'}{c^2}\right)$

$\to x = \gamma(x' + v_0 \cdot t') = \gamma(L_0 + v_0 \cdot t')$

$\to v_k = \frac{x}{t} = \frac{\gamma(L_0 + v_0 \cdot t')}{\gamma\left(t' + \frac{v_0 \cdot L_0}{c^2}\right)} \Rightarrow v_k \cdot t' + \frac{v_k \cdot v_0 \cdot L_0}{c^2} = L_0 + v_0 \cdot t'$

$\to v_k \cdot t' - v_0 \cdot t' = L_0 - \frac{v_k \cdot v_0 \cdot L_0}{c^2}$

$$t' = \frac{L_0\left(1 - \frac{v_k \cdot v_0}{c^2}\right)}{v_k - v_0} = \frac{100\, m\left(1 - \frac{0{,}5 \cdot 0{,}9\, c^2}{c^2}\right)}{0{,}4\, c} = \underline{\underline{46\, \mu s}}$$

5) $v_0 = 0,6c$

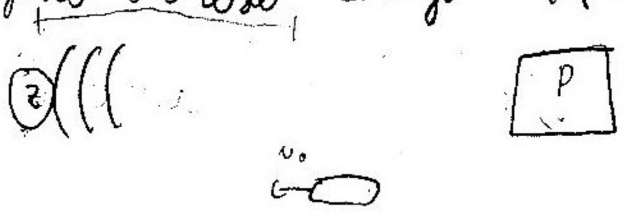
$t_s = 1250 s$

$t = ?$

$t' = ?$

1) Postaja pošlje signal na Zemljo $S(0,0)$, $S'(0,0)$

2) Signal doseže Zemljo $S(0,t_s)$ $S'(L,t')$



3) Ladja prispe do Zemlje $S(\;)$ $S'(\;,\;)$

LORENTZ

$L = c \cdot t_s = 3,75 \cdot 10^{11} m$

$t_z = \frac{x}{v_0} = \frac{L}{v_0} = \underline{\underline{2083 s}}$ ✓ $\Rightarrow$ za opazovalca na Zemlji

$t' = \gamma\left(t - \frac{v_0 \cdot L}{c^2}\right) = \underbrace{\frac{1}{\sqrt{1-(\frac{v}{c})^2}}}_{1,25}\left(t - \frac{v_0 L}{c^2}\right) = 1,25 \cdot t\left(1 - \frac{v_0}{c^2}\right) = \underline{\underline{1666 s}}$ ✓

Za opazovalca na ladji

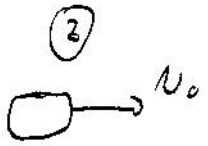
6) $v_0 = 0{,}8\, c_0$

$L = 6{,}66 \cdot 10^8\ km$

$t' = ?$

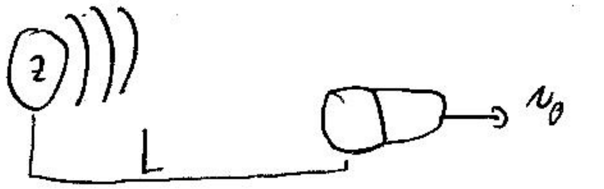
$t = ?$

1) Ladja se oddaljuje $\quad S(0,0) \quad S'(0,0)$

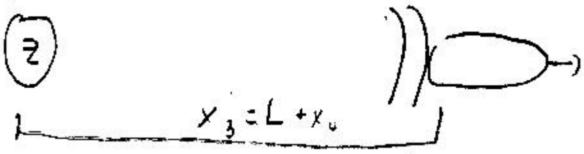


2) Zemlja pošlje signal do ladje $\quad (S(0,t_2),\ S'(x'_2, t'_2))$

$x'_2 = L$



3) Signal pride do ladje: $\quad S(x_3, t_3) \quad S'(0, t'_3)$



LORENTZ

$t_2 = \frac{L}{v_0} = 2750\,s \qquad t_3 = \frac{L + x_4}{v_0} \Rightarrow t_3 - t_2 = t_4 =$

$x_3 = c(t_3 - t_2)$

$c(t_3 - t_2) = L + v_0(t_3 - t_2)$

$c t_3 - c t_2 = L + v_0 t_3 - v_0 t_2$

$t_3 (c - v_0) = L - v_0 t_2 + c\, t_2$

$t_3 = \frac{L - v_0 t_2 + c\, t_2}{c - v_0} = \frac{L + t_2(c - v_0)}{c - v_0} = 13850\,s$

$\rightsquigarrow t_4 = 13850\,s - 2750\,s = \underline{11100\,s} \Rightarrow$ za opazovalca

$c\, t'_3 = \gamma(x_3 - v_0 \cdot t_3) = \gamma(c \cdot t_3 - v_0 t_3) \Rightarrow t'_3 = \gamma(t_3 - \beta \cdot t_3) = \underline{4617\,s}$

$t'_4 = \gamma(t_4 - \beta \cdot t_4) = 9130\,s$

7) $t_2 = 3$ mesece

$L = 0{,}2$ s. l.

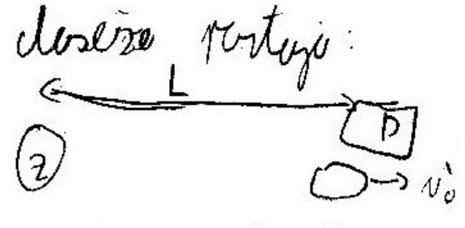
$v_{0_2} = 0{,}98\, c_0$

$t = ?$

$l = ?$

1) Ladja štarta z Zemlje: $S(0,0)$ $S'(0,0)$

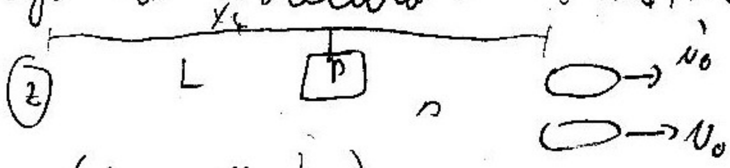
2) Ladja doseže postajo: $S(L, t_2)$ $S'(0, t_2')$



3) Štart druge ladje $S(0, t_3)$ $S'(x_3', t_3')$

$t_3 = t_2$ $x_3' = L$ $t_3' = t_2'$

4) Ladji se srečata: $S(x_4, t_4)$ $S'(0, t_4')$



$$t_2' = \gamma\left(t_2 - \frac{v_{01} L}{c^2}\right)$$

$$v_{01} = \frac{L}{t_2} = \frac{0{,}2\text{ s.l.}}{3\text{ mesece}} = 3{,}15\,\frac{m}{\text{mesece}} \cdot 10^{15} = \frac{7{,}98 \cdot 10^{15}\,m}{7776000\,s} \simeq 2{,}5 \cdot 10^{8}\,\frac{m}{s}$$

$$\simeq 0{,}84\, c_0$$

$x_4 = L + s = L + v_{01}(t_4 - t_3) \Rightarrow$ 1 ladja

$x_4 = v_{02} \cdot t_4$

$L + v_{01}(t_4 - t_3) = v_{02}(t_4 - t_3) \Rightarrow L - v_{01} t_3 + v_{02} t_3 = t_4 (v_{02} - v_{01})$

$$t_4 = \frac{L + t_3 (v_{02} - v_{01})}{(v_{02} - v_{01})} = 40\text{ let}$$

$t = t_4 - t_2 = \underline{1{,}5\text{ mesece}}$

$x_4 = v_{02} \cdot t_4 = \underline{14 \cdot 10^{15}\,m}$

Pravilno rešeno nalogo nekaj strani naprej.

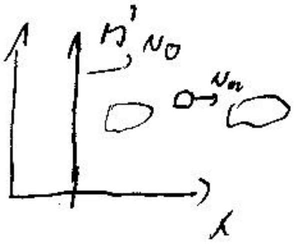
8) $v_{01} = 0{,}8\,c_0$

$t_2 = 10$ min

$v_{02} = 0{,}8\,v_0$

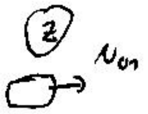
$v_R' = 0{,}4\,c_0$

$t = ?$

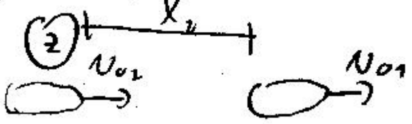


X

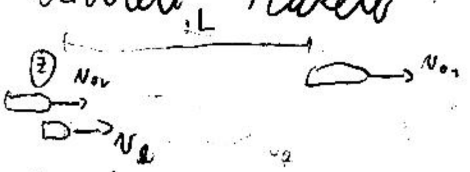
1) 1. ladja pelje mimo Zemlje $S(0,0)$ $S'(0,0)$



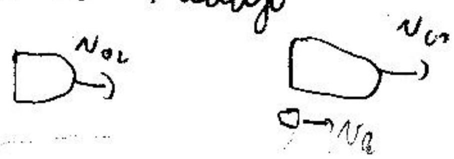
2) 2. ladja pelje mimo Zemlje $S(0,t_2)$ $S'(x_2', t_2')$



3) 2 ladja izstreli raketo $S(0,t_2)$ $S(x_3', t_3')$



4) Raketa pride do 1 ladje $S(x_4,t_4)$ $S'(0,t_4')$



$x_4 = L + s$

$\rightsquigarrow L = v_{01} \cdot t_2 = 1{,}44 \cdot 10^{11}$ m

$\rightsquigarrow X_4 = v_R \cdot t_4 = L + v_{01} \cdot t_4$

$t_4 = \frac{L}{v_R - v_0} = \frac{v_{01} \cdot t_2}{v_R - v_0}$ $\Rightarrow$ Tukaj ne poznamo v_R, poznamo le v_R'

$\rightsquigarrow v_R = \frac{v_R' + v_0}{1 + \frac{v_0 v_R'}{c^2}} = 2{,}72 \cdot 10^{8}\ \frac{m}{s} \simeq 0{,}91c$

$t_4 = \frac{v_0 \cdot t_2}{v_R - v_0} \simeq 4364\,s$ ✓

$\rightsquigarrow$ Čas za opazovalca na 1. ladji

$t_4' = \gamma\left(t_4 - \frac{v_0 \cdot x_4}{c^2}\right) = \frac{1}{\sqrt{1-\beta^2}}\left(t_4 - \frac{v_0 v_R t_4}{c^2}\right) = 1978\,s \simeq 33$ minut ✓

9) $l = 4$ sv. tedni $= 0{,}3$ sv. leta

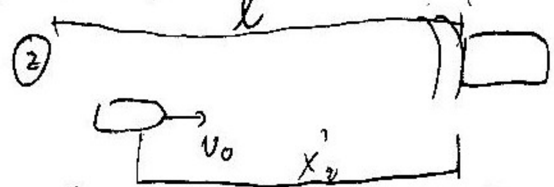
$\underline{t_3' = 6 \text{ tednov}}$

$v_0 = ?$

$t_4' = ?$

1) Ladja odda signal $\quad S(0,0) \quad S'(0,0)$

2) Signal pride do planeta: $S(x_2, t_2) \quad S'(x_2', t_2')$, $x_2 = l$



3) Signal pride nazaj do ladje $\quad S(x_3, t_3) \quad S'(0, t_3')$

4) Ladja doseže planet $\quad S(x_4, t_4) \quad S'(0, t_4')$

$$x_3 = \gamma(0 + v_0 \cdot t_3') = \gamma \cdot v_0 \cdot t_3'$$

$$t_3 = \gamma_3\left(t_3' + \frac{v_0 \cdot 0}{c^2}\right) = \gamma_3 \cdot t_3' \Rightarrow t_3' = \frac{t_3}{\gamma} =$$

$$v_0 = \frac{x_3}{t_3}$$

$$l + l - x_3 = 2l - x_3 = s$$

$$c = \frac{s}{t_2} = \frac{2l - x_3}{t_3} = \frac{2l}{t_3} - v_0 = \frac{2l}{\gamma \cdot t_3'} - v_0$$

$$c = \frac{2l\sqrt{1-\beta^2}}{t_3'} - v_0 \quad)^2$$

$$\left(1 + \frac{v_0}{c}\right)^2 = \frac{4l^2\left(1 - \frac{v_0^2}{c^2}\right)}{t_3'^2} \qquad \beta = \frac{v_0}{c}$$

$$(1+\beta)^2 = \left(\frac{2l}{t_3'}\right)^2(1-\beta^2) = \left(\frac{2l}{t_3'}\right)^2(1-\beta)\cancel{(1+\beta)}$$

$$\frac{\left(\frac{2l}{t_3'}\right)^2 - 1}{1 + \left(\frac{2l}{t_3'}\right)^2} = \beta = \underline{\underline{0{,}28}} \qquad v_0 = \beta \cdot c = \underline{\underline{0{,}28c}}$$

b) $l = x_4 = \gamma(x_4' + v_0 t_4') = \gamma \cdot v_0 t_4'$

$$t_4 = \frac{l}{\gamma \cdot v_0} = 96 \text{ dni}$$

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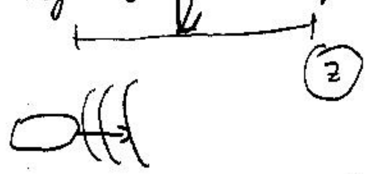
10) $v_0 = 0{,}6\,c$

$t' = 14$ dni

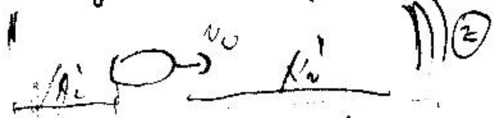
$l = ?$

$t = ?$

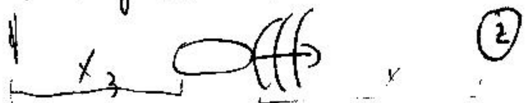
1) Ladja odda signal $S(l,0)$ $S'(0,0)$



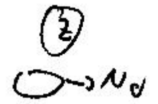
2) Signal dospe do Zemlje $S(0, t_2)$ $S'(x_2', t_2')$



3) Signal pride nazaj do ladje $S(x_3, t_3)$ $S'(0, t_3')$



4) Ladja zleti mimo Zemlje $S(0, t_4)$ $S'(0, t_4')$



Poznamo $t_3' = 14$ dni

$l = ?$

$t = t_4 - t_2 = ?$

$$l + l - x_3 = s = 2l - x_3$$

LORENTZ

$x_3 = \gamma \cdot v_0 \cdot t_3' = 1{,}74 \cdot 10^{14}\ m$

$t_3 = \gamma \cdot t_3' = 17{,}5$ dni

$$c = \frac{2l - x_3}{t_3} = \frac{2l}{t_3} - \frac{x_3}{t_3} \Rightarrow (c + v_0)\frac{t_3}{2} = l = 3{,}63 \cdot 10^{14}\ m$$

1. sv. l ... $4{,}6 \cdot 10^{14}$

x ... $2{,}32 \cdot 10^{14}\ m$

$t = t_1 - t_2$

$t_1 = \frac{l}{v_0} - \frac{l}{c} = l\left(\frac{1}{v_0} - \frac{1}{c}\right) = 23{,}3\,\text{dni} - 14\,\text{dni}$

$= \underline{\underline{9{,}3\,\text{dni}}}$ $l \simeq \underline{\underline{224\,m}}$

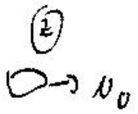
11) $v_{0_1} = \frac{2}{5} c_0$

$t_2' = 15\ min$

$v_{02}' = \frac{3}{5} c_0$

$t_2 = ?$ $\Rightarrow t_2 + t_3 = t_{or}$

1) 1. ladja pelje mimo zemlje $S(0,0)$ $S'(0,0)$



2) 1. ladja se sreča z 2 ladjo $S(x_2, t_2)$ $S'(0, t_2')$ $S''(0,0)$

$$v_{02} = \frac{v_{02}' - v_{01}}{1 - \frac{v_{02}' \cdot v_{01}}{c^2}} = 0{,}26\,c$$

$$l = x_2 = \gamma \cdot v_{01} \cdot t_2' = 1{,}18 \cdot 10^{11}\ m$$

$$t_2 = \gamma \cdot t_2' = 982\ s = 16{,}4\ min$$

3) 2 ladja se giblje mimo zemlje $S(0, t_3)$ $S''(x_3'', t_3'')$

$$t_3 = \frac{l}{v_{02}} = 25{,}5\ min$$

$$t_{or} = t_2 + t_3 \simeq 42\ min$$

12) $l_0 = 50\,m$

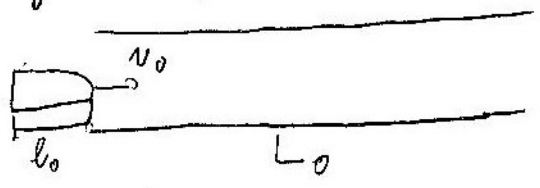
$v_0 = 0{,}6\,c_0$

$L_0 = 200\,m$

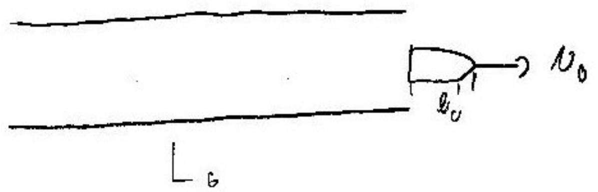
$t = ?$

$t' = ?$

1) Ladja vstopa v postajo $\qquad S(0,0) \quad S'(0,0)$



2) Ladja prepotuje postajo $\qquad S(x_2, t_2) \quad S'(x_2', t_2')$



- Ladja mora prepotovat $L_0 + l_0 = 250\,m$
- Za postajonačelnika ladja prepotuje $240\,m$

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1-\beta^2} = 40\,m$$

- Za pilota $L = \frac{L_0}{\gamma} = 160\,m$; $210\,m$

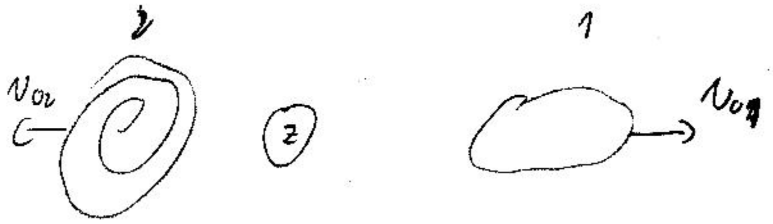
$$t = \frac{s}{v_0} = \underline{\underline{1{,}33\,\mu s}} \checkmark$$

$$t' = \frac{s'}{v_0} = \underline{\underline{1{,}17\,\mu s}} \checkmark$$

13) $v_{01} = 0{,}6\,c$

$v_{02} = 0{,}7\,v_0$

$v_{12}' = ?$



$$v_{12}' = \frac{v_{01} + v_{02}}{1 + \frac{v_{01} \cdot v_{02}}{c^2}} = \underline{\underline{0{,}92\,c}}$$

14) $v_{01} = 0{,}6\,v_0$

$v_{02} = 0{,}9\,v_0$

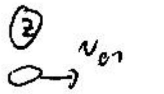
$L_0 = 50\,m$

$t = ?$

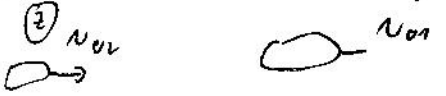
$t' = ?$

$t'' = ?$

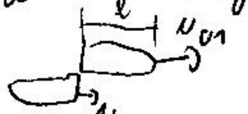
1) 1. ladja leti mimo Zemlje $S(0,0)$ $S'(0,0)$



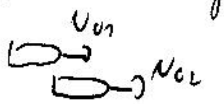
2) 2 ladja leti mimo Zemlje $S(0,t_2)$ $S'(x_2',t_2')$ $S''(0,t_2'')$



3) 2 ladja dohiti 1. ladjo $S'(x_3',t_3')$ $S''(x_3'',t_3'')$



4) 2 ladja prehiti 1 ladjo $S'(x_4',t_4')$ $S''(x_4'',t_4'')$



Dobimo 1 in 2 ladjo kot jo vidi opazovalec na Zemlji

$$l_1 = L_0 \sqrt{1 - \left(\frac{v_{01}}{c}\right)^2} = 40\,m$$

$$l_2 = L_0 \sqrt{1 - \left(\frac{v_{02}}{c}\right)^2} = 22\,m$$

Za opazovalca na Zemlji mora 2. ladja narediti pot 62 m,

$t_4 = \frac{62\,m}{0{,}3\,c} = \underline{0{,}69\,\mu s}$ opazovalec vidi da se druga ladja približuje s hitrostjo $0{,}3c = 0{,}9c - 0{,}6c$

Hitrost 2 ladje glede na 1. ladjo:

$$v_{02}' = \frac{v_{02} - v_{01}}{1 - \frac{v_{02} v_{01}}{c^2}} = \underline{0,65\,c}$$

$$s' = L\sqrt{1 - \left(\frac{v_{02}'}{c}\right)^2} = 40\,m$$

$$t' = \frac{40\,m}{v_{02}'} = \underline{\underline{0,46\,\mu s}}$$

15) $v_{01} = 0,8c$

$v_{02} = 0,9c$

$v'_{02} = ?$

$v'_{01} = ?$

$$v'_{02} = \frac{v_{02} - v_{01}}{1 - \frac{v_{01} \cdot v_{02}}{c^2}} = \underline{\underline{0,36c}} \Rightarrow \text{ra 1-egy}$$

$$v'_{01} = \frac{v_{01} - v_{02}}{1 - \frac{v_{01} \cdot v_{02}}{c^2}} = -\underline{\underline{0,36c}}$$

0.2 sv. let ... = sv. mesecev

7) $t_2 = 3$ mesecev

$l = 0{,}2$ sv. let $= 2{,}4$ sv. mesecev

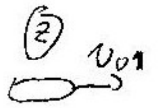
$v_{02} = 0{,}98c$

$t_4 = ?$

$s = ?$

$t_4'' = ?$

1) dogodek: Ladja leti iz Zemlje $S(0,0)$ $S'(0,0)$



2) dogodek: Ladja prispe do postaje $S(x_1, t_2)$ $S'(0, t_2')$

3) Štart 2 ladje $S(0, t_3)$ $S'(0, t_3'')$

4) Ladji se srečata: $S(x_4, t_4)$ $S'(0, t_4)$

$v_{01} = \frac{l}{t_2} = 0{,}8c$

$\rightarrow A = L + s = L + v_{01}(t_4 - t_3) = v_{02} \cdot t_4$

$t_4 = \frac{L + v_{01} t_3}{v_{02} - v_{01}} = 405$ dni $= \underline{1{,}11 \text{ let}}$

$\rightarrow A = v_{02} \cdot t_4 = 1{,}09$ sv. let

$\rightarrow t_4 = 5 t_4' \Rightarrow t_4' = \frac{t_4}{5} = \underline{0{,}22 \text{ let}}$

$t_4' = \gamma\left(t_4 - \frac{A \cdot v_{02}}{c^2}\right) = 5{,}03(34992000 - 33649300) = \underline{0{,}22 \text{ let}}$

Preizkus z 2 enačbo, ce pride enako.

17) $\dfrac{E_k}{W_k} = 1{,}01$

$W_k = \frac{1}{2} m \cdot v^2 \qquad E_k = m \cdot c^2 \cdot \gamma - m \cdot c^2 = mc^2(\gamma - 1)$

$\beta = \dfrac{v}{c}$

$v = ?$

$$\frac{2 m c^2 (\gamma - 1)}{m \cdot v^2} = \frac{2 \cdot (\gamma - 1)}{\beta^2} = \frac{2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)}{\beta^2} =$$

$$\frac{2\left(1 + \binom{-\frac{1}{2}}{1}(-\beta^2) + \binom{-\frac{1}{2}}{2}(-\beta^2)^2 + \dots - 1\right)}{\beta^2}$$

$$= \frac{2\left(\frac{1}{2}\beta^2 + \frac{1}{4}\beta^4 + \dots\right)}{\beta^2} = \frac{\beta^2 + \frac{1}{2}\beta^4}{\beta^2} = \frac{\cancel{\beta^2}\left(1 + \frac{1}{2}\beta^2\right)}{\cancel{\beta^2}} = 1{,}01$$

$1{,}01 - 1 = +\frac{1}{2}\beta^2 \quad / \cdot 2$

$\beta^2 = 0{,}02$

$\beta = 0{,}14$

$v = \beta \cdot c = \underline{\underline{0{,}14\,c}}$

18) $p = 800 \frac{MeV}{c_0}$ $m_p = 938{,}27 \frac{MeV}{c^2}$

$E_k = ?$

$E_p^2 = c^2 \cdot p^2 + m_p^2 c^4 = c^2 \frac{800^2 MeV^2}{c^2} + \frac{938{,}27^2 MeV^2 c^4}{c^2}$

$= 1520350{,}6 \, MeV$

$E_p \simeq 1233 \, MeV$

$E_p = E_k + E_0 \Rightarrow E_k = E_p - m_p \cdot c^2 = \underline{295 \, MeV}$ ✓

---

19) $T = 6000 \, MeV$

$T = m_p \cdot c^2 \cdot (\gamma - 1)$

$\gamma = \frac{T}{m_p \cdot c^2} + 1$

$\frac{1}{\sqrt{1-\beta^2}} = \frac{T}{m_p c^2} \quad /^2 \Rightarrow \frac{1}{1-\beta^2} = \frac{T^2}{m_p \cdot c^2}$

$1 = \frac{T^2}{m_p c^2} - \frac{T^2 \beta^2}{m_p \cdot c^2}$

$\Rightarrow \beta^2 = \frac{\frac{T^2}{m_p \cdot c^2} - 1}{\frac{T^2}{m_p c^2}} = 1 - \frac{m_p \cdot c^2}{T^2} = 0{,}844$

$\beta = 0{,}92$

$v_v = 0{,}92 c = 2{,}76 \cdot 10^8 \frac{m}{s} = \frac{2{,}76 \cdot 10^5 \, km}{s}$

Razlikuje se $\underline{2{,}4 \cdot 10^4 \frac{km}{s}}$ ✓

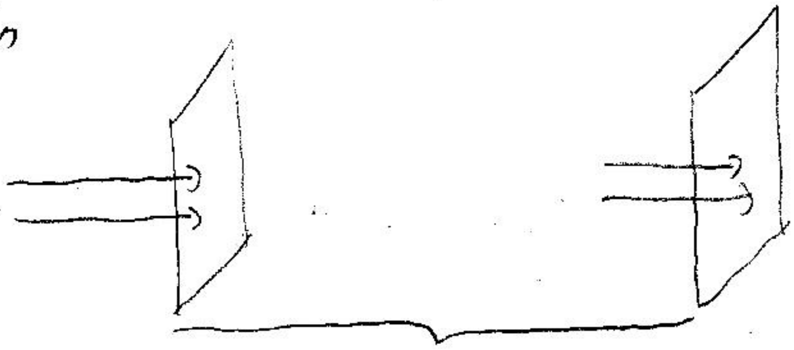
20) $T_\pi = 200\ \mathrm{MeV}$

$A = 10^6/s$

$l = 5\ m$

$\tau = 2{,}6 \cdot 10^{-8}\ s$

$A' = ?$



$l = 5\ m$

$A = \left| \dfrac{dN}{d\tau} \right|$

21)

22) M, $m = \frac{M}{2}$

OHRANITEV ENERGIJE

prej po

$$M\cdot c^2 = \frac{M}{2}\cdot c^2 + T_R + E_f = \frac{M}{2}c^2 + \frac{M}{2}c^2(\gamma-1) + c\,p_f$$

OHRANITEV GIBALNE

$$0 = \frac{M}{2}\cdot\gamma\cdot v - p_f$$

$$p_f = \frac{M}{2}\cdot\gamma\cdot v$$

$$\cancel{M}c^{\cancel{2}} = \frac{\cancel{M}}{2}c^{\cancel{2}} + \frac{\cancel{M}}{2}\cdot c^{\cancel{2}}(\gamma-1) + \frac{\cancel{M}}{2}\cdot\gamma\cdot v\cdot \cancel{c}$$

$$\frac{1}{\cancel{2}}c = \frac{c}{\cancel{2}}(\gamma-1) + \frac{\gamma\cdot v}{\cancel{2}} \quad /:c$$

$$1 = \gamma-1 + \gamma\cdot\beta$$

$$2 = \gamma(1+\beta) = \frac{1}{\sqrt{1-\beta^2}}(1+\beta) \quad /^2$$

$$4 = \frac{(1+\beta)^{\cancel{2}}}{(1-\beta)\cancel{(1+\beta)}} = \frac{1+\beta}{1-\beta} \quad /(1-\beta)$$

$$4-4\beta = 1+\beta$$

$$5\beta = 3$$

$$\beta = \frac{3}{5}$$

$$v = \underline{\underline{0{,}6\,c}}$$

23) $E = 1{,}75\ \frac{kV}{m}$ $\boxed{\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})}$

$t = 1\,\mu s$

a) $v = ?$

b) $s = ?$

c) $T = ?$

$$\frac{dp^\mu}{d\tau} = \left[e\gamma \vec{E}\cdot\frac{\vec{v}}{c},\ e\gamma(\vec{E} + \vec{v}\times\vec{B})\right]$$

$$dp = e\cdot E\cdot dt$$

a)

$$m\cdot d(\gamma\cdot v) = e\cdot E\, dt$$

$$m\cdot\gamma\cdot v = e\cdot E\cdot t \quad /: c\cdot m$$

$$\gamma\cdot\beta = \frac{e\cdot E}{mc}\, t$$

$A = 1{,}10^6$

$$\frac{\beta}{\sqrt{1-\beta^2}} = A\cdot t \quad /^2$$

$$\beta^2 = (A\cdot t)^2 - (At)^2\beta^2$$

$$\beta^2(1 + (At)^2) = (A\cdot t)^2$$

$$\beta^2 = \frac{(At)^2}{1+(At)^2}$$

$$\beta = \frac{(At)}{\sqrt{1+At}} = 0{,}71$$

$$v = \underline{\underline{0{,}71c}}$$

b) $s = v(t)dt = \int_0^t c\,\beta\,dt = \int_0^t \frac{c\cdot At}{\sqrt{1+(At)^2}}\,dt$

$1 + (At)^2 = u$

$du = 2tA^2\,dt$

$$s = \frac{c\cdot \cancel{A}\cdot \cancel{t}}{2\cancel{t}\cdot A^{\cancel{2}}}\int \frac{du}{\sqrt{u}} = \frac{c}{\cancel{2}\cdot A}\,\cancel{2}\sqrt{u} = \frac{c\cdot\sqrt{1+At^2}}{A}\Bigg|_0^t$$

$$= \frac{c\sqrt{1+A^2t^2}}{A} - \frac{c}{A} = \frac{c}{A}\left(\sqrt{1+A^2t^2} - 1\right) = \underline{\underline{124{,}3\,m}}$$

c) $T = e \cdot U = e \cdot E \cdot s = \underline{\underline{0{,}22\ MeV}}$ ✓

---

24) $E = 10^6 \frac{V}{m}$ $\quad v = 0{,}8c$

$s = ?$
$t = ?$

$d(m \cdot \gamma \cdot v) = e \cdot E \cdot dt$

$m \cdot \gamma \cdot v = e \cdot E \cdot t$

$\beta \gamma = \underbrace{\left(\frac{e \cdot E}{m \cdot c}\right)}_{A = 5{,}86 \cdot 10^8} \cdot t$

$$\frac{\beta}{\sqrt{1-\beta^2}} = A \cdot t$$

$$\beta^2 = (At)^2 - (At)^2 \beta^2$$

$$\beta = \frac{At}{\sqrt{1+A^2t^2}} \Rightarrow v = \frac{A \cdot t c}{\sqrt{1-A^2t^2}} = * \quad /^2$$

$$\Rightarrow s = \int_0^t \frac{cAt}{\sqrt{1+A^2t^2}} = \int_0^t \frac{c \cdot A \cdot \cancel{t}\, du}{2 \cdot \cancel{t} A^2 \sqrt{u}} = \frac{c}{2A} 2\sqrt{u} \Big|$$

$u = 1 + A^2t^2$

$du = +2tA^2 dt$

$$= \frac{c \cdot 2\sqrt{1+A^2t^2}}{2A}\Big|_0^t = \frac{c}{A}\left(\sqrt{1+A^2t^2} - 1\right) = **$$

$$* \quad v^2(1+A^2t^2) = A^2t^2c^2$$

$$\frac{v^2}{c^2} + \frac{v^2}{c^2}A^2t^2 = A^2t^2$$

$$\frac{v^2}{c^2} = A^2t^2\left(1 - \frac{v^2}{c^2}\right)$$

$$t = \frac{\beta}{A\sqrt{1-\beta^2}} = \underline{\underline{2{,}3\ ns}}$$ ✓

$** = \underline{\underline{0{,}39\ m = 39\ cm}}$ ✓

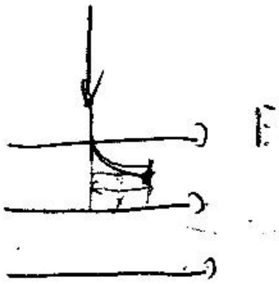
25) $U = 100\ keV$

$E = 1\ \frac{kV}{m}$

$t = 30\ ns$

$\Delta y = ?$

$x = ?$



Treba upoštevati x in y os

$\vec{E} = (E_x, 0)$ $\qquad T = -e \cdot U = -e \cdot E \cdot y$ $\qquad v^2 = v_y^2 + v_x^2$

$\vec{v} = (v_x, v_y)$

$x:$ $m \cdot \gamma \cdot v_x = e \vec{E}_x\, t$

$$\beta_x \gamma = \frac{e \vec{E}_x}{m \cdot c} \cdot t = A \cdot t \Rightarrow 5{,}86 \cdot 10^5$$

$$\beta_x \cdot \gamma = A \cdot t$$

$$\frac{\beta_x^2}{1 - \beta_x^2} = (A^2 \cdot t^2)$$

$$\beta_x^2 = (At)^2 - (At)^2 \cdot \beta_x^2$$

$$\beta_x^2 = \frac{(At)^2}{1 + (At)^2} \Rightarrow \beta_x = \frac{A \cdot t}{\sqrt{1 + (At)^2}} = \underline{\underline{0{,}018\, c}}$$

$y:$ $T = e \cdot U = m \cdot c(\gamma - 1)$

$$\gamma = \frac{e \cdot U}{m \cdot c} + 1 = \frac{1}{\sqrt{1 - \beta_y^2}} \quad /^2$$

$$= \left(\frac{e \cdot U}{m \cdot c} + 1\right)^2 = \frac{1}{1 - \beta_y^2}$$

$$\left(\frac{e \cdot U}{m \cdot c^2} + 1\right)^2 (1 - \beta_y^2) = 1$$

$B = 0{,}195$ $\qquad \beta_y^2 = \frac{\left(\frac{e \cdot U}{m \cdot c^2} + 1\right)^2 - 1}{\left(\frac{e \cdot U}{m \cdot c^2} + 1\right)^2} =$

$$\beta_y = \sqrt{1 - \frac{1}{(B+1)^2}} = \underline{\underline{0{,}547\, c}}$$

$v = \sqrt{v_x^2 + v_y^2} \approx 0,55\,c$

$\ell(t) = \int \frac{c \cdot A \cdot t}{\sqrt{1 + A^2 t^2}}\,dt = \dots = \frac{c}{A}\left(\sqrt{1 + (At)^2} - 1\right) = 7,9\ \mathrm{cm}$

---

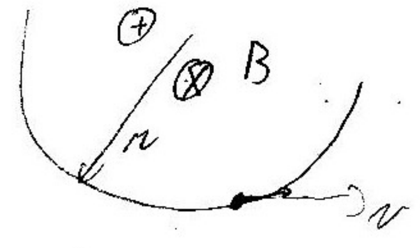
26) $B = 0,5\ \mathrm{T}$

$r = 3\ \mathrm{m}$

$E_0 = 139,6\ \mathrm{MeV}$

$\tau = 2,6 \cdot 10^{-8}\ \mathrm{s}$

$s = ?$



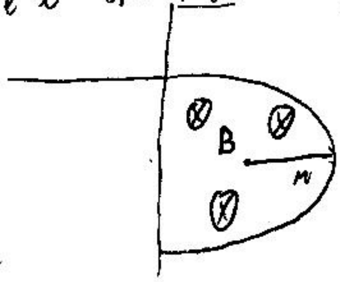
$\vec{F} = e(\vec{v} \times \vec{B})$

$T = m_\pi c^2 (\gamma - 1)$

27) $U = 1{,}02\cdot 10^{6}\,V$ $\quad m_e\cdot c^2 = 0{,}51\,MeV$ $\quad m_e = 0{,}51\,\frac{MeV}{c^2}$

$B = 0{,}048\,T$

$r = ?$



$1{,}6\cdot 10^{-19}\,J \to 1\,eV$

$X \to 1{,}53\cdot 10^{6}\,eV$

$W^2 = c^2 p^2 + m^2 c^4$

$\left(\frac{W}{c}\right)^2 - m_e^2 c^2 = p^2 \Rightarrow \left(\frac{W}{c}\right)^2$

$E = T + E_0 = 1{,}02\,MeV + 0{,}51\,MeV$
$= 1{,}53\,MeV = 2{,}4\cdot 10^{-13}\,J$

$T = e\cdot U = 1{,}02\cdot 10^{6}\,eV = 1{,}6\cdot 10^{-13}\,J$

$p^2 = \left(\frac{W}{c}\right)^2 - \left(\frac{E_0}{c}\right)^2 \Rightarrow p = \sqrt{\left(\frac{W}{c}\right)^2 - \left(\frac{E_0}{c}\right)^2} = \frac{1}{c}\sqrt{E_r^2 - E_0^2} = 1{,}44\,\frac{MeV}{c}$

$= 0{,}0048\,\frac{eVs}{m}$

$p = e r B \Rightarrow r = \frac{p}{e\cdot B} = \underline{\underline{1\,dm}}\ ?$

v rezultat je 2 dm

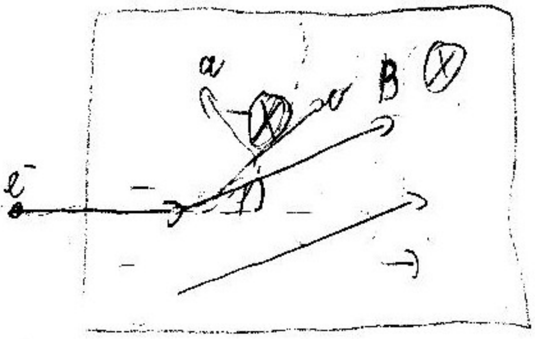
28) $v = 2\cdot 10^{8}\ \frac{m}{s}$

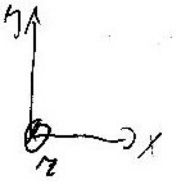
$E_0 = 10^{7}\ \frac{V}{m}$

$\varphi = 30^\circ$

$B = 1\,T$

$\vec{a} = (a_x, a_y, a_z)$





E odklanja elektron

$$\vec{E} = (E_x, E_y, 0) = (E_0 \cos\varphi, E_0 \sin\varphi, 0)$$

$$\vec{B} = (0, 0, B_z)$$

$$\vec{v} = (v_x, v_y, 0)$$

$$\vec{F} = e(\vec{E} + \vec{v}\times\vec{B})$$

$$\vec{v}\times\vec{B} = \begin{vmatrix} i & j & k \\ v_x & v_y & 0 \\ 0 & 0 & B_z \end{vmatrix} = \vec{i}\, v_y \cdot B_z - \vec{j}\, v_x B_z = (v_y B_z, -v_x B_z, 0)$$

$$\vec{F} = e\left((E_0\cos\varphi, E_0 \sin\varphi, 0) + (\vec{v}_y \vec{B}_z, -v_x B_z, 0)\right) = \frac{dP}{dt} = \frac{m\cdot \gamma \cdot \vec{v}}{t}$$

29) $v = 0{,}9\,c_0$
$E = 1000\ \frac{V}{cm}$
$t = ?$

$m \cdot \gamma \cdot v = e \cdot E \cdot t \quad / : c \cdot m$

$\gamma \cdot \beta = \frac{e \cdot E}{c \cdot m} \cdot t$

$$\frac{\beta}{\sqrt{1-\beta^2}} = \frac{e \cdot E}{c \cdot m} \cdot t \Rightarrow t = \frac{c \cdot m}{e \cdot E} \frac{\beta}{\sqrt{1-\beta^2}} = \underline{\underline{3{,}9 \cdot 10^{-8}\ ns}}$$

37) $T_{e^+} = 20\,MeV$

a) $E_{\gamma_1}, E_{\gamma_2}$

Energija

1) $E_{e^+} + E_0 = E_{\gamma_1} + E_{\gamma_2} = E_0 + T_{e^+}$

GIBALNA

2) $p_{e^+} = p_{\gamma_1} - p_{\gamma_2} = \frac{E_{\gamma_1}}{c} - \frac{E_{\gamma_2}}{c} = \frac{1}{c}\sqrt{E_{e^+}^2 - m_e^2 c^2}$

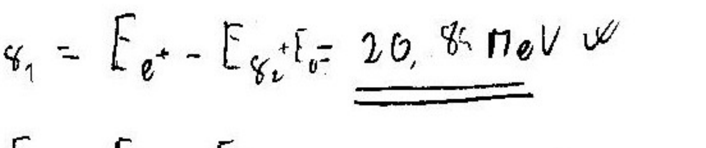
$$\Rightarrow E_{\gamma_1} = E_{\gamma_2} + \sqrt{E_{e^+}^2 - m_e^2 c^2}$$

Vstavimo v prvo enačbo

$E_0 + E_{e^+} = 2E_{\gamma_2} + \sqrt{E_{e^+}^2 - m_e^2 c^2} \Rightarrow E_{\gamma_2} = \frac{E_0 + E_{e^+} - \sqrt{E_{e^+}^2 - m_e^2 c^2}}{2} = \underline{\underline{0{,}26\,MeV}}$ ✓

$E_{\gamma_1} = E_{e^+} - E_{\gamma_2} + E_0 = \underline{\underline{20{,}86\,MeV}}$ ✓

b) $E_{\gamma_1} = E_{\gamma_2} = E_\gamma$



Energija

$\rightarrow E_{e^+} = E_{\gamma_1} + E_{\gamma_2} = 2E_\gamma - E_0$

$$E_\gamma = \frac{E_{e^+} + E_0}{2} = 10{,}51\,MeV$$

Kota sta enaka, ker je energija enaka

$$p_{e^+} = p_{\gamma_1} \cdot \cos\varphi + p_{\gamma_2} \cdot \cos\varphi = 2 p_\gamma \cdot \cos\varphi$$

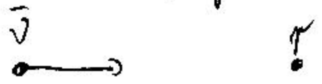
$$\frac{1}{c}\sqrt{\left(\frac{E_e}{c}\right)^2 - (m_e c)^2} = 2 \cdot \frac{E_\gamma}{c} \cdot \cos\varphi$$

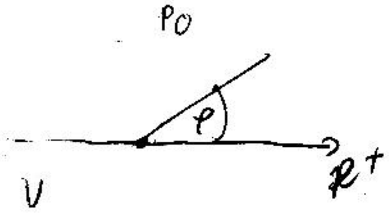
$\frac{1}{c}\sqrt{E^2 - m_e^2 c^4} = 2 \cdot \frac{E_\gamma}{c} \cdot \cos\varphi \Rightarrow \cos\varphi = \frac{\sqrt{E^2 - m_e^2 c^4}}{2E_\gamma} = \underline{\underline{13^\circ}}$ ✓

38) $E_{\bar{\nu}} = 2\,MeV$

$T_{e^+} = ?$

rej





$E_{0e^+} = 0{,}51\,MeV$

$E_{0p} = 938\,MeV$

Energije

$$E_{\bar{\nu}} + E_{0p} = E_{\nu} + E_{e^+}$$

GIBALNA

$$x:\ p_{\bar{\nu}} = p_{e^+} + p_{\nu} \cdot \cos\varphi = p_e - p_{\nu}$$

$$y:\ p_{\nu} \cdot \sin\varphi = 0$$

$$\varphi = \pi$$

$$\frac{E_{\bar{\nu}}}{c} = \sqrt{\frac{T_e^2 + 2T_e \cdot E_0}{c^2}} + \cancel{p_{\nu}}^{\,0}$$

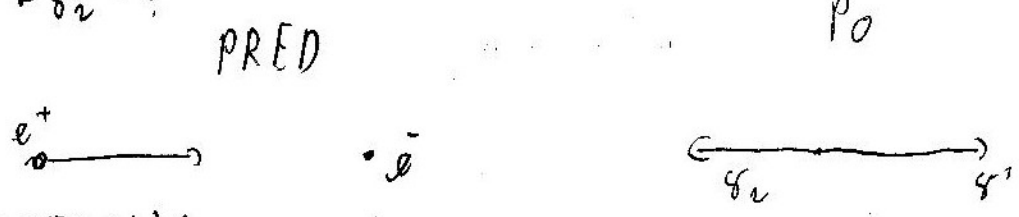
$$\left(\frac{E_{\bar{\nu}}}{c}\right)^2 = \frac{T_e^2 + 2T_e \cdot E_0}{c^2}$$

$$E_{\nu} + m_p \cdot c^2 - m_{\nu} \cdot c^2 - m_e c^2 = T_e = \underline{0{,}19\,MeV}$$

39) $E_{e^+} = 500\ keV$

$E_{\gamma_1} = ?$

$E_{\gamma_2} = ?$



ENERGIJA

$E_{e^+} + E_{e^-} = E_{\gamma_1} + E_{\gamma_2} =$

GIBALNA KOLIČINA

$p_{e^+} = p_{\gamma_1} + p_{\gamma_2} = \frac{E_{\gamma_1}}{c} - \frac{E_{\gamma_2}}{c} = \frac{1}{c}\sqrt{E_{e^+}^2 - m_e^2 c^4}$

$E_{e^+} + E_{0e^-} = 2E_{\gamma_2} + \sqrt{E_{e^+}^2 - m_e^2 c^4}$

$E_{\gamma_2} = \frac{E_{e^+} + E_{0e^-} - \sqrt{E_{e^+}^2 - m_e^2 c^4}}{2} = 0,32\ MeV$ ✓

$E_{\gamma_1} = 1,2\ MeV$ ✓

35)

1) ENERGIJA

$$E_\gamma + E_0 = E_\gamma' + E_e'$$

$$E_\gamma + m_e c^2 = E_\gamma' + E_e'$$

$$E_e' = \sqrt{c^2 p_e^2 + m^2 c^4} \quad E_0$$

$$E_\gamma \gg E_0$$

$$E_e' = \sqrt{c^2 p_e^2} = p_e c$$

2) GIBALNA KOLIČINA

$$x: \; p_\gamma = p_\gamma' \cdot \cos\varphi + p_e' \cdot \cos\beta$$

$$y: \; 0 = p_\gamma' \cdot \sin\varphi - p_e' \cdot \sin\beta$$

$$p_\gamma' \sin\varphi = p_e' \cdot \sin\beta \Rightarrow p_e' = \frac{p_\gamma' \cdot \sin\varphi}{\sin\beta} = \sqrt{\frac{E_e'^2 - m_e^2 c^4}{c^2}}$$

$$\operatorname{ctg}\beta \approx 1$$

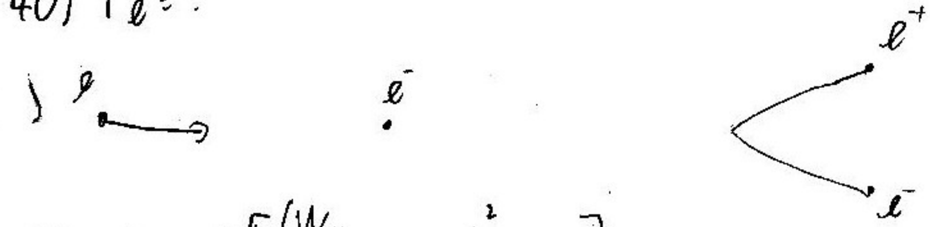
$$\beta \approx 0$$

$$\frac{E_\gamma}{\cancel{c}} = \frac{E_\gamma'}{\cancel{c}} \left(\cos\varphi + \sin\varphi / \operatorname{tg}\beta\right)$$

$$E_\gamma' = \frac{E_\gamma}{\cos\varphi + \sin\varphi \cdot \operatorname{ctg}\beta}$$

$$E_\gamma = E_\gamma' + E$$

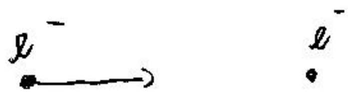
40) $T_e = ?$

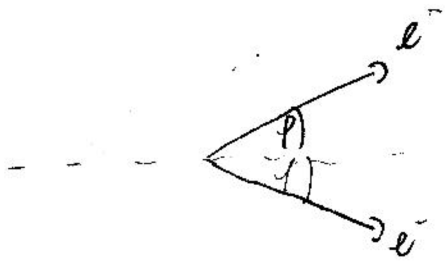


$$T = 2mc^2\left[\left(\frac{W_R}{2mc^2}+1\right)^2-1\right] = 6m_ec^2 = \underline{3{,}06\ MeV}$$

41) $T_e = 20\,MeV$

$\varphi_{min} = ?$





$p_1 = \sqrt{\left(\frac{E_1}{c^2}\right) - m_e^2 c^2}$

1) ENERGIJA

$E_1 + E_2 = E_3 + E_4$

$T_1 + m_e c^2 + m_e c^2 = E_3 + E_4$

$T_1 + 2 m_e c^2 = E_3 + E_4$

$T_1 = T_3 + T_4$

2) GIBALNA KOLIČINA

$p_1 + p_2 = p_3 + p_4 \quad ; \; p_2 = 0$

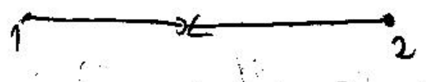
$p_1 = p_3 + p_4$

$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \; /^2$

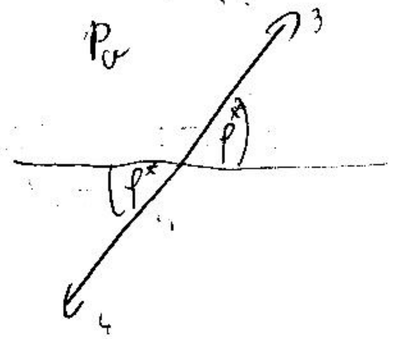
$\gamma^2 - \gamma \cdot \left(\frac{v}{c}\right)^2 \Rightarrow \sqrt{\gamma^2 - 1} \cdot c = v_1$

Zaradi lažjega računanja gledamo težiščni sistem

PRED



P_O



v težiščnem sistemu se gibljeta drug proti drugemu, medtem ko se pri odboju gibljeta vsak v drugo smer.

$p'_{x_1} = p'_3 \cdot \cos\varphi^*$ $\qquad$ $p'_{x_2} = -p'_4 \cdot \cos\varphi^*$

$p'_{y_1} = p'_3 \cdot \sin\varphi^*$ $\qquad$ $p'_{x_2} = -p_4 \cdot \sin\varphi^*$

$E'_1 = E'_3$ $\qquad$ $E'_2 = E'_3$

zdaj izračunajmo hitrost težišča

$$\boxed{P_x' = \gamma_0\left(P_x - v_0 \frac{E}{c^2}\right)} \Rightarrow \text{splošna formula} \quad P_x' = 0$$

$$\gamma_0 = \frac{1}{1-\left(\frac{v_0}{c}\right)^2}$$

$$\boxed{v_0 = \frac{\gamma_0 P_x \cdot c^2}{E}}$$

V našem primeru je

$\rightsquigarrow P_1 = m \cdot v_1 \cdot \gamma = m \cdot c\sqrt{\gamma^2 - 1}$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \Rightarrow \gamma^2 - \gamma^2\frac{v^2}{c^2} = 1$$

$$v_1 = c\sqrt{\gamma^2 - 1}$$

$E_1 = T_e + mc^2$

$E_1 = mc^2(\gamma - 1) + mc^2$

$E_1 = mc^2\gamma$

$\rightsquigarrow E = E_1 + E_2 = T_e + m_e c^2 2$

$$= mc^2(\gamma - 1) + 2m_e c^2 = mc^2\gamma + m_e c^2 = mc^2(\gamma + 1)$$

$$v_0 = \frac{c\, m \cdot c\sqrt{\gamma^2 - 1}}{m c^2 (\gamma + 1)} = c\sqrt{\frac{\gamma - 1}{\gamma + 1}} \Rightarrow \text{hitrost težišča}$$

Uporabimo Lorentza

$\sim P_1' = \gamma_0^*\left(P_1 - v_0 \cdot \frac{E_1}{c^2}\right)$

$$\rightsquigarrow \gamma^* = \frac{1}{\sqrt{1-\left(\frac{v_0}{c}\right)^2}} = \frac{1}{\sqrt{1-\frac{\gamma-1}{\gamma+1}}} = \frac{1}{\sqrt{\frac{\gamma+1-\gamma+1}{\gamma+1}}} = \frac{\sqrt{\gamma+1}}{\sqrt{2}}$$

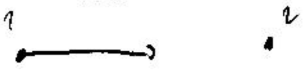
$$\Rightarrow P_1' = \frac{\sqrt{\gamma+1}}{\sqrt{2}}\left(m \cdot c\sqrt{\gamma^2 - 1} - \frac{c}{c^2}\sqrt{\frac{\gamma-1}{\gamma+1}} \cdot mc^2\gamma\right) = \frac{\sqrt{\gamma+1}}{\sqrt{2}}\left(mc\left(\sqrt{\gamma^2-1} - \gamma\sqrt{\frac{(\gamma-1)}{\gamma+1}}\right)\right)$$

$$= \frac{1}{\sqrt{2}} mc(\gamma - 1)^{\frac{1}{2}}$$

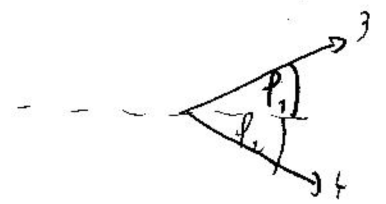
41) $T_e = 20\ \text{MeV}$

$\varphi_1, \varphi_2 = ?$

Pred



Po



→ Energija

$$E_1 + E_2 = E_3 + E_4$$

$$T_1 + mc^2 + mc^2 = T_3 + T_4 + 2mc^2$$

$$T_1 = T_3 + T_4$$

→ gibalna količina

$$p_1 = p_3 + p_4$$

$$x:\ p_{1x} = p_3 \cdot \cos\varphi_1 + p_4 \cos\varphi_2$$

$$y:\ 0 = p_3 \cdot \sin\varphi_1 + p_4 \cdot \sin\varphi_2$$

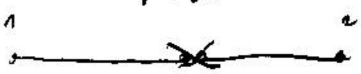
$$p_3 \cdot \sin\varphi_1 = p_4 \cdot \sin\varphi_2$$

$$p_1 = \sqrt{\left(\frac{E_1}{c}\right)^2 - \frac{m^2c^4}{c^2}} = \sqrt{\frac{T^2 + Tmc^2}{c^2}} = 20{,}25\ \frac{\text{MeV}}{c}$$

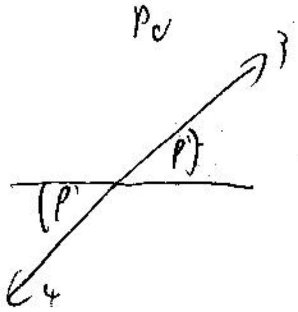
$p_1 =$

Poglejmo v težišče

pred



Po



Pred trkom velja

$$p_1' = -p_2'$$

$$E_1' = E_2'$$

$$p'_{x1} = p_3' \cdot \cos\varphi^* \qquad p'_{x2} = -p_4' \cos\varphi^*$$

$$p'_{y1} = p_3' \cdot \sin\varphi^* \qquad p'_{y2} = -p_4' \cdot \sin\varphi^*$$

$$E_3' = E_1' \qquad E_4' = E_2'$$

Ker gre za simetrijo je $\varphi^* = 90°$

$$\tan\varphi_1 = \tan\varphi_2 = \frac{\sqrt{2}}{\sqrt{8+1}} = \frac{\sqrt{2}}{\sqrt{\frac{T_1}{mc^2} + 1}} = \underline{\underline{12{,}4°}}\ ✓$$

42) $T_p = E_0 = 938{,}27\ MeV$ $\qquad 1\ eV \quad — 1{,}6 \cdot 10^{-19}\ J$

$\varphi_1 = 30°$

$\cancel{p_3 = \sqrt{\left(\frac{E_3}{c}\right)^2}}$ 3,13

$T_4 = ?$

$A = F \cdot s$

$J = N \cdot m \Rightarrow \frac{kg\ m^2}{s^2}$

ENERGIJA

$E_1 + E_2 = E_3 + E_4$

$T_1 = \underline{T_3} + T_4 = 2T$

GIBALNA

$x: P_1 = P_3 \cos\varphi_1 + P_4 \cos\varphi_2$

$y: P_3 \sin\varphi_1 = P_4 \sin\varphi_2$

Pomagamo si s težiščnim sistemom

$\gamma = \frac{1}{\sqrt{1-\left(\frac{v_1}{c}\right)^2}} \Rightarrow \gamma^2 - \gamma^2\left(\frac{v}{c}\right)^2 = 1$

$v = \frac{c\sqrt{\gamma^2-1}}{\gamma}$

$P_x' = 0 \ ; \ P_x' = \gamma^*\left(P_x - v_0 \frac{E}{c^2}\right) = 0$

$$v_0 = \frac{P_1 \cdot c^2}{E} = \frac{m \cdot v_1 \cdot \gamma \cdot c^2}{T_e + 2 m_p c^2} = \frac{m c^2 \cdot c\sqrt{\gamma^2-1} \cdot \cancel{\gamma}}{(m_p c^2 \gamma + m_p c^2)\cancel{\gamma}} =$$

$$v_0 = \frac{c\sqrt{(\gamma-1)}\ \cancel{\sqrt{\gamma+1}}}{\sqrt{(\gamma+1)^2}} = \boxed{c\sqrt{\frac{\gamma-1}{\gamma+1}}} \Rightarrow \text{hitrost težišča}$$

$$\gamma^* = \frac{1}{\sqrt{1-\left(\frac{v_0}{c}\right)^2}} = \frac{1}{\sqrt{1-\frac{\gamma-1}{\gamma+1}}} = \frac{1}{\sqrt{\frac{2}{\gamma+1}}} = \boxed{\sqrt{\frac{\gamma+1}{2}}}$$

Lahko izračunamo $P_1 = \sqrt{\left(\frac{E}{c}\right)^2 - \frac{m_p^2 c^4}{c^2}} = \sqrt{\frac{2 m_p^2 c^4}{c^2} - \frac{m_p^2 c^4}{c^2}} = \underline{m_p \cdot c^2}$

$$P_1 = 3{,}1 \cdot 10^{-6}\ \frac{MeV\, s}{m} = 3{,}13\ \frac{eV \cdot s}{m} = 5{,}008 \cdot 10^{-19}\ \frac{Js}{m}$$

$\frac{kg\ m}{s}$

Sedaj lahko izračunamo hitrost težišča

$$v_0 = \frac{P_1 \cdot \cancel{c}^2}{3 m_p \cancel{c}^2} = \frac{5 \cdot 10^{-19}\ c^2}{3 \cdot 938 \cdot 10^6\ eV} = \underline{0{,}33c}$$

$\gamma^* = \underline{1{,}06}$

Vrnemo Lorentza

$$P_1' = \gamma^*\left(P_1 - \frac{v_0 \cdot E_1}{c^2}\right) = \gamma^*\left(P_1 - \frac{0{,}33c\, 2 m_p c^2}{c^2}\right) = 1{,}06\left(5 \cdot 10^{-19} - 1{,}65 \cdot 10^{-19}\right) = 3{,}6 \cdot 10^{-19}$$

$E_1' = \gamma^*(E_1 - v_0 \cdot P_1) = 2{,}7 \cdot 10^{-10}\ J$ $\qquad P_1' = -P_2'$

$P_2' = \gamma^*\left(P_2 - \frac{v_0 \cdot E_2}{c^2}\right) = -\gamma^* \frac{v_0 \cdot E_2}{c^2}$ $\qquad E_1' = E_2'$

$E_2' = \gamma^*(E_2 - v_0 P_2) -$

$P'_{x_3} = P'_1 \cdot \cos\varphi^*$ $\qquad$ $P'_{x_4} = P'_2 \cos\varphi^* = -P'_1 \cos\varphi^*$

$P'_{y_3} = P'_1 \sin\varphi^*$ $\qquad$ $P'_{y_4} = P'_2 \sin\varphi = -P'_1 \sin\varphi^*$

$E'_1 = E'_3$ $\qquad$ $E'_2 = E'_4$

$$P_{x_3} = \gamma^*\left(P'_{x_1} + \frac{v_0 \cdot E'_1}{c^2}\right) = \gamma^*\left(P'_1 \cdot \cos\varphi^* + \frac{v_0 \cdot E_1}{c^2}\right)$$

$P_{y_3} = P'_{y_3} = P'_1 \cdot \sin\varphi^*$ $\qquad$ $\varphi^* \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right] \in [57°, 58°]$

$\cos\varphi^* = \sin\left(\varphi^* + \frac{\pi}{2}\right)$

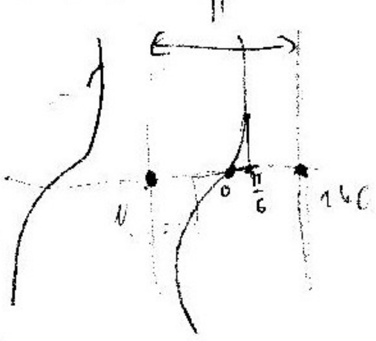
$$\frac{\sqrt{3}}{3} = \tan\varphi_1 = \frac{P'_1 \cdot \sin\varphi^*}{\gamma^*\left(P'_1 \cos\varphi^* + \underbrace{\frac{v_0 E'_1}{c^2}}_{A}\right)}$$

$$\frac{3P'_1 \cdot \sin\varphi^*}{\gamma^*(P_1 \cos\varphi^* + A)} = \sqrt{3} \Rightarrow \sqrt{3}\,\gamma^*(P_1 \cos\varphi^* + A) = 3P_1 \sin\varphi^*$$

$$\sqrt{3} \cdot \gamma^* P'_1 \sin\left(\frac{\pi}{2} + \varphi^*\right) - 3P'_1 \sin\varphi^* = -\sqrt{3} \cdot \gamma^* \cdot A$$

$$\sqrt{3} \cdot \gamma^* \sin\left(\varphi^* + \frac{\pi}{2}\right) - 3\sin\varphi^* = -\frac{\sqrt{3} \cdot \gamma^* \cdot A}{P'_1}$$

$\varphi^* = 57°$



$$P_{x_4} = -\gamma^*\left(P'_{x_4} + \frac{v_0 \cdot E'_2}{c^2}\right) = +\gamma^*\left(-P'_1 \cos\varphi + \frac{v_0 \cdot E'_2}{c^2}\right)$$

$P_{y_4} = +P'_{y_4} = -P'_1 \sin\varphi^*$ $\qquad$ 1461590

$$\tan\varphi_2 = \frac{P'_1 \cdot \sin\varphi^*}{\gamma^*\left(\frac{v_0 E'_1}{c} - P_1 \cos\varphi^*\right)} \Rightarrow \varphi_2 = 20,3°$$

$P_3 = P_4 \cdot \frac{\sin\varphi_2}{\sin\varphi_1}$; $P_1 = P_4 \underbrace{\left(\frac{\sin\varphi_2 \cos\varphi_1}{\sin\varphi_1} + \sin\varphi_1\right)}_{B}$

$$P_4 = \frac{P_1}{B} = 760\ \frac{MeV}{c}$$

$$P_1 = \sqrt{\left(\frac{E}{c}\right)^2 - m^2c^2} = \sqrt{\frac{4E_0^2}{c^2} - \frac{m^2c^4}{c^2}} = 1624,6\ \frac{MeV}{c}$$

$T_3 = E_3 - E_0$ $\qquad$ $P_4^2 = \left(\frac{E}{c}\right)^2 - \frac{m^2c^4}{c^2}$

$T_4 = E_4 - E_0$ $\qquad$ $c^2P_4^2 + m^2c^4 = E_4^2 \Rightarrow E_4 = 1198\ MeV$

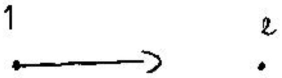
$T_4 = 200\ MeV$

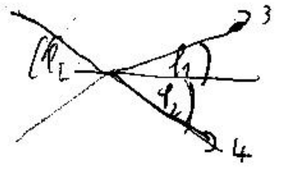
42) $T_p = E_0 = 938{,}27\,MeV$

$E_p = 2E_0 = 1877\,MeV$

$\varphi_1 = 30°$

$T_3 = ?$





1) ENERGIJA

$E_1 + E_2 = E_3 + E_4$

$T_1 + E_0 + E_0 = T_3 + T_4 + 2E_0$

$T_1 = T_3 + T_4$

2) GIBALNA KOLIČINA

$P_1 + \overset{=0}{P_2} = P_3 + P_4$

$X:\ P_1 = P_3 \cdot \cos\varphi_1 + P_4 \cdot \cos\varphi_2$

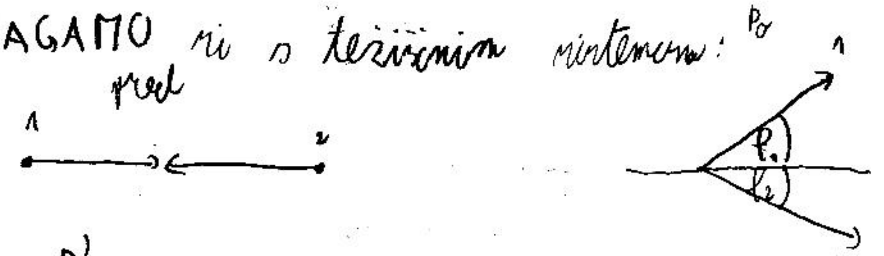
$y:\ P_3 \sin\varphi_1 = P_4 \cdot \sin\varphi_2$

3) Četverec invariante

$$E^2 = c^2 P_1^2 + m^2c^4 \Rightarrow P_1 = \sqrt{\left(\frac{E}{c}\right)^2 - \left(\frac{mc^2}{c}\right)^2} = \frac{1}{c}\sqrt{E^2 - m^2c^4} = \frac{1}{c}\sqrt{3E_0^2}$$

$$\Rightarrow P_1 = 1625 \cdot \frac{MeV}{c}$$

POMAGAMO si s težiščnim sistemom:



$P_1' = -P_2'$

$E_1' = E_2'$

- $P_1' = \gamma^*\left(P_1 - v_0 \cdot \frac{E_1}{c^2}\right) = 6\cancel{55}\ \frac{MeV}{c}$
- $E_1' = \gamma^*\left(E_1 - v_0 \cdot P_1\right) = 1179\,MeV$

$\rightsquigarrow v_0 = \frac{P_1 \cdot c^2}{W_1} = \frac{P_1 \cdot c^2}{3E_0} = \frac{P_1 c^2}{3 \cdot E_0} = 1{,}7 \cdot 10^8\ \frac{MeV\ m}{MeV\ s} = 1{,}7 \cdot 10^8\ \frac{m}{s} = 0{,}58\,c$

$\rightsquigarrow \gamma^* = \frac{1}{\sqrt{1-\left(\frac{v_0}{c}\right)^2}} = 1{,}22$

- $P'_{3x} = P'_1 \cdot \cos\varphi^*$
- $P'_{3y} = P'_1 \cdot \sin\varphi^*$
- $E'_3 = E'_1$

- $P'_{4x} = P'_2 \cos\varphi^* = -P'_1 \cos\varphi^*$
- $P'_{4y} = P'_2 \sin\varphi' = -P'_1 \sin\varphi^*$
- $E'_4 = E'_2$

$\rightsquigarrow P_{3x} = \gamma^*\left(P'_{3x} + v_0 \frac{E'_3}{c^2}\right) = \gamma^*\left(P'_1 \cos\varphi^* + v_0 \frac{E'_1}{c^2}\right)$

$P_{3y} = P'_{3y} = P'_1 \sin\varphi^*$

1762140

$\rightsquigarrow \operatorname{tg}\varphi_1 = \frac{+P'_1 \cdot \sin\varphi^*}{\gamma^*\left(+P'_1 \cos\varphi^* + v_0 \frac{E'_1}{c^2}\right)} = \frac{\sqrt{3}}{3}$

1546652

Poglejmo iz mat. vidika malo $\quad -1 \leq \cos\varphi \leq 1$

$P'_1 \cos\varphi^* + v_0 \frac{E'_1}{c^2} \neq 0 \Rightarrow$ Ni je pol

685157 $\quad P_4 = 827{,}7 \frac{MeV}{c}$

Iščemo φ^*: $\cos\varphi^* = \sin\left(\frac{\pi}{2} + \varphi^*\right)$

$P_3 = 797{,}3$

$\varphi^* = 69^\circ$

7791685

6590538

$\operatorname{tg}\varphi_2 = \frac{-P'_1 \cdot \sin\varphi^*}{\gamma^*\left(v_0 \frac{E'_2}{c^2} - P'_1 \cos\varphi^*\right)}$

~~$\varphi_2 = -50^\circ = 310^\circ$ ali 130°~~ $\quad \boxed{\varphi_2 = 29^\circ}$

$\frac{\operatorname{tg}\varphi_1}{\operatorname{tg}\varphi_2} = \frac{(1+\cos\varphi')}{(1-\cos\varphi')}$

$\operatorname{tg}\varphi_1 - \operatorname{tg}\varphi_1 \cos\varphi' = \operatorname{tg}\varphi_2 + \operatorname{tg}\varphi_2 \cos\varphi'$

- $P_3 = P_4 \cdot \frac{\sin\varphi_2}{\sin\varphi_1}$

$\frac{\operatorname{tg}\varphi_1 - \operatorname{tg}\varphi_2}{\operatorname{tg}\varphi_2 + \operatorname{tg}\varphi_1} = \cos\varphi' =$

$P_1 = P_4\left(\frac{\sin\varphi_2 \cos\varphi_1}{\sin\varphi_1} + \cos\varphi_2\right) \Rightarrow P_4 = \frac{P_1}{A} = 876{,}5 \frac{MeV}{c}$ $\quad \boxed{\varphi' = 88{,}9^\circ}$

$\boxed{T_4 = 313 \text{ MeV}}$ $\quad T_4 = \underline{\underline{306 \text{ MeV}}}$?

44) $T_{\pi^-} = 100\ MeV$ $\qquad E_{\pi_0^-} = 139{,}567\ MeV$

$T_e = ?$ $\qquad p_{\bar{\nu}} = \frac{E_\nu}{c}$

1) ENERGIJA

$$E_{\pi^-} = E_{e^-} + E_{\bar{\nu}}$$

2) GIBALNA

$$p_{\pi^-} = p_{e^-} - p_{\bar{\nu}}$$

$$p_{\pi^-} = \sqrt{\left(\frac{E}{c}\right)^2 - \frac{m^2c^4}{c^2}} - \frac{E_\nu}{c}$$

$$\frac{E_\nu}{\cancel{c}} = \sqrt{\left(\frac{E_{\pi^-}}{\cancel{c}}\right)^2 - \frac{m_\pi^2c^4}{c^2}} + \sqrt{\left(\frac{E_e}{\cancel{c}}\right)^2 - \frac{m_e^2c^4}{\cancel{c}^2}}$$

$$E_{\pi^-} = E_{e^-} + \sqrt{\left(\frac{E_e}{c}\right)^2 - \frac{m_e^2c^4}{c^2}} - \sqrt{\left(\frac{E_{\pi^-}}{c}\right)^2 - \frac{m_\pi^2c^4}{c^2}}$$

$$E_{\pi^-} - E_e + \sqrt{E_\pi^2 - E_{\pi_0}^2} = \sqrt{E_e^2 - E_{e_0}^2} \quad /^2$$

$$E_{\pi^-}^2 + \cancel{E_e^2} + E_\pi^2 - E_{\pi_0}^2 - 2E_{\pi}E_{e^-} + 2E_\pi\sqrt{E_\pi^2 - E_{\pi_0}^2} - 2E_{e^-}\sqrt{E_\pi^2 - E_{\pi_0}^2} = \cancel{E_e^2} - E_{e_0}^2$$

$$E_{e_0}^2\ 2E_\pi^2 - E_\pi^2 + 2E_\pi\sqrt{E_\pi^2 - E_{\pi_0}^2} = 2E_{e^-}\sqrt{E_\pi^2 - E_{\pi_0}^2}$$

$T_e = \underline{\underline{276\ MeV}}$

45) $E_\gamma = 3GeV$ $E_{p_0} = 938,27 MeV$

1) ENERGIJA

$$E_\gamma + E_{p_0} = n E_{\pi^0}$$

$$n = \frac{E_\gamma + E_{p_0}}{E_{\pi^0}}$$

2) GIBALNA KOLIČINA

$$P_\gamma + \overset{=0}{P_p} = n \cdot P_{\pi^0} \; ; \; p = 0$$

$$\frac{E_\gamma}{c} = P_\gamma = n \cdot P_{\pi^0} = n\sqrt{\left(\frac{E_\pi}{c}\right)^2 - \frac{m_{\pi^0}^2 c^4}{c^2}}$$

$$\cancel{E_\gamma} = \frac{\cancel{E_\gamma} \cdot E_{p_0}}{E_{\pi^0}} \sqrt{\left(\frac{E_{\pi^0}}{c}\right)^2 - \frac{m_{\pi^0}^2 c^4}{c^2}} \quad /^2$$

$$E_{\pi^0}^2 = E_{p_0}^2 \cdot E_{\pi^0}^2 - E_{p_0} \cdot E_{\pi_0}$$

$$E_{p_0}^2 \cdot E_{\pi_0}^2 = E_{\pi_0}^2 \left(E_{p_0}^2 - 1\right) \Rightarrow E_{\pi_0} = \frac{E_{p_0} \cdot E_{\pi_0}}{\sqrt{E_{p_0}^2 - 1}} =$$

46) $T_e = ?$

$$e^- + e^- \to e^- \bar{\mu} + \mu$$

$E_{e_0} = 0,51\ MeV$

$E_\mu = 105,7\ MeV$

$$\frac{(E_e + E_0)^2}{c^2} - p_e^2 = \left(\frac{E_\mu + 2E_{\mu_0}}{c^2}\right)^2 =$$

$$E_e^2 + 2E_e \cdot E_0 + E_0^2 - p_e^2 c^2 = (2 m_\mu c^2)^2$$

$$\cancel{E_e^2} + 2E_e E_0 + E_0^2 - \cancel{E_e^2} + E_0^2 = 4 m_\mu^2 c^4$$

$$E_e = \frac{4m^2c^4 - 2E_0^2}{2E_0} = \underline{\underline{44247\ MeV}}$$

$$T_e = E_e - E_0 \approx \underline{44\ GeV}$$

Da lahko

Napačen neutrino:

Uporabiti treba

$$\left(\frac{E_e + E_0}{c}\right)^2 - p_e^2 = (E_{0_0} + 2E_{\mu_0})$$

$$\boxed{E_e = 45\ MeV}$$ ✓

47) $\left(E_1 + E_2\right)^2 - \left(p_1 + p_2\right)^2 c^2 = \left(2 m_\mu c^2\right)^2$ $\qquad E_1 = E_2$

$$\cancel{E_1^2 + 2E_1 E_2 + E_2^2 - p_1^2 + 2p_1 p_2 - p_2^2 = \left(2 m_\mu c^2\right)^2}$$

$$\cancel{4E^2 = \left(2 m_\mu c^2\right)^2}$$

$$\cancel{E = \frac{2 \cdot 105{,}7}{2} =}$$

45) $\gamma p \rightarrow p \cdot \pi^0 \cdot \pi^0$

$E_\gamma = 3\,GeV$

$$\left(E_\gamma + m_p c^2\right)^2 - \left(p_e \cdot c\right)^2 = \left(m_p c^2 + N \cdot m_{\pi^0} c^2\right)^2$$

$$\sqrt{\left(E_\gamma + m_p c^2\right)^2 - E_\gamma^2} = m_p c^2 + N m_{\pi^0} c^2$$

$$\frac{\sqrt{2 E_\gamma m_p c^2 + m_p^2 c^4} - m_p c^2}{m_{\pi^0} c^2} = N$$

$N \approx 12$

48) $E_{p_0} = 938,3\ \text{MeV}$

$E_{\pi^\pm} = 139,6\ \text{MeV}$

$E_{\pi^0} = 135\ \text{MeV}$

$E_\gamma = ?$

$\gamma\, p \to p\, \pi^0 \pi^+ \pi^-$

$$\left(\frac{E_\gamma}{c} + \frac{E_{p_0}}{c}\right)^2 - \left(p_\gamma\right)^2 = \left(m_p \cdot c^2 + m_{\pi^0} c^2 + 2 m_{\pi^\pm} c^2\right)^2$$

$$\left(E_\gamma + m_{p_0} c^2\right)^2 - \left(E_\gamma\right)^2 = \left(m_p \cdot c^2 + m_{\pi^0} \cdot c^2 + 2 m_{\pi^\pm} c^2\right)^2$$

$$\cancel{E_\gamma^2} - \cancel{E_\gamma^2} + 2 E_\gamma \cdot m_{p_0} c^2 + m_{p_0}^2 c^4 = (\ldots)$$

$$E_\gamma = \frac{(\ldots)^2 - m_{p_0}^2 c^4}{m_{p_0} \cdot c^2}$$

$$E_\gamma = \underline{\underline{506\ \text{MeV}}}$$

49) a) $p + p \rightarrow p + n + \pi^+$

$T_p = ?$

$m_{\pi^+} \cdot c^2 = 139{,}6\ MeV$

$m_p \cdot c^2 = 938{,}3\ MeV$

$m_n \cdot c^2 = 939{,}6\ MeV$

$\rightsquigarrow (E_p + m_p c^2)^2 - (P_p c)^2 = (m_p c^2 + m_n c^2 + m_{\pi^+} c^2)^2$

$P_p \cdot c = \sqrt{E_p^2 - m^2 \cdot c^4}$

$\rightsquigarrow (E_p + m_p \cdot c^2)^2 - (E_p^2 - m_p^2 c^4) = (\ldots)^2$

$\cancel{E_p^2} + 2 E_p \cdot m_p \cdot c^2 + m_p^2 c^4 - \cancel{E_p^2} + m_p^2 c^4 = (\ldots)^2$

$2 E_p \cdot m_p \cdot c^2 = (\ldots)^2 - 2 m_p^2 c^4$

$$E_p = \frac{(\ldots)^2 - 2 m_p^2 \cdot c^4}{2 m_p \cdot c^2} = 1231\ MeV$$

$T_p = E_p - E_{p_0} \simeq \underline{\underline{292{,}4\ MeV}}$ ✓

b) $p + n \rightarrow p + p + \pi^-$

$(E_p + m_n c^2)^2 - (P_p c)^2 = (2 m_p c^2 + m_\pi \cdot c^2)^2$

$2 E_p \cdot m_n c^2 + m_n^2 c^4 + m_p^2 \cdot c^4 = (\ldots)^2$

$$E_p = \frac{(\ldots)^2 - m_n^2 \cdot c^4 - m_p^2 \cdot c^4}{2 m_n \cdot c^2} = 1224\ MeV$$

$T_p = E_p - m_p c^2 = \underline{\underline{287\ MeV}}$ ✓

50) $e^- e^+ \to \Psi$ $\qquad e^- \rightarrow \quad \leftarrow e^+$

a) $E_\Psi = 3{,}1\,GeV$

$E_{e^-} = E_{e^+}$

$$(2E)^2 = (m_e c^2 + \Psi)^2$$

$E = \underline{\underline{1550\,MeV}} \approx T_e$ ✓ $\qquad E_0 \lll E_e$

b) $e^- \longrightarrow \qquad e^+ \qquad \dots \qquad e^+ \cdot \Psi \rightarrow$

$$(E_{e^-} + m_e c^2)^2 - (E_{e^-}^2 - m_e^2 c^4) = (m_e c^2 + \Psi)^2$$

$$2E_{e^-} m_e c^2 + 2m_e^2 c^4 = (\dots)^2$$

$$E_{e^-} = \frac{(\dots)^2 - 2m_e^2 c^4}{2 m_e c^2} = 9518\,GeV$$

$T_{e^-} \approx \underline{\underline{9518\,GeV}}$ ✓

36) $\gamma + e^- \rightarrow e^- + (e^- + e^+)$

$$(E_\gamma + m_e c^2)^2 - E_\gamma^2 = (3 m_e c^2)^2$$

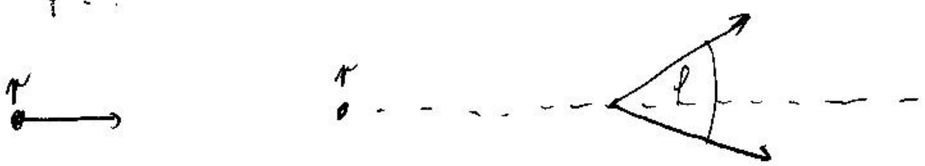
$$2 E_\gamma m_e c^2 + m_e^2 c^4 = 9 m_e^2 c^4$$

$$E_\gamma = 4 m_e c^2 = \underline{\underline{2{,}04 \text{ MeV}}}$$ ✓

51) $T = 1\,GeV$

$T_3 = 0{,}5\,GeV$

$\varphi = ?$



1) ENERGIJA

$E_2 + E_1 = E_3 + E_4$

$T_1 + 2m_p c^2 = T_3 + T_4 + 2m_p \cdot c^2$

$T_1 = T_3 + T_4$

$T_4 = 0{,}5\,GeV$

$p_3 = \sqrt{\left(\frac{E_3}{c}\right)^2 - \frac{m_p^2 \cdot c^4}{c^2}}$

$\parallel$

$p_4 = \sqrt{\left(\frac{E_4}{c}\right)^2 - \frac{m_p^2 \cdot c^4}{c^2}}$

$p_1 = \sqrt{\left(\frac{E_1}{c}\right)^2 - \frac{m_p^2 c^4}{c^2}}$

GIBALNA KOLIČINA

$x:\ p_1 = p_3 \cdot \cos\varphi + p_4 \cdot \cos\varphi$

$y:\ p_3 \cdot \sin\varphi = p_4 \cdot \sin\varphi$

$E_3 = E_4 = E'$

$E_1^2 - m_p^2 c^4 = 4\cos^2\varphi\left(E'^2 - m_p^2 \cdot c^4\right)$

$\cos\varphi = \sqrt{\frac{E_1^2 - m_p^2 c^4}{\left(E'^2 - m_p^2 c^4\right) 4}} = \sqrt{\frac{2876600}{4753200}} = 0{,}77$

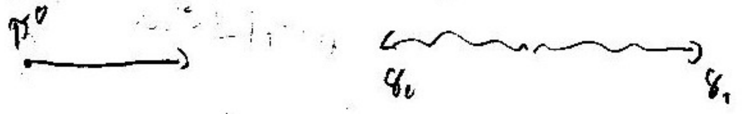
$\varphi = 40^\circ$

$\alpha = 2\varphi = 80^\circ$

52) $T_{\pi^0} = 6\,GeV$ $\quad m_{\pi^0} c^2 = 135\,MeV$

a) $E_{\gamma_1} = ?$

$E_{\gamma_2} = ?$



1) ENERGIJA

$E_{\pi^0} = E_{\gamma_1} + E_{\gamma_2}$

2) GIBALNA

$p_{\pi^0} = p_{\gamma_1} - p_{\gamma_2} = \frac{E_{\gamma_1}}{c} + \frac{E_{\gamma_2}}{c}$

$p_{\pi^0} c + E_{\gamma_2} = E_{\gamma_1}$

$\Rightarrow E_{\pi^0} = p_{\pi^0} c + 2E_{\gamma_2}$ $\qquad p_{\pi^0} c = \sqrt{E_{\pi^0}^2 - m_{\pi^0}^2 c^4}$

$$\frac{E_{\pi^0} - \sqrt{E_{\pi^0}^2 - m_{\pi^0}^2 c^4}}{2} = E_{\gamma_2} = 0{,}74\,MeV$$

$$E_{\gamma_1} = E_{\pi^0} - E_{\gamma_2} = 6134\,MeV$$

b) Če odletita simetrično je $E_{\gamma_1} = E_{\gamma_2} = E_\gamma$

$$2E_\gamma = E_{\pi^0}$$

$$\frac{E_{\pi^0}}{2} = E_\gamma = 3068\,MeV$$

c) $\varphi = ?$

$$p_1 = 2p_\gamma \cos\varphi \Rightarrow \cos\varphi = \frac{p_1}{2p_\gamma} \Rightarrow \varphi = 1{,}6°$$

55) $T_{\pi^+} = 100\,MeV$ $\quad m_{\mu}c^2 = 105{,}7\,MeV$

$T_{\mu^+} = 80\,MeV$ $\quad m_{\pi^+}c^2 = 139{,}6\,MeV$

$\varphi_1 = ?$

$\varphi_2 = ?$

E_ν

1) Energije

$$E_{\pi^+} = E_{\mu^+} + E_\nu \Rightarrow E_{\pi^+} - E_\mu = E_\nu$$

$$T_{\pi^+} + m_{\pi^+}c^2 - T_{\mu^+} - m_\mu c^2 = E_\nu = \underline{\underline{53{,}9\,MeV}}$$

2) GIBALNA KOLIČINA

$$x: \; P_{\pi^+} = P_\nu \cos\varphi_1 + P_\mu \cos\varphi_2$$

$$y: \; P_\nu \sin\varphi_1 = P_\mu \sin\varphi_2$$

$$P_{\pi^+} = \sqrt{\left(\frac{E_\pi}{c}\right)^2 - \frac{m^2c^4}{c^2}} = 194{,}7\,\frac{MeV}{c}$$

$$P_{\mu^+} = 152{,}7\,\frac{MeV}{c}$$

Poglejmo v težišče

$$v_0^* = \frac{c^2 \cdot P_{\pi^+}}{E_{\pi^+}} = \frac{c^2 \cdot P_{\pi^+}}{T + m_\pi c^2} = \frac{194{,}7\,MeV \; c^2}{c\,(100\,MeV + 139{,}6\,MeV)} = \underline{\underline{0{,}81\,c}}$$

$$\gamma_1^* = \frac{1}{\sqrt{1 - \left(\frac{v^*}{c}\right)^2}} = 1{,}7$$

$$E_{\pi^+}^* = E_\nu^* + E_{\mu^+}^*$$

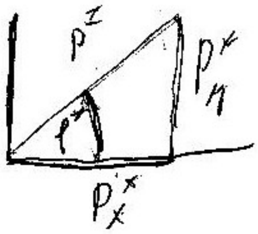
$$0 = P_\nu^* + P_{\mu^+}^*$$

$$E_{\pi^+}^* = \gamma^*\left(E_{\pi^+} - v_*^* \cdot P_\pi\right) = 139{,}2\,MeV$$

$$E_{\mu^+} = \gamma^*\left(E_{\mu^+} - v^* \cdot P_{\mu^+}\right) = 105{,}4\,MeV$$

$$E_\nu = 33{,}8\,MeV$$

$P_{\nu}^{*} = -P_{\mu^+}^{*}$



$P_{\mu}^{*} = \gamma^{*}\left(P_{\mu} - v_0 \frac{E_{\mu}}{c}\right) = 3,8 \ \frac{MeV}{c}$

$P_{\nu}^{*} = -3,8 \ \frac{MeV}{c}$

$P_{x_\mu}^{*} = 0,76 \cdot m_{\pi} \cdot c \cdot \cos\varphi^{*}$ $\qquad P_{x_\nu}^{*} = -0,24 \cdot m_{\pi} \cdot c \cdot \cos\varphi^{*}$

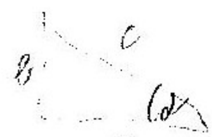
$P_{y_\mu}^{*} = 0,76 \cdot m_{\pi} \cdot c \cdot \sin\varphi^{*}$ $\qquad P_{y_\nu} = -0,24 \cdot m_{\pi} \cdot c \cdot \sin\varphi^{*}$

$P_{x_\mu} = \gamma^{*}\left(P_{x_\mu}^{*} + \frac{v_0 E_{\mu}^{*}}{c^2}\right) =$ $\qquad P_{x_\nu} = +\gamma^{*}\left(P_{x_\nu}^{*} + \frac{v_0 \cdot E_{\nu}^{*}}{c^2}\right)$

$P_{y_\mu} = P_{y}^{*} = 0,76 \cdot m_{\pi} \cdot c \cdot \sin\varphi^{*}$ $\qquad P_{y_\nu} = -P_{y_\nu}$

$$\mathrm{tg}\,\varphi_1 = \frac{0,76\, m_{\pi} \cdot c \cdot \sin\varphi^{*}}{\gamma^{*}\left(0,76\, m_{\pi} \cdot c \cdot \cos\varphi^{*} + 85,37 \frac{MeV}{c}\right)}$$

$$\mathrm{tg}\,\varphi_2 = \frac{-0,24 \cdot m_{\pi} \cdot c \cdot \sin\varphi^{*}}{\gamma^{*}\left(v_0 \cdot \frac{E_{\nu}^{*}}{c^2} - 0,24\, m_{\pi}\, c \cos\varphi^{*}\right)}$$



$\sin\alpha = \frac{a}{c}$

$\cos\alpha = \frac{b}{c}$

$P_{\mu^+}^{*2} = P_{x_\mu}^{2} + P_{y_\mu}^{2} = 2 \cdot 0,76^2\, m_{\pi}^2 \cdot c^2 : 150 \ \frac{MeV}{c}$

$\sin\varphi^{*} = \frac{P_x^{*}}{P^{*}} = \frac{0,76\, m_{\pi}\, c \cos\varphi^{*}}{P^{*}}$

$\mathrm{tg}\,\varphi^{*} = \frac{0,76\, m_{\pi} \cdot c}{P^{*}} \Rightarrow 35^{\circ} = \varphi^{*}$

$\underline{\varphi_1 = 17,7^{\circ}}$ ✓

$\sin\varphi_2 = \frac{P_{\mu}}{P_{\nu}} \cdot \sin\varphi_1$

$\varphi_2 = \underline{34,7^{\circ}}$ ✓

56) $T_{\pi^+} = 100\,MeV$

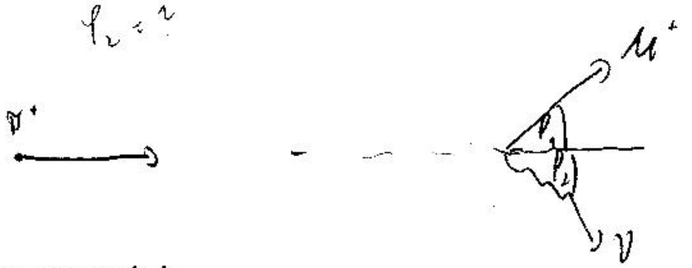
$\varphi_1 = 10^\circ$

$T_\mu = ?$

$T_\nu = ?$

$\varphi_2 = ?$

$$p_1 c = \sqrt{E_{\pi^+}^2 - m_\pi^2 c^4} = 194\,MeV$$



1) ENERGIJA

$E_{\pi^+} = E_\mu + E_\nu$

2) GIBALNA

$p_{\pi^+} = p_{\mu^+} + p_\nu$

$x: p_{\pi^+} = p_\mu \cos\varphi_1 + p_\mu \cdot \cos\varphi_2$

$y: p_\mu \cdot \sin\varphi_1 = p_\nu \cdot \sin\varphi_2$

V težišču

$$v^* = \frac{c^2 \cdot p_1}{T_{\pi^+} + m_\pi c^2} = \underline{0{,}81\,c} \quad , \quad \gamma^* = 1{,}7$$

$$E^*_{\pi^+} = E^*_\mu + E^*_\nu$$

$$0 = p^*_\mu + p^*_\nu$$

$$p_1^\mu = \left(\frac{E_\pi}{c}, p_\pi\right), \quad p_2^\mu = \left(\frac{E_\mu}{c}, p_\mu\right), \quad p_3^\mu = \left(\frac{E_\nu}{c}, p_\nu\right)$$

$$\left(\frac{E_\pi}{c}\right)^2 - p_\pi^2 = \left(\frac{E_\mu}{c} + \frac{E_\nu}{c}\right)^2 - (p_\mu + p_\nu)^2$$

$$\cancel{\left(\frac{E_\pi}{c}\right)^2} - \cancel{\left(\frac{E_\pi}{c}\right)^2} + \frac{m_\pi^2 c^4}{c^2} = \cancel{\left(\frac{E_\mu}{c}\right)^2} + 2\frac{E_\mu \cdot E_\nu}{c^2} + \cancel{\left(\frac{E_\nu}{c}\right)^2} - p_\mu^2 - 2 p_\mu \cdot p_\nu \cdot \cos\vartheta - \cancel{\left(\frac{E_\nu}{c}\right)^2}$$

$$p_\mu^2 = \cancel{\left(\frac{E_\mu}{c}\right)^2} + \frac{m_\mu^2 c^4}{c^2}$$

$$\frac{m_\pi^2 c^4}{c^2} = 2\frac{E_\mu \cdot E_\nu}{c^2} + \frac{m_\mu^2 c^4}{c^2} - 2\frac{E_\nu \sqrt{E_\mu^2 - m_\mu^2 c^4}}{c^2} \cos\vartheta$$

$$\frac{m_\pi^2 c^4}{c^2} - \frac{m_\mu^2 c^4}{c^2} = \frac{2E_\nu}{c^2}\left(E_\mu - \sqrt{E_\mu^2 - m_\mu^2 c^4}\cos\vartheta\right)$$

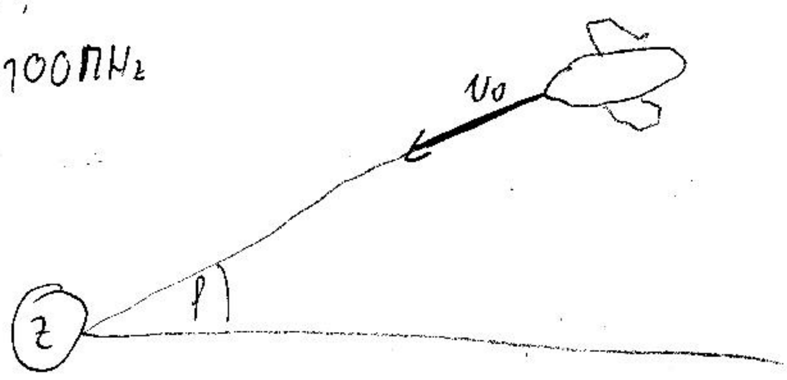
62) $v_0 = 0{,}6c$

$\vartheta = 30^\circ$

$\nu' = ?$

$\vartheta' = ?$

$\nu = 100\,MHz$



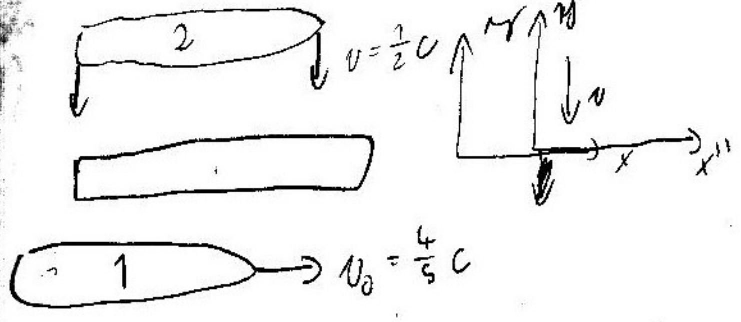
$$\nu' = \frac{\nu}{\sqrt{1-\beta^2}}\left(1 - v_0 \cdot \cos\vartheta / c\right)$$

$$\nu' = \nu\,\frac{1-\beta\cdot\cos\vartheta}{\sqrt{1-\beta^2}} = \underline{\underline{190\,MHz}}$$

$$\operatorname{tg}\vartheta' = \frac{\sin\vartheta}{\gamma(\cos\vartheta - \beta)} =$$

$$\vartheta' = \underline{\underline{15{,}26^\circ}}$$

Primer iz Strnada pri vesoljskih ladjah



$t'' = \gamma\left(t - \frac{v\cdot y}{c^2}\right)$

$x'' = x$

$y'' = \gamma(y - v\cdot t)$

$t = \gamma\left(t'' + \frac{v\cdot y}{c^2}\right)$

$x = x''$

$y = \gamma(y'' + v\cdot t'')$

Prehod iz S v S''

Prehod iz S v S'

$t' = \gamma\left(t - \frac{v_0\cdot x}{c^2}\right)$ $\qquad t = \gamma\left(t' + \frac{v_0\cdot x}{c^2}\right)$

$y = y'$

$x' = \gamma(x - v_0\cdot t)$ $\qquad x = \gamma(x' + v_0\cdot t)$

$t' = \gamma_0\left(\gamma\left(t'' + \frac{v\cdot y}{c^2}\right) - \frac{v_0\cdot x''}{c^2}\right)$

$x' = \gamma_0\left(x'' - v_0\gamma\left(t'' + \frac{v\cdot y}{c^2}\right)\right)$

$y' = \gamma(y'' + v\, t'')$

v S'' prvo krajišče miruje pri $x''_a = 0,\ y''_a = 0$

$x''_l = l\ ,\ y''_l = 0$

$t' = \gamma_0\left[\gamma\left(t'' + \frac{v\cdot 0}{c^2}\right) - \frac{v_0 x''}{c^2}\right] \Rightarrow$

$t' = \gamma\cdot\gamma_0\cdot t'' - \frac{\gamma_0\cdot v_0 x''}{c^2} \Rightarrow t'' = \frac{t' + \frac{\gamma_0\cdot v_0\cdot x''}{c^2}}{\gamma\cdot\gamma_0}$

$x' = \gamma_0 x'' - v_0\cdot\gamma\cdot t''\cdot\gamma_0 = \gamma x'' - \frac{v_0\cdot\left(t' + \frac{\gamma_0\cdot v_0 x''}{c^2}\right)}{\gamma_0} = -\frac{v_0\cdot t\cdot\gamma_0}{\gamma_0} = -v_0\cdot t$

$y' = \gamma\cdot v\cdot t'' = v\cdot\frac{t' + \frac{\gamma_0\cdot v_0\cdot x''}{c^2}}{\gamma_0} = \boxed{\frac{v\cdot t'}{\gamma_0} = y'}$

$\boxed{x'_a = -v_0 t}$

$y'_l = v_0 v l/c^2 + v t'/\gamma_0$

$\boxed{x'_l = \frac{l}{\gamma_0} - v_0\cdot t}$

$$v_x' = \frac{dx'}{dt'} = -v_0 \qquad v_y = \frac{dy'}{dt'} = \frac{v}{\gamma_0}$$

$$= \frac{\gamma_0 x'' - \gamma_0 \gamma v_0 t''}{\gamma_0 \gamma t'' - \gamma_0 v_0 x''/c^2} = \frac{\gamma v t''}{\gamma_0 \gamma t'' - \gamma_0 v_0 x''/c^2} = \frac{v}{\gamma_0}$$

$$= \frac{-\gamma v_0 t''}{\gamma t''} = -v_0 \qquad t'' = \frac{v_0 x''}{c^2 \gamma}$$

$$\tan\theta' = \frac{v_x'}{v_y} = -\frac{v_0 \gamma_0}{v}$$

$$x' = \gamma_0 [x'' - v_0 \gamma (t'' + 0)]$$

$$x' = \gamma_0 \left[x'' - \frac{v_0 \gamma \cdot v_0 x''}{c^2 \gamma}\right]$$

$$y' = \gamma v \frac{v_0 x''}{c^2 \gamma} = \frac{v_0 v x''}{c^2}$$

3) KVANTNA FIZIKA

str. 12 / 1 – 10
str. 13 / 11 – 18
str. 14 / 19 – 27
str. 15 / 28 – 33
str. 16 / 34 – 41
str. 17 / 42 – 50
str. 18 / 51 – 57
str. 19 / 58 – 61
str. 20 / 63 – 70

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1) $\lambda = 588{,}995\ nm$

$\lambda_c = 0{,}0024\ nm$

$\vartheta = 180°$

$N = ?$

sipana vpadne

$\Delta\lambda = \lambda_1 - \lambda = \lambda_c(1-\cos\vartheta)$

$= \lambda_1 - \lambda = \lambda_c$

$\lambda = \lambda_1 - \lambda_c = 588{,}9926\ nm$

$N = \frac{\lambda}{\Delta\lambda} = \frac{588{,}995}{0{,}0048} \cong \underline{1{,}2 \cdot 10^5}$

max. razmik bo ko je $\vartheta = 180°$
$(\cos 180 = -1)$

---

2) λ =

$W_{el} = e \cdot U$

$\lambda = \lambda_c = 0{,}0024\ nm$

$U = ?$

$\lambda = \frac{c \cdot h}{e_0 \cdot U} \Rightarrow U = \frac{c \cdot h}{e_0 \cdot \lambda} \cong \underline{\underline{516\ keV}}$

3) $\vartheta = 180^\circ$

$E_\gamma = ?$

$$E_\gamma + mc^2 = E_\gamma + W_0$$

$$E_\gamma + mc^2 = \frac{E_\gamma}{2} + W_0$$

$$\frac{h\cdot c}{\lambda_1} + mc^2 = \frac{h\cdot c}{\lambda_1\cdot 2} + W_0$$

$$\lambda_1 - \lambda_2 = \lambda_c\,(1 - \cos\vartheta)$$

$$\lambda_1 = \frac{h\cdot c}{E_\gamma}$$

$$\lambda_2 = \frac{h\cdot c\cdot 2}{E_\gamma}$$

$$\frac{2h\cdot c}{E_\gamma} - \frac{2\,h\cdot c}{E_\gamma} = \lambda_c\cdot(1+1) \qquad /\cdot E_\gamma$$

$$2h\cdot c - 2\cdot h\cdot c = 2\lambda_c\cdot E_\gamma$$

$$E_\gamma = \frac{h\cdot c}{2\lambda_c} \approx \underline{\underline{258\ \text{keV}}}$$

4) $\lambda = 0,154\ nm$

$\vartheta = 30°$

$\varphi = ?$

$T_e = ?$

$E_{\gamma_1} = \frac{h \cdot c}{\lambda} = 8036\ eV$

$E_{\gamma_2} = 8020\ eV$

$\rightsquigarrow \frac{h \cdot c}{\lambda} + m \cdot c^2 = \frac{h \cdot c}{\lambda_1} + E_e \quad ; \quad \lambda_1 - \lambda = \lambda_c (1 - \cos\vartheta)$

$E_{\gamma_1} + m \cdot c^2 = E_{\gamma_2} + E_e \qquad \lambda_1 = \lambda_c (1 - \cos\vartheta) + \lambda$

$\lambda_1 = 0,15432\ nm$

$T_e = E_{\gamma_1} - E_{\gamma_2} = \underline{\underline{16\ eV}}$

GIBALNA

$\longrightarrow x: \frac{h}{\lambda} = \frac{h}{\lambda_1} \cdot \cos\vartheta - p_e \cos\varphi$

$y: \frac{h}{\lambda_1} \cdot \sin\vartheta = p_e \cdot \sin\varphi \Rightarrow p_e = \frac{h}{\lambda} \frac{\sin\vartheta}{\sin\varphi}$

$\frac{h}{\lambda} = \frac{h}{\lambda_1} \cdot \cos\vartheta - \frac{h}{\lambda_1} \sin\vartheta \cdot \operatorname{ctg}\varphi$

$\frac{\frac{h}{\lambda} - \frac{h}{\lambda_1} \cos\vartheta}{\frac{h}{\lambda_1} \cdot \sin\vartheta} = \operatorname{ctg}\varphi = 0,27$

$\operatorname{tg}\varphi = \frac{1}{\operatorname{ctg}\varphi} = \frac{1}{0,27} \Rightarrow \varphi \cong \underline{\underline{75°}}$ ✓

5) $E_\gamma = 255\,keV$

$\underline{\vartheta = 180^\circ}$

$v = ?$

Lahko izračunamo prvo vpadno val. dolžino

$$E_\gamma = \frac{h\cdot c}{\lambda} \Rightarrow \lambda = \frac{h\cdot c}{E_\gamma} = 4{,}85\,10^{-12}\,m \quad ; \quad \lambda_c = 0{,}0024\,nm = 2{,}4\cdot 10^{-12}\,m$$

$$\lambda_1 - \lambda = \lambda_c\,(1-\cos\vartheta)$$

$$\lambda_1 = 2\lambda_c + \lambda = 9{,}65\cdot 10^{-12}\,m \Rightarrow E_{\gamma_2} = \frac{h\cdot c}{\lambda_1} \doteq 128\,keV$$

$$E_{\gamma_1} + \cancel{mc^2} = E_{\gamma_2} + T_e + \cancel{m_e c^2}$$

$$E_{\gamma_1} - E_{\gamma_2} = T_e = 127\,keV$$

$$T_e = mc^2(\gamma - 1) \Rightarrow \gamma = \frac{T_e + m_e c^2}{m_e c^2} = 1{,}25$$

$$\gamma^2 = \frac{1}{1-\beta^2} \Rightarrow \gamma^2 - \gamma^2\beta^2 = 1$$

$$\beta = \sqrt{\frac{-1+\gamma^2}{\gamma^2}}$$

$$v = \underline{0{,}6\,c}$$

$A = F \cdot n$

6) $B = 0{,}002\,T$ $\qquad U = I \cdot R$ $\qquad B = \frac{F}{I \cdot l}$ $\qquad \frac{m}{n} e T m = e \frac{N}{A} \frac{m}{n}$

$\vartheta = 90°$ $\qquad T = \left[\frac{N}{A \cdot m}\right]$ $\qquad = \frac{e\,J}{e} = J$

$r = 2\,cm$ $\qquad eV$

$\lambda = ?$

$p_e c = e \cdot r \cdot B c = 12\ keV$

$E_\gamma + m_0 c^2 = E_\gamma' + T_e + m_e c^2$ $\qquad 1\,eV \qquad 1{,}6 \cdot 10^{-19}\,eV$

$6{,}6 \cdot 10^{-24}\,J$

$p_e c = 0{,}000$

$\frac{h \cdot c}{\lambda} = \frac{h \cdot c}{\lambda_1} + T_e$; $\lambda_1 - \lambda_1 = \lambda_c$

$\lambda_1 = \lambda + \lambda_c$

$\frac{h \cdot c}{\lambda} = \frac{h \cdot c}{\lambda + \lambda_c} + T_e$; $(p_e \cdot c)^2 + m^2 c^4 = E^2$

$h c (\lambda + \lambda_c) = h c \lambda + T_e \cdot \lambda(\lambda + \lambda_c)$ $\qquad E = \sqrt{(p_e \cdot c)^2 + m^2 c^4} =$

$h c \lambda + h c \lambda_c = h c \lambda + T_e \lambda^2 + T_e \lambda \lambda_c$ $\qquad E_e = \sqrt{2{,}60244 \cdot 10^{11}} = 510141\ eV$

$h c \lambda - h c \lambda = T_e \lambda^2 + T_e \cdot \lambda \cdot \lambda_c - h c \lambda_c$ $\qquad T_e = \underline{141\,eV} = 0{,}14\,keV$

$T_e \lambda^2 + T_e \cdot \lambda \cdot \lambda_c - h c \lambda_c = 0$

$D = (T_e \lambda_c)^2 + 4 \cdot h c T_e \cdot \lambda_c$

$$\lambda_1 = \frac{-T_e \cdot \lambda_c + \sqrt{(T_e \cdot \lambda_c)^2 + 4 h c T_e \cdot \lambda_c}}{2}$$

$$\lambda_1 = \frac{-141\,eV \cdot 0{,}0024 \cdot 10^{-9} + \sqrt{(141 \cdot 0{,}0024 \cdot 10^{-9})^2 + 4 \cdot 6{,}6 \cdot 10^{-34}\,Js \cdot 3 \cdot 10^8 \frac{m}{s} \cdot 141\,eV \cdot 0{,}0024 \cdot 10^{-9}}}{2 \cdot 141}$$

$\lambda_1 = \underline{\underline{0{,}144\,nm}}$

7) $E_\gamma = 1\,MeV$

$n = 1,33$

$\sin\vartheta = \left(\frac{c}{n}\right)dt / v\,dt = \frac{c}{nv}$

$$E_\gamma = \frac{h\cdot c}{\lambda} \Rightarrow \lambda = \frac{h\cdot c}{E_\gamma} = 1,2375\cdot 10^{-12}\,m$$

$v \to c$

$v = \frac{c}{n} \quad /:c$

$\frac{v}{c} = \beta = \frac{1}{n}$

$$\lambda_1 - \lambda = \lambda_c(1-\cos\vartheta)$$

1) ENERGIJA

$$\frac{h\cdot c}{\lambda} + \cancel{mc^2} = \frac{hc}{\lambda_1} + T_e + \cancel{mc^2}$$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$\gamma = \frac{1}{\sqrt{1-\frac{1}{n^2}}} = 1,5$

2) GIBALNA

$x: p_\gamma = p_\gamma'\cdot\cos\vartheta + p_e\cdot\cos\varphi$

$y: p_\gamma'\sin\vartheta = p_e\sin\varphi$

$E = mc^2\cdot\gamma = \boxed{0,76\,MeV}$

$E_\gamma + mc^2 - E_e = E_\gamma'$

$$E_\gamma + mc^2 = E_\gamma' + mc^2(\gamma - 1) = E_\gamma' + mc^2\gamma - mc^2$$

$\boxed{E_\gamma' = 0,75\,MeV}$

$$\boxed{E_\gamma = E_\gamma' + \frac{m\cdot c^2}{\sqrt{1-\frac{1}{n^2}}} - mc^2}$$

$p_\gamma' = 1,2\cdot 10^{-13}$

$p_\gamma = 1,6\cdot 10^{-13}$

$p_e = 8,96\cdot 10^{-14}$

$$E_\gamma' = p_\gamma'\cdot c \Rightarrow p_\gamma' = \frac{E_\gamma'}{c} \quad ; \quad p_e = \sqrt{\left(\frac{E}{c}\right)^2 - \frac{m^2c^4}{c^2}}$$

$$p_\gamma^2 - 2p_\gamma p_\gamma'\cdot\cos\vartheta + p_\gamma'^2\cos^2\vartheta = p_e^2\cos^2\varphi$$
$$p_\gamma'^2\sin^2\vartheta = p_e^2\sin^2\varphi$$

$$p_e^2 = p_\gamma^2 - 2p_\gamma p_\gamma'\cos\vartheta + p_\gamma'^2$$

$$\boxed{\cos\vartheta = \frac{p_\gamma'^2 - p_e^2 + p_\gamma^2}{2p_\gamma p_\gamma'}}$$

Kot ki se odkloni foton

$\underline{\vartheta = 34^\circ}$ ✓

$$p_\gamma^2 - 2p_\gamma p_e\cos\varphi + p_e^2\cos^2\varphi = p_\gamma'^2\cos^2\vartheta$$

$$p_\gamma'^2 = p_\gamma^2 - 2p_\gamma p_e\cos\varphi + p_e^2$$

$$\boxed{\cos\varphi = \frac{p_e^2 + p_\gamma^2 - p_\gamma'^2}{2p_\gamma p_e}}$$

$\varphi = \underline{48^\circ}$ ✓

8) $\lambda_B = \lambda_C$

$v = ?$

$p = m \cdot v \qquad p = \gamma\, m \cdot v$

$\lambda = \frac{h}{m \cdot v}\sqrt{1-\beta^2} \quad / : c$

$\lambda_C = \lambda_B$

$\frac{h}{m c} = \frac{h}{m \cdot v}\sqrt{1-\beta^2}$

$\beta = \sqrt{1-\beta^2} \quad /^2 \Rightarrow 2\beta^2 = 1$

$\beta^2 = \frac{1}{2} \Rightarrow \beta = \sqrt{\frac{1}{2}} \Rightarrow v = \underline{\underline{2{,}12 \cdot 10^8 \frac{m}{s}}}$

---

9) $\lambda_1 = 0{,}071\ nm$

$\vartheta = 45°$

$p_e = ?$

$U_{min} = ?$

$\lambda_1 - \lambda = \lambda_C (1 - \cos\vartheta)$

$\lambda_1 - \lambda_C(1-\cos\vartheta) = \lambda = 0{,}0703\ nm$

$E_\gamma = E_\gamma' + T_e \Rightarrow T_e = E_\gamma - E_\gamma' = \frac{hc}{\lambda} - \frac{hc}{\lambda_1} = 17{,}603\ keV - 17{,}430\ keV$

$T_e = 0{,}173\ keV$

$E_e = 510{,}173\ keV$

$p_e = \frac{\sqrt{E^2 - m^2c^4}}{c} = \underline{\underline{13{,}3 \frac{keV}{c}}}$ ✓

$\lambda_1 = \frac{c \cdot h}{e_0 \cdot U} \Rightarrow U = \frac{c \cdot h}{e_0 \cdot \lambda_1} = \underline{\underline{17{,}4\ kV}}$ ✓

OBRATNI COMPTONOV POJAV

$$p_1 + \frac{h\nu_1}{c} = p - \frac{h\cdot\nu}{c} \;/\cdot c\,; \quad W_1 + h\nu_1 = W + h\nu$$

$$p_1\cdot c + h\cdot\nu_1 = pc - h\cdot\nu$$

$$p_1 c + \cancel{h\cdot\nu_1} - W_1 - \cancel{h\nu_1} = p\cdot c - h\nu - W - h\nu$$

$$p_1\cdot c = W_1 - (W - pc + 2h\cdot\nu) \;\;/^2$$

$$(p_1\cdot c)^2 = (W_1 - (W - p c + 2h\cdot\nu))^2 \;; \quad c^2p^2 = W^2 - m^2c^4$$

$$(cp_1)^2 = W_1^2 - m^2c^4$$

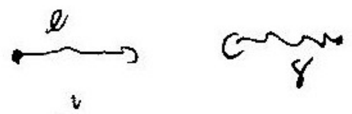
$$\cancel{W_1^2} - m^2c^4 = \cancel{W_1^2} - 2W_1(W - p\cdot c + 2h\cdot\nu) + (W - pc + 2h\nu)^2$$

$$-m^2c^4 = -2W_1\cdot W + 2W_1\cdot pc - 4h\nu\cdot W_1 + W^2 + (pc)^2 + 4(h\nu)^2$$

$$W - W_1 = \frac{2h\cdot\nu(cp - h\nu)}{W - cp + 2h\nu}$$

$$h\nu\,\frac{W+cp}{W-cp} = h\nu\,\frac{1+(1-m^2c^4/W^2)^{\frac{1}{2}}}{1-(1-m^2c^4/W^2)^{\frac{1}{2}}} \approx h\cdot\nu\left(\frac{2W}{mc^2}\right)^2 \text{ ili } \lambda_1 = \lambda\left(\frac{mc^2}{2W}\right)^2$$

S'



$$W = \gamma_0\cdot mc^2$$

11) $\lambda = 435{,}8\ nm$

$E_i = 1{,}9\ eV$

$E = ?$

$$E_{Hg} = \frac{hc}{\lambda} = 2{,}8\ eV$$

$$E_{Hg} = E_{me} + E_i \Rightarrow E_{me} = \underline{0{,}9\ eV}$$

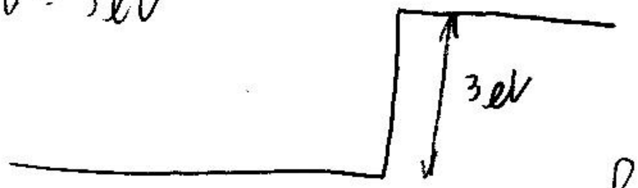
12)

λ [nm]	U [V]	h [Js]
254	1,95	$2{,}6 \cdot 10^{-34}$
313	0,98	$1{,}6 \cdot 10^{-34}$
365	0,5	$9{,}7 \cdot 10^{-35}$
405	0,14	$3 \cdot 10^{-35}$

$$\lambda = \frac{c \cdot h}{e \cdot U} \Rightarrow \boxed{h = \frac{\lambda \cdot e \cdot U}{c}}$$

23) $E = 10\,eV$ $\qquad V < E$

$V = 3\,eV$



$$k_1 = \frac{\sqrt{2m\cdot E}}{\hbar} = 1{,}7\cdot 10^{10}$$

$$k_2 = \frac{\sqrt{2m(E-V)}}{\hbar} = 1{,}4\cdot 10^{10}$$

odbojnost

$$R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2}$$

$$R = \underline{\underline{0{,}009}}$$

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24) $E = 40\,eV$ $\qquad n_2 = 1$

$V = 10\,eV$

$n_k = 1$

$$k_1 = \sqrt{2mE}/\hbar = 3{,}4\cdot 10^{10}$$

$$k_2 = \sqrt{2m(E-V)}/\hbar = 2{,}9\cdot 10^{10}$$

$$\frac{(k_1-k_2)^2}{(k_1+k_2)^2} = \frac{(n_k-n_2)^2}{(n_k+n_2)^2}$$

$$(k_1-k_2)(n_k+n_2) = (n_2-n_1)(k_1+k_2)$$

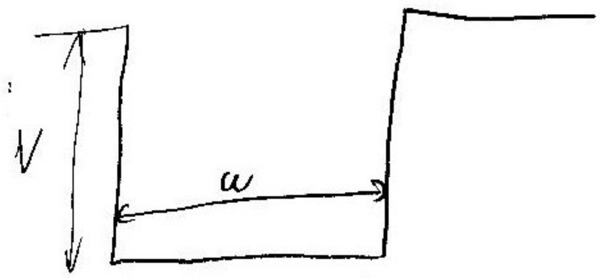
$$\cancel{k_1 n_2} + k_1 n_1 - k_2 n_2 - \cancel{k_2 n_1} = \cancel{n_2 k_1} + n_2 k_2 - n_1 k_1 - \cancel{n_1 k_2}$$

$$2 k_1 n_1 = 2 n_2 k_2$$

$$n_2 = n_1 \frac{k_1}{k_2} = \underline{\underline{1{,}17}}$$

25) $a = 0,1\ nm$

$\underline{V = 10\ eV}$



$$T = \frac{1}{1 + \frac{1}{4}\left(k_2/k_1 - k_1/k_2\right)^2 \sin^2(k_2 a)}$$

Za prepustnost velja $\boxed{k_2\, a = n \cdot \pi}$

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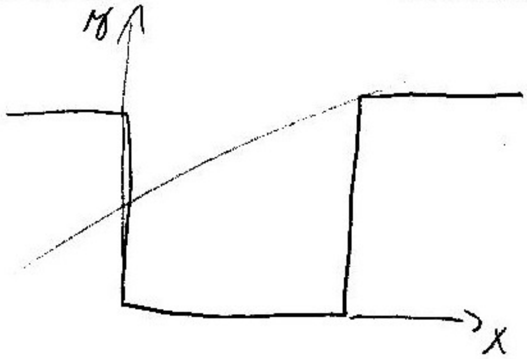
26) $N = 50\%$

$T = 3\ eV$

$V = 5\ eV$

$\underline{x = 0,3\ nm}$

$T = ?$



$\psi_{k_1} = a_1 \cdot e^{ik_1x} + b_1 \cdot e^{-ik_1x}$; ψ

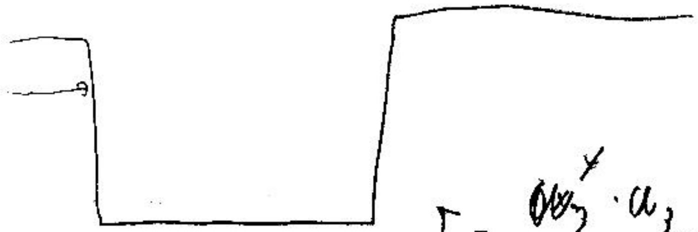
26) $A = 500 \frac{e}{n}$

$E = 3eV$

$V = 5eV$

$x_0 = 0{,}3\,nm$

$A_T = ?$



$$T = \frac{a_3^* \cdot a_3}{a_1^2}$$

$$\psi_{k_1} = a_1 \cdot e^{ik_1x} + b_1 \cdot e^{-ik_1x}$$

$$\psi_{k_2} = a_2 \cdot e^{ik_2x} + b_2 \cdot e^{-ik_2x}$$

$$\psi_{k_3} = a_3 \cdot e^{ik_3x} + b_3 \cdot e^{-ik_3x} \qquad b_3 = 0$$

Nastavek: $\psi_{k_1}(0) = \psi_{k_2}(0)$

1) $\boxed{a_1 + b_1 = a_2 + b_2}$

$\psi_{k_2}(x_0) = \psi_{k_3}(x_0)$

2) $\boxed{a_2 \cdot e^{ik_2x_0} + b_2 e^{-ik_2x_0} = a_3 \cdot e^{ik_3x_0}} + b_3 \cdot e^{-ik_3x_0}$

$b_2 e^{-ik_2x_0} = a_3 e^{ik_3x_0} - a_2 e^{ik_2x_0}$

$a_2 e^{ik_2x_0} = a_3 e^{ik_3x_0} - b_2 e^{-ik_2x_0}$

$$\left(\frac{\partial \psi_{k_1}}{\partial x}\right)_{x=0} = a_1 i k_1 - b_1 i k_1$$

$$\left(\frac{\partial \psi_{k_2}}{\partial x}\right)_{x=0} = a_2 i k_2 - i b_2 k_2$$

$a_1 i k_1 - b_1 i k_1 = a_2 k_2 i - i b_2 k_2 \quad / : i$

3) $\boxed{k_1 (a_1 - b_1) = k_2 (a_2 - b_2)}$

$a_1 k_1 - b_1 k_1 = k_2 a_2 - k_2 b_2$

$$\left(\frac{\partial \psi_{k_2}}{\partial x}\right)_{x=x_0} = \left(\frac{\partial k_3}{\partial x}\right)_{x=x_0}$$

$$\boxed{a_2 \cdot k_2 \cdot e^{ik_2x_0} - b_2 \cdot k_2 e^{-ik_2x_0} = a_3 \cdot k_3 \cdot e^{ik_3x_0}}$$

$$\Rightarrow a_2 k_2 \cdot e^{ik_2x_0} - k_2 a_3 \cdot e^{ik_3x_0} + k_2 a_2 \cdot e^{ik_2x_0} = a_3 k_3 e^{ik_3x_0}$$

$$2 a_2 \cdot k_2 \cdot e^{ik_2x_0} = a_3 \cdot e^{ik_3x_0} (k_3 + k_2)$$

$$\boxed{a_2 = \frac{a_3 \cdot e^{ix_0(k_3 - k_2)} (k_3 + k_2)}{2 k_2}}$$

$$a_3 k_2 \cdot e^{ik_3x_0} - 2 k_2 b_2 e^{-ik_2x_0} = a_3 k_3 e^{ik_3x_0}$$

$$2 k_2 b_2 \cdot e^{-ik_2x_0} = a_3 e^{ik_3x_0} (k_2 - k_3) \Rightarrow \boxed{b_2 = \frac{a_3 e^{ix_0(k_3 + k_2)} (k_2 - k_3)}{2 k_2}}$$

$$b_1 = a_2 + b_2 - a_1 \qquad k_3 = k_1$$

$$\underline{a_1 \cdot k_1} - a_2 k_1 - b_2 k_1 + \underline{a_1 \cdot k_1} = k_2 a_2 - k_2 b_2$$

$$2 k_1 a_1 = a_2 (k_2 + k_1) - b_2 (k_2 - k_1)$$

$$2 k_1 a_1 = \frac{a_3 \cdot e^{i x_0 (k_3 - k_2)} (k_3 + k_2)(k_2 + k_1)}{2 k_2} - \frac{a_3 e^{i x_0 (k_3 + k_2)} (k_2 - k_3)(k_2 - k_1)}{2 k_2}$$

$$a_3 = \frac{4 k_1 \cdot k_2 \cdot a_1}{e^{i x_0 (k_3 - k_2)} \cdot (k_3 + k_2)(k_2 + k_1) - e^{i x_0 (k_3 + k_2)} (k_2 - k_3)(k_2 - k_1)}$$

$a_3^* \Rightarrow$ pri e spremeniš v potenci predznak $\quad e^{i x_0 (k_3 - k_2) - i x_0 (k_3 + k_2)} = e^{-2 i x_0 k_2}$

$$a_3^* a_3 = \frac{16 (k_1 k_2 a_1)^2}{[(k_3 + k_2)^2 (k_2 + k_1)]^2 - (k_2 - k_3)^2 (k_2 - k_1)^2 - 2 (k_3 + k_2)(k_2 + k_1)(k_2 - k_3)(k_2 - k_1) e^{2 x_0 i k_2}}$$

$$e^{-i k_2 x_0 \cdot 2} = \cos(2 k_2 x_0) + 2 \sin(k_2 x_0) \qquad V < 0$$

$$e^{i\varphi} = \cos\varphi + i \sin\varphi$$

$$\boxed{T = \frac{a_3^* a_3}{a_3}} \qquad k_1 = \frac{\sqrt{2 m \cdot W}}{\hbar} = 9{,}3 \cdot 10^9$$

$$k_2 = \frac{\sqrt{2 m (W - V_0)}}{\hbar} = 1{,}52 \cdot 10^{10}$$

$$T = \frac{a_3^* \cdot a_3}{a_3} = \frac{1}{1 + \frac{1}{4} \left(\underbrace{\frac{k_2}{k_1}}_{1{,}63} - \underbrace{\frac{k_1}{k_2}}_{0{,}61} \right)^2 \sin^2(k_2 x_0)} = 0{,}99$$

$$A_2 = 0{,}99 \cdot 500 = 495 \, \frac{e}{s}$$

b) $\frac{T = 1}{E = ?}$

$$T = \frac{1}{1 + \frac{1}{4}\left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)^2 \sin^2(k_2 x_0)}$$

To bo 1 ko bo $\sin^2(k_2 x_0) = 0$

$$k_2 x_0 = n \cdot \pi \quad , n = 1, 2$$

$$\frac{\sqrt{2m(E-V)}}{\hbar} x_0 = n \cdot \pi \quad /^2$$

$$E = \frac{n^2 \pi^2 \hbar^2}{2 x_0^2 m} + |V|$$

$$E = 3{,}76\,eV + 5\,eV = \underline{\underline{8{,}76\,eV}}$$

27) $x_0 = 0{,}41\ nm$

$E = 0{,}7\ eV$

$V = ?$

$$T \to 1 \quad : \quad T = \frac{1}{1 - \frac{1}{4}\left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)^2 \sin^2(k_2 x_0)}$$

$$, k_2 x_0 = n \cdot \pi$$

$$\frac{2m(E-V)}{\hbar^2} x_0^2 = \pi^2 \Rightarrow E - \frac{\pi^2 \hbar^2}{2 m x_0^2} = V = -1{,}32\ eV$$

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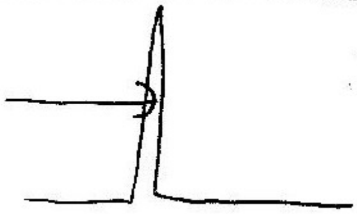
28) $E = 10\ eV$

$V = 12\ eV$



a) $x_0 = 0{,}1\ nm$

$V > E$

$$K_2 = \frac{k_2}{i} = \frac{\sqrt{2m(E-V)}}{\hbar i} = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$k_2 = \frac{\sqrt{2m(E-V)}}{\hbar}$$

$k_1 = 1{,}7 \cdot 10^{10}$ $\qquad 7{,}6 \cdot 10^{9}$

$$T_1 = \frac{a_3^* \cdot a_3}{a_1^2} = \frac{1}{1 + \underbrace{\frac{1}{16}\left(\frac{K_2}{k_1} + \frac{k_1}{K_2}\right)^2}_{0{,}45}\left(e^{-K_2 x_0} - e^{K_2 x_0}\right)^2} = 44\%$$

b) $x_0 = 1\ nm$

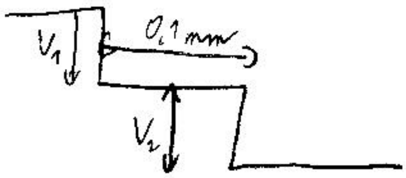
$T = 10^{-6}$

c) $x_0 = 1\ \mu m$

$T = 0$

29) $\underline{R = ?}$

$V_1 = 25 \quad V_2 = 50$



$V = 100\,V$

$E = 100\,eV$

$$k_1 = \frac{\sqrt{2m \cdot E}}{\hbar} = 5{,}4 \cdot 10^{10}$$

$$k_2 = \frac{\sqrt{2m(E - V_1)}}{\hbar} = 4{,}67 \cdot 10^{10}$$

$$k_3 = \frac{\sqrt{2m\,50\,eV}}{\hbar} = 3{,}8 \cdot 10^{10}$$

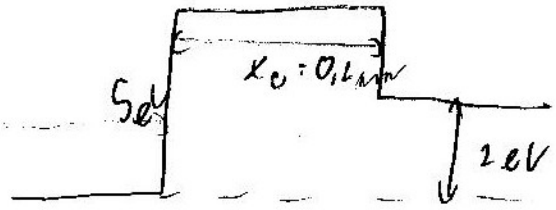
$$T = \frac{4 \cdot k_1 \cdot k_2^2 \cdot k_3}{\underbrace{(k_1 k_2 + k_2 k_3)^2}_{1846} - (k_1^2 - k_2^2)(k_2^2 - k_3^2) \underbrace{\sin^2(k_2 x_0)}_{0{,}35}} = 0{,}97$$

-1790

$$R = 1 - T = \underline{\underline{0{,}03}}$$

30) $\underline{E = 10\,eV}$

$T = ?$



$$k_1 = \frac{\sqrt{2mE}}{\hbar} = 1{,}7\cdot 10^{10}$$

$$k_2 = \frac{\sqrt{2m(E-5eV)}}{\hbar} = 1{,}2\cdot 10^{10}$$

$$k_3 = \frac{\sqrt{2m\,8eV}}{\hbar} = 1{,}52\cdot 10^{10}$$

$$T = \frac{\overbrace{4\,k_1\cdot k_2^2\cdot k_3}^{\to\ 1{,}48\cdot 10^{41}}}{\underbrace{(k_1\cdot k_2 + k_2\cdot k_3)^2}_{1{,}493\cdot 10^{41}} - \underbrace{\underbrace{(k_1^2-k_2^2)}_{1{,}45\cdot 10^{20}}\underbrace{(k_2^2-k_3^2)}_{\cdot\,(-8{,}7\cdot 10^{19})}}_{2{,}2\cdot 10^{37}}\underbrace{\sin^2(k_2 a)}_{0{,}00175}} = \underline{\underline{0{,}95}}$$

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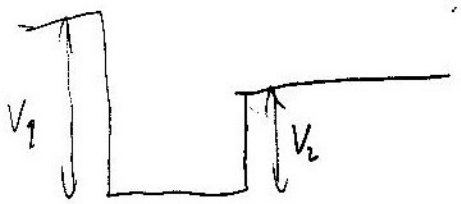
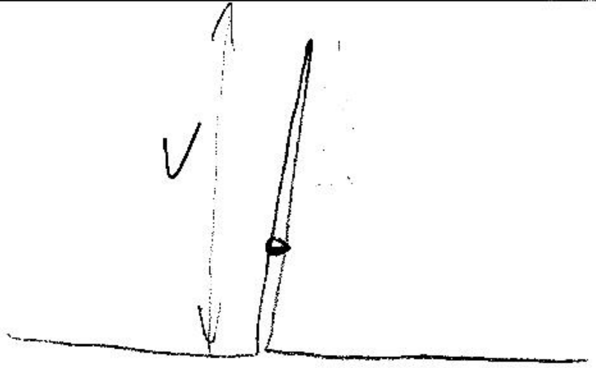
31)



32) $v = 2000 \frac{m}{s}$

$V \cdot x_0 = 10^{-3}$ eV·nm

$T = ?$



$V \cdot x_0$

$E = \frac{1}{2} m \cdot v^2 = 1{,}1 \cdot 10^{-5}\, eV \approx 10^{-5}\, eV$ $\qquad k_2 \gg k_1$

$$T = \frac{a_3^* \cdot a_3}{a_1^2} = \frac{1}{1 + \frac{1}{4}\left(i\frac{K_2}{k_1} + \frac{k_1}{i K_2}\right)^2 \left(e^{-K_2 x_0} - e^{K_2 x_0}\right)^2}$$

$$k_2 = i K_2 \Rightarrow K_2 = \frac{k_2}{i} = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$\lim_{k_1 \ll K_1} T = \lim_{k_1 \to K_2} \frac{1}{1 + \frac{1}{4}\left(i\frac{K_2}{k_1} - \frac{k_1}{i K_2}\right)^2 \sin i K_2 x_0}$$

$$= \frac{1}{1 + \frac{1}{16}\left(\frac{K_2}{k_1}\right)^2 \cdot \underbrace{\left(e^{-K_2 x_0} - e^{K_2 x_0}\right)}_{i \sin kK x_0 \ll 1 \quad x \ll 1}}$$

$$= \left[1 + \frac{1}{16} \frac{2m(V-E)}{\hbar \cdot k_1^2} \cdot \left(\left(K_2 \cdot x_0\right)^2\right)\right]^{-1}$$

$$= \left[1 + \frac{1}{8} \frac{m(V-E)\hbar^2}{\hbar^2 2m \cdot E} \cdot \frac{2m(V-E)}{\hbar^2} \cdot x_0^2\right]^{-1} = \left[1 + \frac{1}{2} \frac{E_0}{E} \frac{m x_0^2}{\hbar^2}\right]^{-1}$$

$= \underline{0{,}631}$

33) $E = 10\,eV$

$R = 1$

$T = 0$

$$k_1 = \frac{\sqrt{2mE}}{\hbar} = 1{,}7 \cdot 10^{10}$$

$$k_2 = \frac{\sqrt{2m(E-V}}{\hbar} = 1{,}9 \cdot 10^{10}$$

$$k_3 = i K_3 \Rightarrow K_3 = \frac{k_3}{i} = \frac{\sqrt{2m(E-V)}}{i\hbar} = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$T = \lim_{k_3 \to \infty} \frac{4 k_1 \cdot k_2 \cdot k_3}{(k_1 k_2 + k_2 k_3)^2 - (k_1^2 - k_2^2)(k_2^2 - k_3^2)\sin^2(k_2 a)} \begin{matrix} : k_3^4 \\ : k_3^4 \end{matrix}$$

$$\lim_{k_3 \to 0} \frac{\frac{4 k_1 \cdot k_2}{k_3}}{\left(\frac{k_1 k_2}{k_3^2} + \frac{k_2}{k_3}\right)^2 - \left(\frac{k_1^2}{k_3^2} - \frac{k_2^2}{k_3^2}\right)\left(\frac{k_2^2}{k_3^2} - 1\right)\sin^2(k_2 a)} = 0$$

$$\boxed{R = 1 - 0 = 1}$$ ✓

34) $m = 1\ kg$
$x_0 = 1\ cm$

a) $E = ?$

b) $E = 10^{-3}\ J =$
$n = ?$

$$\frac{J\,s^2}{kg\ m^2} = \frac{N^2 \cdot m^2 \cdot s^2}{kg\ m^2} = \frac{kg^2\, m^2\, m\, s^2}{kg\ m^2 \cdot s^4}$$

$$= \frac{kg\ m}{s^2} = N$$

$$E_1 = \frac{\hbar^2 \cdot \pi^2}{2 m x_0^2} = \underline{\underline{3 \cdot 10^{-39}}}$$

$$E_2 = \frac{\hbar^2 \cdot \pi^2 \cdot 4}{2 m x_0^2} = \frac{2 \hbar^2 \pi^2}{m x_0^2} = J$$

$$E_n = \frac{\pi^2 \cdot \hbar^2 \cdot n^2}{2 m x_0^2} \Rightarrow \boxed{n = \frac{\sqrt{E_n \cdot 2m}\, x_0}{\pi \cdot \hbar}} =$$

---

35) $E_1 = ?$
$x_0 = 0{,}3\ nm$

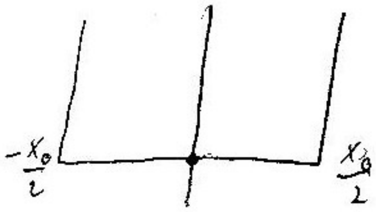
$$E_1 = \frac{\hbar^2 \cdot \pi^2}{2 m x_0^2} = \underline{3{,}76\ eV}$$

Valovna funkcija za osnovno stanje je:

$$\boxed{\Psi = \sqrt{\frac{2}{x_0}}\ \sin\left(\pi \frac{x}{x_0} + \frac{\pi}{2}\right)}$$

36) $2x_0 = 0{,}25$ nm

$\Psi(x) = A\left(x_0^6 - x^6\right)$



$\frac{x_0}{2} = 0{,}0625$

$P\left(-\frac{x_0}{2} < p < \frac{x_0}{2}\right)$

NORMIRAMO:

$$1 = \int_{-\infty}^{\infty} \Psi(x)\cdot\Psi^*(x)\,dx = A^2\int (x_0^6 - x^6)^2\,dx = A^2 \int_{-x_0}^{x_0} (x_0^{12} - 2x_0^6x^6 + x^{12})\,dx$$

$$A^2\left(\left[x_0^{12}\cdot x\right]_{-x_0}^{x_0} - \left.\frac{2x_0^6\cdot x^7}{7}\right]_{-x_0}^{x_0} + \left.\frac{x^{13}}{13}\right|\right)$$

$$A^2\left(2x_0^{13} - \frac{4x_0^{13}}{7} + \frac{2x_0^{13}}{13}\right)$$

$$A^2 x_0^{13}\left(\frac{2\cdot 13\cdot 7 - 52 + 14}{7\cdot 13}\right) = A^2 x_0^{13}\left(\frac{144}{91}\right) =$$

$$\boxed{A = \sqrt{\frac{91}{144\,x_0^{13}}}}$$

$$P = A^2\int_{-\frac{x_0}{2}}^{\frac{x_0}{2}} \left(x_0^{12} - 2x_0^6x^6 + x^{12}\right)dx = A^2\left(x_0^{12}\cdot x - \frac{2x_0^6\cdot x^7}{7} + \frac{x^{13}}{13}\right)\Bigg|_{-\frac{x_0}{2}}^{\frac{x_0}{2}}$$

$$= A^2\left(x_0^{13} - \frac{2x_0^{13}}{7\cdot 2^6} + \frac{x_0^{13}}{13\cdot 2^{12}}\right) = \frac{91\,\cancel{x_0^{13}}}{144\,\cancel{x_0^{13}}}\left(1 - \underbrace{\frac{2}{7\cdot 2^6} + \frac{1}{13\cdot 2^{12}}}_{\to 0}\right)$$

$$= \underline{\underline{0{,}63}}$$

37) $2x_0 = 1\,nm$

$\Psi_{(x)} = A(x_0^2 - x^2)$

$\langle E \rangle = ?$

$$1 = A\int_{-\infty}^{\infty} \Psi^*_{(x)} \cdot \Psi_{(x)}\, dx = A^2 \int_{-x_0}^{x_0} (x_0^4 - 2x_0^2x^2 + x^4)\, dx = A^2\left(x_0^4 x - \frac{2x_0^2x^3}{3} + \frac{x^5}{5}\right)\Big|_{-x_0}^{x_0}$$

$$= A^2\left(2x_0^5 - \frac{4x_0^5}{3} + \frac{2x_0^5}{5}\right) = A^2 x_0^5\left(\frac{30-20+6}{15}\right) \Rightarrow A^2 x_0^5 \frac{16}{15} = 1$$

$$\boxed{A = \sqrt{\frac{15}{16x_0^5}}}$$

$$\langle E \rangle = \left(\sqrt{\frac{15}{16x_0^5}}\right)^2 \int \Psi^*_{(x)} \cdot \hat{H} \cdot \Psi_{(x)}\, dx = A^2 \int_{-\frac{x_0}{2}}^{\frac{x_0}{2}} (x_0^2 - x^2) \cdot \frac{\hbar^2}{m}\, dx = *$$

$$\hat{H} = \hat{T} + \underset{0}{V} = \hat{T} = -\frac{\hbar}{2m}\frac{\partial^2 \Psi(x)}{\partial x^2}$$

$$\hat{H} = -\frac{\hbar^2 A}{2m}(-2) = \frac{\hbar A}{2m}$$

$$* = \frac{15\hbar^2}{16x_0^5 m}\int_{-\frac{x_0}{2}}^{\frac{x_0}{2}} (x_0^2 - x^2)\, dx = \frac{15\hbar}{16x_0^5 m}\left(x_0^2 x - \frac{x^3}{3}\right)\Big|_{-\frac{x_0}{2}}^{\frac{x_0}{2}}$$

$$= \frac{15\hbar^2 x_0^3}{16x_0^5 m}\left(\left(\frac{1}{2} + \frac{1}{2}\right) - \frac{1}{3}\left(\frac{1}{2^3} + \frac{1}{2^3}\right)\right)$$

$$\left(1 - \frac{1}{12}\right) = \frac{11}{12}$$

$$\langle E \rangle = \frac{15\hbar^2}{16x_0^2 m} \cdot \frac{11}{12} = \underline{0{,}24\, eV}$$

38) $2x_0 = 0{,}6\ nm$

$$\Psi(x) = \sqrt{\frac{1}{10x_0}}\left(\cos\frac{\pi x}{2x_0} + 3\left(\frac{\sin\pi x}{x_0}\right)\right)$$

a) $\langle\hat{p}\rangle = ?$

b) $\langle\hat{T}\rangle = ?$

$$\langle\hat{p}\rangle = \frac{1}{10x_0}\int_{-x_0}^{x_0}\left(\cos\frac{\pi x}{2x_0} + \frac{3\sin\frac{\pi x}{x_0}}{x_0}\right)\cdot\frac{\hbar}{i}\frac{\partial\Psi(x)}{\partial x}\,dx$$

$$\frac{\partial\Psi(x)}{\partial x} = \sqrt{\frac{1}{10x_0}}\left(-\sin\left(\frac{\pi x}{2x_0}\right)\cdot\frac{\pi}{2x_0} + 3\cos\left(\frac{\pi x}{x_0}\right)\cdot\frac{\pi}{x_0}\right)$$

$$\langle\hat{p}\rangle = \frac{1}{10x_0}\frac{\hbar}{i}\int_{-x_0}^{x_0}\left(\cos\frac{\pi x}{2x_0} + \frac{3\sin\frac{\pi x}{x_0}}{x_0}\right)\left(3\cos\left(\frac{\pi x}{x_0}\right)\cdot\frac{\pi}{x_0} - \sin\left(\frac{\pi x}{2x_0}\right)\frac{\pi}{2x_0}\right)dx$$

- $$\int_{-x_0}^{x_0} 3\cos\left(\frac{\pi x}{2x_0}\right)\cdot\cos\left(\frac{\pi x}{x_0}\right)\cdot\frac{\pi}{x_0}\,dx = \frac{3\pi}{2x_0}\int_{-x_0}^{x_0}\left(\cos\left(-\frac{\pi x}{2x_0}\right) + \cos\left(\frac{3\pi x}{2x_0}\right)\right)dx =$$

$$t = -\frac{\pi x}{2x_0} \qquad u = \frac{3\pi x}{2x_0}$$

$$-\frac{2x_0}{\pi}dt = dx \qquad \frac{2x_0\,du}{3\pi} = dx$$

$$= \frac{3\pi}{2x_0}\left(\int -\frac{2x_0}{\pi}\cos t\,dt + \frac{2x_0}{3\pi}\int\cos u\,du\right) = \frac{3\pi}{2x_0}\left(-\frac{2x_0}{\pi}\cdot\sin t + \frac{2x_0}{3\pi}\cdot\sin u\right)$$

$$= \frac{3\pi}{2x_0}\left(-\frac{2x_0}{\pi}\sin\left(-\frac{\pi x}{2x_0}\right) + \frac{2x_0}{3\pi}\left(\sin\frac{3\pi x}{2x_0}\right)\right)\Bigg|_{-x_0}^{x_0} = \frac{3\pi}{2x_0}\left[+\frac{4x_0}{\pi} - \frac{4x_0}{3\pi}\right] = [6-2] = \underline{\underline{4}}$$

$$-1-(1)$$

- $$\frac{3\pi}{2x_0}\int\left(\sin\frac{\pi x}{x_0}\cdot\sin\left(\frac{\pi x}{2x_0}\right)\right)dx = \frac{3\pi}{4x_0}\int\left(\cos\left(\frac{\pi x}{2x_0}\right) - \cos\left(\frac{3\pi x}{2x_0}\right)\right)dx$$

$$\frac{3\pi}{4x_0}\left(\int\cos\left(\frac{\pi x}{2x_0}\right)dx - \int\cos\left(\frac{3\pi x}{2x_0}\right)dx\right) = \frac{3\pi}{4x_0}\left(\frac{2x_0}{\pi}\sin\frac{\pi x}{2x_0} - \frac{2x_0}{3\pi}\cdot\sin\left(\frac{3\pi x}{2x_0}\right)\right)\Bigg|_{-x_0}^{x_0}$$

$$= \frac{3\pi}{4x_0}\left(\frac{4x_0}{\pi} - \frac{2x_0}{3\pi}(-1-1)\right) = 3\left(1+\frac{1}{3}\right) = 3+1 = \underline{\underline{4}}$$

$$\langle\hat{p}\rangle = \frac{\hbar}{10x_0 i}(4-4) = \underline{\underline{0}}$$

$$\boxed{\hat{T} = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}} \Rightarrow \text{Po tem izračunaš}$$

39) $\Psi = A \cdot \sin kx$

$k \cdot a = \frac{2\pi}{3}$

$E_1 = \frac{3}{4} \cdot V(x)$

Verjetnost da naletimo na delec zunaj jame je 0

$$1 = A^2 \int \sin^2(k \cdot x) = A^2 \frac{1}{2} \int (1 - \cos 2kx)\, dx = A^2 \frac{1}{2} \left(x - \frac{\sin 2kx}{2k} \right) \Big|_0^a$$

$$1 = A^2 \frac{1}{2} \left(a + \frac{\sqrt{3}}{2 \cdot 2} \right) = A^2 \left(\frac{1}{2} a - \frac{\sqrt{3}}{4} \right)$$

$$A^2 = \frac{1}{\left(\frac{1}{2} a - \frac{\sqrt{3}}{4} \right)}$$

$$P = A^2 \int_0^a \sin^2(kx) = \frac{1}{\cancel{\left(\frac{1}{2} a - \frac{\sqrt{3}}{4} \right)}} \cancel{\left(\frac{1}{2} a - \frac{\sqrt{3}}{4} \right)} = \underline{\underline{1}}$$

42) $\Psi = \psi_1 + 3\psi_3$

$a = 0{,}6\,nm$

$$\Psi(x) = A(\psi_1 + 3\psi_3)$$

$$1 = A^2\int_0^a (\psi_1 + 3\psi_3)(\psi_1^* + 3\psi_3^*)\,dx = A^2\int_0^a (\psi_1 + 3\psi_3)^2\,dx$$

$$= A^2\int_0^a (\psi_1^2 + 6\underbrace{\psi_1\psi_3}_{0} + 9\psi_3^2)\,dx = A^2(1+9) = 10A^2$$

$$A = \sqrt{\frac{1}{10}}$$

$$\psi_\perp = \alpha\psi_1 + \beta\psi_3$$

$$A\int_0^a (\psi_1 + 3\psi_3)(\alpha\psi_1 + \beta\psi_3)\,dx = A\int_0^a (\alpha\psi_1^2 + \psi_1\beta\psi_3 + \alpha 3\psi_1\psi_3 + 3\beta\psi_3^2)\,dx$$

$$A\int_0^a (\alpha\psi_1^2 + (\alpha + 3\beta)\psi_1\psi_3 + 3\beta\psi_3^2)\,dx$$

$$A(\alpha + 3\beta) = 0 \qquad \alpha^2 + \beta^2 = 1$$

$$\alpha + 3\beta = 0 \qquad 9\beta^2 + \beta^2 = 1$$

$$\alpha = -3\beta \qquad \beta = \frac{1}{\sqrt{10}},\ \alpha = -\frac{3}{\sqrt{10}}$$

$$\boxed{\psi_\perp = \frac{1}{\sqrt{10}}(-3\psi_1 + \psi_3)}$$

$$\langle E\rangle_\Psi = \sum c_n^2 \cdot E_n = \left(\frac{1}{\sqrt{10}}\right)^2 \frac{1}{2m}\left(\frac{\hbar\pi}{x_0}\right)^2 + \frac{9}{10}\frac{1\cdot 9}{2m}\left(\frac{\hbar\pi}{x_0}\right)^2 = \left(\frac{\hbar\pi}{x_0\cdot m}\right)^2\left(\frac{1}{10} + \frac{81}{10}\right)\frac{1}{2m}$$

$$= \left(\frac{\hbar\pi}{x_0\cdot m}\right)^2 \cdot \frac{82}{20\,m} = \frac{41}{10m}\left(\frac{\hbar\pi}{x_0}\right)^2 = \underline{17{,}6\,eV}$$

$$\langle E\rangle_{\psi_\perp} = \left(\left(-\frac{3}{\sqrt{10}}\right)^2\frac{\pi^2}{x_0^2} + \left(\frac{1}{\sqrt{10}}\right)^2\frac{\pi^2 9}{x_0^2}\right)\frac{\hbar}{2m} = \left(\frac{9\pi^2}{10x_0^2} + \frac{9\pi^2}{10x_0^2}\right)\frac{1}{2m} = \frac{18\pi^2\hbar^2}{20\,x_0^2\,m}$$

$$= \frac{9\pi^2}{10\,m\,x_0^2} = \underline{7{,}6\,eV}$$

43) $a = 0{,}2\ nm$

$$\Psi(x) = \Psi_1 + \frac{\Psi_3}{3} + \frac{\Psi_5}{5}$$

$$1 = A^2\int_0^a\left(\Psi_1 + \frac{\Psi_3}{3} + \frac{\Psi_5}{5}\right)^2 dx = A^2\int_0^a\left(\Psi_1^2 + \frac{\Psi_3^2}{9} + \frac{\Psi_5^2}{25} + \frac{2\Psi_1\Psi_3}{3} + \frac{2\Psi_1\Psi_5}{5} + \frac{2\Psi_3\Psi_5}{15}\right)dx$$

$$1 = A^2\left(1 + \frac{1}{9} + \frac{1}{25}\right) = A^2(225 + 25 + 9) = A^2 \cdot 259$$

$$A^2 = \frac{1}{259} \Rightarrow \boxed{A = \frac{25}{\sqrt{259}}}$$

$$\langle E\rangle = \sum_{n=1}^{\infty} |c_n|^2 \cdot E_n = \left(\frac{1}{\sqrt{10}}\right)^2 \frac{\hbar^2 \pi^2}{2 m x_0^2}\left(1 + \frac{9}{3} + \frac{25}{5}\right) = \frac{1}{20}\frac{\hbar^2\pi^2}{m}\left(\frac{15+45+75}{15}\right)$$

$$= \frac{\hbar^2\pi^2}{20 m}\cdot\frac{135}{15} = \frac{27\,\hbar^2\pi^2}{60\, m x_0^2} = \underline{7{,}6\ eV}$$

$$\Psi_\perp(x) = A\int_0^a\left(\Psi_1 + \frac{\Psi_3}{3} + \frac{\Psi_5}{5}\right)(\alpha\Psi_1 + \beta\Psi_3)\,dx$$

$$= A\int_0^a\left(\alpha\Psi_1^2 + \beta\Psi_1\cdot\Psi_3 + \alpha\frac{\Psi_3\Psi_1}{3} + \beta\frac{\Psi_3^2}{3} + \alpha\frac{\Psi_5\Psi_1}{5} + \beta\frac{\Psi_3\Psi_5}{5}\right)dx$$

$$= A\int_0^a\left(\alpha\Psi_1^2 + \Psi_1\cdot\Psi_3\left(\beta + \frac{\alpha}{3}\right) + \beta\frac{\Psi_3^2}{3} + \frac{\alpha\,\Psi_5\cdot\Psi_1}{5} + \beta\frac{\Psi_3\cdot\Psi_5}{5}\right)dx$$

$$\Psi_1\Psi_3\left(\beta + \frac{\alpha}{3}\right) = 0$$

$$\alpha^2 + \beta^2 = 1$$

$$\beta + \frac{\alpha}{3} = 0 \qquad \frac{\alpha^2}{9} + \alpha^2 = \frac{10\alpha^2}{9} = 1 \Rightarrow \frac{9}{10} = \alpha^2 \Rightarrow \boxed{\alpha = \sqrt{\frac{9}{10}}}$$

$$\beta = -\frac{\alpha}{3}$$

$$\beta = -\frac{1}{\sqrt{10}}$$

$$\Psi_\perp(x) = \frac{3}{\sqrt{10}}\Psi_1 - \frac{1}{\sqrt{10}}\Psi_3$$

$$\langle E\rangle_\Psi = \sum_{n=1}^{\infty}|c_n|^2 E_n = \left(\frac{3}{\sqrt{10}}\right)^2\frac{\hbar^2\pi^2}{2 m x_0^2} + \left(-\frac{1}{\sqrt{10}}\right)^2\frac{\hbar^2\pi^2\cdot 9}{2 m x_0^2} = \frac{12\,\hbar^2\pi^2}{20\, m x_0^2} = \frac{3}{5}\frac{\hbar^2\pi^2}{m x_0^2} = \underline{10{,}2\ eV}$$

44) $\nu = 10^{10}\, s^{-1}$
$m = 10^{-30}\, kg$
$E = \frac{3\hbar\omega}{2}$

Vse razo stanje $n=1$

$\omega = 2\pi\nu = 6{,}2\cdot 10^{10}$

$$E = \frac{1}{2} m\omega x_0^2$$

$$\psi_1 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-m\omega x^2/2\hbar} \cdot 2^{-\frac{1}{2}} \cdot 2\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}} \cdot x$$

$$P = \int_{-\infty}^{\infty} \psi_1 \cdot \psi_1^* dx = \int_{-\infty}^{\infty} \left(\frac{m\cdot\omega}{\pi\cdot\hbar}\right)^{\frac{1}{2}} \cdot e^{-\frac{m\omega x^2}{\hbar}} \cdot \frac{1}{2} \cdot 4 \frac{m\omega}{\hbar} \cdot x^2$$

$$= \left(\frac{m\cdot\omega}{\pi\cdot\hbar}\right)^{\frac{1}{2}} \cdot \frac{4 m\cdot\omega}{\hbar} \int_0^{\infty} e^{-\frac{m\omega x^2}{\hbar}} \cdot x^2\, dx = 4\left(\frac{m^3\cdot\omega^3}{\hbar^3}\right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{2+1}{2}\right)}{2\cdot\frac{m\omega}{\hbar}\left(\frac{2+1}{2}\right)}$$

$$= 4\, \frac{m\cdot\omega}{\hbar}\left(\frac{m\cdot\omega}{\pi\cdot\hbar}\right)^{\frac{1}{2}} \frac{\Gamma(\frac{3}{2})\,\hbar}{3 m\omega} = \frac{4}{3}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}} \frac{1}{2}\sqrt{\pi} = \frac{2}{3}\left(\frac{m\cdot\omega}{\hbar}\right)^{\frac{1}{2}}$$

45) $\langle x^2 \rangle^{\frac{1}{2}}$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \left(\frac{m\omega}{\pi \cdot \hbar}\right)^{\frac{1}{2}} e^{-2m\omega x^2/2\hbar} \cdot x^2 \, dx$$

$$= \left(\frac{m\omega}{\pi \cdot \hbar}\right)^{\frac{1}{2}} \int x^2 \cdot e^{-\frac{m\omega x^2}{\hbar}} \, dx$$

---

49) $\Psi(x) = A \exp\left(\frac{-(x-x_0)^2}{2a^2} + \frac{i p_0 x}{\hbar} \right)$

a) Izračunajmo normo:

$$1 = A^2 \int_{-\infty}^{\infty} \Psi^*(x) \Psi(x) \, dx = A^2 \int_{-\infty}^{\infty} \exp\left(\frac{-(x-x_0)^2}{2a^2} + \frac{i p_0 x}{\hbar}\right) \cdot \exp\left(\frac{-(x-x_0)^2}{2a^2} - \frac{i p_0 x}{\hbar}\right) dx$$

$$= A^2 \int_{-\infty}^{\infty} \exp\left(\frac{-(x-x_0)^2}{a^2}\right) dx = A^2 \cdot a \int_{-\infty}^{\infty} \exp(-t^2) \, dt = A^2 a \cdot \sqrt{\pi}$$

$$t = \frac{x-x_0}{a} \qquad\qquad = 2A^2 a \sqrt{\pi}$$

$$a \, dt = dx$$

$$1 = A^2 a \sqrt{\pi} \Rightarrow \boxed{A^2 = \frac{1}{a\sqrt{\pi}}}$$ Rešitev je $\boxed{A^2 = (a\sqrt{\pi})^{-1}}$

b) $$\langle x \rangle = (a \cdot \sqrt{\pi})^{-1} \int_{-\infty}^{\infty} \exp\left(\frac{-(x-x_0)^2}{2a^2}\right) \exp\left(\frac{-i p_0 x}{\hbar}\right) \cdot x \, \exp\left(\frac{-(x-x_0)^2}{2a^2}\right) \cdot \exp\left(\frac{i p_0 x}{\hbar}\right) dx$$

$$= (a\sqrt{\pi})^{-1} \int_{-\infty}^{\infty} \exp\left(-\left(\frac{x-x_0}{a}\right)^2\right) \cdot x \, dx = \frac{a}{a\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-t^2)(ta + x_0) \, dt$$

$$t = \frac{x-x_0}{a} \Rightarrow (ta + x_0) = x$$

$$a \, dt = dx$$

$$= \frac{1}{\sqrt{\pi}} \left(a \underbrace{\int_{-\infty}^{\infty} t \exp(-t^2) \, dt}_{0} + x_0 \int_{-\infty}^{\infty} \exp(-t^2) \, dt \right) = \frac{1}{\sqrt{\pi}} \left(0 + x_0 \sqrt{\pi} \right)$$

$$\frac{1}{\sqrt{\pi}} x_0 \sqrt{\pi} = x_0 \Rightarrow \boxed{\langle x \rangle = x_0}$$

c) $\phi(P) = ?$

$\Psi_p = e^{-ipx/\hbar}$; $p = \frac{\hbar}{i}\left(\frac{\partial}{\partial x}\right)$

$$\Psi(x) = \sum_n c_n \Psi_n = \sum c_p \cdot \Psi_n = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) e^{ipx/\hbar} dp$$

$$\frac{1}{a\sqrt{\pi}} \exp\left(\frac{-(x-x_0)^2}{a^2} + \frac{ip_0 x}{\hbar}\right) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) e^{ip_2 x/\hbar} dp \quad / \cdot e^{ip_1 x/\hbar}$$

$$\int_{-\infty}^{\infty} \Psi(x) \cdot e^{ip_1 x/\hbar} dx = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p_2) dp_2 \int_{-\infty}^{\infty} e^{i(p_2-p_1)x/\hbar}$$

$$\int_{-\infty}^{\infty} \Psi(x) e^{-ip_1 x/\hbar} dx = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \phi(p_2) dp_2 \delta(p_2 - p_1) = \sqrt{\frac{1}{2\pi\hbar}} \cdot \phi(p_1)$$

$$f(0) = \int_{-\infty}^{\infty} f(P)\delta(P) dP = f(0)$$

$$\phi(P) = \frac{1}{(2\pi\hbar)^{\frac{1}{2}}} \int_{-\infty}^{\infty} \Psi(x) e^{ipx/\hbar} dx = \frac{1}{\sqrt{2\pi\hbar}}$$

d) $(\sigma_p)^2 = \, = \langle p^2 \rangle - \langle p \rangle^2 \qquad p = \frac{\hbar}{i}\frac{\partial}{\partial x}$

$$\langle p \rangle = \frac{1}{a\sqrt{\pi}} \int_{-a}^{a} \psi^*(x) \, \frac{\hbar}{i} \frac{\partial \psi(x)}{\partial x} \, dx$$

$$\frac{\partial \psi(x)}{\partial x} = \exp\left(-\frac{(x-x_0)^2}{2a^2}\right) e^{\frac{i p_0 x}{\hbar}} = \frac{-2(x-x_0)}{2a^2} \cdot \exp\left(\frac{-(x-x_0)^2}{2a^2}\right) e^{\frac{i p_0 x}{\hbar}} + \exp\left(\frac{-(x-x_0)^2}{2a^2}\right) \cdot \frac{p_0 \cdot i}{\hbar} \exp^{\frac{i p_0 x}{\hbar}}$$

$$\exp\left(\frac{-(x-x_0)^2}{2a^2} + \frac{i p_0 x}{\hbar}\right)\left[\frac{x-x_0}{a^2} + \frac{p_0 \cdot i}{\hbar}\right]$$

$$\frac{1}{a\sqrt{\pi}} \frac{\hbar}{i} \int_{-\infty}^{\infty} \exp\left(\frac{-(x-x_0)^2}{2a^2} - \frac{i p_0 x}{\hbar}\right) \exp\left(\frac{-(x-x_0)^2}{2a^2} + \frac{i p_0 x}{\hbar}\right)\left[\frac{x-x_0}{a^2} + \frac{p_0 i}{\hbar}\right] dx$$

$$= \frac{\hbar}{i a \sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(\frac{-(x-x_0)^2}{a^2}\right)\left[\frac{x-x_0}{a^2} + \frac{p_0 i}{\hbar}\right] dx$$

$$= \frac{\hbar}{a i \sqrt{\pi}} \left(\underbrace{\int \frac{x-x_0}{a^2} \exp\left(\frac{-(x-x_0)^2}{a^2}\right) dx}_{0} + \frac{p_0 \cdot i}{\hbar} \int_{-\infty}^{\infty} \exp\left(\frac{-x-x_0}{a^2}\right)^2 dx \right)$$

$$t = \frac{x-x_0}{a}$$

$$a\,dt = dx$$

$$= \frac{\cancel{\hbar}}{\cancel{a}\cancel{i}\sqrt{\pi}} \left(\frac{\cancel{a} p_0 \cancel{i} \sqrt{\pi}}{\cancel{\hbar}} \right) = \boxed{p_0 = \langle p \rangle}$$

$$\frac{\partial^2 \psi(x)}{\partial x} = \frac{1}{a^2} \exp\left(\frac{-(x-x_0)^2}{2a^2} + \frac{i p_0 x}{\hbar}\right) + \frac{(x-x_0)}{a^2 \, a^2}$$

50) $\Psi(x, t=0) = A\left(\frac{2x^2}{a^2} + \frac{ix}{a}\right) \exp\left(\frac{-x^2}{2a^2}\right)$ $\qquad \frac{x}{a} = z$

Lastna valovna funkcija: $\Psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-m\omega x^2/2\hbar} \left(\frac{1}{2^n n!}\right)^{\frac{1}{2}} H_n\left[\sqrt{\frac{m\omega}{\hbar}}\, x\right]$

$$\boxed{\Psi(x, t=0) = \sum_{n=0}^{\infty} c_n \Psi_n(x)} \qquad B = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}$$

$$A\left(\frac{2x^2}{a^2} + \frac{ix}{a}\right) \exp\left(\frac{-x^2}{2a^2}\right) = B_1\left(c_0 \cdot 1 + c_1 \frac{2x}{a}\left(\frac{1}{2\cdot 1!}\right)^{\frac{1}{2}} + c_2\left(\frac{1}{8}\right)^{\frac{1}{2}} \cdot \left(4\left(\frac{x}{a}\right)^2 - 2\right)\right)e$$

$$A\left(\frac{2x^2}{a^2} + \frac{ix}{a}\right) \exp\left(\frac{-x^2}{2a^2}\right) = Be^{-\frac{m\omega x^2}{2\hbar}}\left[c_0 \cdot 1 + \frac{2}{\sqrt{2}}\frac{x}{a} c_1 + c_2\left(\frac{1}{\sqrt{8}}\left(4\left(\frac{x}{a}\right)^2 - 2\right)\right)\right]$$

$$-\|- \qquad = \qquad -\|- \qquad + c_2 \frac{4}{\sqrt{8}}\frac{x^2}{a^2} - \frac{2c_2}{\sqrt{8}}$$

$$\frac{2x^2}{a^2} = c_2 \frac{4}{\sqrt{8}} \frac{x^2}{a^2}$$

$$2 = c_2 \frac{4}{\sqrt{8}} \Rightarrow \boxed{c_2 = \frac{\sqrt{8}}{2} = \sqrt{2}}$$

$$\frac{ix}{a} = \frac{2}{\sqrt{2}} \frac{x}{a} c_1 \Rightarrow \boxed{c_1 = \frac{\sqrt{2}\, i}{2}}$$

$$c_0 - \frac{2c_2}{\sqrt{8}} = 0$$

$$\boxed{c_0 = \frac{+2\sqrt{2}}{\sqrt{2}\sqrt{4}} = +1}$$

$$\Psi(t=0) = C\left(\Psi_0 + \frac{\sqrt{2}}{2} i \Psi_1 + \sqrt{2}\, \Psi_2\right) \qquad \int \Psi_n \Psi_{n'}\, dx = \delta_{nn'} = \begin{matrix} 1 & n = n' \\ 0 & n \neq n' \end{matrix}$$

Normiramo:

$$\int \Psi^* \Psi = 1 \qquad C^2\left(|\Psi_0|^2 + \frac{2}{4}|\Psi_1|^2 + 2|\Psi_2|^2\right) = 1$$

$$C^2\left(1 + \frac{1}{2} + 2\right) = 1$$

$$\boxed{C^2 = \frac{2}{7}}$$

$$C = \pm\sqrt{\frac{2}{7}}$$

$$\langle E \rangle = \sum_{n=0}^{\infty} |c_n|^2 \cdot E_n$$

$$\langle E \rangle = \frac{2}{7}\left(1 \cdot \frac{1}{2}\hbar\omega + \frac{1 \cdot 3}{2 \cdot 2}\hbar\omega + 2 \cdot \frac{5}{2}\hbar\omega\right) = \frac{2}{7}\hbar\omega\left(\frac{1}{2} + \frac{3}{4} + 5\right)$$

$$= \frac{2}{7}\hbar\omega\left(\frac{1}{2} + \frac{3}{4} + 5\right) = \frac{\cancel{2}}{7} \cdot \frac{27}{\cancel{4}_2}\hbar\omega = \boxed{\frac{25}{14}\hbar\omega}$$

$e^{-\frac{m\omega x^2}{2\hbar}}$

$$\boxed{\Psi(x,t) = \frac{2}{7}\left(\psi_0 \cdot e^{-iW_0 t/\hbar} + \frac{1}{\sqrt{2}} e^{-i\frac{3}{2}\hbar\omega t/\hbar}\psi_1 + 2\psi_2 e^{-i\frac{5}{2}\omega t}\right).}$$

5.1) $\Psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_3)$

$e^{i\varphi} = \cos\varphi + i\sin\varphi$

$$\boxed{\Psi(t) = \frac{1}{\sqrt{2}}\left(\psi_1 \cdot e^{-iE_1 t/\hbar} + \psi_3 e^{-iE_3 t/\hbar}\right)}$$

$$|\Psi(t)|^2 = \frac{1}{2}\int_{-\infty}^{\infty}\left(\psi_1 e^{-iE_1 t/\hbar} + \psi_3 e^{-iE_3 t/\hbar}\right)\left(\psi_1^* \cdot e^{iE_1 t/\hbar} + \psi_3^* e^{iE_3 t/\hbar}\right)$$

$$= \frac{1}{2}\left(\psi_1^2 + \psi_3^2 + 2\psi_1 \cdot \psi_3 \cdot e^{-i(E_1-E_3)t/\hbar}\right)$$

$$= \boxed{\frac{1}{2}\left(\psi_1^2 + \psi_3^2 + 2\psi_1\psi_3 \cdot \cos\frac{(E_1-E_3)t}{\hbar}\right)}$$

$$\langle x(t)\rangle = \frac{1}{2}\int_0^a \left(\psi_1 \cdot e^{iE_1 t/\hbar} + \psi_3 e^{-iE_3 t/\hbar}\right) x \left(\psi_1^* \cdot e^{iE_1 t/\hbar} + \psi_3^* e^{+iE_3 t/\hbar}\right) dx$$

$$= \frac{1}{2}\int_0^a \left(\psi_1^2 x + \psi_3^2 x + 2\psi_1 \cdot \psi_3 \cdot x \cos\left(\frac{(E_3-E_1)t}{\hbar}\right)\right) dx$$

$$\boxed{= \frac{1}{2}\left(\left(\frac{x^2}{2} + \frac{x^2}{2}\right)\right) = \frac{a^2}{2}}$$

odd curve

neodvisno

$\langle x(t)\rangle = 0$

$$\int_{-\infty}^{\infty} \psi_1 \cdot \psi_3 \cos\frac{(E_1-E_3)t}{\hbar}$$

52) $\Psi_{100} = \frac{1}{\sqrt{\pi r_B^3}} \exp\left(\frac{-r}{r_B}\right)$

$$\langle r \rangle = \int_0^\infty \Psi^* \hat{r} \Psi \, dV = 4\cancel{\pi}\int_0^\infty r^3 \cdot \frac{1}{\cancel{\pi} r_B^3} \exp\left(-\frac{2r}{r_B}\right) \cdot dr =$$

$$= \frac{4}{r_B^3} \int_0^\infty r^3 \exp\left(-\frac{2r}{r_B}\right) dr = \frac{\cancel{4} r_B \cdot \cancel{r_B^3}}{\cancel{r_B^3} \cdot \cancel{2} \cdot 8_4} \int_0^\infty t^3 e^{-t} \cdot dt =$$

$$t = \frac{2r}{r_B} \Rightarrow r = \frac{t \cdot r_B}{2}$$

$$\frac{r_B \, dt}{2} = dr$$

$$= \frac{r_B}{4} \Gamma(3+1) = \frac{r_B}{4} \cdot 3! = \frac{6}{4} r_B = \boxed{\frac{3}{2} r_B}$$

$$\langle r^2 \rangle = 4\int_0^\infty r^4 \frac{1}{r_B^3} \exp\left(-\frac{2r}{r_B}\right) dr = \frac{\cancel{4} r_B \cdot r_B^{\cancel{4}}}{r_B^3 \cdot 2 \cdot 2^{\cancel{4}}} \int_0^\infty t^4 e^{-t} dt$$

$$t = \frac{2r}{r_B} \Rightarrow r = \frac{t \cdot r_B}{2}$$

$$dr = \frac{r_B}{2} dt$$

$$= \frac{r_B^2}{8} 4! = \frac{r_B^2}{8} (4 \cdot 3 \cdot 2 \cdot 1) = \frac{24 r_B^2}{8} = 3 r_B^2$$

$$\langle r^2 \rangle^{\frac{1}{2}} = \boxed{\sqrt{3} r_B}$$ ✗

53) $\langle V \rangle = ?$ $\qquad V(r) = -\dfrac{Z \cdot e^2}{4\pi \cdot \varepsilon_0 \cdot r}$; $Z = 1$

$$\psi_{100} = \frac{1}{\sqrt{\pi r_B^3}} \cdot \exp\left(\frac{-r}{r_B}\right)$$

$$\langle V \rangle = \int \psi_{100}^* \cdot \hat{V}(r)\, \psi_{100}\, dV = 4\pi \int_0^\infty \frac{1}{\pi r_B^3}\left(\frac{-e^2}{4\pi \cdot \varepsilon_0 r}\right) r^2\, dr\, \exp\left(-\frac{2r}{r_B}\right) =$$

$$= \frac{-\cancel{4}\, e^2}{r_B^3 \cdot \cancel{4}\pi\, \varepsilon_0} \int r \exp\left(-\frac{2r}{r_B}\right) dr = -\frac{e^2 r_B^2}{r_B^3 \cdot 2 \cdot 2} \int_0^\infty t \exp(-t)\, dt = -\frac{e^2}{r_B \cdot 4\pi \varepsilon_0} = -27{,}3$$

$$t = \frac{2r}{r_B} \Rightarrow r = \frac{t \cdot r_B}{2}$$

$$\frac{r_B\, dt}{2} = dr$$

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54) $\psi = \dfrac{1}{2\sqrt{\pi}} f(r,\vartheta)(\cos\varphi + i\sqrt{3} \sin\varphi)$

$\langle l_z \rangle = ?$ $\qquad dV = r^2 \sin\vartheta \cdot d\varphi$

$\hat{l}_z = -i\hbar \dfrac{\partial}{\partial \varphi}$

$$\langle l_z \rangle = -i\hbar \int_0^\infty \psi^* \cdot \frac{\partial}{\partial \varphi} \psi\, dV = \underset{1}{?}$$

$$= -\frac{i\hbar}{4\pi} \int_0^\infty \boxed{f^*(r,\vartheta)\, f(r,\vartheta) \cdot r^2\, d(\cos\vartheta)} (\cos\varphi - i\sqrt{3} \sin\varphi)(-\sin\varphi + i\sqrt{3} \cdot \cos\varphi)\, d\varphi$$

$$= -\frac{i\hbar}{4\pi} \int_0^{2\pi} (-\cos\varphi \sin\varphi + i\sqrt{3}\cos^2\varphi + i\sqrt{3}\sin^2\varphi + 3 \sin\varphi \cdot \cos\varphi)\, d\varphi$$

$$= -\frac{i\hbar}{4\pi} \int_0^{2\pi} (\overset{0}{\widetilde{2\cos\varphi \sin\varphi}} + i\sqrt{3})\, d\varphi = -\frac{i\hbar}{\cancel{4\pi}} i\sqrt{3} \cdot \cancel{2\pi} = +\frac{\hbar\sqrt{3}}{2}$$

$t = \sin\varphi$

$dt = \cos\varphi\, d\varphi$

55) $\Psi_{100} = \frac{1}{\sqrt{\pi r_B^3}} \exp\left(-\frac{r}{r_B}\right)$

$$\langle r \rangle = \frac{4\pi}{\pi r_B^3} \int_0^\infty r^3 \exp\left(-\frac{2r}{r_B}\right) dr = \frac{\cancel{4}\, r_B^4}{r_B^3 \cdot 2 \cdot 8_2} \int_0^\infty t^3 e^{-t}\, dt = \frac{r_B}{2} \cdot 6 = \boxed{3 r_B}$$

$$t = \frac{2r}{r_B} \Rightarrow r = \frac{t \cdot r_B}{2}$$

$$\frac{r_B}{2} dt = dr$$

$$\langle P(r_B \leq r) \rangle = \frac{4\pi}{\pi r_B^3} \int_{3r_B}^\infty r^2 \exp\left(-\frac{2r}{r_B}\right) dr = \frac{4 r_B r_B^2}{r_B^3 \cdot 2 \cdot 4} \int_{3r_B}^\infty t^2 \exp(-t)\, dt$$

$$t = \frac{2r}{r_B} \Rightarrow r = \frac{t r_B}{2}$$

$$\frac{r_B dt}{2} = dr$$

$$= \frac{1}{2}\left(\int_0^\infty t^2 \exp(-t)\, dt - \int_0^3 t^2 \exp(-t)\, dt \right) = \frac{1}{2}\left[2 - 2 + 17 e^{-3}\right] = \underline{\frac{17}{2} e^{-3}} = \underline{0{,}42}$$

$$\left.\begin{array}{l} u = t^2 \Rightarrow du = 2t \\ dv = e^{-t} dt \Rightarrow v = -e^{-t} \end{array}\right\} -t^2 e^{-t}\Big|_0^3 + 2\int t e^{-t} dt = -t^2 e^{-t}\Big|_0^3 - 2t e^{-t}\Big|_0^3 - 2e^{-t}\Big|_0^3$$

$$u = t \quad du = dt$$
$$dv = e^{-t} \quad v = -e^{-t}$$

$$-9e^{-3} - 6e^{-3} - e^{-3} = -17e^{-3}$$

56) $\Psi_{100} = \frac{1}{\sqrt{\pi r_B^3}} \exp\left(-\frac{r}{r_B}\right)$

$$P\left(-\tfrac{1}{2} r_B < r < \tfrac{3}{2} r_B\right) = \frac{4\pi}{\pi r_B^3} \int_{+\frac{1}{2} r_B}^{\frac{3}{2} r_B} r^2 \exp\left(-\frac{2r}{r_B}\right) dr = \frac{4 r_B^3}{r_B^3 \cdot 2 \cdot 4} \int_1^3 t^2 \cdot e^{-t} dt$$

$$t = \frac{2r}{r_B} \Rightarrow r = \frac{t \cdot r_B}{2}$$

$$\frac{r_B dt}{2} = dr$$

$$= \frac{1}{2}\left(-t^2 e^{-t} - 2te^{-t} - 2e^{-t}\right)\Big|_1^3 = \frac{1}{2}\left(-9e^{-3} - 6e^{-3} - 2e^{-3} + e^{-1} + 2e^{-1} + 2e^{-1}\right)$$

$$= \frac{1}{2}\left(5e^{-1} - 17e^{-3}\right) = \underline{0{,}50}$$

$$\psi_{210} = \frac{1}{2\sqrt{8\pi r_B^3}} \frac{r}{r_B} \exp\left(-\frac{r}{2r_B}\right)\cos\vartheta \quad ; \quad V = -\frac{e^2}{4\pi\cdot\varepsilon_0\cdot r}$$

$$\langle V\rangle = \frac{-\cancel{2}\cancel{\pi} e^2}{\varepsilon_0 32\pi r_B^5 4\pi} \int \frac{r^4}{r} \exp\left(-\frac{r}{r_B}\right) dr \int_0^{\pi} \cos^2\vartheta \, d\vartheta$$

$$\langle V\rangle = -\frac{e^2}{64\pi\cdot\varepsilon_0 r_B^5} \int_0^{\infty} r^3 \exp\left(-\frac{r}{r_B}\right) dr \int_0^{\pi} \frac{1}{2}\left(1+\cos 2\vartheta\right) d\vartheta$$

$$= \frac{-e^2\cdot\cancel{\pi}}{128\cdot\cancel{\pi}\cdot\varepsilon_0 r_B^5} \int_0^{\infty} r^3 \exp\left(-\frac{r}{r_B}\right) dr = \frac{-e^2 r_B^4}{128\cdot\varepsilon_0 r_B^6} \int_0^{\infty} t^3 e^{-t} dt = \frac{-e^2\cdot 6}{128\,\varepsilon_0\cdot r_B} =$$

$$t = \frac{r}{r_B} \Rightarrow r = r_B\cdot t$$
$$dr = r_B\cdot dt$$

$$= -\frac{3e^2}{64\,\varepsilon_0\cdot r_B\pi} = \underline{\underline{-9{,}1\,eV}}$$

Pričakovana da bo $-3{,}4$ eV

58) $R_{20} = 2\left(\frac{1}{\sqrt{2r_B}}\right)^3\left(1-\frac{r}{2r_B}\right)\exp\left(\frac{-r}{2r_B}\right)$

$$\int_0^\infty R_{nl}^2\, r^2\, dr = 1$$

$$\int_0^\infty 4\left(\frac{1}{\sqrt{2r_B}}\right)^6 \cdot \left(1-\frac{r}{2r_B}\right)^2 \exp\left(-\frac{r}{r_B}\right)\cdot r^2\, dr = 1$$

$$\frac{4}{8r_B^3}\int_0^\infty \left(1-\frac{r}{2r_B}\right)^2 \cdot \exp\left(-\frac{r}{r_B}\right) r^2\, dr$$

$$\frac{1}{2r_B^3}\int_0^\infty \left(r^2 - \frac{2r^3}{2r_B} + \frac{r^4}{4r_B^2}\right)\exp\left(-\frac{r}{r_B}\right) dr$$

$$\frac{1}{2r_B^3}\left[\int_0^\infty r^2\exp\left(-\frac{r}{r_B}\right) - \int_0^\infty \frac{r^3}{r_B}\exp\left(-\frac{r}{r_B}\right) + \int_0^\infty \frac{r^4}{4r_B^2}\exp\left(-\frac{r}{r_B}\right)\right] dr$$

$$t = \frac{r}{r_B} \Rightarrow r = t\cdot r_B$$

$$r_B\, dt = dr$$

$$= \frac{1}{2\cancel{r_B^3}}\left[\cancel{r_B^3}\int_0^\infty t^2 e^{-t}\, dt - \cancel{r_B^3}\int_0^\infty t^3 e^{-t}\, dt + \frac{\cancel{r_B^3}}{4}\int_0^\infty t^4 e^{-t}\, dt\right]$$

$$= \frac{1}{2}\left[\int_0^\infty t^2 e^{-t}\, dt - \int_0^\infty t^3 e^{-t}\, dt + \frac{1}{4}\int_0^\infty t^4 e^{-t}\, dt\right]$$

$$\frac{1}{2}\left[2 - 6 + \frac{24}{4}\right] = 1$$

$$\underline{\underline{1 = 1}}\ \checkmark$$

$$\langle \Psi_{200} = 2\left(\frac{1}{\sqrt{2r_B}}\right)^3\left(1-\frac{r}{2r_B}\right)\exp\left(\frac{-r}{2r_B}\right)\left(\frac{1}{4\pi}\right)^{\frac{1}{2}}$$

$$\Psi_{200} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{\sqrt{2r_B}}\right)^3\left(1-\frac{r}{2r_B}\right)\exp\left(\frac{-r}{2r_B}\right)$$

$$r\rangle = \frac{\cancel{4\pi}}{\cancel{\pi}\cdot 8 r_B^3}\int_0^\infty\left(1-\frac{r}{2r_B}\right)^2\exp\left(-\frac{r}{r_B}\right)r^3\,dr$$

$$= \frac{1}{2r_B^3}\left[\left(\int_0^\infty r^3\exp\left(-\frac{r}{r_B}\right)\right)-\int_0^\infty\frac{r^4}{r_B}\exp\left(-\frac{r}{r_B}\right)+\frac{r^5}{4r_B^2}\exp\left(-\frac{r}{r_B}\right)\right]dr$$

$$= \frac{1}{2r_B^3}\left[r_B^4\int_0^\infty t^3e^{-t}\,dt - r_B^4\int_0^\infty t^4e^{-t}\,dt + \frac{r_B^4}{4}\int_0^\infty t^5e^{-t}\,dt\right]$$

$$= \frac{r_B}{2}\left[6-24+120\right] = \underline{\underline{51\,r_B}}$$

$$r^2\rangle = \frac{1}{2r_B^3}\int_0^\infty\left[r^4\exp\left(-\frac{r}{r_B}\right)-\frac{r^5}{r_B}\exp\left(-\frac{r}{r_B}\right)+\frac{r^6}{4r_B^2}\exp\left(-\frac{r}{r_B}\right)\right]dr$$

$$= \frac{1}{2r_B^3}\left[r_B^4\int_0^\infty t^4e^{-t}\,dt - r_B^4\int_0^\infty t^5e^{-t}\,dt + \frac{r_B^4}{4}\int_0^\infty t^6e^{-t}\,dt\right]$$

$$= \frac{r_B^2}{2}\left[24-120+720\right] = \underline{312\,r_B^2}$$

$$\langle r^2\rangle^{\frac{1}{2}} = \sqrt{312}\,r_B$$

$$t = \frac{r}{r_B},$$

$$= \frac{1}{2r_B^3}\int_{51r_B}^{\sqrt{312}r_B}\left(r^2-\frac{r^3}{r_B}+\frac{r^4}{4r_B}\right)\exp\left(-\frac{r}{r_B}\right)dr$$

$$= \frac{1}{2r_B^3}\left[r_B^3\int t^2e^{-t}\,dt - r_B^3\int_{51}^{\sqrt{312}} t^3e^{-t}\,dt + \frac{r_B^3}{4}\int t^4e^{-t}\,dt\right]$$

$$\int_{51}^{\sqrt{312}} t^2e^{-t}\,dt = \left(-t^2e^{-t}-2te^{-t}-e^{-t}\right)\Big|_{51}^{\sqrt{312}} = -\left(312e^{-\sqrt{312}}+2\sqrt{312}e^{-\sqrt{312}}+e^{-\sqrt{312}}-51^2e^{-51}-102e^{-51}-e^{-51}\right.$$

$$\int t^3e^{-t}\,dt = -t^3e^{-t}-3t^2e^{-t}-6te^{-t}-6e^{-t} =$$

$$= -\left((\sqrt{312})^3e^{-\sqrt{312}}+3\cdot 312\cdot e^{-\sqrt{312}}+6\sqrt{312}e^{-\sqrt{312}}+6e^{-\sqrt{312}}-51^3e^{-51}-3\cdot 51^2e^{-51}-6\cdot 51e^{-51}-6e^{-51}\right.$$

$$\int t^4e^{-t}\,dt = -t^4e^{-t}-4t^3e^{-t}-12t^2e^{-t}-24te^{-t}-24e^{-t}$$

$$= \frac{1}{2}\left[-t^2e^{-t}-2te^{-t}-e^{-t}+\cancel{t^3e^{-t}}+\cancel{3t^2}e^{-t}+6te^{-t}+6e^{-t}-\frac{t^4}{4}e^{-t}-\cancel{t^3e^{-t}}-3\cancel{t^2}e^{-t}-6te^{-t}-6e^{-t}\right]$$

$$\frac{1}{2}\left[-t^2e^{-t}-2te^{-t}-\frac{t^4}{4}e^{-t}-e^{-t}\right]$$

$$= \frac{1}{2}\left[-t^2 e^{-t} - 2t e^{-t} - \frac{t^4}{4} e^{-t} - e^{-t}\right]\Bigg|_{51}^{\sqrt{312}} =$$

$$- \frac{1}{2}\left[312 \cdot e^{-\sqrt{312}} + 2\cdot\sqrt{312}\, e^{-\sqrt{312}} + \frac{312^2}{4} e^{-\sqrt{312}} + e^{-\sqrt{312}} - 51^2 \cdot e^{-51} - 102 \cdot e^{-51} - \frac{51^4}{4} e^{-51} - e^{-51}\right]$$

$$= \frac{1}{2}\left[e^{-\sqrt{312}}\left(312 + 2\sqrt{312} + \frac{312^2}{4} + 1\right) - e^{-51}\left(51^2 + 102 + \frac{51^4}{4} + 1\right)\right]$$

$P = 0{,}0002$

$$P = \frac{1}{2 r_B^3} \int_{10 r_B}^{\infty} \left(r^2 - \frac{r^3}{r_B} + \frac{r_B^4}{4 r_B}\right) e^{-\frac{r}{r_B}}\, dr$$

$$= \frac{1}{2 r_B^3}\left[r_B^3 \int_{10 r_B}^{\infty} t^2 e^{-t}\, dt - r_B^3 \int_{10 r_B}^{\infty} t^3 e^{-t} + \frac{r_B^3}{4} \int_{10 r_B}^{\infty} t^4 e^{-t}\, dt\right.$$

$$= -\frac{1}{2}\left[+t^2 e^{-t} + 2t e^{-t} + \frac{t^4}{4} e^{-t} + e^{-t}\right]\Bigg|_{10}^{\infty} = +\frac{1}{2}\left[e^{-10}\left(100 + 20 + \frac{10000}{4} + 1\right)\right.$$

$= \underline{\underline{0{,}06}}$ ✓

$$\Delta W = \frac{e_0^2}{8\pi\varepsilon_0 m^2 c_0^2}\left\langle\frac{1}{r^3}\right\rangle\langle\hat{l}\cdot\hat{s}\rangle$$

$$\psi(r,\vartheta,\varphi) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{\sqrt{r_B}}\right)^3\frac{r}{r_B}\exp\left(-\frac{r}{2r_B}\right)\sin\vartheta\, e^{-i\varphi}$$

$$\rangle = \int_0^\infty \psi^*_{211}\cdot\frac{1}{r_1^3}\cdot\psi_{211}\,dV \quad ; \quad dV = r^2\sin\vartheta\, d\vartheta\, d\varphi\, dr$$

$$\frac{1}{64\pi r_B^5}\int_0^\infty\frac{1}{r^2}\exp\left(-\frac{r}{2r_B}\right)r^2 dr\int_0^\pi\sin^2\vartheta\,d\vartheta\int_0^{2\pi}d\varphi$$

$$\frac{2\pi}{64\pi r_B^5}\int_0 r\exp\left(-\frac{r}{r_B}\right)dr\int_0^\pi\frac{1}{2}(1+\cos 2\vartheta)\,d\vartheta$$

$$\frac{1}{64}\frac{r_0^2}{r_B^5}\int_0^\infty t\,e^{-t}\,dt \qquad = \frac{1}{\quad}\,\Gamma(1+1)$$

$$\left[\frac{1}{24r_B}\right]^3 \Rightarrow \text{v rezultat}$$

$$= \vec{l}+\vec{s}$$

$$= (\vec{l}^2 + 2\vec{l}\vec{s} + \vec{s}^2) \Rightarrow \hat{l}\vec{s} = \frac{\hat{j}^2-\hat{l}^2-\hat{s}^2}{2} =$$

$n = 2$ $\qquad s = \frac{1}{2}$ $\qquad l - s < j < l + s$

$l = 1$ $\qquad l = 1$ $\qquad \frac{1}{2} < j < \frac{3}{2}$

$m_l = 1$

$$\langle l^2\rangle = l(l+1)\hbar^2 = 2\hbar^2$$
$$\langle s^2\rangle = s(s+1)\hbar^2 = \frac{3}{4}\hbar^2$$
$$\langle j\rangle = j(j+1)\hbar = \frac{3}{4}\hbar^2$$

ali pa

$\frac{15}{4}\hbar^2$

$$\hat{l}\hat{s} = \begin{cases} -\hbar^2 & ; j = \frac{1}{2} \\ \frac{\hbar^2}{2} & ; j = \frac{3}{2}\end{cases}$$

$$\Delta W = \boxed{\frac{e_0^2}{8\pi\varepsilon_0 m^2 c^2}\,\frac{1}{16 r_0^3}\,\hbar^2}\begin{cases}\frac{1}{2} & 20\ \mu\text{eV} \\ -1 & ; -41\ \mu\text{eV}\end{cases}$$

655,7

61) $m = 10^{-31}$ kg
$V = \frac{1}{2} k \cdot x^2$
$k = 50 \frac{eV}{nm^2}$

$$\int_{-\infty}^{\infty} H_n(y) \exp\left(-y^2 + y y_0 - \frac{y_0^2}{2}\right) dy = \sqrt{\pi}\, y_0^n \exp\left(-\frac{y_0^2}{4}\right)$$

$$\Psi(x) = \sqrt{\frac{\alpha}{\sqrt{\pi}}} \exp\left(-\frac{\alpha^2 (x-a)^2}{2}\right) = \sqrt{\frac{\alpha}{\sqrt{\pi}}} \exp\left(-\frac{\alpha^2 x^2}{2} + \frac{2x\alpha^2 a - a^2\alpha^2}{2}\right)$$
$$= \sqrt{\frac{\alpha}{\sqrt{\pi}}} \exp\left(-\frac{\alpha^2 x^2}{2} + x a \alpha^2 - \frac{a^2}{2}\right)$$

Upoštevamo le $n = 0$, ker nimamo nobenega z višjo stopnjo

$H_0 = 1$ $\quad x = y \quad y_0 = a$

- Razvoj je, $n = 0$

$$\Psi(0) = \sqrt{\pi}\, \alpha^2 \exp\left(-\frac{\alpha^2 a}{4}\right) \quad , E_0 = \frac{1}{2} \hbar \omega$$

$$\langle E \rangle = \sum_{n=0}^{\infty} |c_n|^2 E_n \qquad c_0 = \sqrt{\pi} \cdot \alpha^2$$

$$\langle E \rangle = c_0^2 \cdot E_0 = c_0^2 \cdot \frac{1}{2} \hbar \omega = \pi \cdot \alpha^4 \hbar \omega \frac{1}{2} = \pi \cdot \frac{m \cdot k}{\hbar^2} \hbar \cdot \omega \cdot \frac{1}{2}$$
$$= \frac{\pi \cdot m \cdot k\, \omega}{\hbar \cdot 2} = \frac{\pi\, m k^2}{\hbar \cdot m} = \frac{\pi k^2}{\hbar}$$

2) Matrični element za neskon. potenc. jamo

$$\boxed{A_{nn'} = \int \Psi_n^* \hat{A}\, \Psi_{n'}\, dx = \langle n | \hat{A} | n' \rangle}$$

$A \quad \langle$

$\Psi_n = \psi_n e^{-i W_n t/\hbar} = \sqrt{\frac{2}{x_0}} \sin\left(n\left(\pi \cdot \frac{x}{x_0} + \frac{\pi}{2}\right)\right) \cdot \exp\left(-\frac{i \hbar \pi^2 n^2 t}{2 m x_0^2}\right)$

$x_{nn} = \langle n | \hat{x} | n \rangle = \frac{2}{x_0} \int x |\psi_{n}|^2 dx = 0$

$x_{nn'} = e^{-i(W_{n'} - W_n)t/\hbar} \int \psi_n x \psi_{n'} dx =$

$\frac{2}{x_0} e^{-\frac{i \pi^2 \hbar t (W_{n'} - W_n) t}{2 x_0^2}} \int_{-\frac{x_0}{2}}^{\frac{x_0}{2}} \sin\left(n\left(\pi \cdot \frac{x}{x_0} + \frac{1}{2}\pi\right)\right) x \cdot \sin\left(n'\left(\pi \frac{x}{x_0} + \frac{1}{2}\pi\right)\right) dx$

$$\boxed{\frac{4 n n' x_0}{\pi^2 (n'^2 - n^2)^2} \left[(-1)^{n+n'} - 1\right] e^{-i \pi^2 (n'^2 - n^2) \hbar t / 2 x_0^2} = x_{nn'}}$$

$\langle n | \hat{p}_x | n' \rangle = \frac{2 n n' \hbar}{i x_0 (n'^2 - n^2)} \left[(-1)^{n+n'} - 1\right] e^{-i \pi (n'^2 - n^2) \hbar t / 2 m x_0^2}$

$\hat{p}_x = \frac{\hbar \partial}{i \partial x}$

64) $n = 2$

$a = 0{,}3\,nm$

$$p_e = e \cdot \langle n | x | n' \rangle$$

$$\langle n | p_e | n' \rangle = \langle 1 | p_e | 2 \rangle = -e_0 \langle 1 | x | 2 \rangle$$

$$\langle 1 | x | 2 \rangle = \frac{4 \cdot 1 \cdot 2\, x_0}{\pi^2 (4-1)^2} \left[(-1)^{1+2} - 1 \right] = \frac{8 \cdot x_0}{\pi^2 \cdot 9} (-2) = \frac{-16\, x_0}{9 \pi^2}$$

$$\boxed{\langle 1 | p_e | 2 \rangle = \frac{16\, x_0 \cdot e_0}{9 \pi^2}} = 8{,}64 \cdot 10^{-30} \qquad E_n = \frac{\pi^2 \cdot n^2 \cdot \hbar^2}{2 x_0^2 m}$$

$$\frac{1}{\tau} = \frac{\omega_{12}^3\, p_{e12}^2}{3 \pi \varepsilon_0 c^3 \cdot \hbar} =$$

$$\omega_{12} = \frac{(W_2' - W_1)}{\hbar} = \frac{\pi^2 \cdot \hbar}{2 x_0^2 m} (4-1) = \frac{3 \pi^2 \cdot \hbar}{2 x_0^2 m} = 1{,}8 \cdot 10^{16}\, s^{-1}$$

$$\frac{1}{\tau} = \frac{27 \pi^6 \cdot \hbar^3 \cdot 16^2\, x_0^2 \cdot e_0^2}{8 \cdot x_0^6 \cdot m^3 \cdot 81 \pi^4 \cdot 3 \pi \varepsilon_0 c^3 \hbar} = \frac{\hbar^2\, e_0^2 \cdot \pi \cdot 32}{x_0^4 \cdot m^3 \cdot \varepsilon_0 \cdot c^3 \cdot 9}$$

$$W_{12} = \frac{\hbar}{\tau} = \boxed{\frac{\hbar^3 \cdot e_0^2 \cdot \pi \cdot 32}{x_0^4 \cdot m^3 \cdot \varepsilon_0 \cdot c^3 \cdot 9}} = 2\, \mu eV$$

$\tau = ?$

$n = 2$

$l = 1$

$m_l = 0$

Pri tej nalogi je glede na izbirna pravila možni prehod v osnovno stanje, ima 3 magnetna stanja

$$\frac{1}{\tau} = \frac{\omega_{12}^3 |r_{e12}|^2}{3\pi \cdot \varepsilon_0 \cdot c^3 \hbar} \qquad \omega_{12} = \frac{W_2 - W_1}{\hbar} = \frac{W_R}{\hbar}\left(\frac{1}{1} - \frac{1}{4}\right) = \boxed{\frac{3}{4}\frac{W_R}{\hbar}}$$

$$|r_{e12}|^2 = \frac{2^{15}}{3^{10}} \cdot e_0^2 r_B^2$$

$$\frac{1}{\tau} = \frac{27 W_R^3 \cdot 2^{15} \cdot e_0^2 \cdot r_B^2}{64 \hbar^3 \cdot 3\pi \varepsilon_0 c^3 \hbar \cdot 3^{10}} = \frac{3^3 \cdot 2^{15} W_R^3 e_0^2 r_B^2}{2^6 \cdot 3^{11} \cdot \hbar^4 \pi \varepsilon_0 c^3} = \boxed{\frac{2^9 \cdot W_R^3 \cdot e_0^2 \cdot r_B^2}{3^8 \cdot \hbar^4 \cdot \pi \cdot \varepsilon_0 \cdot c^3}}$$

$$W_{12} = \frac{\hbar}{\tau} = \underline{\underline{354\ \mathrm{neV}}}$$

66) $n = 10$

$n' = 9$

$\nu = ?$

$$\omega = 2\pi \cdot \nu \Rightarrow \nu = \frac{\omega}{2\pi}$$

$$\nu_{n-1\,n} = \nu_{9\,10} = \frac{W_{10} - W_9}{h} = \frac{W_R}{h}\left[\frac{1}{9^2} - \frac{1}{10^2}\right] = \frac{W_R}{h} \cdot 0{,}0023 =$$

$$= \underline{\underline{7{,}7 \cdot 10^{12}\,Hz}}$$

$$\frac{1}{t_0} = \frac{v}{2\pi r} \quad ; \quad W_{10} = \frac{W_R}{10^2} = 0{,}136\,eV$$

$$W_9 = 0{,}17\,eV$$

$$\frac{1}{2} m \cdot v^2 = \frac{W_R}{n^2} \quad ; \quad -\frac{e_0^2}{4\pi\varepsilon_0 r} = 2W_n = -2\frac{W_R}{n^2}$$

$$\boxed{v = \sqrt{\frac{2W_R}{m \cdot n^2}}} \qquad \boxed{r = \frac{2n^2 e_0^2}{2W_R \cdot 4\pi\varepsilon_0} = \frac{n^2 \cdot e_0^2}{4W_R \pi \cdot \varepsilon_0}}$$

$$\nu = \frac{1}{t_0} = \left(\frac{2W_R}{m \cdot n^2}\right)^{\frac{1}{2}} \frac{1}{2\pi} \left(\frac{4W_R \pi \cdot \varepsilon_0}{n^2 \cdot e_0^2}\right) =$$

Klasične frekvence:

$$\nu = \begin{cases} 6{,}8\,THz & n = 10 \\ 9{,}2\,THz & n = 9 \end{cases}$$

$$P_r = \frac{\hbar\,\omega_{9\,10}}{\tau} = \frac{\omega_{9\,10}^4 \cdot p_{e\,9\,10}^2}{3\pi \cdot \varepsilon_0 \cdot c^3}$$

$$|^2_{9\,10}$$

γ) $V = \frac{1}{2} k x^2$, $k = 50 eV/nm^2$ $\qquad n = 1$

$\gamma = ?$

$$= \frac{\omega_{01}^3 \, \mu_{e01}^2}{3\pi \varepsilon_0 c^3 \hbar}$$

$= -e_0 \cdot \langle n | x | n' \rangle = -e_0 \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n}\, \delta_{k,n-1} + \sqrt{n+1}\, \delta_{k,n+1} \right)$

$= -e_0 \langle 0 | x | 1 \rangle = -e_0 \sqrt{\frac{\hbar}{2m\omega}} \Big(1 \cdot$

$\frac{N_R}{\hbar}$

89) $n = 80$

$\underline{n' = 79}$

$\lambda_{79\,80} = ?$

$$\omega_{79,80} = \frac{W_R}{\hbar}\left(\frac{1}{79^2} - \frac{1}{80^2}\right) = 8{,}67 \cdot 10^{10}$$

$$\frac{1}{\omega_{79,80}} = \frac{\tau}{T} = 1{,}15 \cdot 10^{-11}\, s^{-1}$$

$$\lambda_{79,80} = \frac{\lambda^2}{2\pi \cdot c \cdot T} = \frac{\lambda^2}{1{,}63 \cdot 10^{20}} = 6{,}4 \cdot 10^{16}$$

$$\frac{1}{\lambda} = Z^2 R_{H} \left(\frac{1}{n'^2} - \frac{1}{n^2}\right) = 39\, m^{-1}$$

$$\boxed{\lambda = 0{,}025\, m}$$

$a = 0{,}3\,nm$ $\qquad \omega = 2\pi \cdot \nu$

$\dfrac{\tau_{45}}{\tau_{34}} = ?$

$\mu_{e\,ln} = -e_0 \langle l | x | n \rangle =$

$\langle 4|x|5\rangle = \dfrac{4 \cdot 4 \cdot 5 x_0}{\pi^2 (25-16)^2}\left[(-1)^{4+5} - 1\right] = \dfrac{80 x_0}{81\pi^2}(-2) = \dfrac{-160 x_0}{81\pi^2}$

$\langle 3|x|4\rangle = \dfrac{4 \cdot 3 \cdot 4 x_0}{\pi^2 (16-9)^2}\left[(-1)^{3+4} - 1\right] = \dfrac{-96 \cdot x_0}{49\pi^2}$

$\gamma = \dfrac{\omega_{45}^3 \, |\mu_{e45}|^2}{3\pi\varepsilon_0 c^3 \hbar} \Rightarrow \tau_1 = \dfrac{3\pi \cdot \varepsilon_0 \cdot c^3 \hbar}{|\omega_{45}|^3 \cdot |\mu_{e45}|^2}$

$= \dfrac{|\omega_{34}|^3 \, |\mu_{e34}|^2}{|\omega_{45}|^3 \, |\mu_{e45}|^2}$

$\omega_{34} = \dfrac{(W_4 - W_3)}{\hbar} = (4^2 - 9)\dfrac{\pi^2\hbar^2}{2m x_0^2 \cdot \hbar} = \dfrac{7}{2}\dfrac{\pi^2 \hbar}{m x_0^2}$

$\omega_{45} = \dfrac{W_5 - W_4}{\hbar} = (25-16)\dfrac{\pi^2 \hbar^2}{2m x_0^2 \hbar} = \dfrac{9}{2}\dfrac{\pi^2\hbar}{m x_0^2}$

$= \dfrac{7^3 \pi^6 \cdot \hbar^3 \cdot 2^3 \, m^3 x_0^6}{2^3 m^3 \cdot x_0^6 \cdot 9^3 \pi^6 \hbar^3} \cdot \dfrac{96^2 \cdot x_0^2 \quad 81^2 \pi^4}{49^2 \cdot \pi^4 \cdot 160^2 x_0^2} = \dfrac{7^3 \cdot 81^2 \cdot 96^2}{9^3 \cdot 160^2 \cdot 49^2} = \underline{\underline{0{,}46}}$ ✓

4) ATOMI Z VEČ ELEKTRONI

str. 21 / 1 - 8
str. 22 / 8 - 18
str. 23 / 19 - 26

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1) ${}^{2S+1}L_j$

$n_1, l_1 = 1$ $\quad$ $n_2, l_2 = 2$

$s_1 = \frac{1}{2}$ $\quad$ $s_2 = \frac{1}{2}$

$l - s < j < l + s$

$\frac{1}{2} < j_1 < \frac{3}{2}$

$\frac{3}{2} < j_2 < \frac{5}{2}$

$|l_1 - l_2| < L < l_1 + l_2$

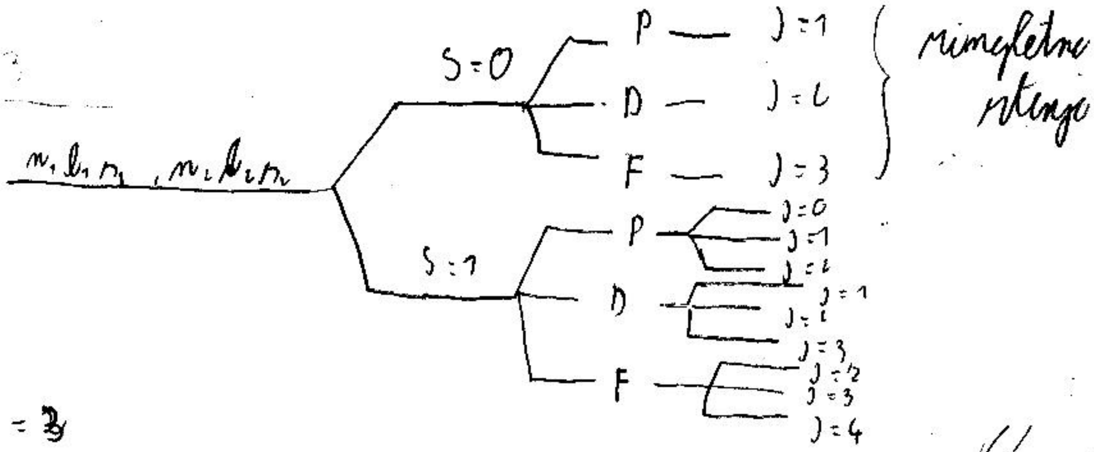
$\boxed{1 < L < 3} \Rightarrow L = 1, 2, 3$

$|L - S| \leq J \leq L + S$

$J = 1, 2, 3, 4$

$s_1 - s_2 < S < s_1 + s_2$

$0 < S < 1$ $\quad$ $S = 0, 1$



$2S + 1 = 3$

$S = 0$

$n_1, l_1 = 1$ $\qquad$ $j_1 = l_1 + s_1$

$n_2, l_2 = 2$ $\qquad$ $j_2 = l_2 + s_2$

$\frac{1}{2} \leq j_1 \leq \frac{3}{2}$ $\qquad$ $\frac{3}{2} < j_2 < \frac{5}{2}$

$j_1 = \left(\frac{1}{2}, \frac{3}{2}\right)$ $\qquad$ $j_2 = \left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right)$

$|j_1 - j_2| \leq J \leq j_1 + j_2$

$$\boxed{J = 0, 1, 2, 3, 4}$$

za kombinacije

$\left(\frac{1}{2}, \frac{1}{2}\right)$ $\quad$ $\left[\frac{3}{2}, \frac{1}{2}\right]$

$\left(\frac{1}{2}, \frac{3}{2}\right)$ $\quad$ $\left(\frac{3}{2}, \frac{3}{2}\right)$

$\left(\frac{1}{2}, \frac{5}{2}\right)$ $\quad$ $\left(\frac{3}{2}, \frac{5}{2}\right)$

Skupaj imamo 12 stanj

$L = 0, 1, 2, 3$

$S = \frac{3}{2}$

$|L - S| \leq J \leq |L + S|$

$L = 0 \quad J = \frac{3}{2},$

$L = 1: \quad J = \frac{1}{2}, \frac{5}{2}, \frac{3}{2}$

$L = 2: \quad J = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$

$L = 3 \quad J = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$

5) $p_{3/2}$ $\quad l_1 = 1$, $s_1 = \frac{1}{2}$

$d_{5/2}$ $\quad l_1 = 2$ $\quad s_2 = \frac{1}{2}$

$|l_1 - l_2| \leq L \leq |l_1 + l_2|$ $\qquad s_1 - s_2 \leq S \leq s_1 + s_2$

$L = 1, 2, 3$ $\qquad S = 0, 1$

$|L - S| \leq J \leq |L + S|$

$(1,0)$ $\quad J = 1$

$(1,1)$ $\quad J = 0, 1, 2$

$(2,0)$ $\quad J = 2$

$(2,1)$ $\quad J = 1, 2, 3$

$(3,0)$ $\quad J = 3$

$(3,1)$ $\quad J = 2, 3, 4$

$$\boxed{J = 0, 1, 2, 3, 4}$$

$\Psi = \frac{1}{2}(R_1 + R_2 + R_3 + R_4)$ $\qquad l_z = \frac{\hbar}{i}\frac{\partial}{\partial \varphi}$

$R_2 = 2^{-\frac{1}{2}}(|2,1,1\rangle + |2,1,-1\rangle)$ $\qquad$ Če sta $\delta_{ll'} = \begin{cases} 1, & l = l' \\ 0, & l \neq l' \end{cases}$

$R_3 = 2^{-\frac{1}{2}} i(|2,1,1\rangle - |2,1,-1\rangle))$ $\qquad l^2 = l(l+1)\hbar^2$

$R_1 = |n=2, l=0, m_l=0\rangle = |2,0,0\rangle$

$R_4 = |2,1,0\rangle$ $\qquad \langle n l, m_l | n l m_l\rangle = \delta_{ll'}\delta_{m_l m_l'}$

$\langle \hat{l}^2\rangle = ?$

$\langle\Psi|\hat{l}^2|\Psi\rangle = \frac{1}{4}\Big(|2,0,0\rangle + \frac{1}{\sqrt{2}}(|2,1,1\rangle + |2,1,-1\rangle) + \frac{1}{\sqrt{2}} i(|2,1,1\rangle - |2,1,-1\rangle) +$

$+ |2,1,0\rangle\Big)\hat{l}^2\Big(|2,0,0\rangle + \frac{1}{\sqrt{2}}(|2,1,1\rangle + |2,1,-1\rangle) - \frac{1}{\sqrt{2}} i(|2,1,1\rangle - |2,1,-1\rangle) + |2,1,0\rangle$

$\Big(\langle 2,0,0|\hat{l}^2|2,0,0\rangle + \langle 2,1,0|\hat{l}^2|2,1,0\rangle + \frac{1}{2}(\langle 2,1,-1|\hat{l}^2|2,1,-1\rangle) +$

$\cancel{\frac{1}{2} i \langle 2,1,1|\hat{l}^2|2,1,1\rangle + \frac{1 \cdot i}{2}\langle 2,1,-1|\hat{l}^2|2,1,-1\rangle} + \langle 2,1,0|\hat{l}^2|2,1,0\rangle$

$\cancel{\frac{1}{2} i \langle 2,1,1|\hat{l}^2|2,1,1\rangle - \frac{1}{2} i\langle 2,1,-1|\hat{l}^2|2,1,-1\rangle} + \frac{1}{2}\langle 2,1,1|\hat{l}^2|2,1,1\rangle + \frac{1}{2}\langle 2,1,-1|\hat{l}^2|2,1,-1\rangle$

$\frac{1}{4}\hbar^2(0 + 2 + 1 + 2, +1 + 1) = \frac{6}{4}\hbar^2 = \boxed{\frac{3}{2}\hbar^2}$

$\langle\Psi|l_z|\Psi\rangle \Rightarrow$ dobimo vse isto

Upoštevamo $\hat{l}_z|n l m_l\rangle = m_l \hbar |n l m_l\rangle$

$\frac{1}{4}\hbar(0, + 0 + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2}) = 0$

7) $2p \to 2s$

$\lambda = 670{,}8\ nm$

$E_i = -5{,}39\, eV$

$$E = \frac{h \cdot c}{\lambda} = 1{,}84\, eV$$

$$W = E_i + E = \underline{\underline{-3{,}55\, eV}}$$

$\omega = 2\pi \cdot \nu$,

---

8) $E_1 = -5{,}11\, eV$

$E_2 = -3{,}02\, eV$

$$\Delta E = E_1 - E_2 = 2{,}09\, eV$$

$$\Delta E = \frac{h \cdot c}{\lambda} \Rightarrow \lambda = \frac{h \cdot c}{\Delta E} = \underline{\underline{592\ nm}}$$

$$\nu = \frac{c}{\lambda} \Rightarrow \omega = 2\pi \cdot \frac{c}{\lambda}$$

$$\omega = 3{,}18 \cdot 10^{15}\, s^{-1}$$

$$\frac{1}{\tau} = \frac{\omega_{01}^3 \, |p_{e_{01}}|^2}{3\pi \cdot \varepsilon_0 \cdot \hbar c^3}$$

$$\alpha = \sqrt{\frac{m\omega}{\hbar}}$$

$$p_{e_{01}} = -e_0 \langle 1 | x | 0 \rangle = -e_0 \, \alpha \, (\cdot 1) = \boxed{-\frac{e_0}{\alpha\sqrt{2}}}$$

$$\omega_{01} = \frac{(W_n - W_{n-1})}{\hbar} = \frac{3}{2}\omega - \frac{1}{2}\omega = \omega$$

$$\frac{1}{\tau} = \frac{\omega^3 \cdot e_0^2}{\alpha^2 \cdot 2 \cdot 3\pi \varepsilon_0 \hbar \cdot c^3} = \frac{\omega^3 \cdot e_0^2 \cancel{\hbar}}{m \cancel{\omega} \cdot 6\pi \cdot \varepsilon_0 \cancel{\hbar} c^3} = \frac{e_0^2 \omega^2}{6 m \pi \varepsilon_0 c^3}$$

$$\tau = \frac{6 \cdot m \cdot \pi \cdot \varepsilon_0 \cdot c^3}{e_0^2 \cdot \omega^2} = \underline{\underline{1{,}6\ ns}}$$

$$W_{01} = \frac{\hbar}{\tau} = \underline{\underline{40\ neV}}$$

9) $T = 300K$

$\underline{W_{01} = ?}$

$$\nu_D = \left(\frac{8k\cdot T}{m\cdot c^2}\right)^{\frac{1}{2}} \cdot \nu \quad ; \quad m = \frac{M}{N_A}$$

$3{,}7\cdot 10^{-10}$; $9{,}10^{13}$; $m = 3{,}8\cdot 10^{-26}\,kg$

$$\nu = \frac{c}{\lambda} = 5{,}06\cdot 10^{14}\,Hz$$

$$\boxed{\nu_D = 3{,}7\ GHz}$$

$m\,c^2$

${}^{23}_{11}N_{A\,12}$

$m = 2m_H + N\cdot m_n - W_v(N_A)$

$30\,\frac{kN}{cm^2}$

$T = 50°C$

$\lambda = 600\,nm$

$r = 0{,}27\,nm$

$\underline{M = 86\,\frac{kg}{kmol}}$

$\nu_p = ?$

$$\nu_p = \frac{1}{2\pi(\langle l\rangle/\langle v\rangle)} = \left(\frac{2}{\pi\cdot m_1 k\cdot T}\right)^{\frac{1}{2}}(2r_1)^2 p$$

$$\langle l\rangle = \frac{1}{\pi n(2r_1)^2 n} = \frac{k\cdot T}{\pi(2r_1)^2\cdot p}$$

$$\langle v\rangle = \left(\frac{8k\cdot T}{\pi\cdot m_1}\right)^{\frac{1}{2}}$$

$$\nu_p = \left(\frac{2}{\pi\cdot m_1\cdot k T}\right)^{\frac{1}{2}}\cdot(2r_1)^2 p \quad ; \quad m_1 = \frac{M}{N_A} = 1{,}42\cdot 10^{-25}\,kg$$

$$\underline{\underline{\nu_p = 1{,}6\ GHz}}\ \checkmark$$

$\underline{p = ?}$

$\nu = \nu_p\,\frac{\lambda}{\lambda_p}$

$\nu = \nu_p\cdot\frac{\lambda\cdot c}{\lambda^2\nu_p}$

$\nu = \frac{c}{\lambda}$

$$\nu_D = \left(\frac{8k\cdot T}{m_1 c^2}\right)^{\frac{1}{2}}\cdot\nu_1 = \left(\frac{1}{\pi\cdot m_1 k\cdot T}\right)^{\frac{1}{2}}(2r_1)^2\cdot p$$

$$\left(\frac{8k\cdot T}{m\cdot c^2}\right)^{\frac{1}{2}}\frac{c}{\lambda} = \left(\frac{1}{\pi\cdot m\cdot k\cdot T}\right)^{\frac{1}{2}}(2r_1)^2\cdot p$$

$$p = \frac{\left(\frac{8k\cdot T}{m\,c^2}\right)^{\frac{1}{2}}\frac{c}{\lambda}}{\left(\frac{2}{\pi\cdot m\cdot k\cdot T}\right)^{\frac{1}{2}}(2r_1)^2}$$

$$p = \left(\frac{8\cdot k\cdot T\ \pi\cdot \cancel{m}\,k\cdot T}{m\cdot c^2\cdot 2}\right)^{\frac{1}{2}}\frac{\frac{c}{\lambda}}{(2r_1)^2}$$

$$p = \frac{\sqrt{4\pi}\ k\cdot T\cdot c}{c\ 4r_1^2\ \lambda} = \frac{2k_B\cdot T}{4r_1^2}\sqrt{\pi} = \underline{\underline{150\,kPa}}\ \checkmark$$

11) $B = 0{,}8\,T$

$3p \rightarrow 3s$

$$W = W_0 + \left(g_{L'S'J'} M_J' - g_{LSJ} M_J\right) \frac{\mu_B \cdot B}{\hbar} \rightarrow \nu = 7{,}4 \cdot 10^{10}$$

$l_2 = 0$ $\qquad {}^{2S+1}L_J$ $\qquad L =$

$l_1 = 1$ $\qquad\qquad\qquad L = 1$

$${}^2P_{\frac{3}{2}} \longrightarrow {}^2S_{\frac{1}{2}}$$

IZBIRNA PRAVILA

$L' - L = \pm 1$

$J - J' = \pm 1, 0$

$M_J - M_J' = \pm 1, 0$

$S - S' = 0$

$2S+1 = 2$ $\qquad S' = \frac{1}{2}$

$S = \frac{1}{2}$

$J = \frac{3}{2}$ $\qquad J' = \frac{1}{2}$

$L = 1$ $\qquad L' = 0$

$M_J = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ $\qquad M_J' = -\frac{1}{2}, +\frac{1}{2}$

dovoljeni prehodi

$\left(-\frac{3}{2}, -\frac{1}{2}\right)$ ✓ $\quad \left(-\frac{1}{2}, -\frac{1}{2}\right)$ ✓ $\quad \left(\frac{1}{2}, -\frac{1}{2}\right)$ ✓ $\quad \left(\frac{3}{2}, -\frac{1}{2}\right)$ //

$\left(-\frac{3}{2}, \frac{1}{2}\right)$ // $\quad \left(-\frac{1}{2}, \frac{1}{2}\right)$ ✓ $\quad \left(\frac{1}{2}, \frac{1}{2}\right)$ ✓ $\quad \left(\frac{3}{2}, \frac{1}{2}\right)$ ✓

$$g_{LSJ} = g_{1\frac{1}{2}\frac{3}{2}} = \frac{3}{2} - \frac{1}{2}\,\frac{1(1+1) - \frac{1}{2}\left(\frac{3}{2}\right)}{\frac{3}{2}\left(\frac{5}{2}\right)} = \frac{3}{2} - \frac{1 \cdot 2 - \frac{3}{4}}{2 \cdot \frac{15}{4}}$$

$$= \frac{3}{2} - \frac{\frac{5}{4}}{\frac{15}{2}} = \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \underline{\underline{\frac{4}{3}}}$$

$$g_{l'S'J'} = g_{0\frac{1}{2}\frac{1}{2}} = \frac{3}{2} - \frac{1\cdot 0 - \frac{1}{2}\left(\frac{3}{2}\right)}{2\cdot\frac{1}{2}\cdot\frac{3}{2}} = \frac{3}{2} + \frac{1}{2}\frac{\frac{3}{4}}{\frac{3}{4}} = \underline{\underline{2}}$$

$$= \omega_{01} \pm \left(2\cdot\frac{1}{2} - \frac{4}{3}\cdot\frac{3}{4}\right)\sigma = \underline{\underline{\omega_{01} \pm \sigma}}$$

$$= \omega_{02} \pm \left(2\cdot\frac{1}{2} - \frac{4}{3}\cdot\frac{1}{2}\right)\sigma = \omega_{02} \pm \frac{1}{3}\sigma$$

$$= \omega_{03} \pm \left(2\cdot\frac{1}{2} + \frac{4}{3}\cdot\frac{1}{2}\right)\sigma = \underline{\underline{\omega_{03} \pm \frac{5}{3}\sigma}}$$

...limo 6 črt

$P_{\frac{1}{2}} \longrightarrow 3S_{\frac{1}{2}}$

$= \frac{1}{2}$ $\qquad S' = \frac{1}{2}$

$= \frac{1}{2}$ $\qquad J' = \frac{1}{2}$

$= 1$ $\qquad L' = 0$

$= -\frac{1}{2}, \frac{1}{2}$ $\qquad M_J' = -\frac{1}{2}, \frac{1}{2}$

...ovoljeni prehodi

$\left(-\frac{1}{2}, -\frac{1}{2}\right)$ ✓ $\quad \left(\frac{1}{2}, -\frac{1}{2}\right)$ ✓ $\quad$ Ima 4 črte

$\left(-\frac{1}{2}, \frac{1}{2}\right)$ ✓ $\quad \left(\frac{1}{2}, \frac{1}{2}\right)$ ✓

$$g_{LSJ} = g_{1\frac{1}{2}\frac{1}{2}} = \frac{3}{2} - \frac{1}{2}\frac{\left(2 - \frac{3}{4}\right)}{\frac{3}{4}} = \frac{3}{2} - \frac{1}{2}\frac{\frac{5}{4}}{\frac{3}{4}} = \frac{3}{2} - \frac{5}{6} = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$$

$$g_{L'S'J'} = 2$$

$$\omega_1 = \omega_{01} \pm \left(2\cdot\frac{1}{2} - \frac{2}{3}\cdot\frac{1}{2}\right)\sigma = \underline{\underline{\omega_{01} \pm \frac{2}{3}\sigma}} \ ✓$$

$$\omega_2 = \omega_{02} \pm \left(2\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{1}{2}\right)\sigma = \underline{\underline{\omega_{02} \pm \frac{4}{3}\sigma}} \ ✓$$

12) ${}^4F_{3/2} \longrightarrow {}^4D_{5/2}$

$2S+1=4$ $\qquad S' = \frac{3}{2}$ $\qquad S - S' = 0$

$S = \frac{3}{2}$ $\qquad J' = \frac{5}{2}$ $\qquad L - L' = \pm 1, 0$

$J = \frac{3}{2}$ $\qquad L' = 2$ $\qquad M_j - M_j' = 0, \pm 1$

$L = 3$

$M_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ $\qquad M_j' = -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

$(-\frac32, -\frac52)$ ✓	$(-\frac12, -\frac52)$ //	$(\frac12, -\frac52)$ //	$(\frac32, -\frac52)$ //
$(-\frac32, -\frac32)$ ✓	$(-\frac12, -\frac32)$ ✓	$(\frac12, -\frac32)$ ✓//	$(\frac32, -\frac32)$ //
$(-\frac32, -\frac12)$ ✓	$(-\frac12, -\frac12)$ ✓	$(\frac12, -\frac12)$ ✓	$(\frac32, -\frac12)$ //
$(-\frac32, \frac12)$ //	$(-\frac12, \frac12)$ ✓	$(\frac12, \frac12)$ ✓	$(\frac32, \frac12)$ ✓
$(-\frac32, \frac32)$ //	$(-\frac12, \frac32)$ //	$(\frac12, \frac32)$ ✓	$(\frac32, \frac32)$ ✓
$(-\frac32, \frac52)$ //	$(\frac12, \frac52)$ //	$(\frac12, \frac52)$ //	$(\frac32, \frac52)$ ✓

✓

✓

Imamo 12 možnih prehod (12 črt)

$$g_{LSJ} = g_{3\frac32\frac32} = \frac32 - \frac12 \frac{(3\cdot 4 - \frac32\cdot\frac52)}{\frac32 \frac52} = \frac32 - \frac12 \frac{(12 - \frac{15}{4})}{\frac{15}{4}}$$

$$= \frac32 - \frac12 \frac{\frac{33}{\cancel{4}}}{\frac{15}{\cancel{4}}} = \frac32 - \frac12 \cdot \frac{33}{15} = \frac{12}{30} = \boxed{\frac25}$$ ✓

$$g_{L'S'J'} = g_{2\frac32\frac52} = \frac32 - \frac12 \frac{(6 - \frac{15}{4})}{\frac52 \cdot \frac72} = \frac32 - \frac12 \frac{\frac{9}{\cancel{4}}}{\frac{35}{\cancel{4}}} = \frac32 - \frac12 \cdot \frac{9}{35}$$

$$= \frac32 - \frac{9}{70} = \frac{105 - 9}{70} = \frac{96}{70} = \frac{48}{35}$$ ✓

$$\omega_1 = \omega_{01} \pm \left(\frac{\overset{24}{\cancel{48}}}{{}_7\cancel{35}}\cdot\frac{5}{\cancel{2}} - \frac{2}{5}\cdot\frac{3}{\cancel{2}}\right)\sigma = \omega_{01} \pm \left(\frac{24}{7} - \frac35\right) = \omega_{01} \pm \frac{99}{35}\sigma$$ ✓

$$\omega_2 = \omega_{02} \pm \left(\frac{\overset{24}{\cancel{48}}}{35}\cdot\frac{3}{\cancel{2}} - \frac{\cancel{2}}{5}\cdot\frac{3}{\cancel{2}}\right)\sigma = \omega_{02} \pm \left(\frac{72}{35} - \frac35\right)\sigma = \underline{\underline{\omega_0 \pm \frac{51}{35}\sigma}}$$ ✓

$$\omega_3 = \omega_{03} \pm \left(\frac{\cancel{48}}{35}\cdot\frac{1}{\cancel{2}} - \frac35\right)\sigma = \omega_{02} \pm \left(\frac{24}{35} - \frac35\right)\sigma = \underline{\underline{\omega_0 \pm \frac{3}{35}\sigma}}$$ ✓

$$\omega_4 = \omega_{04} \pm \left(\frac{48}{35}\cdot\frac32 - \frac25\cdot\frac12\right)\sigma = \omega_{04} \pm \left(\frac{72}{35} - \frac15\right)\sigma = \underline{\underline{\omega_{04} \pm \frac{65}{35}\sigma}}$$ ✓

$$w_5 = w_{05} \pm \left(\frac{48}{35} \cdot \frac{1}{2} - \frac{1}{5} \right) \sigma = w_{05} \pm \left(\frac{24}{35} - \frac{1}{5} \right) \sigma = \underline{\underline{w_{05} \pm \frac{17}{35} \sigma}}$$

$$w_6 = w_{06} \pm \left(- \frac{24}{35} - \frac{1}{5} \right) \sigma = \underline{\underline{w_{06} \pm \frac{31}{35} \sigma}}$$

13) $^3D_1 \longrightarrow {}^3P_1$

$2S+1=3$

$S=1$ $\quad$ $S'=1$

$L=2$ $\quad$ $L'=1$

$J=1$ $\quad$ $J'=1$

$M_J=-1,1,0$ $\quad$ $M_J'=-1,1,0$

Dovoljeni prehod

$(-1,-1)$ ✓ $\quad (1,-1)$ // $\quad (0,-1)$ ✓ $\quad$ 9 prehodov

$(-1,1)$ // $\quad (1,1)$ ✓ $\quad (0,1)$ ✓ $\quad$ 6 črt

$(-1,0)$ ✓ $\quad (1,0)$ ✓ $\quad (0,0)$ ✓

$$g_{LSJ} = \frac{3}{2} - \frac{1}{2}\frac{(6-2)}{2} = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

$$g_{L'S'J'} = \frac{3}{2} - \frac{1}{2}\frac{(3-2)}{2} = \frac{3}{2} - \frac{1}{4} = \boxed{\frac{5}{4}}$$

$$\omega_1 = \omega_{01} \pm \left(\frac{5}{4}\cdot 1 - \frac{1}{2}\right)\sigma = \underline{\omega_{01} \pm \frac{3}{4}\sigma}$$

$$\omega_2 = \omega_{02} \pm \left(\frac{5}{4}\cdot 0 - \frac{1}{2}\right)\sigma = \underline{\omega_{02} \pm \frac{2}{4}\sigma}$$

$$\omega_3 = \omega_{03} \pm \left(\frac{5}{4} \cdot 0\right)\sigma = \underline{\omega_{03} \pm \frac{5}{4}\sigma}$$

4) $^3P_1 \longrightarrow {}^3D_2$

$M_J'' - M_J' = \pm 1, 0$

$S = 1 \qquad S' = 1$

$L = 1 \qquad L' = 2$

$J = 1 \qquad J' = 2$

$M_J = -1, 0, 1 \qquad M_J' = -2, -1, 0, 1, 2$

$(-1,-2)$ $(0,-2)$ $(1,-2)$

$(-1,-1)$ $(0,-1)$ $(1,-1)$

$(-1,0)$ $(0,0)$ $(1,0)$

$(-1,1)$ $(0,1)$ $(1,1)$

$(-1,2)$ $(0,2)$ $(1,2)$

9 čar

$g(LSJ) = g_{1\,1,1} = \frac{3}{2} - \frac{1}{2}\frac{(2-2)}{2} = \boxed{\frac{3}{2}}$

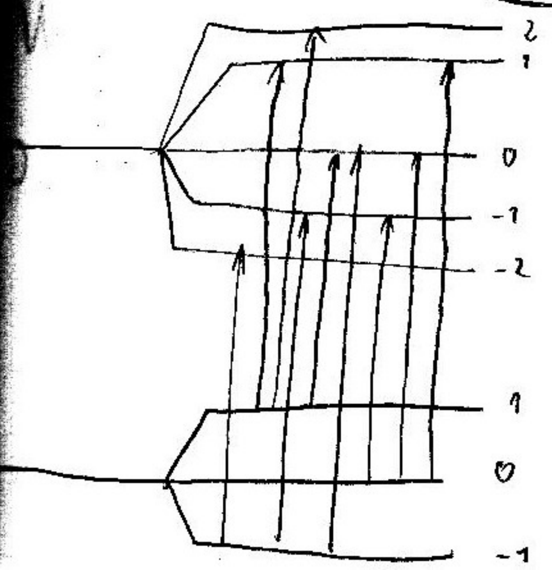
$g(L'S'J') = g_{2\,1,2} = \frac{3}{2} - \frac{1}{2}\frac{(3-2)}{3} = \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \boxed{\frac{4}{3}}$

$\omega_1 = \omega_{01} \pm \left(\frac{4}{3}\cdot 2 - \frac{3}{2}\cdot 1\right)\sigma = \omega_{01} \pm \left(\frac{8}{3} - \frac{3}{2}\right)\sigma = \omega_{01} \pm \frac{7}{6}\sigma$

$\omega_2 = \omega_{02} \pm \left(\frac{4}{3} - \frac{3}{2}\right)\sigma = \omega_{02} \pm \frac{1}{6}\sigma$

$\omega_3 = \omega_{03} \pm \left(-\frac{3}{2}\right)\sigma = \omega_{03} \pm \frac{9}{6}\sigma$

$\omega_4 = \omega_{04} \pm \left(\frac{4}{3}\sigma\right) = \omega_{04} \pm \frac{8}{6}\sigma$



15) $B = 0{,}02\,T$

$^3P_2 \longrightarrow {}^3S_1$

$L = 1 \qquad L' = 0$

$S = 1 \qquad S' = 1$

$J = 2 \qquad J' = 1$

$M_J = -2, -1, 0, 1, 2 \qquad M_J' = -1, 0, 1$

(-2, -1) ✓ (-1, -1) ✓ (0, -1) ✓ (1, -1) // (2, -1) //
(-2, 0) // (-1, 0) ✓ (0, 0) ✓ (1, 0) ✓ (2, 0) //
(-2, 1) // (-1, 1) ✓ (0, 1) ✓ (1, 1) ✓ (2, 1) ✓

g ert

$g_{112} = \frac{3}{2} - \frac{1}{2}\,\frac{(2-2)}{3} = \frac{3}{2}$

$g_{011} = \frac{3}{2} - \frac{1}{2}\left(\frac{0-2}{2}\right) = \underline{2}$

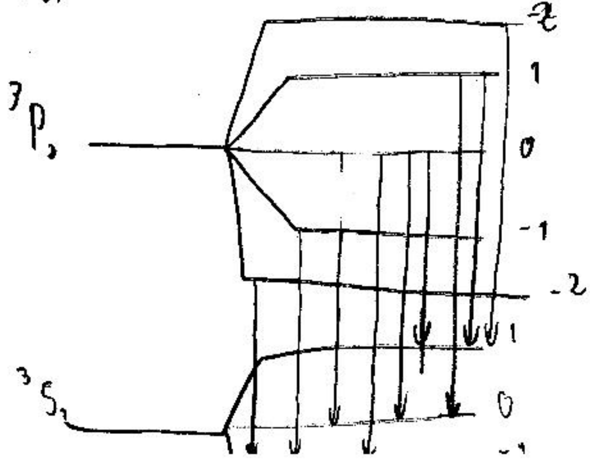
$\omega_1 = \omega_{01} \pm \left(2.1 - \frac{3}{2}.2\right)\sigma = \underline{\omega_{01} \pm \frac{2}{2}\sigma}$ ✓

$\omega_2 = \omega_{02} \pm \left(2 - \frac{3}{2}\right)\sigma = \underline{\omega_{02} \pm \frac{1}{2}\sigma}$ ✓

$\omega_3 = \omega_{03} \pm \left(-\frac{3}{2}\sigma\right) = \underline{\omega_{03} \pm \frac{3}{2}\sigma}$ ✓

$\omega_4 = \omega_{04} \pm (2) = \underline{\omega_{03} \pm \frac{4}{2}\sigma}$ ✓

$\omega_5 = \omega_{05}$ ✓



✓

✓

$B = T$

6) $\lambda = 18,49\,nm$; zbog spinske

$^2S_0 \longrightarrow * \; ^2P_{\phi}$

$J = 0$ $\quad$ $J' = 0$

$L = 0$ $\quad$ $L' = 1$

$S = \frac{1}{2}$ $\quad$ $S' = \frac{1}{2}$

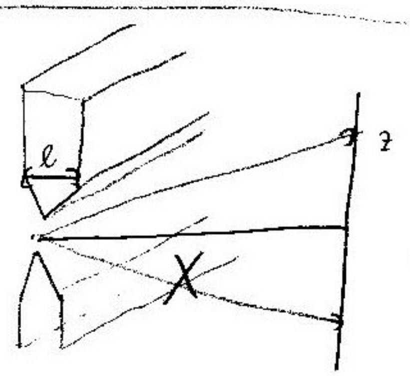
$l = 1,\; j = \frac{3}{2}$

$E = 1eV$

$\frac{\partial B}{\partial x} = 10^3 \frac{T}{m}$

$l = 10cm$

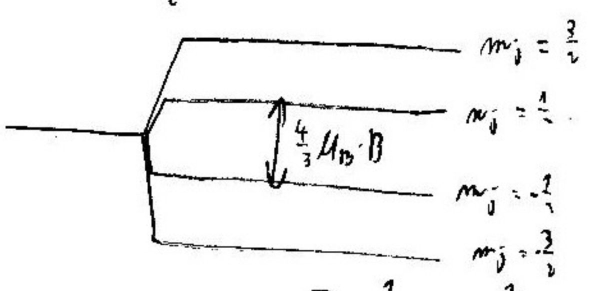
$x = 20$



$m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$; Dobimo 4 curke

$= g_{lj} \cdot m_j \cdot \mu_B$

$g_{\frac{3}{2}} = \frac{3}{2} - \frac{2 - \frac{3}{4}}{2 \cdot \frac{3}{2}\left(+\frac{5}{2}\right)} = \frac{3}{2} - \frac{\frac{5}{4}}{\frac{15}{2}} = \frac{3}{2} - \frac{1}{2 \cdot 3} = \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \boxed{\frac{4}{3}}$



$W_{ej} = \frac{4}{3} \cdot \mu_B \cdot B$ $\qquad$ $T = \frac{1}{2} m \cdot v^2$

$D_z = \mu_B \cdot \frac{\partial B}{\partial z} \frac{l \cdot x}{m \cdot v_0^2} = \mu_B \cdot \frac{\partial B}{\partial z} \frac{l \cdot x}{2T} \left(\frac{4}{3}\right) = \underline{7,7 \cdot 10^{-6} m}$ ✓

18) $^2S_{\frac{1}{2}}$

$v = 500 \frac{m}{s}$

$l = 10\,cm$

$\frac{\partial B}{\partial z} = 100\ \frac{T}{m}$

$x = 2\,m$

$M = 201\,kg$

$L = 0$

$2S+1 = 2$

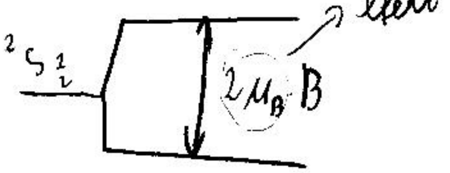
$S = \frac{1}{2}$

$J = \frac{1}{2}$

$M_J = -\frac{1}{2}, \frac{1}{2}$ $\Rightarrow$ Dobimo dva curka

$g_{0\frac{1}{2}} = \frac{3}{2} - \frac{\left(-\frac{3}{4}\right)}{2\cdot\frac{1}{2}\cdot\frac{3}{2}} = \frac{3}{2} + \frac{3}{4}\cdot\frac{2}{3} = 2$

$\boxed{W_{0\frac{1}{2}} = 2\cdot\mu_B\cdot B}$



$\partial z = 2\cdot\mu_B\cdot\frac{\partial B}{\partial z}\ \frac{l\cdot x}{m\cdot v_0^2}$; $m = \frac{M}{N_A} = 3{,}34\cdot 10^{-25}\,kg$

$\partial z = \underline{\underline{4{,}4\,mm}}$

9) $^2P_{1/2}$ $\quad l = 1 \quad j = \frac{1}{2}$ $\quad {}^{11}_{5}B_{6}$

$v = 500 \frac{m}{s}$

$l = 10\,cm$

$\frac{\partial B}{\partial z} = 500 \frac{m}{s}$

$x = 1\,m$

$\partial z = ?$

2 curka $\quad m_j = \pm \frac{1}{2}$

$m = 5 \cdot m_H + 6 \cdot m$

$$g_{1\frac{1}{2}} = \frac{3}{2} - \frac{\left(2 - \frac{3}{4}\right)}{2 \cdot \frac{1}{2} \frac{3}{2}} = \frac{3}{2} - \frac{\frac{5}{4}}{\frac{3}{2}} = \frac{3}{2} - \frac{5}{4} \cdot \frac{2}{3} = \frac{3}{2} - \frac{5}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\mu_{1\frac{1}{2}} = \frac{2}{3} \mu_B \cdot B \cdot m_j$$

$$\boxed{\partial z = \left(\frac{2}{3}\right) \mu_B \cdot \frac{\partial B}{\partial z} \cdot \frac{l \cdot x}{m \cdot v_0^2}} = \underline{6,7\ mm}$$

20) $4\,^2D_{\frac{5}{2}} \longrightarrow 4\,^2P_{3/2}$

$\lambda = 315{,}8\,nm$

$B = 0{,}2\,T$

$L = 2 \qquad L' = 1 \qquad M_J - M_J' = \pm 1, 0$

$S = \frac{1}{2} \qquad S' = \frac{1}{2}$

$J = \frac{5}{2} \qquad J' = \frac{3}{2}$

$M_J = -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \qquad M_J' = -\frac{3}{2}, \frac{3}{2}, -\frac{1}{2}, \frac{1}{2}$

$(\frac{3}{2}, -\frac{3}{2})$ // $(\frac{5}{2}, -\frac{3}{2})$ //

$(-\frac{5}{2}, -\frac{3}{2})$ ✓ $(-\frac{3}{2}, -\frac{3}{2})$ ✓ $(-\frac{1}{2}, -\frac{3}{2})$ ✓ $(\frac{1}{2}, -\frac{3}{2})$ // $(\frac{5}{2}, \frac{3}{2})$ ✓

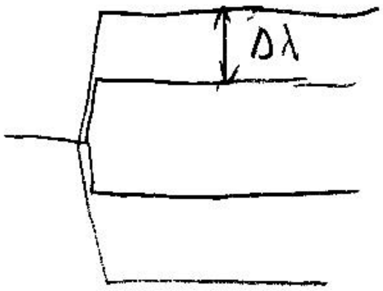
$(-\frac{5}{2}, -\frac{1}{2})$ // $(-\frac{3}{2}, -\frac{1}{2})$ ✓ $(-\frac{1}{2}, -\frac{1}{2})$ ✓ $(\frac{1}{2}, -\frac{1}{2})$ ✓ $(\frac{3}{2}, -\frac{1}{2})$ //

$(\frac{3}{2}, \frac{1}{2})$ ✓ $(\frac{5}{2}, -\frac{1}{2})$ //

$(-\frac{5}{2}, \frac{1}{2})$ // $(-\frac{3}{2}, \frac{1}{2})$ // $(-\frac{1}{2}, \frac{1}{2})$ ✓ $(\frac{1}{2}, \frac{1}{2})$ ✓

$(-\frac{5}{2}, \frac{3}{2})$ // $(-\frac{3}{2}, \frac{3}{2})$ // $(-\frac{1}{2}, \frac{3}{2})$ // $(\frac{1}{2}, \frac{3}{2})$ ✓ $(\frac{3}{2}, \frac{3}{2})$ ✓ $(\frac{5}{2}, \frac{1}{2})$ //

Imamo 12 črt

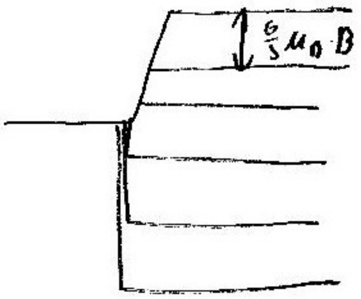


$g_{2\frac{1}{2}\frac{5}{2}} = \frac{3}{2} - \frac{1}{2}\frac{(6 - \frac{3}{4})}{\frac{5}{2}\cdot\frac{7}{2}} = \frac{3}{2} - \frac{\frac{21}{4}}{\frac{35}{2}} = \frac{3}{2} - \frac{21}{2\cdot 35} = \frac{84}{70} = \frac{42}{35} = \frac{6}{5}$

$g_{1\frac{1}{2}\frac{3}{2}} = \frac{3}{2} - \frac{1}{2}\frac{(2 - \frac{3}{4})}{\frac{3}{2}\cdot\frac{5}{2}} = \frac{3}{2} - \frac{\frac{5}{8}}{\frac{15}{2}} = \frac{3}{2} - \frac{5}{4}\cdot\frac{2}{15} = \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \boxed{\frac{4}{3}}$

$$\left.\begin{aligned}
W_1 &= W_{01} \pm \left(\frac{4}{3}\cdot\frac{3}{2} - \frac{6}{5}\cdot\frac{5}{2}\right)\sigma = W_{01} \pm (2 - 3)\sigma = W_{01} \pm \frac{15\sigma}{15} \\
W_2 &= W_{02} \pm \left(\frac{4}{3}\cdot\frac{3}{2} - \frac{6}{5}\cdot\frac{3}{2}\right)\sigma = W_{02} \pm \left(2 - \frac{9}{5}\right)\sigma = W_{02} \pm \frac{3\sigma}{15} \\
W_3 &= W_{03} \pm \left(\frac{4}{3}\cdot\frac{1}{2} - \frac{9}{5}\right)\sigma = W_{03} \pm \left(\frac{10}{15} - \frac{27}{15}\right) = W_{03} \pm \frac{17}{15}\sigma \\
W_4 &= W_{04} \pm \left(\frac{4}{3}\cdot\frac{3}{2} - \frac{6}{5}\cdot\frac{1}{2}\right)\sigma = W_{04} \pm \left(2 - \frac{3}{5}\right)\sigma = W_{04} \pm \frac{21}{15}\sigma \\
W_5 &= W_{05} \pm \left(\frac{4}{3}\cdot\frac{1}{2} - \frac{6}{5}\cdot\frac{1}{2}\right)\sigma = W_{04} \pm \left(\frac{10}{15} - \frac{9}{15}\right)\sigma = W_{05} \pm \frac{\sigma}{15} \\
W_6 &= W_{06} \pm \left(-\frac{4}{3}\cdot\frac{1}{2} - \frac{6}{5}\cdot\frac{1}{2}\right)\sigma = W_{05} \pm \left(-\frac{10}{15} - \frac{9}{15}\right)\sigma = W_{05} \pm \frac{19\sigma}{15}
\end{aligned}\right\} ✓$$

$W_{LSJB} = \frac{6}{5} \cdot \mu_B \cdot B = \boxed{4,1\ meV}$



$W_{12} = \frac{\hbar}{\tau} \Rightarrow \frac{1}{\tau} = \frac{W_{LS}}{\hbar}$

$\Delta\lambda = \frac{\lambda^2}{2\pi \cdot \tau \cdot c} = \frac{\lambda^2 W_{LSJ}}{2\pi \cdot \hbar c}$

$\Delta\lambda = ?$

$W_{1\frac{1}{2}\frac{3}{2}} = \frac{4}{3}\mu_B \cdot B$

$\mu_B \cdot B \left(\frac{6}{5} + \frac{4}{3}\right) = \mu_B \cdot B \cdot \frac{38}{15}$

$\lambda = \frac{\lambda^2}{h \cdot c} \cdot \Delta E = \underline{\underline{2,4\ pm}}$ ✓

$= \frac{6}{5}$

21) $^2P_{\frac{1}{2}} \longrightarrow {}^2S_{\frac{1}{2}}$ $\lambda = 769{,}9\,nm$

$^2P_{3/2} \longrightarrow {}^2S_{\frac{1}{2}}$ $(764{,}5\,nm)$

$B = 0{,}08\,T$

Izborna pravila

$S - S' = 0$

$L - L' = \pm 1, 0$

$M_J - M_J' = \pm 1, 0$

$J - J' = \pm 1, 0$

1) $L = 1$ $\quad L' = 0$

$J = \frac{1}{2}$ $\quad S' = \frac{1}{2}$

$S = \frac{1}{2}$ $\quad J' = \frac{1}{2}$

$M_J = \pm \frac{1}{2}$ $\quad M_J' = \pm \frac{1}{2}$

$\left.\begin{matrix} \left(-\frac{1}{2}, -\frac{1}{2}\right) & \left(\frac{1}{2}, -\frac{1}{2}\right) \\ \left(-\frac{1}{2}, \frac{1}{2}\right) & \left(+\frac{1}{2}, \frac{1}{2}\right) \end{matrix}\right\}$ 4 črte

$$g_{LSJ} = \frac{3}{2} - \frac{1}{2}\,\frac{2 - \frac{3}{4}}{\frac{3}{4}} = \frac{3}{2} - \frac{\frac{5}{4}}{\frac{3}{2}} = \frac{3}{2} - \frac{5}{4}\cdot\frac{2}{3} = \frac{3}{2} - \frac{5}{6} = \frac{4}{6} = \frac{2}{3}$$

$$g_{L'S'J'} = \frac{3}{2} - \frac{1}{2}\,\frac{\left(-\frac{3}{4}\right)}{\frac{3}{4}} = \boxed{2}$$

$$\omega_1 = \omega_{01} \pm \left(2\cdot\frac{1}{2} - \frac{2}{3}\,\frac{1}{2}\right)\sigma = \underline{\omega_{01} \pm \frac{2}{3}\sigma}$$

$$\omega_2 = \omega_{02} \pm \left(-1 - \frac{1}{3}\right)\sigma = \underline{\omega_{02} \pm \frac{4}{3}\sigma}$$

2) $L = 1$ $\quad L' = 0$

$J = \frac{3}{2}$ $\quad J' = \frac{1}{2}$

$S = \frac{1}{2}$ $\quad S' = \frac{1}{2}$

$M_J = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ $\quad M_J' = -\frac{1}{2}, \frac{1}{2}$

$\left(-\frac{3}{2}, -\frac{1}{2}\right)$ ✓ $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ ✓ $\left(\frac{1}{2}, -\frac{1}{2}\right)$ ✓ $\left(\frac{3}{2}, -\frac{1}{2}\right)$ // $\qquad$ 6 črt

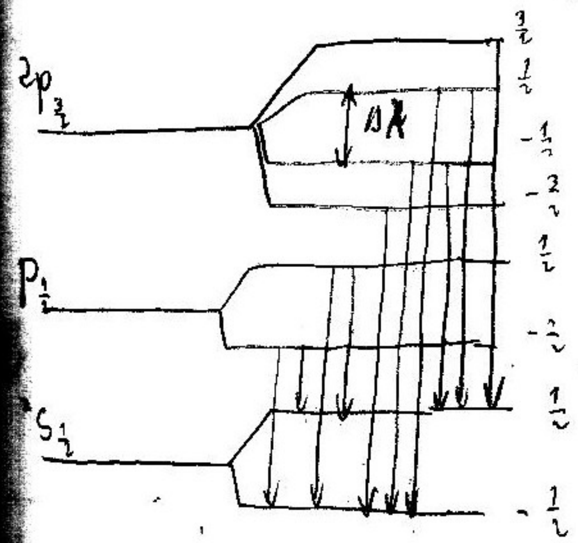
$\left(-\frac{3}{2}, \frac{1}{2}\right)$ // $\left(-\frac{1}{2}, \frac{1}{2}\right)$ ✓ $\left(\frac{1}{2}, \frac{1}{2}\right)$ ✓ $\left(\frac{3}{2}, \frac{1}{2}\right)$ ✓

$$g_{LSJ} = \frac{3}{2} - \frac{1}{2}\,\frac{\left(2 - \frac{3}{4}\right)}{\frac{15}{4}} = \frac{3}{2} - \frac{\frac{5}{8}}{\frac{15}{4}} = \frac{3}{2} - \frac{1}{2\cdot 3} = \frac{9-1}{6} = \boxed{\frac{4}{3}}$$ ✓

$$\omega_1 = \omega_{01} \pm \left(2\cdot\frac{1}{2} - \frac{4}{3}\,\frac{3}{2}\right)\sigma = \underline{\omega_{01} \pm 3\sigma/3}$$

$$\omega_2 = \omega_{02} \pm \left(2\cdot\frac{1}{2} - \frac{4}{3}\,\frac{1}{2}\right)\sigma = \omega_{02} \pm \left(1 - \frac{2}{3}\right) = \underline{\omega_{02} \pm \frac{1}{3}\sigma}$$ ✓

$W_3 = W_{03} \pm \left(-2\cdot\frac{1}{2} - \frac{2}{3}\right)\vartheta = \underline{\underline{W_{03} \pm \frac{5}{3}\vartheta\hbar}}$, $\vartheta = \frac{\mu_B \cdot B}{\hbar}$



$W_{\frac{2}{3}\frac{1}{2}\frac{1}{2}} = \frac{2}{3}\mu_B \cdot B$

$W_{2\frac{1}{2}\frac{1}{2}} = 2\mu_B \cdot B$

$\Delta W = \frac{2}{3}\mu_B \cdot B + 2\mu_B B = \frac{8}{3}\mu_B B$

$\Delta\lambda = \frac{\lambda^2}{h c}\Delta W = \underline{\underline{6\ pm}}$ (za prehod med $^2P_{\frac{1}{2}} \to {}^2S_{\frac{1}{2}}$)

b) $W_{\frac{4}{3}\frac{1}{2}\frac{3}{2}} = \frac{4}{3}\mu_B B$

$\Delta W = \left(\frac{4}{3} + 2\right)\mu_B \cdot B = -\frac{10}{3}\mu_B B$

$\Delta\lambda = \frac{\lambda^2 \Delta W}{h c} = \underline{\underline{7{,}3\ pm}}$

22) $^2S_{\frac{1}{2}} \longrightarrow {}^2P_{\frac{3}{2}}$ $(\lambda = 657{,}83\,nm)$

$B = 0{,}1\,T$

$L = 0$ $\quad L' = 1$

$J = \frac{1}{2}$ $\quad J' = \frac{3}{2}$

$S = \frac{1}{2}$ $\quad S' = \frac{1}{2}$

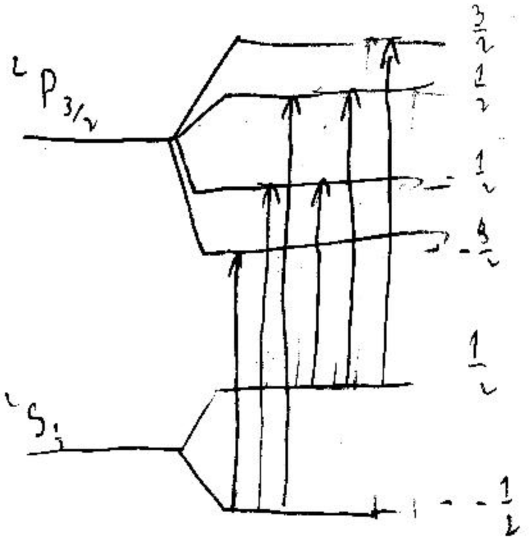
$M_J = \pm \frac{1}{2}$ $\quad M_J' = \pm \frac{3}{2}, \pm \frac{1}{2}$

$(-\frac{1}{2}, -\frac{3}{2})$ ✓ $\quad (\frac{1}{2}, -\frac{3}{2})$ //

$(-\frac{1}{2}, -\frac{1}{2})$ ✓ $\quad (\frac{1}{2}, -\frac{1}{2})$ ✓ $\qquad$ 6 črt

$(-\frac{1}{2}, \frac{1}{2})$ ✓ $\quad (\frac{1}{2}, \frac{1}{2})$ ✓

$(-\frac{1}{2}, \frac{3}{2})$ // $\quad (\frac{1}{2}, +\frac{3}{2})$ ✓



$$g_{LSJ} = \frac{3}{2} - \frac{1}{2}\frac{(0 - \frac{3}{4})}{\frac{3}{4}} = \frac{3}{2} - \frac{1}{2} = \underline{\underline{2}}$$

$$g_{L'S'J'} = \frac{3}{2} - \frac{1}{2}\frac{(2 - \frac{3}{4})}{\frac{15}{4}} = \frac{3}{2} - \frac{\frac{5}{4}}{\frac{15}{2}} = \frac{3}{2} - \frac{5}{4}\cdot\frac{2}{15} = \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \boxed{\frac{4}{3}}$$

$$W = W_{01} \pm \left(\frac{4}{3}\cdot\frac{3}{2} - 2\cdot\frac{1}{2}\right)\nu = \underline{\underline{W_{01} \pm \frac{3}{3}\nu}}$$

$$W = W_{02} \pm \left(\frac{4}{3}\cdot\frac{1}{2} - 2\cdot\frac{1}{2}\right)\nu = \underline{\underline{W_{02} \pm \frac{\nu}{3}}}$$

$$W = W_{03} \pm \left(-\frac{4}{3}\cdot\frac{1}{2} - 2\cdot\frac{1}{2}\right)\nu = \underline{\underline{W_{03} \pm \frac{5\nu}{3}}}$$

✓

$W_{LSJ} = 2\mu_B \cdot B$

$W_{L'S'J'} = \frac{4}{3}\mu_B \cdot B$

$\Delta E = \left(2 + \frac{4}{3}\right)\mu_B B = \boxed{\frac{10}{3}\mu_B \cdot B}$

$$\Delta\lambda = \frac{N \cdot \lambda^2 \cdot \Delta E}{h \cdot c} = \underline{\underline{6{,}75\,pm}}$$

$$N = \frac{\lambda}{\Delta\lambda} = \underline{\underline{97456}}$$

$\lambda_{K_\alpha} = 0{,}1\ nm$

$$\frac{1}{\lambda_{K_\alpha}} = \frac{(Z-1)^2}{\lambda_0}$$

$$\boxed{\frac{1}{\lambda_0} = 121{,}6\ nm}$$

$$\sqrt{\frac{\lambda_0}{\lambda_{K_\alpha}}} + 1 = Z = \underline{\underline{36}}$$

$$\sqrt{\frac{\lambda_0}{\lambda_{K_\alpha}}} + 1 = Z = \underline{\underline{35}}$$

5) MOLEKULE

str. 23 / 27
str. 24 / 28 - 36
str. 25 / 37 - 40

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27) $R_0 = 0{,}251\,nm$

$W_d = -4{,}24\,eV$

$W_i = 5{,}14\,eV$

$W_A = 3{,}81\,eV$

$V_{odl} = ?$

$$V_e(R) = -W_d - W_i + W_a + W_c$$

$$= -|V(R_0)| - W_i + W_A + \frac{e_0^2}{4\pi\varepsilon_0 \cdot R} = -4{,}24\,eV - 5{,}14\,eV + 3{,}81\,eV + 5{,}7\,eV$$

$$= \underline{\underline{0{,}16\,eV}}$$

---

28) $Ca^+ O^-$

$Ca^{++} O^{--}$; $Z = 2$

$R = 0{,}1822\,nm$

$W_A(O^-) = 2{,}26\,eV$

$W_A(O^{--}) = -9\,eV$

$W_i(Ca^+) = 6{,}1\,eV$

$W_i(Ca^{++}) = 11{,}9\,eV$

$W_d = ?$

$$|V(R_0)| = V_c(R) + V_e(R) + W_i - W_a$$

$$= -\frac{e^2}{4\pi\varepsilon_0 R} + 6{,}1\,eV - 2{,}26\,eV$$

$$= -7{,}9\,eV + 6{,}1\,eV - 2{,}26\,eV = \underline{\underline{-4{,}06\,eV}}$$

$$V(R_0) = -7{,}9\,eV + (W_{i_1} + W_{i_2}) - (W_{a_1} + W_{a_2})$$

$$= \underline{\underline{-6{,}86\,eV}}$$

29) $W = \hbar^2 l(l+1)/m_\mu R^2$

$l \to l-1$

$W_{l\,l-1} = \frac{\hbar}{J} \cdot l$

$W_{l+1\,l} = \frac{\hbar}{2J}(l+1)$

---

30) H_2

HD

D_2

$R = 0{,}074$ nm

$u = 1{,}66 \cdot 10^{-27}$ kg

H_2

$$m_\mu = \frac{m_H \cdot m_H}{m_H + m_H} = \frac{m_H}{2}$$

$$J = m_\mu \cdot R^2 = \frac{m_H}{2} \cdot R^2 = 4{,}58 \cdot 10^{-48}\ \text{kg m}^2$$

$$W_l = \frac{\hbar}{2J}\, l(l+1) \Rightarrow l = 1 \quad \boxed{W_{l'} = \frac{2\hbar}{m_H R^2}}$$

$$l = 2 \quad \boxed{W_2 = \frac{6\hbar}{m_H R^2}}$$

HD

$$m_\mu = \frac{m_H \cdot m_D}{m_H + m_D}$$

$$J = \frac{m_H m_D}{m_H + m_D} R^2$$

$$W_l' = \frac{\hbar (m_H + m_D)}{2 m_H m_D R^2} \cdot l(l+1) \Rightarrow l = 1 \quad \boxed{W_l' = \frac{\hbar (m_H + m_D)}{m_H \cdot m_D R^2}}$$

$$l = 2 \quad \boxed{W_l' = \frac{3\hbar (m_H + m_D)}{m_H \cdot m_D \cdot R^2}}$$

31) $\lambda = 4{,}8 \cdot 10^{-4}\, m$

$R = ?$

HCl

$$\omega_{l,l-1} = \frac{\hbar}{J} \cdot l$$

$$\boxed{\omega_{10} = \frac{\hbar}{J} \cdot 1}$$

$\frac{\lambda}{c}$ $\quad \omega = 2\pi \nu$

$\nu = \frac{\omega}{2\pi}$

$$J = \frac{m_H \cdot m_{Cl}}{m_H + m_{Cl}} \cdot R^2$$

m_N

$$\nu = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{\nu} = \frac{c \cdot 2\pi}{\omega}$$

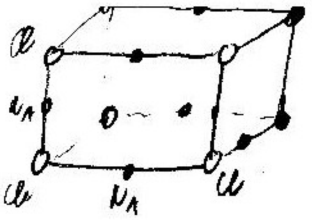
$$\omega_{10} = \frac{\hbar (m_H + m_{Cl})}{m_H \cdot m_{Cl}\ R^2} = \frac{2\pi \cdot c}{\lambda} \quad / R^2 : \left(\frac{2\pi \cdot c}{\lambda}\right)$$

$$R^2 = \frac{\hbar (m_H + m_{Cl}) \cdot \lambda}{m_H\, m_{Cl} \cdot 2\pi \cdot c} \quad / \sqrt{\ }$$

$m_N = 1{,}62 \cdot 10^{-27}\, kg$

$$R = \sqrt{\frac{\hbar (m_H + m_{Cl})\, \lambda}{m_H \cdot m_{Cl} \cdot 2\pi \cdot c}} = \sqrt{\frac{\hbar \cdot \lambda}{2\pi \cdot c \cdot m_N}} = \underline{0{,}125\, nm}$$

32) $\rho = 0{,}83 \frac{g}{cm^3} = 830 \frac{kg}{m^3}$ $a = 0{,}281\,nm$



$\frac{0{,}00083}{1000000}$?

$l \to l-1$ $(1 \to 0$

$\lambda = ?$

$W_l' = \frac{\hbar}{2J} \cdot l(l+1)$; $\boxed{W_1 = \frac{\hbar}{2J}}$

$\rho = \frac{m_R}{V} \Rightarrow \boxed{m_R = \rho \cdot V} = \boxed{\rho a^3}$

$a = R$

$\omega_{10} = \frac{2\hbar}{J}$; $J = m_R \cdot R^2 = \rho \cdot a^3 \cdot R^2 = \rho \cdot a^5$

$\frac{c \cdot 2\pi}{\lambda} = \frac{\hbar}{\rho \cdot a^5} \Rightarrow \lambda = \frac{2\pi \cdot c \cdot \rho a^5}{\hbar} = \underline{\underline{0{,}03\,m}}$

---

33) $^{12}C\,^{16}O$ $\quad ^{x}C\,^{y}O$ $\quad ^{A}_{Z}B_N$ $\quad J = \frac{m_c \cdot m_o}{m_c + m_o} \cdot R^2$ $\quad m_1 = \frac{m_o \cdot m_c}{m_c + m_o}$

$\nu_1 = 115{,}3\,GHz$

$\nu_2 = 104{,}7$

$m_c = 12 \cdot 1{,}66 \cdot 10^{-27} = 1{,}99 \cdot 10^{-26}\,kg$

$m_o = 2{,}66 \cdot 10^{-26}\,kg$

$m_1 = 1{,}14 \cdot 10^{-26}\,kg$

$\omega_1 = 2\pi \cdot \nu_1 \cong 724 \cdot 10^3\,s^{-1}$

$\omega_2 = 2\pi \cdot \nu_2 \cong 658 \cdot 10^3\,s^{-1}$

$\omega_{10} = \frac{\hbar}{J_1} \Rightarrow J_1 = \frac{\hbar}{\omega_{10}} = 1{,}38 \cdot 10^{-40}$

$\omega_{20} = \frac{\hbar}{J_2} \Rightarrow J_2 = \frac{\hbar}{\omega_{20}} = 1{,}52 \cdot 10^{-40}$

$\frac{\omega_{10}}{\omega_{20}} = \frac{J_2}{J_1} = \frac{m_2 \cdot \cancel{R^2}}{m_1 \cdot \cancel{R^2}} \Rightarrow m_2 = m_1 \frac{\omega_{10}}{\omega_{20}} = 1{,}25 \cdot 10^{-26}$

$m_2 = \frac{X m_c \cdot Y m_o}{X m_c + Y m_o} \Rightarrow {}^{13}C\,O^{18}$

34) $k = 480 \frac{N}{m}$ HCl

$l \to l-1 \quad (1 \to 0)$

$m_r = \frac{m_{Cl} \cdot m_H}{m_{Cl} + m_H} = 1{,}62 \cdot 10^{-27}$

a) $\omega_{10} = \frac{\hbar}{J}$; $\omega = \sqrt{\frac{k}{m_r}}$

$\omega_{10} = \sqrt{\frac{480 N}{m \cdot 1{,}60 \cdot 10^{-27}}} = \underline{\underline{5{,}4 \cdot 10^{14} s^{-1}}}$

$\nu_{10} = \frac{\omega}{2\pi} = \underline{\underline{86 \, THz}}$ ✓

b) DCl

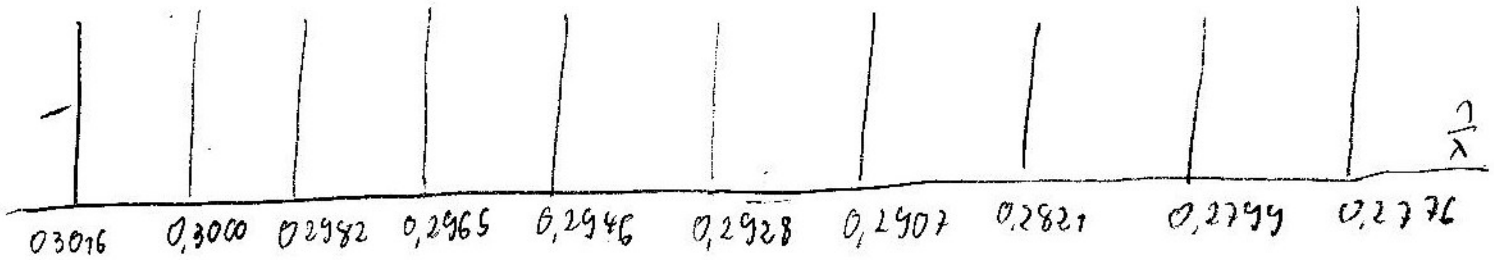
$m_{r_2} = \frac{m_{Cl} \cdot 2m_H}{m_{Cl} + 2m_H} = 3{,}17 \cdot 10^{-27} kg$

$\omega_{20} = \sqrt{\frac{K}{m_{r_2}}} = 3{,}89 \cdot 10^{14} s^{-1}$

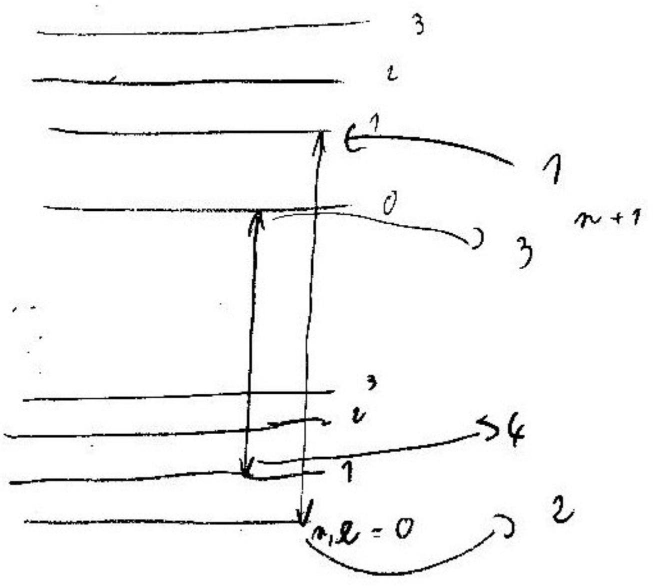
$\nu_{20} = 62 \, THz$

$\Delta = \frac{\nu_{20}}{\nu_{10}} = \underline{\underline{28\%}}$ ✓

35) H O $\quad K = ?,\ R = ?$



Prehodi $\quad l - l' = \pm 1$



$$E = E_{VIB} + E_{ROT}$$

$$E = \hbar\omega_0\left(n + \tfrac{1}{2}\right) + \frac{\hbar^2}{2J}\, l(l+1)$$

$$(1)\quad E = \hbar\omega_0\left(n + 1 + \tfrac{1}{2}\right) + \frac{\hbar^2}{2J}\cdot 2 = \hbar\omega_0\left(n + \tfrac{3}{2}\right) + \frac{\hbar^2}{J}$$

$$(2)\quad E = \hbar\left(n + \tfrac{1}{2}\right) + 0$$

$\Big\}\ \lambda_1$

$$(3)\quad E = \hbar\omega_0\left(n + \tfrac{3}{2}\right) + 0$$

$$(4)\quad E = \hbar\omega\left(n + \tfrac{1}{2}\right) + \frac{\hbar^2}{J}$$

$\Big\}\ \lambda_2$

$$\frac{h \cdot c}{\lambda_1} = \hbar\omega\left(n + \tfrac{3}{2}\right) + \frac{\hbar^2}{J} - \hbar\omega_0\left(n + \tfrac{1}{2}\right)$$

$$\frac{hc}{\lambda_2} = \hbar\omega\left(n + \tfrac{1}{2}\right) + \frac{\hbar}{J} - \hbar\omega_0\left(n + \tfrac{3}{2}\right)$$

38) $V(r) = V_0 \left(e^{-2(r-r_0)/a} - 2 e^{-(r-r_0)/a} \right)$

$V_0 = 3\,eV$

$a = 0{,}12\,nm$

$E_0 = \frac{1}{2} \hbar \omega$

$E_1 = \frac{3}{2} \hbar \omega$

$$V'(r) = V_0 \left(-\frac{2}{a} e^{-2(r-r_0)/a} + \frac{2}{a} e^{-(r-r_0)/a} \right)$$

$$V''(r) = V_0 \left(\frac{4}{a^2} e^{-2(r-r_0)/a} - \frac{2}{a^2} e^{-(r-r_0)/a} \right)$$

$r = r_0$

$$V''(r) = V_0 \left[\frac{2}{a^2} \right] = \frac{2 V_0}{a^2}$$

Taylorjev razvoj

$$V(r) = V(r_0) + V'(r_0)(r - r_0) + \frac{1}{2} V''(r_0)(r-r_0)^2$$

$$\boxed{\frac{1}{2} V''(r_0)\underbrace{(r-r_0)^2}_{x^2} = \frac{1}{2} k x^2}$$

$V''(r_0) = k$

$\frac{2 V_0}{a^2} = k \Rightarrow 67 \frac{N}{m}$

$\omega = \sqrt{\frac{k}{m}} = 4{,}9 \cdot 10^{13}\,s^{-1}$

$E_0 = \underline{0{,}016\,eV}$

$E_1 = \underline{0{,}046\,eV}$

39) $V_{odb}(r) = \frac{C}{r^n}$

$n = 35$

$-W_d = ?$

$W_i = 5{,}14\,eV$

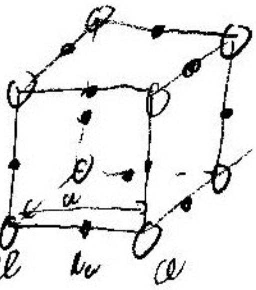
$W_a = 3{,}81\,eV$

~~$r = 0{,}350\,nm$~~

$M_{Na} = 23\,kg$

$M_{Cl} = 35\,kg$

$\rho = 2160\,\frac{kg}{m^3}$



$M_r = \frac{m_{Cl}\, m_{Na}}{m_{Cl} + m_{Na}} = 2{,}3 \cdot 10^{-26}\,kg$

$\rho = \frac{m}{V} \Rightarrow a^3 = \frac{m}{\rho}$

$a = \sqrt[3]{\frac{m_r}{\rho}} = 2{,}8 \cdot 10^{-9}\,m$

$\boxed{a' = \frac{a}{2} = 1{,}4 \cdot 10^{-10}\,m}$

$X = 0{,}89 \cdot a' = 9{,}29 \cdot 10^{-11}\,m$

$V_{odb} = -\frac{e_0^2}{4\pi\varepsilon_0 r} + \frac{C}{r^{35}}$

$V'(r) = \frac{e_0^2}{4\pi\varepsilon_0 r^2} - \frac{35C}{r^{36}} = 0$

$\Rightarrow \frac{e_0^2}{4\pi\varepsilon_0 r^2} = \frac{35C}{r^{36}}$

$C = \frac{e_0^2\, r^{34}}{140\pi\,\varepsilon_0}$

$V_{odb} = \frac{e_0^2 \cdot r^{34}}{140\pi\,\varepsilon_0\, r^{35}} - \frac{e_0^2}{140\pi\,\varepsilon_0\, r} = -0{,}42\,eV$

$W_d = V_c(R) + V_R + W_i - W_a = -0{,}42\,eV + 5{,}14\,eV - 3{,}81\,eV = \underline{0{,}91\,eV}$

6) OSNOVE KVANTNE STATISTIKE

str. 26 / 1 – 8

str. 27 / 9 – 17

str. 28 / 18 – 20

---

1) $x_0 = 0{,}9\,nm$

$T = 1500\,K$

$\frac{N_3}{N_2} = ?$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2 m x_0^2} = 0{,}46\,eV\,n^2$$

$$E_2 = 1{,}84\,eV \quad ; \quad E_3 = 4{,}14\,eV$$

$$F_n = \frac{1}{e^{E_n/k_B T}} = e^{-E_n/k_B T}$$

$$\frac{N_3}{N_2} = \frac{F_3}{F_2} = \frac{e^{E_2/k_B T}}{e^{E_3/k_B T}} = \exp\left(\frac{1}{k_B T}(E_2 - E_3)\right) = \underline{1{,}9 \cdot 10^{-9}}$$

---

2) $n = 1, 2, 3, 4$

$T = 6000\,K$

$$k_B \cdot T = 0{,}517\,eV/K$$

$$W_2 = \frac{-13{,}6\,eV}{4} = -3{,}4\,eV \qquad W_3 = -\frac{13{,}6}{9} = -1{,}51\,eV \qquad W_4 = -\frac{13{,}6}{16} = 0{,}85\,eV$$

$$F(W) = \exp\left(-\frac{E_n}{k_B T}\right)$$

$$\frac{F_1}{F_2} = \exp\left(\frac{1}{k_B T}(E_2 - E_1)\right) =$$

$$\frac{F_3}{F_4} = \exp\left(\frac{1}{k_B T}(E_4 - E_3)\right)$$

3) $r = 0{,}113\,nm$ CO

$\frac{N_2}{N_1} = 1{,}55$

$T = ?$

$m_r = \frac{16 \cdot 12}{16+12}\,u = 1{,}14 \cdot 10^{-26}\,kg$

$$\frac{N_2}{N_1} = \frac{5}{3}\exp\left(-\frac{\hbar^2}{2J}[6-2]/k_B T\right) / \ln$$

$$\ln \frac{3}{5}\frac{N_2}{N_1} = -\frac{\hbar}{2J}\frac{4}{k_B T}$$

$$T = \frac{\hbar^2 \cdot 2}{\ln \frac{5}{3}\frac{N_1}{N_2}\,(2J)} = \frac{2\hbar^2}{h\,\frac{5}{3}\frac{N_1}{N_2}\cdot m_r \cdot r^2 k_B} = \underline{\underline{137\,K}}$$ ✓

4) $T = 300K$; HCl $m_r = \frac{m_H \cdot m_{Cl}}{m_H + m_{Cl}} = 1{,}63 \cdot 10^{-27} kg$

$l = 1$

$l = 0$

$r = 0{,}129 nm$

$\frac{N_1}{N_0} = ?$

degeneracija je $2l+1$

$g_{deg} = 2l+1$

$$\frac{N_l}{N_{l'}}(l > l') = \frac{2l+1}{2l'+1} \exp\left(-\frac{\hbar^2}{2J}\frac{(l(l+1) - l'(l'+1))}{k_B T}\right)$$

$$\frac{N_1}{N_0} = 3 \exp\left(-\frac{\hbar \cdot 2}{2 J k_B T}\right) = 3 \exp\left(-\frac{\hbar^2}{J k_B \cdot T}\right) \quad ; \quad J = m_r \cdot r^2 = 2{,}7 \cdot 10^{-47} kg m^2$$

$$= \underline{\underline{2{,}7}}$$

---

5) $T = 500K$ $J = m \cdot r^2 = 9{,}2 \cdot 10^{-48} kg m^2$

$r = 0{,}0742 nm$

$l_0 = ?$

$$\frac{N_l}{N_{l'}} = \frac{2l+1}{2l'+1} \exp\left(-\frac{\hbar^2}{2 J k_B \cdot T}(l(l+1) - l'(l'+1))\right)$$

$$\frac{d\left(\frac{N_l}{N_0}\right)}{dl} = 0 = 2 \cancel{\exp\left(-\frac{\hbar^2 (l^2+l)}{2 J k_B T}\right)} + (2l+1)\left(-\frac{\hbar^2}{2 J k_B T}\right)(2l+1) \cancel{\exp}(\ldots$$

$$= 2 - (2l+1)^2 \left(-\frac{\hbar^2}{2 J k_B \cdot T}\right)$$

$$\frac{4 J k_B T}{\hbar^2} = (2l+1)^2$$

$$l = \frac{\sqrt{\frac{4 J k_B \cdot T}{\hbar^2}}}{2} - \frac{1}{2} = \sqrt{\frac{J k_B T}{\hbar^2}} - \frac{1}{2} = \underline{\underline{2}}$$

6) $B = 1T$

$T = 1300°C$

$N_\uparrow - N_\downarrow = N'$

$\mu_b = 1 \mu_B$

Imamo dva spina. En bi kazal v smeri B in drugi bi je nasproten B

$W_{m\uparrow} = -\mu_B \cdot B$

$$N = N_\uparrow + N_\downarrow$$

$$N_\uparrow = \frac{1}{\exp\left(\frac{E_\uparrow T}{k_B \cdot T}\right)} \qquad N_\downarrow = \frac{1}{\exp\left(\frac{E_\downarrow}{k_B \cdot T}\right)}$$

$$\frac{N_\uparrow}{N_\downarrow} = \exp\left(\frac{1}{k_B T}\left(E_\downarrow - E_\uparrow\right)\right) = \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)$$

$$N_\uparrow = N_\downarrow \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right) \quad ; \quad N_\downarrow = \frac{N_\uparrow}{\exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)}$$

$$N = N_\downarrow\left(1 + \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)\right) \qquad N = N_\uparrow\left(1 + \frac{1}{\exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)}\right)$$

$$\boxed{N_\downarrow = \frac{N}{1 + \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)}} \qquad \boxed{N_\uparrow = \frac{N}{1 + \frac{1}{\exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)}}}$$

$$\frac{N \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)}{1 + \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)} - \frac{N}{1 + \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)} = N\left(\frac{\exp\left(\frac{2\mu_B \cdot B}{k_B T}\right)}{1 + \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)} - \frac{1}{1 + \exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right)}\right)$$

$$= \boxed{N\left(\frac{\exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right) - 1}{\exp\left(\frac{2\mu_B \cdot B}{k_B \cdot T}\right) + 1}\right)} \; \checkmark$$

$M = 1 \,\text{kil.}$

$m = \frac{m}{N}$

$N = \frac{M}{m} = 2{,}6 \cdot 10^{20}$

7) $j = \frac{1}{2}$

$E = E_0 + 0{,}102\,eV$

$j = \frac{3}{2}$

$\frac{N_2}{N_1} = 1{,}1$

$T = ?$

$S_n\, g_n = 2n^2$

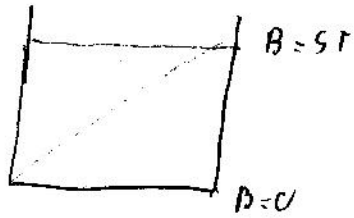
$$\frac{N_2}{N_1} = 4\, e^{-W_R\left(\frac{1}{4} - \frac{1}{1}\right)/k_B T}$$

8) $\mu = 1\mu_B$

$B = 0 - 5T$

$T = ?$ $B_1 = 0T$

$B_2 = 5T$



Tako na podu kot na vrhu imamo opravka s spinoma dol in navzgor

Dno: $B = 0$

$$\frac{N_\uparrow}{N_\downarrow} = \frac{A e^{-\left(\frac{W}{k_B T}\right)}}{A e^{-\left(\frac{W}{k_B T}\right)}} = 1 \qquad W_1 = -\mu_B \cdot B$$

$$\boxed{N_\uparrow = N_\downarrow}$$

Vrh: $N_\uparrow \neq N_\downarrow$

Tiste, ki gledajo gor imajo energijo $W_m = N_\uparrow \cdot W$ tiste kaj dol $N_\downarrow W_m$

$$\Delta W = W_m (N_\uparrow - N_\downarrow)$$

$$\frac{N_\uparrow}{N_\downarrow}(h) \quad \frac{e^{-\frac{W_1(h)}{kT}}}{e^{-\frac{W(h)}{kT}}} = \exp\left(-\frac{2W_1(h)}{kT}\right)$$

$$\frac{N_\uparrow(h)}{N_\uparrow(0)} = e^{-\frac{W_1(h)}{kT}} \quad ; \quad \frac{N_\downarrow(h)}{N_\downarrow(0)} = e^{\frac{W_1(h)}{k \cdot T}}$$

$$N(0) = N_\uparrow + N_\downarrow = 2N_\downarrow \qquad N_\uparrow(h) + N_\downarrow(h) = N(h)$$

$$\frac{N(h)}{N_0} = \frac{N_\uparrow + N_\downarrow}{N_\uparrow(0) + N_\downarrow(0)} = \frac{N_\uparrow(h) + N_\downarrow(h)}{N_\downarrow(h) \cdot \exp\left(-\frac{W_1(h)}{k \cdot T}\right) + N_\uparrow(h) \exp\left(\frac{W_1(h)}{k \cdot T}\right)} =$$

$$\frac{\frac{N_\uparrow(h)}{N_\downarrow(h)} + 1}{\frac{N_\uparrow(h)}{N_\downarrow(h)} \exp\left(\frac{W_1(h)}{k_B T}\right) + \exp\left(-\frac{W_1(h)}{k_B T}\right)} = \frac{\exp\left(-\frac{2W_1(h)}{k \cdot T}\right) + 1}{2 \exp\left(-\frac{W_1(h)}{k \cdot T}\right)} = \frac{\exp\left(-\frac{W_1(h)}{k T}\right) + \exp\left(\frac{W_1(h)}{kT}\right)}{2}$$

$$\Rightarrow \operatorname{ch} \frac{\mu_B \cdot B}{k \cdot T} \Longrightarrow \boxed{1 + \frac{1}{2}\left(\frac{\mu_B B}{k T}\right)^2}$$

11) $\lambda = 100\,nm$

$T = 5500\,K$

$$\nu^3 = \frac{c^3}{\lambda^3}$$

$$\frac{dj}{d\lambda} = \frac{dj}{d\nu}\frac{d\nu}{d\lambda} = \frac{2\pi \cdot h}{c^2}\frac{\nu^3 c}{\lambda^2 \exp\left(\frac{h\nu}{k_B \cdot T}\right) - 1} = \frac{2\pi h c^2}{\lambda^5 \left(\exp\left(\frac{hc}{\lambda k_B \cdot T}\right) - 1\right)}$$

$$dj = \frac{2\pi h c^2}{\lambda^5 \left(\exp\left(\frac{hc}{\lambda k_B T}\right) - 1\right)} d\lambda$$

$$j = 2\pi h c^2 \int_0^\infty \frac{d\lambda}{\lambda^5 \left(\exp\left(\frac{hc}{\lambda k_B T}\right) - 1\right)} = \frac{2\pi h c^2 k_B \cdot T}{hc} \int_0^\infty \frac{dt}{\lambda^3 (e^t - 1)} =$$

$$t = \frac{hc}{\lambda k_B \cdot T} =$$

$$\frac{k_B T\, dt}{h \cdot c} = -\frac{d\lambda}{\lambda^2}$$

$$= \frac{2\pi c \cdot k_B^4 \cdot T^4}{h^4 c^3} \int \frac{t^3 dt}{t^3 (e^t - 1)} = \frac{2\pi k_B^4 \cdot T^4}{h^3 c^3} \underbrace{\int_0^\infty t^3 \cdot e^{-t} dt}_{\frac{\pi^4}{15}} = \underline{\sigma \cdot T^4}$$

lahko zanemarimo

$$-t^3 e^{-t} - 3t^2 e^{-t} - 6t e^{-t} - 6e^{-t}\Big|_{\frac{hc}{\lambda k_B T}}^{\infty} =$$

$$e^{-\frac{hc}{\lambda k_B \cdot T}} \left(\left(\frac{hc}{\lambda k_B T}\right)^3 + 3\left(\frac{hc}{\lambda k_B T}\right)^2 + 6\frac{hc}{\lambda k_B \cdot T} + 6 \right) \quad ; \; u = \frac{hc}{\lambda k_B \cdot T} = 26$$

$$j = 3 \cdot 10^{-9} \frac{W}{m^2}$$

$$\frac{j_\infty}{j_{<100\,nm}}$$

12) $\lambda = 1\,\mu m$

$T_1 = 1473\,K$

$T_2 = 798\,K$

$$j_1 = \frac{2\pi k_B^4 \cdot T^4}{h^3 \cdot c^2} \int_0^{\frac{hc}{\lambda k_B T}} t^3 e^{-t}\,dt$$

$$\frac{j_1}{j_2} = \frac{\cancel{2\pi k_B^4} \cdot T_1^4 \cancel{h^3 c^2} \int t^3 e^{-t}\,dt}{\cancel{h^3 c^2}\, \cancel{2\pi k_B^4} \cdot T_2^4 \int t^3 e^{-t}}$$

$$\int_0^{\frac{hc}{\lambda k_B T}} t^3 e^{-t} = \left[-t^3 e^{-t} - 3t^2 e^{-t} - 6t e^{-t} - 6 e^{-t}\right]\Big|_0^{\frac{hc}{\lambda k_B T_1}}$$

$$= \left[-\left(\frac{hc}{\lambda k_B T}\right)^3 - 3\left(\frac{hc}{\lambda k_B T}\right)^2 e^{-t} - 6\,\frac{hc}{\lambda k_B T} e^{-t} - 6 e^{-t} + 6\right.$$

13) $T = 1200°C$

$$j_\infty = \sigma \cdot \tilde{T}^4 = 347502 \ \frac{W}{m^2}$$

$$j = \int_{\lambda_1}^{\lambda_2} \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \left(\exp(hc/\lambda k_B T) - 1\right)} \, d\lambda = 2\pi h c^2 \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^5 \exp\left(\frac{hc}{\lambda k_B \cdot T}\right)}$$

$$t = \frac{hc}{\lambda k_B T} \Rightarrow \lambda = \frac{hc}{t k_B \cdot T}$$

$$\frac{-\lambda^2 k_B \cdot T \, dt}{h \cdot c} = d\lambda$$

$$= \frac{2\pi \cdot h c^{\cancel{2}} T^4 k_B^3}{h c^2 \, h^3 c^3} \int t^3 \, e^{-t} \, dt = \underbrace{\frac{2\pi T^4 \cdot k_B^3}{h^3 c^2}}_{A} \left[-t^3 e^{-t} - 3t^2 e^{-t} - 6 t e^{-t} - 6 e^{-t} \right]$$

$$= A \Big[\Big(\frac{hc}{\lambda^3}$$

7) ELEKTRONI V KRISTALIH

str. 29 / 1 - 7
str. 30 / 8 - 15
str. 31 / 16 - 26
str. 32 / 28 - 37
str. 33 / 38 - 44
str. 34 / 45 - 50

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1) $R = 0{,}281\,nm$

$\alpha = 1{,}75$

$W_i = 5{,}19\,eV$

$W_A = 3{,}82\,eV$

$$W_D = V_c(R) + W_i - W_a = -8{,}96\,eV + 5{,}19\,eV - 3{,}82\,eV = \underline{\underline{-7{,}59\,eV}}$$

$$V = -\frac{\alpha \cdot e_0^2}{4\pi \varepsilon R} = -8{,}96\,eV$$

2) BaO (NaCl)

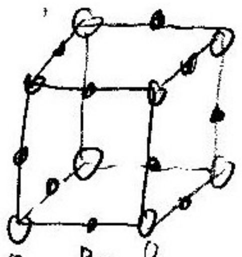
$r = 0{,}276\, nm$

$W_i^{+} = 5{,}19\, eV$

$W_i^{++} = 9{,}65\, eV$

$W_a^{-} = 2{,}2\, eV$

$W_a^{--} = -9\, eV$



$$W_d = V_c(R) + (W_i^{+} + W_i^{++}) - (W_a^{-} + W_a^{--}) =$$

$$= -\frac{Ze^2 \cdot 2}{4\pi\varepsilon_0 R} + (W_i^{+} + W_i^{++}) - (W_a^{-} + W_a^{--}) = \underline{\underline{14{,}14\, eV}}$$

$$W_d = V_c(R) + W_i^{+} - W_a^{-} = \underline{\underline{-6{,}7\, eV}}$$

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5) V zvezku, ko nem je že rešil.

6) $r = 0{,}315\,nm$ $\quad m = 3{,}09 \cdot 10^{-26}\,kg$

$W_{odb}(r) = \frac{C}{r^{8,69}}$

$\mathcal{L} = 1{,}75$

$$\int_r r^{-2}\,dr = \frac{r^{-1}}{-1} = \frac{1}{r^{+1}}$$

$$\int \frac{1}{r^{9,69}}\,dr = r^{-9,69}\,dr = \frac{r^{-8,69}}{-8,69}$$

$\omega = \sqrt{\frac{K}{m}}$ $\quad W''(r) = k$ $\quad \frac{1}{2} V''(r)(r-r_0)^2 = \frac{1}{2} k x^2$

$$\boxed{V(r) = V(r) + V'(r)(r-r_0) + \frac{1}{2} V''(r)(r-r_0)^2 + \dots}$$

$$V(r) = -\frac{e_0^2}{4\pi\varepsilon_0 r} + \frac{C}{r^{8,69}}$$

$$V_0 = \frac{\mathcal{L} \cdot e_0^2}{4\pi\varepsilon_0 r} = 7{,}99\,eV$$

$$V'(r) = +\frac{e_0^2 \mathcal{L}}{4\pi\varepsilon_0 r^2} - \frac{8{,}69\,C}{r^{9,69}} = 0$$

$$\frac{e_0^2 \mathcal{L}}{4\pi\varepsilon_0 r^2} = \frac{8{,}69\,C}{r^{9,69}} \Rightarrow \boxed{C = \frac{e_0^2\, r^{7,69}}{4\pi\varepsilon_0 \cdot 8{,}69}}$$

$$V(r) = -\frac{e_0^2 \mathcal{L}}{4\pi\varepsilon_0 r} + \frac{e_0^2}{4\pi\varepsilon_0 8{,}69 r} = \frac{e_0^2 \mathcal{L}}{4\pi\varepsilon_0 r}\left[-1 + \frac{1}{8{,}69}\right] = -V_0 \cdot \frac{7{,}69}{8{,}69}$$

$$V'(r) = \frac{e_0^2 \mathcal{L}}{4\pi\varepsilon_0 r^2}\ \frac{7{,}69}{8{,}69}\ ;\ V''(r) = +\frac{2 e_0^2 \cdot 7{,}69}{4\pi\varepsilon_0 r^3 \cdot 8{,}69} = \frac{V_0 \cdot 2 \cdot 7{,}69}{r^2 \cdot 8{,}69} = k$$

$$\omega = \sqrt{\frac{2 V_0 \cdot 7{,}69}{8{,}69\, r^2 \cdot m}} = \underline{\underline{2{,}7 \cdot 10^{13}\,s^{-1}}}$$

7) $W(r) = -\frac{\alpha \cdot e_0^2}{4\pi \cdot \varepsilon_0 \cdot r} + \frac{C}{r^m}$ $\qquad r^{-m} = \cancel{m\, r^{m-1}}$

$\alpha = 1{,}75$; $r_0 = 0{,}281\,nm$ $\qquad r^{-m} = -m\, r^{-m-1}$

b) $\frac{1}{\chi_0} = \frac{1}{18 r_0} \left(\frac{d^2W}{dr^2} \right)$

$\chi_0 = 4{,}17 \cdot 10^{-11} / Pa$

$$W'(r) = \frac{\alpha e_0^2}{4\pi \cdot \varepsilon_0 \cdot r^2} - \frac{m \cdot C}{r^{m+1}} = 0$$

$$\frac{\alpha \cdot e_0^2}{4\pi \cdot \varepsilon_0 \cdot r^2} = \frac{m}{r^{m+1}} C \Rightarrow C = \frac{\alpha \cdot e_0^2 r^{m+1}}{4\pi \varepsilon_0 r^2 \cdot m} = \frac{\alpha \cdot e_0^2 \cdot r^{m-1}}{4\pi \varepsilon_0 \cdot m}$$

$$W(r) = -\frac{\alpha e_0^2}{4\pi \cdot \varepsilon_0 \cdot r} + \frac{\alpha \cdot e_0^2 r^{m} \cdot r^{-1 \cdot}}{4\pi \varepsilon_0 \cdot m \cdot \cancel{r^m}} = \frac{\alpha e_0^2}{4\pi \cdot \varepsilon_0 r}\left[\frac{1}{m} - 1\right] = \frac{\alpha e_0^2}{4\pi \varepsilon_0 \cdot r}\left[\frac{1-m}{m}\right]$$

$$\frac{\partial W(r)}{\partial r} = -\frac{\alpha e_0^2}{4\pi \varepsilon_0 \cdot r^2}\left[\frac{1-m}{m}\right]$$

$$\frac{\partial^2 W(r)}{\partial r^2} = \frac{\alpha \cdot e_0^2 \cdot 2}{4\pi \varepsilon_0 \cdot r^3}\left[\frac{1-m}{m}\right]$$

$$\frac{1}{\chi_0} = \frac{\cancel{2}}{\cancel{18} r} \cdot \frac{\alpha e_0^2}{4\pi \cdot \varepsilon_0 r^3}\left[\frac{1-m}{m}\right] = \frac{\alpha e_0^2 (1-m)}{36\pi \varepsilon_0 r^4 \cdot m}$$

$$m = \frac{\alpha e_0^2 \chi_0}{36\pi \varepsilon_0 r^4} - \frac{\alpha e_0^2 \cdot \chi}{36\pi \varepsilon_0 r^4} m \Rightarrow \delta$$

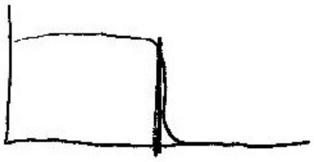
$\delta = 0{,}3$

$$m(1 + \delta) = \delta$$

$$m = \frac{\delta}{1+\delta}$$

11) $F = \frac{1}{4}$

$F = \frac{3}{4}$



$$F(W) = \frac{1}{\exp\left(\frac{-W_F + W}{k_B \cdot T}\right) + 1} \quad / \ln$$

$$\ln F(W) = \frac{1}{\frac{W_F}{k_B \cdot T}} = \frac{k_B \cdot T}{W_F}$$

$$W - W_{F_1} = k_B \cdot T \cdot \ln 4 = \underline{\underline{0{,}035\,eV}}$$

$$W - W_{F_2} = k_B \cdot T \cdot \ln \frac{4}{3} = \underline{\underline{0{,}0073\,eV}}$$

b) $0{,}01\,eV = W - W_F$

$0{,}1\,eV = W - W_F$

$1\,eV = W - W_F$

$$F_1 = \frac{1}{\exp\left(\frac{0{,}01}{k_B \cdot T}\right) + 1} = \underline{\underline{0{,}4}} \checkmark$$

$$F_2 = \frac{1}{\exp\left(\frac{0{,}1}{k_B T}\right) + 1} = \underline{\underline{0{,}02}} \checkmark$$

$$F_3 = \frac{1}{\exp\left(\frac{1}{k_B \cdot T}\right) + 1} \approx \underline{\underline{10^{-18}}} \checkmark$$

12) a) ^{23}Na

$\rho_{Na} = 0{,}97\ \frac{g}{cm^3}$

$s = \frac{1}{2}$

$W_F = ?$

$S = 2s + 1 = 2$

$m = 3{,}3 \cdot 10^{-26}\ kg$

$$F = \frac{dN}{dg} \Rightarrow dN = \int S \cdot F\, dg = \int F \frac{V 2 \cdot 2\pi (2m)^{3/2} W^{1/2}\, dW}{h^3}$$

$$N = \int_0^{W_F} \frac{4\pi V (2m)^{3/2} W^{1/2}\, dW}{h^3} = \frac{4\pi \cdot V (2m)^{3/2}}{h^3} \int_0^{W_F} W^{1/2} dW$$

$$= \frac{4\pi \cdot V (2m)^{3/2}}{h^3} \frac{2 \cdot W^{3/2}}{3}\Big|_0^{W_F} = \frac{8\pi \cdot V (2m)^{3/2}}{3 h^3} W_F^{3/2}$$

$$\frac{3 h^3 N}{8\pi V (2m)^{3/2}} = W_F^{3/2} \Rightarrow W_F = \sqrt[3]{\left(\frac{3 h^3 N}{16\pi V (2m)^{3/2}}\right)^2} = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

$$\mathcal{N} = \frac{N}{V} \qquad W_F = \frac{h^2}{2m}\left(\frac{3\mathcal{N}}{8\pi}\right)^{2/3}$$

$$\boxed{W_F = \frac{h^2}{2m}\left(\frac{3\mathcal{N}}{8\pi}\right)^{2/3} = \frac{h^2}{2m}\left(\frac{3 \cdot N_A \rho}{M \cdot 8\pi}\right)^{2/3}}$$

$(2^1)^{\frac{2}{3}} = 2^{\frac{6}{3}} = 2^2$

$(2^{\frac{3}{2}})^{\frac{4}{3}}$

$\boxed{W_{F_1} = 3{,}13\ eV} \Rightarrow$ za natrij

b) $^{63}Cu \Rightarrow$ Enaka enačba, $\rho_{Cu} = 8{,}9\ \frac{g}{cm^3} = 8900\ \frac{kg}{m^3}$

$$W_{F_2} = \frac{h^2}{2m}\left(\frac{3 N_A \cdot \rho_{Cu}}{M_{Cu} \cdot 8\pi}\right)^{2/3} = \underline{\underline{7\ eV}}$$

$s = \pm \frac{1}{2}$

c) ^{27}Al, $\rho_{Al} = 2{,}7\ \frac{g}{cm^3} = 2700\ \frac{kg}{m^3}$

$S = 2s + 1 = 2$

Ker imamo 3 elektrone je vsaki elektron v svojem stanju

$$N = \int S\, F\, dg = \int_0^{W_F} \frac{3 \cdot 2 \cdot 2\pi (2m)^{3/2} V W^{1/2}\, dW}{h^3}$$

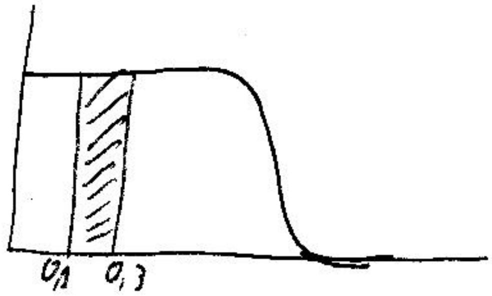
$F = 1 \cdot 1 + 1 + 1$

$$N = \frac{V 12\pi (2m)^{3/2}}{h^3} \cdot \frac{2}{3} W_F^{3/2} = \frac{24\pi \cdot V \cdot (2m)^{3/2}}{3 h^3} W_F^{3/2}$$

$$\Rightarrow W_F = \sqrt[3]{\left(\frac{3 h^3 \cdot N}{24\pi \cdot V (2m)^{3/2}}\right)^2} = \frac{h^2}{4 \cdot 2m} \sqrt[3]{\left(\frac{N}{\pi \cdot V}\right)^2} = \frac{h^2}{8m} \sqrt[3]{\left(\frac{N_A \rho}{\pi \cdot M_A}\right)^2}$$

17) $W_F = 11{,}7\ eV$

$W = 0{,}1 - 0{,}3\ eV$



$T \to 0$

$$\frac{dN}{N} = \frac{\int_{W_1}^{W_2} F(W)\, dg \cdot S}{\int_0^{W_F} F(W)\, dg \cdot S} = \frac{\int_{W_1}^{W_2} \frac{4\pi V}{\cancel{h}^3} (2m)^{\frac{3}{2}} W^{\frac{1}{2}} dW}{\frac{8\pi V (2m)^{3/2}}{3 \cancel{h}^3} W_F^{\frac{3}{2}}}$$

$$= \frac{\cancel{4\pi} \cancel{V} \cancel{(2m)^{\frac{3}{2}}} \cdot \frac{2}{3} \left(W_2^{\frac{3}{2}} - W_1^{\frac{3}{2}}\right)}{\frac{\cancel{8\pi} \cancel{V} \cancel{(2m)^{3/2}}}{\cancel{3}} \cdot W_F^{3/2}} = \left(\frac{W_2^{\frac{3}{2}} - W_1^{\frac{3}{2}}}{W_F^{\frac{3}{2}}}\right) = \underline{\underline{0{,}33\%}}$$

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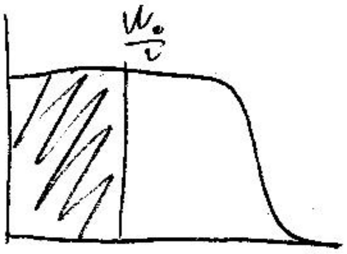
18) $W_0 = 7eV$

$W_1 = 0$

$W_2 = \frac{W_0}{2}$



$T \to 0$

$$\frac{dN}{N} = \frac{\left(W_2^{\frac{3}{2}} - W_1^{\frac{3}{2}}\right)}{W_F^{\frac{3}{2}}} = \left(\frac{W_0}{2 \cdot W_0}\right)^{\frac{3}{2}} = \underline{\underline{0{,}35}} \Rightarrow \underline{\underline{35\%}}$$

19) $W_F = 7\,eV$

$T \to 0$

$\rho_{Cu} = 8{,}9 \frac{g}{cm^3} = 8900 \frac{kg}{m^3}$

$M = 63\,kg$

$\langle v \rangle = \frac{3}{4} v_F \,;\qquad W_F = \frac{1}{2} m v_F^2$

$v_F = \sqrt{\frac{2 W_F}{m}}$

$\langle v \rangle = \frac{3}{4}\sqrt{\frac{2 \cdot W_F}{m}} = \underline{\underline{1{,}2 \cdot 10^6 \frac{m}{s}}}$ ✓

b) $p = ?$

$p = -\frac{\partial W}{\partial V} = \frac{h^2}{20m}\left(\frac{3}{\pi}\right) N^{\frac{5}{3}} = \frac{h^2}{20m}\left(\frac{3}{\pi}\right)\left(\frac{N_A \cdot \rho}{M_A}\right)^{\frac{5}{3}} = \underline{\underline{3{,}75 \cdot 10^{10}\,Pa}}$ ✓

c) $\chi = ?$

$\chi = \frac{1}{p} = \underline{\underline{2{,}66 \cdot 10^{-11}\,Pa^{-1}}}$

20) $T_1 = 0°C$

$T_2 = 100°C$

$\rho = 8960 \frac{kg}{m^3}$

$\alpha = 1{,}67 \cdot 10^{-5}\ K^{-1}$

$\boxed{\Delta l = \alpha \cdot l \cdot \Delta T}$

$\Delta T = \frac{\Delta l}{\alpha \cdot l}$

$\boxed{\Delta W_F = \alpha \cdot W_F \cdot \Delta T}$

23) $\langle l \rangle = ?$

$\sigma_{Cu} = 5{,}9 \cdot 10^7 / \Omega m$

$\sigma_{Na} = 2{,}2 \cdot 10^7 / \Omega m$

$\rho_{Cu} = 8900 \frac{kg}{m^3}$

$\rho_{Na} = 970 \frac{kg}{m^3}$

$W_{F Na} = 3{,}13\ eV$

$W_F(Cu) = 7 eV$

$\langle l \rangle = v_F \cdot \tau$

$$\boxed{\sigma = \frac{e_0^2}{m} \cdot N \cdot \tau} \Rightarrow \tau_{Na} = \frac{\sigma_{Na} \cdot m \cdot M_{Na}}{e_0^2 \cdot N_A \cdot \rho} = 3{,}08 \cdot 10^{-14}\ s$$

$$\tau_{Cu} = 2{,}41 \cdot 10^{-14}\ s$$

$$v_F = \sqrt{\frac{2 W_F}{m}} = 1{,}049 \cdot 10^6 \frac{m}{s} \quad [Na]$$

$$v_F = \sqrt{\frac{2 W_F}{m}} = 1{,}57 \cdot 10^6 \frac{m}{s} \quad [Cu]$$

$\langle l \rangle_{Na} = 0{,}3\ nm$ ✓

$\langle l \rangle_{Cu} = 0{,}4\ nm$ ✓

24) $E = 1 \frac{V}{m}$

$\frac{v}{v_F} = ?$

$\mu = 0{,}0032 \frac{m^2}{Vs}$

$W_F = 7 eV$

$$v_F = \sqrt{\frac{2 W_F}{m}} = \underline{\underline{1{,}57 \cdot 10^6 \frac{m}{s}}}$$

$$\mu_e = \frac{e_0}{m} \cdot \tau_i \Rightarrow \tau = \frac{\mu_e\, m}{e_0} = \underline{\underline{1{,}82 \cdot 10^{-14}\, s}}$$

$$\langle v_x \rangle = \mu_e \cdot E = \underline{0{,}0032 \frac{m}{s}}$$

$$\frac{\langle v_x \rangle}{v_F} \sim \underline{\underline{10^{-9}}}$$

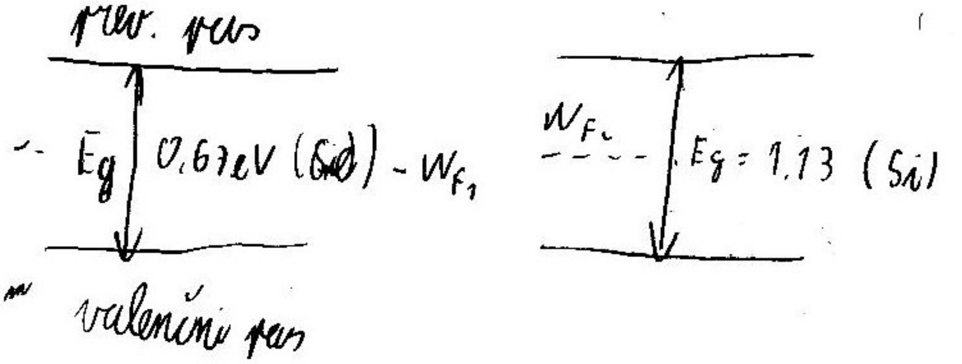
25) $n_e = ?$

$n_V = ?$

$T = 294K$

$E_g = 0,67 eV$

$m_V^* = 0,35 m$ $m_e^* = 0,56 m$



čisto polprevodnik

$$\boxed{\begin{array}{l} n_e = n_V = n_i \\ W_F = \frac{1}{2} W_g \end{array}}$$

Fermijeva energija je v čistem polprevodniku na sredini

Germanij

$$n_i = N_0 \cdot \exp\left(-\frac{1}{2} W_g / k_B T\right) = 2,5 \cdot 10^{25}/m^3 \left(\frac{293}{300}\right)^{\frac{3}{2}} \cdot 0,032$$

$$N_0 = 2\left(\frac{2\pi m^* k_B T}{h^2}\right)^{\frac{3}{2}}, \quad \left(\frac{m^*}{m}\right)^{\frac{3}{2}} = 0,032 \text{ za Germanij}$$

$$m^* = (0,032)^{\frac{2}{3}} \cdot m$$

$$N_0 = 2 \cdot 0.032 \left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}}$$

$$\underline{\underline{n_i = 1,36 \cdot 10^{18} m^3}}$$

Silicij: $\left(\frac{m^*}{m}\right) = 0,089$

$$\underline{\underline{n_i = 4,17 \cdot 10^{14}/m^3}}$$

27) $\beta_e = 0{,}38 \frac{m^2}{Vs}$

$\beta_v = 0{,}18 \frac{m^2}{Vs}$

$E_g = 0{,}72\,eV$

$I = \frac{U}{R} \Rightarrow R = \frac{U}{I}$

$As \cdot \frac{m^2}{Vs} \cdot \frac{1}{m^3}$

$\sigma = ?$

$\sigma = e_0 \cdot n_e (\beta_e + \beta_v)$

Ker gre za čist polprevodnik velja enačba

$n_e = n_i = n_v = N_0 \exp(-\frac{1}{2} W_g / k_B T)$

$n_i = 2{,}5 \cdot 10^{25}/m^3 \cdot 0{,}032 \cdot \left(\frac{293}{300}\right)^{\frac{3}{2}} \cdot \exp\left(-\frac{\frac{1}{2} W_g}{k_B \cdot T}\right) = 4{,}98 \cdot 10^{17}/m^3$

$\sigma = \underline{\underline{0{,}045\ (\Omega m)^{-1}}}$ ✓

29) $T = 293\,K$

$E_g = 0{,}72\,eV$

$\frac{dR}{R} = 0{,}1\,\% = 0.001$

$$\frac{d\sigma}{\sigma} = 0{,}1 = \left(\frac{3}{2}T^{-1} + \frac{1}{2}\frac{E_g \cdot T^{-2}}{k_B}\right) dT$$

$$dT = \frac{0{,}001}{\left(\frac{3}{2}T^{-1} + \frac{1}{2}\frac{E_g T^{-2}}{k_B}\right)} = \underline{\underline{0{,}02\ K}}$$

---

30) $T = 300\,K$

$\mu_e = 0{,}38\,\frac{m^2}{Vs}$

$\mu_h = 0{,}18\,\frac{m^2}{Vs}$

$\underline{N_d = 10^{21}/m^3\ (10^{22}, 10^{23}, 10^{24})}$

$\sigma = ?$

Ker gre za donorske primesi lahko zanemarimo n_h in upoštevamo le n_e

$\sigma_1 = e_0 \cdot N_d \cdot \mu_e = \underline{\underline{60{,}8\,(\Omega m)^{-1}}}$

$\sigma_2 = 608\,(\Omega m)^{-1}$

$\sigma_3 = 6080\,(\Omega m)^{-1}$

$\sigma_4 = \underline{60800\,(\Omega m)^{-1}}$

31) $N_d = 10^{23}/m^3$

$N_a = 0{,}5 \cdot 10^{23}/m^3$

$T = 300\,K$

$\mu_e = 0{,}16 \frac{m^2}{Vs}$

$\mu_v = 0{,}05 \frac{m^2}{Vs}$

$E_g = 1{,}13\,eV$

$\sigma = ?$

$$\sigma = e_0 \cdot n_e \cdot \mu_e + e_0 \cdot n_v \cdot \mu_v = e_0 (N_d - N_a)\mu_e = \underline{1120 (\Omega m)^{-1}}$$

---

33) $\rho_n = 10^2 \frac{\Omega mm^2}{m}$

$\rho_p = 10^3 \frac{\Omega mm^2}{m}$

$\mu_e = 0{,}14 \frac{m^2}{Vs}$

$\mu_v = 0{,}048 \frac{m^2}{Vs}$

$\rho_{Si} = 2{,}36 \frac{g}{cm^3}$

$M_B = 10{,}8\,g$

$$\sigma_1 = \frac{1}{\rho_n} = e_0 \cdot \mu_e \cdot n_e \qquad (n_e \approx N_d)$$

$$N_d = \frac{1}{\rho_n \cdot e_0 \mu_e}$$

$$\sigma_2 = e_0 (N_a - N_d) \cdot \mu_v = \underline{e_0 N_a \mu_v - \frac{e_0 \sigma_1 \cdot \mu_v}{e_0 \cdot \mu_e}} =$$

$$= N_a - \frac{\sigma_1 \cdot \mu_v}{e_0 \cdot \mu_v \mu_e} \Rightarrow N_a = \frac{\sigma_2}{e \mu_v} + \frac{\sigma_1}{e_0 \mu_e} = \underline{5{,}8 \cdot 10^{23}/m^3}$$

$$\frac{m_B}{m_{Si}} = \frac{N_a \cdot V \cdot M_B}{N_{Si} \cdot V \cdot M_{Si}} = \frac{M_{Si} \cdot N_a \cdot M_B}{\rho_{Si} \cdot N_A \cdot M_{Si}} = \boxed{\frac{N_a M_B}{\rho_{Si} \cdot N_A}}$$

34) $T = 300K$

$m_e^* = 0,56\, m$

$m_v^* = 0,35\, m$

$E_g = 0,67\, eV$

$$n_v = n_0 \cdot e^{-W_F/k_B \cdot T} = 2,5 \cdot 10^{25}/m^3\ 0,35^{\frac{3}{2}}\ e^{-W_F/k_B \cdot T} =$$

$$n_e = n_0 \cdot e^{-(W_G - W_F)/k_B \cdot T} = 2,5 \cdot 10^{25}/m^3\ 0,56^{\frac{3}{2}}\ e^{-(W_G - W_F)/k_B T}$$

$$n_i^2 = n_v \cdot n_e = (2,5 \cdot 10^{25}/m^3)^2\ (0,35)^{\frac{3}{2}} (0,56)^{\frac{3}{2}} \cdot exp\left(-\frac{1}{2}\frac{W_g}{k_B \cdot T}\right) = 2,29 \cdot 10^{44}/m^6$$

$$\boxed{n_i = 1,51 \cdot 10^{22}/m^3}$$

$$W_F = \frac{1}{2} E_g + \frac{1}{2} k_B \cdot T\ \ \frac{n_e - n_v}{n_i}$$

34) $T = 300K$

$m_e^* = 0{,}56\,m$

$m_v^* = 0{,}35\,m$

$E_g = 0{,}67\,eV$

$W_F = ?$

$n_e = n_v = n_i$

$$n_e = 2\left(\frac{2\pi \cdot m^* \cdot k_B \cdot T}{h^2}\right)^{\frac{3}{2}} \cdot e^{-(W_g - W_F)/k_B \cdot T}$$

$$n_v = 2\left(\frac{2\pi \cdot m^* \cdot k_B T}{h^2}\right)^{\frac{3}{2}} \cdot e^{-W_F/k_B \cdot T}$$

$$\cancel{2}\left(\frac{\cancel{2\pi} \cdot m_e^* \cancel{k_B T}}{\cancel{h^2}}\right)^{\frac{3}{2}} e^{-(W_g - W_F)/k_B \cdot T} = \cancel{2}\left(\frac{\cancel{2\pi} \cdot m_v^* \cancel{k_B T}}{\cancel{h^2}}\right)^{\frac{3}{2}} e^{-\frac{W_F}{k_B T}}$$

$$\left(\frac{m_e^*}{m_v^*}\right)^{\frac{3}{2}} = \exp\left(-\frac{2W_F}{k_B \cdot T} + \frac{W_g}{k_B \cdot T}\right) \quad / \ln$$

$$\frac{3}{2} \ln \frac{m_e^*}{m_v^*} = -\frac{2W_F + W_g}{k_B \cdot T}$$

$$2W_F = W_g + \frac{3}{2} \ln\left(\frac{m_e^*}{m_v^*}\right)^{-1} \cdot k_B \cdot T$$

$$W_F = \frac{E_g}{2} + \frac{3}{4} \ln\left(\frac{m_e^*}{m_v^*}\right) k_B \cdot T = \underline{\underline{0{,}34\,eV}}$$

36) $T = 20\,K$

$N_d = 10^{23}/m^3$

$W_F = ?$

$N_d \approx n_e$

$E_g = 0{,}67\,eV$

$\varepsilon = 16$

$N_0 = 1{,}37 \cdot 10^{22}$

$$n_e = N_0 \cdot \exp\left(-\frac{(W_g - W_F)}{k_B \cdot T}\right) = 2\left(\frac{2\pi \cdot m^* \cdot T k_B}{h^2}\right)^{\frac{3}{2}} \cdot \exp\left(-\frac{W_g - W_F}{k_B \cdot T}\right) \quad / \ln$$

$$k_B T \ln \frac{N_d}{N_0} = -W_g + W_F$$

$E_F = E_g - \Delta E$

$$W_F = W_g + k_B \cdot T \ln \frac{N_d}{N_0} = \underline{\underline{0{,}67396\,eV}}$$

---

36) $T = 20\,K$

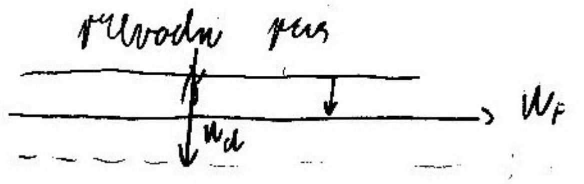
$N_d = 10^{23}\,m^{-3}$

$W_F = ?$

$$\boxed{E_d = \frac{W_R\, m^*}{\varepsilon^2 \cdot m} = 0{,}005}$$

$W_F = E_g - \Delta E_d$

$W_F = 0{,}67 - 0{,}005 = \boxed{0{,}665\,eV}$



37) $N_d = 5 \cdot 10^{23} /m^3$

$T = 300\,K$

$E_g = 0{,}67\,eV$

$\Delta E_d = 0{,}0127\,eV$

$m_e^* = 0{,}56\,m$

$m_v^* = 0{,}35\,m$

$\beta_e = 0{,}38\,\frac{m^2}{Vs}$

$\beta_v = 0{,}18\,\frac{m^2}{Vs}$

$$-0{,}08 = kT \ln \frac{N_e}{N_d}$$

$$\exp\left(-\frac{0{,}08}{kT}\right) N_d = N_e = 1{,}1 \cdot 10^{25}$$

$$N_d^+ = N_d \left\{1 + \tfrac{1}{2} \exp\left[-(W_F - E_d)/kT\right]\right\}^{-1}$$

$$n_e = n_0 \cdot \exp\left(-\beta(E_g - E_F)\right)$$

$$n_v = n_0 \exp(-\beta E_F)$$

$N_{0e} = 1{,}\cdot 10^{25}$

$N_{0v} = 5{,}4 \cdot 10^{24}$

$$n_e n_v = n_i^2 = n_{0e} n_{0v} \exp\left(-\frac{W_g}{k_B T}\right) \Rightarrow n_i = 1{,}75 \cdot 10^{19}$$

$$\Delta n = n_e - n_v = n_0 \cdot \exp\left(-\frac{\beta E_g}{2}\right)\left[\exp\left(-\beta\left(\frac{E_g}{2} - E_F\right)\right) - \exp\left(\beta\left(\frac{E_g}{2} - E_F\right)\right)\right]$$

$$= 2\underbrace{n_0 \cdot \exp\left(-\frac{\beta E_g}{2}\right)}_{n_i} \sinh\left[\beta\left(E_F - \frac{E_g}{2}\right)\right]$$

$$E_F = \frac{1}{2} E_g + kT \ln\left\{\frac{\Delta n}{2 n_i} + \sqrt{\frac{\Delta n_i^2}{4 n_i^2} + 1}\right\} \doteq \frac{1}{2} E_g + \frac{1}{2} kT \ln \frac{\Delta n}{n_i}$$

Predpostavimo, da je

$n_e \approx N_d \quad , N_a \approx 0$

$n_0 =$

$$E_F = \frac{1}{2} E_g + \frac{1}{2} k_B T \ln \frac{N_d}{n_i} = \frac{1}{2} E_g - \underbrace{k_B T \ln\left(\frac{n_0}{N_d}\right)}_{0{,}08}$$

$$\frac{\sigma^{n-tip}}{\sigma^{int}} = \frac{e_0 \cdot N_d \cdot \beta_e}{e_0 \beta_e n_e + \beta_v e_0 n_v} = \frac{e_0 N_d \beta_e}{e_0 n_i (\beta_e + \beta_v)} = \frac{N_d \beta_e}{n_i (\beta_e + \beta_v)}$$

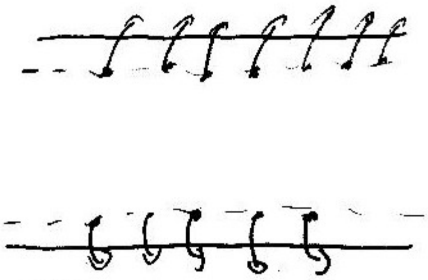
38) $E_F = ?$ $\qquad E_g = 0{,}67\,eV$

$N_d = 10^{23}/m^3 \qquad m_e^* = 0{,}56\,m$

$N_a = 5\cdot 10^{22}/m^3$

$T = 300\,K$

$\sigma = ?$



Svi su ionizirani pri sobnoj T

$$\boxed{E_F = E_g - k_B T \ln \frac{n_0}{N_d - N_a}}$$

$n_e = n_0 \cdot \exp\left(-(E_g - E_F)/k_B T\right)$

$$n_0 = 2\left(\frac{2\pi \cdot m_e^* \cdot k_B \cdot T}{h^2}\right)^{\frac{3}{2}} = 1{,}05\cdot 10^{25}/m^3$$

$$E_F = 0{,}67\,eV - k_B \cdot 300 \ln\left(\frac{1{,}05\cdot 10^{25}}{10^{23} - 5\cdot 10^{22}}\right)$$
$$= \underline{\underline{0{,}53\,eV}}$$

Za 0,14eV pod prevodnim rubom.

$\sigma = \mu_e (N_d - N_a)\cdot e_0 = 3040\,(\Omega m)^{-1}$

39) $N_d = 10^{23}/m^3$ $\quad E_g = 0{,}67\,eV$

$n_e = 2N_d$

$p = N_d$

$$n_e \cdot p = n_i^2 = 2N_d^2 = N_0^2 \cdot \exp\left(-\frac{E_g}{k_B \cdot T}\right)$$

$$N_0 = 2\left(\frac{2m^* k_B T \pi}{h^2}\right)^{\frac{3}{2}} = 2\left(\frac{2m^* k_B \pi}{h^2}\right) T^{\frac{3}{2}}$$

A

$$N_0 = A \cdot T^{\frac{3}{2}} \qquad A = 2\left(\frac{2\pi \cdot m^* \cdot k_B \cdot T}{h^2}\right)^{\frac{3}{2}}$$

$$2N_d^2 = A^2 \cdot T^3 \cdot \exp\left(-\frac{E_g}{k_B \cdot T}\right) \quad / \ln$$

$$\ln\frac{2N_d^2}{A^2 T^3} = -\frac{E_g}{k_B \cdot T} \Rightarrow k_B \cdot T \ln\left(\frac{\overset{C^2}{A^2} T^3 e^3}{2N_d^2}\right) = E_g$$

$$k_B \cdot T \ln\left(C \cdot T k^3\right) = E_g$$

$$C^2 = \frac{4\left(\frac{2\pi \cdot m^*}{h^2}\right)^{\frac{3}{2}}}{2N_d^2}$$

$T = \underline{\underline{698\,K}}$

40) N_d, $\varepsilon = 16$ $\quad r_B$

$m_e^* = 0,2\,m$

$r = 0,246\,nm$

$$W_d = \frac{m^* \cdot e_0^2}{32\pi^2 \cdot \varepsilon^2 \cdot \varepsilon_0^2 \hbar^2} = \frac{W_R \cdot m^*}{\varepsilon^2 \cdot m} = \underline{\underline{0,0106\,eV}}$$

$$r_1 = \frac{4\pi \cdot \varepsilon \cdot \varepsilon_0 \cdot \hbar^2}{m^* \cdot e_0^2} = \frac{r_B \cdot \varepsilon \cdot m}{m^*} = \underline{\underline{4\,nm}}$$

To je 16 razmikov med sosednjimi atomi. $\boxed{\frac{r_1}{r} = 16}$

---

42) $N_a \cdot 10^{24}/m^3$

$\underline{N_d = 8 \cdot 10^{27}/m^3}$

$x_d = ?$

$\phi_0 = 0,63\,eV$

$\varepsilon = 16$

$$x_n = \sqrt{\frac{2\varepsilon \cdot \varepsilon_0 \cdot N_a \cdot \Delta\phi_0}{e \cdot N_d (N_a + N_d}} = \underline{\underline{0,37\,\mu m}}$$

$$x_p = \sqrt{\frac{2\varepsilon \cdot \varepsilon_0 \, N_d \cdot \Delta\phi_0}{e \cdot N_a (N_a + N_d}} = 2\,nm$$

$x_p << x_n$

$x_d = \underline{\underline{0,37\,\mu m}}$ ✓

43) $T = 300K$

$\varrho_p = 0{,}01 \Omega m$

$\varrho_n = 0{,}00 \Omega m$

$E_g = 0{,}67 eV$

$m_e^* = 0{,}56 m$

$m_v^* = 0{,}35 m$

$\beta_e = 0{,}39 \frac{m^2}{Vs}$

$\beta_v = 0{,}19 \frac{m^2}{Vs}$

$\varepsilon = 15{,}8$

$\Delta\phi = ?$

$x_d = ?$

$$e_0 \cdot \Delta\phi = k \cdot T \ln \frac{N_d \cdot N_a}{n_i^2}$$

Pri sobni T : $n_e \approx N_d$
$n_v \approx N_a$

$$\sigma = \frac{1}{\varrho} = e_0 \cdot \beta \cdot n$$

$$\sigma_n = e_0 \cdot n_e \cdot \beta_e = e_0 N_d \cdot \beta_e \Rightarrow N_d = \frac{\sigma_n}{e_0 \beta_e} = 1{,}6 \cdot 10^{22} / m^3$$

$$N_a = \frac{\sigma_p}{e_0 \beta_v} = 3{,}23 \cdot 10^{21} / m^3$$

$$n_e \cdot n_v = n_i^2 = N_{0e} \cdot N_{0v} \cdot \exp\left(-\frac{E_g}{k_0 T}\right) = \left(2{,}5 \cdot 10^{25} / m^3\right)^2 \cdot \left(\frac{m_e^*}{m}\right)^{3/2} \left(\frac{m_v^*}{m}\right)^{3/2} \exp\left(-\frac{E_g}{k_0 \cdot T}\right)$$

$$= 3{,}04 \cdot 10^{38} / m^3$$

$$e_0 \Delta\phi = k \cdot T \ln \left(\frac{N_d \cdot N_a}{n_i^2}\right) = 0{,}31 eV$$

$$\Delta\phi = \underline{\underline{0{,}31 V}}$$

$$x_n = \sqrt{\frac{2 \varepsilon \cdot \varepsilon_0 \cdot N_a \cdot \Delta\phi_0}{e \cdot N_d (N_a + N_d)}} = 7{,}5 \cdot 10^{-8} m$$

$$x_p = \sqrt{\frac{2 \varepsilon \cdot \varepsilon_0 \cdot N_d \cdot \Delta\phi_0}{e \cdot N_a (N_a + N_d)}} = 3{,}74 \cdot 10^{-7} m$$

$$x_d = x_n + x_p = \boxed{4{,}49 \cdot 10^{-7} m}$$

44) $I = I_0 (e^{eU/kT} - 1)$

$U_1 = 0{,}5\,V$

$U_2 = -0{,}5\,V$

$T = 300\,K$

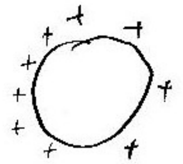
$\frac{I_1}{I_2} = ?$

$$\frac{I_1}{I_2} = \frac{\cancel{I_0}(e^{e \cdot U/k \cdot T} - 1)}{\cancel{I_0}(e^{-eU/k \cdot T} - 1)} = \frac{2{,}49 \cdot 10^{8}}{4 \cdot 10^{-9}} \approx \underline{\underline{2{,}5 \cdot 10^{8}}}$$

II) JEDRA IN DELCI

str. 35

1) a)



$$dV = \frac{e \cdot de}{4\pi \cdot \varepsilon_0 r} = \frac{1}{4\pi \cdot \varepsilon_0 r} \cdot \overbrace{\frac{4\pi \cdot r^3 \cdot \rho}{3}}^{e} \cdot \overbrace{\rho\, 4\pi \cdot r^2 \cdot dr}^{de}$$

$$V = \frac{4\pi}{3\varepsilon_0} \rho^2 \int_0^R r^4\, dr = \frac{4\pi}{3\varepsilon_0} \cdot \rho^2 \, \frac{R^5}{5} = \left(\frac{4\pi}{3}\right)^2 \frac{3\,\rho^2}{4\pi \cdot \varepsilon_0 r} \, \frac{R^5}{5}$$

$$= \left(\frac{4\pi \cdot R^3}{3} \cdot \rho\right)^2 \frac{1}{4\pi \cdot \varepsilon_0 \cdot R} \cdot \frac{3}{5} = \boxed{\frac{e^2}{4\pi \cdot \varepsilon_0 \cdot R} \cdot \frac{3}{5}}$$ ✓

b) $U(r<R) = \frac{e}{4\pi \varepsilon_0 R^3}\left(\frac{r^2}{2} - \frac{3R^2}{2}\right)$ ✓

---

2) e_0, R

$V = ?$

$$\rho = \frac{e \cdot 3}{4\pi \cdot R^3} \Rightarrow \rho\, 4\pi \cdot r^2 \cdot dr = de$$

$$dV = \frac{e \cdot de}{4\pi \cdot \varepsilon_0 \cdot R} = \frac{1}{4\pi \cdot \varepsilon_0 R} \cdot \frac{4\pi \cdot R^3 \cdot \rho}{3} \cdot \rho\, 4\pi \cdot R^2 \cdot dR$$

$$= \frac{4\pi}{3} \cdot \rho^2 \cdot \frac{R^5}{\varepsilon_0 5} = \left(\frac{4\pi R^3}{3} \cdot \rho\right)^2 \cdot \frac{3}{4 \cdot 5 \varepsilon_0 \cdot R \cdot \pi} = \boxed{\frac{3}{20} \frac{e^2}{\varepsilon_0 R \pi}}$$ ✓

3) $^{56}_{26}Fe$; $A = 56$

$Z = 26$

$N = 30$

Gre za α razpad

$r_j = r_1 \cdot A^{\frac{1}{3}}$

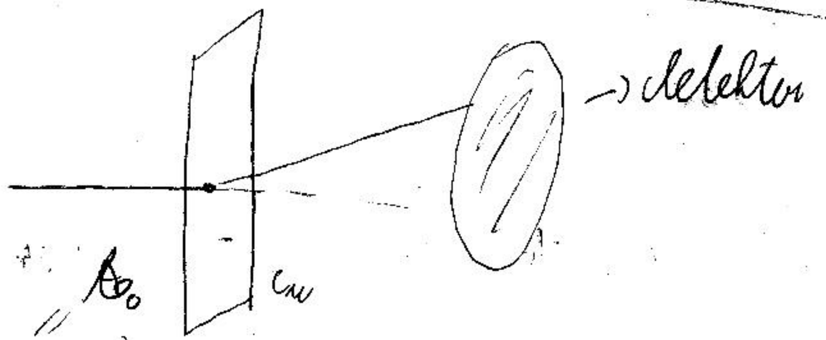
$$W = \frac{Z \cdot e^2}{4\pi \varepsilon_0 r_j} = \frac{Z^2 \cdot e^2}{4\pi \varepsilon_0 r_1 \cdot A^{\frac{1}{3}}} = \frac{Z_{Fe} \cdot Z_{\alpha} \cdot e\, e_0}{4\pi \varepsilon_0 r_1 A^{\frac{2}{3}}} = \underline{17,78\,MeV}$$

---

4) $\underline{E = 5,3\,MeV}$

$P_1(45 < \vartheta < 90)$

$\underline{P_2\,(90 < \vartheta < 150}$



$$P_1 = \frac{d\sigma}{d\Omega} \cdot d\Omega = \int_0^{2\pi}\int_{45}^{90} \left(\frac{Z \cdot z \cdot e_0^2}{16\pi \varepsilon_0 \cdot E_0}\right)^2 \cdot \frac{\sin\vartheta \cdot d\vartheta\, d\varphi}{\sin^4\frac{\vartheta}{2}}$$

$$= 2\pi \cdot \sigma_0 \int_{45}^{90} \frac{\sin\vartheta \cdot d\vartheta}{\sin^4\frac{\vartheta}{2}} = 2\pi\sigma_0 \int_{22,5}^{45} \frac{\sin 2t \cdot 2dt}{\sin^4 t} = 4\pi\sigma_0 \int_{22,5}^{45} \frac{2 \sin t \cos t}{\sin^4 t}$$

$t = \frac{\vartheta}{2}$

$2\,dt = d\vartheta$

$$= 8\pi\sigma_0 \int \frac{\cos t}{\sin^3 t}\, dt = 8\pi\sigma_0 \int \frac{du}{u^3} = -8\pi\sigma_0\, u^{-2} = -4\pi\sigma_0 \left[\frac{1}{(\sin 45)^2} - \frac{1}{(\sin\ldots}\right.$$

$u = \sin t$

$du = \cos t\, dt$

$$= 4\pi\sigma_0 \left[\frac{1}{(\sin 22,5)^2} - \frac{1}{(\sin 45)^2}\right]$$

$$P_2 = \ldots = 8\pi\sigma_0 \left[\frac{1}{(\sin 45)^2} - \frac{1}{(\sin 75)^2}\right]$$

$$\frac{P_1}{P_2} = \frac{\frac{1}{(\sin 22,5)^2} - \frac{1}{(\sin 45)^2}}{\frac{1}{(\sin 45)^2} - \frac{1}{(\sin 75)^2}} = \underline{5,2 > 1}$$

5) ${}_{2}^{4}He$, ${}_{6}^{12}C$, ${}_{26}^{56}Fe$, ${}_{28}^{63}Ni$, ${}_{37}^{93}Rb$, ${}_{55}^{141}Cs$, ${}_{82}^{208}Pb$, ${}_{84}^{210}Po$, ${}_{91}^{229}Pa$, ${}_{92}^{235}U$, ${}_{92}^{238}U$

$W_v = ?$

$$W_v = -w_0 \cdot A + w_1 \cdot A^{\frac{2}{3}} + w_2 \cdot \frac{Z^2}{A^{\frac{1}{3}}} + w_3 \cdot \frac{(A-2Z)^2}{A} + w_4 \cdot \frac{d_{Z,N}}{A^{3/4}}$$

$w_0 = 15{,}6\ MeV$
$w_1 = 17{,}3\ MeV$
$w_2 = 0{,}7\ MeV$
$w_3 = 23{,}3\ MeV$
$w_4 = 33{,}5$

$$W_v(He) = -15{,}6\ MeV \cdot 4 + 17{,}3\ MeV \cdot \sqrt[3]{4^2} + 0{,}7 \frac{4}{\sqrt[3]{4}} + 23{,}3 \frac{(4-2\cdot 2)^2}{4} - \frac{w_4}{\sqrt[3]{4^4}}$$
$$= \underline{-22{,}3\ MeV}$$

b) $$W_v(C) = -15{,}6\ MeV \cdot 12 + 17{,}3 \cdot \sqrt[3]{12^2} + 0{,}7 \frac{36}{\sqrt[3]{12}} + 23{,}3 \frac{(12-2\cdot 6)^2}{4} - \frac{w_4}{\sqrt[3]{12^4}}$$
$$= \underline{-86{,}7\ MeV}$$

$W_v(Fe)$ ....

6) $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{141}_{55}Cs + ^{95}_{37}Rb$

$E = ?$

$$W_v(U) = -15{,}6\,MeV \cdot 235 + 17{,}3\,MeV \cdot \sqrt[3]{235^2} + 0{,}7\,MeV\,\frac{92^2}{\sqrt[3]{235}} + 23{,}3\,MeV\,\frac{(235-2\cdot92)^2}{235} + C$$

$$= \underline{-1789\,MeV}$$

$W_v(Cs) = -1165{,}3\,MeV$

$W_v(Rb) = -803{,}6\,MeV$

$$Q = -(\underbrace{Z\cdot m_p + N\cdot m_n + W_v(U)}_{U}) + (\underbrace{Z\cdot m_p + N\cdot m_n + W_v(Cs)}_{Cs} + \underbrace{Z\,m_p + N\cdot m_n + W_v(Rb)}_{Rb}$$

$$Q = -\cancel{92\,m_p} - 144\cdot m_n - W_v(U) + \cancel{55\,m_p} + 86\,\cancel{m_n} + W_v(Cs) + \cancel{37\,m_p} + 58\,\cancel{m_n} + W_v(Rb)$$

$$|Q| = W_v(Cs) + W_v(Rb) - W_v(U) = \underline{182\,MeV}$$

7) ${}^{A}_{Z}X_N \longrightarrow {}^{A-4}_{Z-2}Y_{N-2} + {}^{4}_{2}He_2 \qquad T_2 = ?$

${}^{229}_{91}Pa_{138} \longrightarrow {}^{225}_{89}Ac_{136} + {}^{4}_{2}He_2$

$$\boxed{M(A,Z) = Z\, m_H + N \cdot m_n + \frac{W_v(A,Z)}{c^2}}$$

$$-Q = \left[M(A,Z) - M(A-4,Z-2) - M(4,2) \right] \cdot c^2$$

$$-Q = \left[91\, m_H + 138\, m_n + \frac{W_v(A,Z)}{c^2} - 89\, m_H - 136\, m_n - W_v(Ac)/c^2 - 2\, m_H - 2\, m_n - \frac{W_v(He)}{c^2}\right]$$

$$W_v(Pa) = -15{,}6\,MeV \cdot 229 + 17{,}3\,MeV \cdot \sqrt[3]{229^2} + 0{,}7\,MeV \frac{91^2}{\sqrt[3]{229}} + 23{,}3\,MeV \frac{(229-2\cdot 91)}{229}$$

$$= -1753\,MeV$$

$W_v(Ac) = -1730\,MeV$

$W_v(He) = -29\,MeV$

$-Q = -7\,MeV$

$\boxed{Q = 7\,MeV}$

$m_{Ac} \cdot v_{Ac} = -m_{He} \cdot v_{He}$

$v_Y = -\frac{m_2}{m_Y} \cdot v_2$

$Q = \frac{1}{2} m_{Ac} \cdot v_{Ac}^2 + \frac{1}{2} m_2 \cdot v_2^2$

$Q = \frac{1}{2} \frac{m_2^2}{m_Y} v_2^2 + \frac{1}{2} m_2 \cdot v_2^2$

$Q = \underbrace{\frac{1}{2} m_2 v_2^2}_{T_2} \left[\frac{m_2}{m_Y} + 1 \right]$

$$T_2 = \frac{Q}{1 + \frac{m_2}{m_Y}} = \underline{\underline{6{,}9\,MeV}}$$

8) ${}^{56}_{26}Fe_{30} + \gamma \longrightarrow {}^{55}_{25}Y_{30} + {}^{1}_{1}p$

M_x

$Q_{min} = ?$

$W_v(Fe) \cong -491{,}6\,MeV$

$W_v(Y) = -482\,MeV$

$-Q = W_v(Fe) - W_v(Y) = -10\,MeV$

$Q = 9{,}6\,MeV$

$W_v(Fe) + W_\gamma = W(Y) + W_p + W_e(Y)$

$p_\gamma = p_Y + p_p$ Proton bo absorbiral ker je lažji

$p_\gamma = p_Y = \frac{E_\gamma}{c}$

$p_p = 0$
$W_p = 0$

$E_\gamma + \Delta W_v^{=Q} = \frac{p^2}{2m_x} = \frac{E_\gamma^2}{c^2 \cdot 2m_Y}$

$E_\gamma^2 - 2m_x c^2 \cdot E_\gamma - Q \cdot 2m_x c^2 = 0$

$a = 1$
$b = -2m_x c^2 \cdot E_\gamma$
$c = -2m \cdot Q \cdot c^2$

$D = 4m_x^2 c^4 - 8 m_x c^2 Q$

$E_{\gamma_{1,2}} = \frac{+2m_x c^2 \pm \sqrt{4m_x^2c^4 - 8m_xc^2Q}}{2} = m_x c^2 \pm \sqrt{m_x^2c^4 - 2m_xc^2 Q}$

$E_{\gamma_{1,2}} = m_x c^2 \cdot \frac{Q}{m_x c^2}$

$E_{\gamma_1} = 9{,}5\,MeV$

$${}^{56}_{26}Fe_{30} + \gamma \longrightarrow {}^{A-1}_{26}X_{N-1} + {}^{1}_{0}n_1$$

$${}^{55}_{26}Y_{29} = {}^{55}_{26}Fe_{29}$$

$W_v(Fe) = -491.6\,MeV$

$W_v(Fe') = -479\,MeV$

$|Q| = 9\ldots4\,MeV$

$E_0(Fe) + E_\gamma = E_x + E_r(x) + E_n$

$+ E_\gamma + \Delta Q = E_X + E_n$

$p_\gamma = p_x + p_n$ nevtron obmiruje ker je lažji

$p_\gamma = p_X = \frac{E_\gamma}{c}$ $\qquad E_X = \frac{p^2}{2m_X}$

$E_\gamma + \Delta Q = \frac{E_\gamma^2}{c^2 \cdot 2m_x}$

$E_\gamma^2 - 2m_x \cdot c^2 \cdot E_\gamma - 2c^2 m_x Q = 0$

$D = 4m_x^2 c^4 \pm 8 m_x c^2 Q$

$E_{\gamma_1} = m_x c^2 - \sqrt{m_x^2 c^4 - 2m_x c^2 Q}$ ⇒ Razvijemo $= 14,\ldots\,MeV$

$E_{\gamma_1} = \underline{12,1\,MeV}$

$\frac{W_\gamma}{\Delta W_v}$. $\qquad W_r = \frac{p_i^2}{2m_i} = \frac{W_\gamma^2}{2m_i c^2}$

9) ${}^{210}_{84}Po_{126} \longrightarrow {}^{206}_{82}Pb_{124} + {}^{4}_{2}He_{2}$ $\quad T_2 = ?$ $\quad$ Pb

$W_v(Po) = -1638,7 \text{ MeV}$

$W_v(Pb) = -1674,3 \text{ MeV}$

$W_v(He) = -28,9 \text{ MeV}$

$Q = m_e - m_z = -4,5 \text{ MeV}$

$-Q = \frac{1}{2} m_{Pb} \cdot v_{Pb}^2 + \frac{1}{2} m_2 \cdot v_2^2$

$m_x v_x = -m_2 \cdot v_2$

$v_x = -\frac{m_2}{m_x} \cdot v_2$

$-Q = \frac{1}{2} \frac{m_2^2}{m_x} \cdot v_2^2 + \frac{1}{2} m_2 \cdot v_2^2$

$-Q = \frac{1}{2} m_2 \cdot v_2^2 \left(\frac{m_2}{m_x} + 1 \right)$

$T_2 = \frac{-Q}{1 + \frac{m_2}{m_x}} = \frac{4,5 \text{ MeV}}{1 + \frac{4}{207}} = \underline{4,4 \text{ MeV}}$ ✓

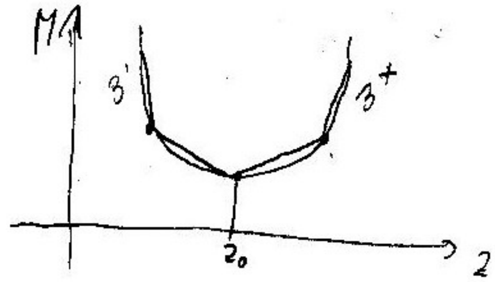
10.) $M(Z,A) = C_A + \frac{1}{2} B_A (Z - Z_A)^2 + D_A = w_3' Z^2 - w_2' Z + w_1'$

$$W_V = -w_0 A + w_1 \cdot A^{2/3} + w_2 \frac{Z^2}{A^{1/3}} + w_3 \cdot \frac{(A-2Z)^2}{A} + w_4 \frac{\delta_{Z,N}}{A^{3/4}}$$

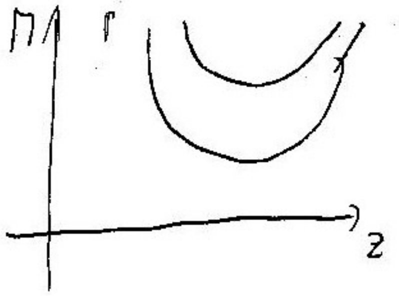
$$M(A,Z) = Z \cdot m_H + N \cdot m_n + \frac{W_V}{c} = w_3' Z^2 - w_2' Z + w_1'$$

$$= \left(\frac{w_2}{A^{\frac{1}{3}}} + \frac{4 w_3}{A} \right) \cdot Z^2 - Z \left(4 w_3 + \frac{(m_n + m_H)}{c^2} \right) + \left(A \cdot m_n c^2 - w_0 \cdot A + w_1 A^{\frac{2}{3}} + w_3 A \pm w_4 \frac{\delta_{Z,N}}{A^{3/4}} \right)$$

a) $A = \text{lih}$, $\delta_{Z,N} = 0$



b) $A = \text{sod}$ $\delta_{Z,N} = \pm 1$



11) $A = 97, 169, 80, 194$

$$W_V = -w_0 \cdot A + w_1 \cdot A^{\frac{2}{3}} + w_2 \cdot \frac{Z^2}{A^{\frac{1}{3}}} + w_3 \frac{(A-2Z)^2}{A} + w_4 \cdot \frac{d_{ZN}}{A^{\frac{3}{4}}}$$

$$M(A,Z) = w_3' Z^2 - w_2' \cdot Z + w_1' \quad \Big/ \frac{\partial}{\partial Z}$$

$$\frac{\partial M(A,Z)}{\partial Z} = 2Z w_3' - w_2' Z + 0$$

Minimum bo $\frac{\partial M(A,Z)}{\partial Z} = 0$

$$2Z w_3' - w_2' = 0$$

$$Z = \frac{w_2'}{2 \cdot w_3'} =$$

94,09

$$\boxed{Z = \frac{4 w_3 + (m_n - m_H) c^2}{2 \cdot \left(\frac{w_2}{A^{1/3}} + \frac{4 w_3}{A} \right)}}$$

$\rightarrow A = 97,\ Z = 42 \Rightarrow$ Mo ✓

$\rightarrow A = 169,\ 69 = Z \Rightarrow$ Tm ✓

$\rightarrow A = 80,\ Z = 35 \Rightarrow$ Br ✓

$\rightarrow A = 194,\ Z = 78 \Rightarrow$ Pt ✓

12) Rozpad β^+: ${}^{A}_{Z}X_N \rightarrow {}^{A}_{Z-1}Y_{N+1} + e^+ + \nu$

$$+Q = [M(A,Z) - M(A,Z-1) - 2m]c^2$$

Rozpad β^-: ${}^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}Y_{N-1} + e^- + \bar{\nu}$

$$+Q = [M(A,Z) - M(A,Z+1)]c^2$$

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13) ${}^{65}_{28}Ni$

$E_e = ?$

$${}^{65}_{28}Ni_{37} \rightarrow {}^{65}_{29}Cu_{36} + e^- + \bar{\nu}$$

$$Q = [M(A,Z) - M(A,Z+1)]c^2$$

$$Q = [28 \cdot m_H + 37 \cdot m_n + W_v(Ni) - 29 m_H - 36 m_n - W_v(Cu)]c^2$$

$$Q = [m_n - m_H + W_v(Ni) - \frac{W_v(Cu)}{c^2}]c^2$$

$W_v(Ni) = -568,79 \text{ MeV}$

$W_v(Cu) = -570,34 \text{ MeV}$

$$Q = [(1,00886 - 1,00789) 931,5 \text{ MeV} + 1,55 \text{ MeV} \cong 3,1 \text{ MeV}$$

$$T_e = \frac{|Q|}{\left(1 + \frac{m_e}{m_Y}\right)} = 2,44 \text{ MeV}$$

$m_e \cong 10^{-4}\, u$

14) ${}^{107}_{48}Cd_{59}$

a) $M(A,Z) = w_1' - w_2' Z + w_3' Z^2$

$$\frac{\partial M(A,Z)}{\partial Z} = 2 Z \cdot w_3' - w_2' = 0$$

94,04

$$Z_0 = \frac{w_2'}{2 \cdot w_3'} = \frac{4 w_3 + (m_n - m_H) c^2}{2\left(\frac{w_4}{A^{1/3}} + \frac{4 w_3}{A}\right)} = \underline{\underline{46}}$$

Ta zadeva razpade z β^+

b) ${}^{107}_{48}Cd_{59} \longrightarrow {}^{107}_{47}Ag_{60} + e^+ + \nu$

$$-Q = [M(A,Z) - M(A,Z-1) - 2 m_e] c^2$$

$$= \left[48 \cdot m_H + 59 m_n + \frac{W_v(Cd)}{c^2} - 47 \cdot m_H + 60 m_n - \frac{W_v(Ag)}{c^2} - 2 m_e\right] c^2$$

$$= (m_H - m_n) c^2 + W_v(Cd) - W_v(Ag) - 2 m_e c^2$$

$W_v(Cd) = -913,22 \text{ MeV}$

$W_v(Ag) = -916,77 \text{ MeV}$

$-Q \cong 1 \text{ MeV}$

$Q = \underline{-1 \text{ MeV}}$

$$T_e = \frac{|Q|}{1 + \frac{m_e}{m_x}} = \underline{\underline{1,64 \text{ MeV}}}$$

15) $v_e = ?$ $\quad {}^1_0n_1 \rightarrow {}^1_1p_0 + e^- + \bar{\nu} \quad \rightarrow \beta^-$

$$-Q = [\quad] c^2$$

$$-Q = [m_n - m_p - m_e] c^2$$

$$Q = -1{,}045 \text{ MeV}$$

$$T_{e^-} = \frac{|Q|}{1 + \frac{m_e}{m_p}} \approx 1{,}03 \text{ MeV}$$

Treba računat relativistično

$$T = mc^2(\gamma - 1)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$T = mc^2 \cdot \gamma - mc^2 \Rightarrow \frac{T + mc^2}{mc^2} = \gamma \cong 3$$

$$\gamma^2 - \gamma^2\beta^2 = 1$$

$$\beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \Rightarrow \underline{\underline{0{,}94c = v}}$$

16) ${}^{12}_{7}N_5 \rightarrow {}^{12}_{6}C_6 + e^+ + \nu$. β^+

$Q = 16{,}7\ MeV$

$T_{cmax}(C)$

Da dobimo $T_{c\,max}$ bo elektron miroval

$0 = p_\nu + p_c + p_e = 0$

$p_\nu = p_c = \frac{E_\nu}{c} \Rightarrow$ GIBALNA

$[M(12,7) - M(12,6) - 2m]c^2$

$m_N c^2 = T_c + m_e c^2 + m_e c^2 + p_c \cdot c$

$[7m_H + 5m_n - 6m_c - 6m_n]c^2 + Q - m_e c^2 = T_c + \sqrt{E_c^2 - m_e^2 c^4}$

$[m_H - m_p]c^2 + Q - T_c = \sqrt{(T_c + m_c c^2)^2 - m_c^2 c^4}$

$-59{,}12\ MeV - T_c = \dots \quad /^2$

$333\ (MeV)^2 + 36{,}54\, T_c + \cancel{T_c^2} = \cancel{T_c^2} + 2T_c \cdot m_c \cdot c^2$

$2T_c \cdot m_c \cdot c^2 - 36{,}54\, T_c = 333{,}6\ (MeV)^2$

$$T_c = \frac{333\ (MeV)^2}{2m_c c^2 - 36{,}54\ MeV} = \underline{\underline{0{,}0149\ MeV}}$$

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2 NAČIN

$m_N c^2 = T_c + m_c c^2 + m_e c^2 + E_\nu$

$Q = T_c + E_\nu = T_c + p_\nu \cdot c = T_c + p_c \cdot c = T_c + \sqrt{(T_c + m_c c^2)^2 - (m_c c^2)^2}$

$(Q - T_c)^2 = T_c^2 + 2T_c \cdot m_c c^2$

$Q^2 - 2T_c Q + \cancel{T_c^2} = \dots$

$2T_c \cdot m_c \cdot c^2 + 2T_c Q = Q^2$

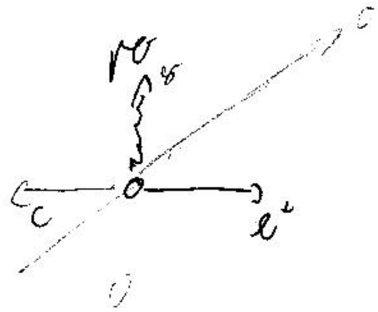
$$T_c = \frac{Q^2}{2m_c c^2 + 2Q} = \underline{\underline{0{,}0125\ MeV}}$$

17) $T_e = 6,5$ MeV

$\varphi = 90^\circ$

pred

N



1 $\cos\beta \, p_c = p_\gamma \cdot \cos\varphi + p_e \cdot \cos\varphi$

$\sin\beta \cdot p_c = p_\gamma \cdot \sin\varphi + p_e \cdot \sin\varphi$

$p_c = p_\gamma + p_e = \frac{E_\gamma}{c} + p_e$

$m_N \cdot c^2 = T_c + m_c \cdot c^2 + E_e + E_\nu$

18) $^{95}Tc + e^- \longrightarrow {}^{95}_{42}Mo_{53} + \nu_e$

$Q = 1{,}6$ MeV

$E_\nu = ?$

Težišćni sistem

$Q = T_{Mo} + E_\nu$

$p_{Mo} = p_\nu = \frac{E_\nu}{c}$, $T_{Mo} = \frac{p_{Mo}^2}{2 m_{Mo}}$

$Q = \frac{p_{Mo}^2}{2 m_{Mo}} + E_\nu = \frac{E_\nu^2}{2 m_{Mo} \cdot c^2} + E_\nu$

$-2 m_{Mo} Q \cdot c^2 + 2 m_{Mo} c^2 \cdot E_\nu + E_\nu^2 = 0$

$D = 4 m_{Mo}^2 c^4 + 8 m_{Mo} \cdot Q \cdot c^2$

$E_{\nu_{1,2}} = \frac{-2 m_{Mo} c^2 \pm 2\sqrt{m_{Mo}^2 c^4 + 2 m_{Mo} Q c^2}}{2} = -m_{Mo} c^2 \pm \sqrt{m_{Mo}^2 c^4 + 2 m_{Mo} Q c^2}$

$E_{\nu_1} = -m_{Mo} c^2 + \sqrt{1 + \frac{2Q}{m_{Mo} c^2}}\, m_{Mo} c^2 = m_{Mo} c^2 \left(\sqrt{1 + \frac{2Q}{m_{Mo} c^2}} - 1\right)$

$M_{Mo}(95,42)c^2 = \underbrace{(42 \cdot m_H + 53\, m_n)c^2}_{89212 \text{ MeV}} + \underbrace{W_\sigma(Mo)}_{-821{,}5} = 88390$ MeV

$E_\nu = 1{,}6$ MeV

19) ${}^{3}_{1}T_{2} + {}^{3}_{1}T_{2} \rightarrow {}^{4}_{2}He + 2n$; $T_T = 15\,MeV$

$\rightarrow \leftarrow$

$$Q = 2T_T' - 2T_n - T_{He}$$ $-29{,}2\,MeV$

$$2T_T' + 2m_T \cdot c^2 = 2m_n \cdot c^2 + 2T_n + T_\alpha + m_\alpha \cdot c^2$$

$$2T_T' - 2T_n - T_\alpha = Q = 2m_n c^2 + m_\alpha \cdot c^2 - 2m_T \cdot c^2$$

$$Q = \underline{\underline{12{,}27\,MeV}}$$

$\rightsquigarrow$ $2m_n v_n = m_\alpha \cdot v_\alpha$

$$2m_n \cdot \sqrt{\frac{T_n \cdot 2}{m_n}} = m_\alpha \cdot \sqrt{\frac{T_\alpha 2}{m_\alpha}} \quad /^2 \qquad v = \sqrt{\frac{2T}{m}}$$

$$4m_n \cdot 2 \cdot T_n = m_\alpha \cdot T_\alpha \cdot 2$$

$$\cancel{4}m_n\left(2T_T' - T_\alpha - |Q|\right) = m_\alpha \cdot T_\alpha \cdot \cancel{2}$$

$$m_n 4T_T' - 2T_\alpha m_n - 2m_n \cdot |Q| = m_\alpha \cdot T_\alpha$$

$$T_\alpha = \frac{4T_T' m_n - 2m_n|Q|}{m_\alpha + 2 \cdot m_n} = \frac{2m_n(T_T' \cdot 2 - |Q|)}{m_\alpha + 2m_n} = \underline{\underline{5{,}9\,MeV}}$$

20) $E_d = 35\ MeV$ $\qquad W_{vz}$

a) $d + {}^{31}P \longrightarrow {}^{31}P + d$

$W_{vz} = (m_d^* - m_d)c^2$ $\qquad a + X \longrightarrow b_p + Y_p^*$

$Q + W_{vz} = W_{ka} - W_{kb} - W_{kY}$

$Q + W_{vz} = W_{ka}\left(1 - \frac{m_a}{m_Y}\right) - W_{kb}\left(1 + \frac{m_b}{m_Y}\right) + 2\left(\frac{m_a m_b W_{ka} W_{kb}}{m_Y^2}\right)^{\frac{1}{2}} \cos\vartheta$

1 a) $d + {}^{31}P \longrightarrow {}^{31}P + d$

$Q = 0\ ;\ W_{vz} = W_{kd}\left(1 + \frac{m_d}{\ }\right.$

21) $^{238}_{92}U \longrightarrow {}^{96}_{39}Y + {}^{140}_{53}I + 2n$

$W_v(U) = -1807{,}4\ MeV$

$W_v(Y) = -822{,}6\ MeV$

$W_v(I) = -1145{,}7\ MeV$

$+Q = [m(A,Z) - m(A',Z') - m(A-A', Z-Z')]c^2$

$+Q = [2m_n + m_Y + m_I - m_U]c^2$

$+Q = 2\cancel{m_n}c^2 + [39\cancel{m_H} + 57\cancel{m_n} + W_v(Y) + 53\cancel{m_H} + 87\cancel{m_n} + W_v(I) - 92\cancel{m_H} - 146m_n - W_U$

$+Q = W_v(Y) + W_v(I) - W_v(U)$

$+Q = -160{,}9$

$Q = \underline{-161\ MeV}$

22) $n + {}^{14}_{7}N_{7} \to {}^{13}_{7}N_{6} + 2n$

$W_{prag} = ?$

$W_V(^{14}N) = -99{,}04\ MeV$

$W_V(^{13}N) = -90{,}37\ MeV$

$-Q = [m_n + M(^{14}N) - M(^{13}N - 2m_n]c^2$

$-Q = [M(^{14}N) - M(^{13}N) - m_n]c^2$

$-Q = [7\cancel{m_H} + 7\cancel{m_n} + W(V)(^{14}N) - 7\cancel{m_H} - 6m_n - W_V(^{13}N) - m_n]c^2$

$Q = 8{,}26\ MeV$

$$W_{prag} = \frac{Q(m_a + m_X)}{m_X} = Q\left(\frac{m_a}{m(N)} + 1\right) = \underline{\underline{8{,}9\ MeV}}$$

23) a) $p + {}^{7}_{3}Li_{4} \rightarrow {}^{7}_{4}Be_{3} + n$

$m_p \approx m_n$

$W_{k_p} = ?$

$W_v(Li) = -39{,}27\,MeV$ $\quad M(Li) = 6533{,}4\,MeV$

$W_v(Be) = -36{,}71\,MeV$ $\quad M(Be) = 6535{,}4\,MeV$

$Q = [m_p + M(Li) - M(Be) - m_n]c^2$

$Q = -3{,}9\,MeV$

$W_{prag} = Q\left(\frac{m_n}{m({}^{7}Li)} + 1\right) = 3{,}77\,MeV$

b) $n + {}^{13}_{6}C_{7} \rightarrow {}^{4}_{2}He_{2} + {}^{10}_{4}Be_{6}$

$W_v(C) = -94{,}64\,MeV$ $\quad M(C) = 12112{,}8\,\frac{MeV}{c^2}$

$W_v(He) = -29\,MeV$ $\quad M(He) = 3226{,}9\,\frac{MeV}{c^2}$

$W_v(Be) = -67{,}2\,MeV$ $\quad M(Be) = 9324{,}1$

$Q = (m_n + M(C) - M(He) - M(Be))c^2 =$

$Q = -1{,}3\,MeV$

$W_{prag} = |Q|\left(\frac{m_n}{m_C} + 1\right) = 1{,}4\,MeV$

24) ${}^{2}_{1}D_1 + {}^{3}_{1}T_2 \rightarrow {}^{4}_{2}He_2 + n$ $\qquad m_D = 2{,}014$

$Q = ?$ $\qquad m_T = 3{,}016$

$$-Q = (m_D + m_T - m_{He} - m_n)\,c^2 = 17{,}45\ \mathrm{MeV}$$

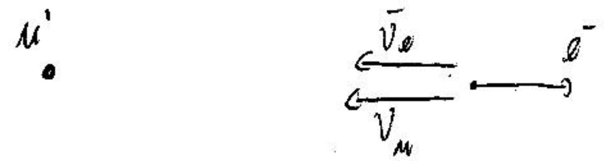
$$Q = \underline{\underline{-17{,}45\ \mathrm{MeV}}}$$

25) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

$T_{e\,max} = ?$

$E_0 = 105{,}7\ \mathrm{MeV}$

$Q = ?$



1) $Q = (m_\mu - m_e)\,c^2 = \underline{\underline{105{,}2\ \mathrm{MeV}}}$ ✓

2) $E_{\mu^-} = E_{e^-} + E_{\bar{\nu}_e} + E_{\nu_{\mu^-}}$

$$m_\mu \cdot c^2 = T_e + m_e c^2 + E_{\bar{\nu}_e} + E_{\nu_{\mu^-}}$$

$$p_{e^-} = p_{\bar{\nu}_e} + p_{\nu_\mu}$$

$$p_{e^-} = \frac{E_{\nu_e}}{c} + \frac{E_{\nu_\mu}}{c} \Rightarrow p_e \cdot c - E_{\nu_{\mu^-}} = E_{\bar{\nu}_e}$$

$$m_\mu \cdot c^2 = T_e + m_e c^2 + p_e \cdot c$$

$$m_\mu \cdot c^2 = \underbrace{T_e + m_e c^2}_{E} + \sqrt{E_e^2 - m_e^2 c^4}$$

$$(m_\mu \cdot c^2 - E_e)^2 = E_e^2 - m_e^2 c^4$$

$$m_\mu^2 c^4 - 2E_e m_\mu c^2 + \cancel{E_e^2} = \cancel{E_e^2} - m_e^2 c^4$$

$$E_e = \frac{m_\mu^2 c^4 + m_e^2 c^4}{2\, m_\mu c^2} = 52{,}95\ \mathrm{MeV}$$

$T_e = 52{,}34\ \mathrm{MeV}$ ✓

2.6) $m_\mu \cdot c^2 = 105{,}7\ MeV$ $\quad \pi^\pm \to \mu^\pm + \nu_\mu$

$m_\pi \cdot c^2 = 139{,}6\ MeV$

$T_\mu = ?$

$p_\mu = ?$

$Q = ?$

a) $Q = (m_{\pi^\pm} - m_\mu)c^2$

$Q = -33{,}9\ MeV$

b) π $\quad \nu_\mu \longleftarrow \cdot \longrightarrow \mu$

- $m_{\pi^\pm} \cdot c^2 = E_\mu + E_{\nu_\mu}$
- $p_\mu = \frac{E_{\nu_\mu}}{c} \Rightarrow E_{\nu_\mu} = p_\mu \cdot c$

$m_\pi \cdot c^2 = E_\mu + p_\mu \cdot c$

$(m_\pi \cdot c^2 - E_\mu)^2 = E_\mu^2 - m_\mu^2 \cdot c^4$

$m_\pi^2 \cdot c^4 - 2 m_\pi \cdot c^2 E_\mu + \cancel{E_\mu^2} = \cancel{E_\mu^2} - m_\mu^2 \cdot c^4$

$$E_\mu = \frac{m_\pi^2 \cdot c^4 + m_\mu^2 \cdot c^4}{2 m_\pi \cdot c^2} = 109{,}8\ MeV$$

$$T_\mu = 4{,}1\ MeV$$

$$c\, p_\mu = \sqrt{E_\mu^2 - m_\mu^2 c^4} = 29{,}7\ MeV$$

27) a) $p + p \rightarrow p + n + \pi^+$; $\gamma + p = n + \pi^+$

a) $Q = ?$

b) $W_{prag} = ?$

$$-Q = (2m_p - m_p - m_n - m_{\pi^+})c^2 = -140{,}9$$

$$Q = 140{,}9 \text{ MeV}$$

$$-Q = (m_p - m_n - m_{\pi^+})c^2 = -140{,}9 \text{ MeV}$$

$$Q = 141 \text{ MeV}$$

$$p_a^2 = p_v^2$$

$$W_{vz} = -Q_u + W_{ka} - W_{kv} = -Q_u + \frac{p_a^2}{2m_a} - \frac{p_v^2}{2m_u^*} = -Q_v + \frac{p_a^2}{2m_a} - \frac{p_a^2}{2m_v}$$

$$= -Q_u + \frac{p_a^2}{2}\left(\frac{1}{m_a} - \frac{1}{m_u^*}\right) = \frac{p_a^2}{2m_a}\left(1 - \frac{m_a}{m_v^*}\right) - Q_v$$

$$= -Q_v + W_{ka}\left(1 - \frac{m_a}{m_a + m_x}\right) = -Q_v + \frac{m_x \cdot W_{ka}}{m_a + m_x}$$

$$m_v^* = m_x + m_a$$

28) $\;{}^{1}_{1}p + {}^{2}_{1}D \rightarrow 2\,{}^{1}_{1}p + {}^{1}_{0}n$ $\qquad m_n \approx m_p$

$W_v = 2{,}226\ \mathrm{MeV}$ $\qquad M(D) =$

$\Rightarrow -Q = (m_p + M(D) - 2m_p - m_n)\,c^2 =$

$-Q = (M(D) - 2m_p)\,c^2$

$\Rightarrow M(D) = Z \cdot m_H + N \cdot m_n + W_V(D)$

27) 1) $p + p \Rightarrow p + n + \pi^+$

2) $\gamma + p \rightarrow n + \pi^+$

a) $Q = ?$

b) $W_{prag} = ?$

a) $-Q_1 = (m_p - m_n - m_{\pi^+})c^2 \cong -141\ \text{MeV}$ ✓ $\Rightarrow Q = 141\ \text{MeV}$

$-Q_2 = (m_p - m_n - m_{\pi^+})c^2 \cong -141\ \text{MeV}$ ✓

$Q = 141\ \text{MeV}$

b) 1) S'

$$2T_p + 2m_pc^2 = m_pc^2 + m_{\pi}c^2 + m_nc^2$$

$$Q = 2T_p \Rightarrow T_p = \frac{|Q|}{2} = 70{,}5\ \text{MeV}$$

$W_{prag} =$

2) $\overset{\gamma}{\longrightarrow} \quad \overset{p}{\longleftarrow}$

$\circ^{\pi^+}$

$\circ_n$

$$E_\gamma + T_p + m_p c^2 = m_\pi c^2 + m_n c^2$$

$$p_\gamma^2 = \frac{E_\gamma^2}{c^2} \Rightarrow E_\gamma$$

$$p_\gamma = p_p$$

$$T_p = \frac{p^2}{2m_p}$$

$$E_\gamma + \frac{E_\gamma^2}{2m_p c^2} + m_p c^2 = m_\pi c^2 + m_n c^2$$

$$E_\gamma + \frac{E_\gamma^2}{2m_p c^2} - Q = 0 \qquad / \, 2m_p c^2$$

$$E_\gamma^2 + 2m_p c^2 E_\gamma - Q \cdot 2m_p c^2 = 0$$

$$D = 4m_p^2 c^4 + 8m_p c^2 Q$$

$$E_\gamma = -m_p c^2 + \sqrt{m_p^2 c^4 + 2m_p c^2 Q} = \underline{\underline{132\,\mathrm{MeV}}} \quad ?$$

29) $j = 1{,}3\ kW/m^2$ $4\ {}^1H \rightarrow {}^4He + 2e^+ + 2\nu$

$R = 1{,}5 \cdot 10^{11}\ m$ $-Q = (4m_H - m_\alpha - 2m_{e^+})c^2 = 26\ MeV$

$N = \frac{m}{t} = ?$ $\underline{\underline{Q = -26\ MeV}} \Rightarrow \frac{6{,}5\ MeV}{H} = Q$

$j = P/S \Rightarrow P = j \cdot S = j \cdot 4\pi r^2 = 3{,}68 \cdot 10^{26}\ W$

$N = \frac{P}{Q} = \frac{3{,}68 \cdot 10^{26}\ W \cdot 1{,}0079\ u}{6{,}5\ MeV} = \underline{\underline{5{,}92 \cdot 10^{11}\ \frac{kg}{s}}}$

---

30) $\tau = 10^{31}$ let

$m_\nu = ?$

$A = 1/dan$ $A = \frac{N_0 e^{-\frac{t}{\tau}}}{\tau}$; ob času $t = 0$ je N_0

$N(t) = N_0 \cdot e^{-\frac{t}{\tau}}$ $A = \frac{N_0}{\tau}$

$N_0 = A\tau = 10^{31} \cdot 365$

Število nukleonov, ki jih vsebuje H_2O (18)

$$2\,{}^1_1H_0 + {}^{16}_8O_8 \rightarrow H_2O$$

$N_{H_2O} = \frac{N_0}{18} = 2{,}02 \cdot 10^{33}$

$m_{H_2O} = N_{H_2O} \cdot \frac{M_{H_2O}}{N_A} \cong \underline{\underline{6000\ t}}$

31) $^{238}_{92}U_{146} \longrightarrow {}^{234}_{90}Th_{144} \longrightarrow {}^{234}_{91}Pa_{143} \longrightarrow {}^{234}_{92}U_{142} \longrightarrow {}^{230}_{90}Th_{140} \longrightarrow {}^{226}_{88}Ra_{138}$

$\boxed{T = 6{,}5 \cdot 10^9 \, l}$ $36 \, d$ $1{,}7 \, m$ $3{,}4 \cdot 10^5 \, l$ $1{,}2 \cdot 10^5 \, l$ $\boxed{2\,352 \, l}$

$$\frac{m_{Ra}}{m_U} = ?$$

$m_U = 1 \, kg$

$$t = 0 \quad \left| \frac{dN_U}{dt} \right| = \left| \frac{dN_{Ra}}{dt} \right|$$

$$\frac{dN_U}{dt} = \frac{N_{0U}}{T_1} \quad ; \quad \frac{dN_{Ra}}{dt} = \frac{N_{0Ra}}{T_2}$$

$$\frac{1}{T_1} N_{0U} = \frac{N_{0Ra}}{T_2}$$

$$\frac{T_2}{T_1} N_{0U} = N_{0Ra} \quad ; \quad m_{Ra} = N_{0Ra} \frac{M_{Ra}}{N_A}$$

$$m_U = N_{0U} \frac{M_U}{N_A}$$

$$\frac{m_{Ra}}{m_U} = \frac{T_2 \cdot \cancel{N_{0U}} \; M_{Ra} \cdot \cancel{N_A}}{T_1 \; \cancel{N_{0U}} \; \cancel{N_A} \; M_U} = \frac{T_2 \; M_{Ra}}{T_1 \cdot M_U} = \frac{3{,}4 \cdot 10^{-7} \, kg}{1 \, kg}$$

34) $$\frac{\left|\frac{dN_1}{dt}\right|_{(I)}}{\left|\frac{dN_2}{dt}\right|_{(T)}} = 2{,}14 \quad ; \quad \frac{\left|\frac{dN_1}{dt}\right|}{\left|\frac{dN_2}{dt}\right|} = 0{,}1$$

$T_1\,(^{133}I) = 20{,}8\,ur\,(\ln 2)^{-1}$

$T_2\,(^{131}I) = 8\,dn\,(\ln 2)^{-1}$

$T_1 = 30\,h$

$T_2 = 11{,}5\,d = 277\,h$

a) =

$$t=0 \quad \frac{\frac{N_1}{T_1}}{\frac{N_2}{T_2}} = \frac{\frac{N_0 \cdot e^{-\frac{t}{T_1}}}{T_1}}{\frac{N_0 \cdot e^{-\frac{t}{T_2}}}{T_2}} = \frac{T_2}{T_1} \cdot e^{t\left(\frac{1}{T_2}-\frac{1}{T_1}\right)} = 2{,}14 = \frac{T_2}{T_1}$$

$$t = t: \quad \frac{T_2}{T_1} \cdot e^{t\left(\frac{1}{T_2}-\frac{1}{T_1}\right)} = 0{,}1$$

$$\frac{1}{e^{t\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}} = 21{,}4$$

$$e^{t\left(\frac{1}{T_2}-\frac{1}{T_1}\right)} = \frac{0{,}1}{2{,}14} \quad / \ln$$

$$t = \frac{\ln\left(\frac{0{,}1}{2{,}14}\right)}{\left(\frac{1}{T_2}-\frac{1}{T_1}\right)} = \underline{\underline{4{,}3\,dni}}$$

b) $T = 7\,dn$

$$T = \frac{\ln \frac{x}{2{,}14}}{\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}$$

$$T\left(\frac{1}{T_2}-\frac{1}{T_1}\right) = \ln\frac{x}{2{,}14} \quad / e$$

$$2{,}14 \cdot \exp\left(T\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right) = x = \underline{\underline{0{,}015}}$$

35) $^{208}Pb^{*} \rightarrow \gamma + Pb$ Prehod iz prvega stanja v osnovno

$E = 0{,}78\ MeV$

$\nu = ?$

$p_{Pb} = \sqrt{2 \cdot T_{Pb} \cdot m} \Rightarrow T_{Pb} = \frac{p_{Pb}^2}{2m}$

1) $W_0(Pb) = W_\gamma + T_{Pb}$

2) $0 = p_\gamma - p_{Pb} = \frac{E_\gamma}{c} + \sqrt{2 T_{Pb} \cdot m_{Pb}} = 0$

$\frac{E_\gamma}{c} - \left((W_0 - E_\gamma) 2 m_{Pb}\right)^{\frac{1}{2}} = 0$

$\frac{E_\gamma}{c} = \left((W_0 - E_\gamma) 2 m_{Pb}\right)^{\frac{1}{2}} \quad / ^2$

$\frac{E_\gamma^2}{c^2} = \left(W_0(Pb) - E_\gamma\right) 2 m_{Pb} \quad / \cdot c^2$

$E_\gamma^2 - W_0(Pb) c^2 \cdot 2 m_{Pb} + 2 m_{Pb} \cdot E_\gamma \cdot c^2 = 0$

$W_0(Pb) \cdot c^2 \cdot 2 m_{Pb} = E_\gamma^2 + 2 m_{Pb} \cdot E_\gamma \cdot c^2$

$W_0 = \frac{E_\gamma^2}{2 m_{Pb} \cdot c^2} + E_\gamma = E_\gamma \left(1 + \frac{E_\gamma}{2 m_p \cdot c^2}\right)$

$\boxed{\nu = \nu_0 \left(1 + \frac{v}{c}\right)} \quad / \cdot h$

$h \cdot \nu = h \nu_0 \left(1 + \frac{v}{c}\right)$

$\frac{v}{c} = \frac{E_\gamma}{2 m_p \cdot c^2}$

$\boxed{v = \frac{E_\gamma}{2 m_p \cdot c} =}$ ✓

$\frac{E_\gamma}{2 m_{Pb} \cdot c^2} =$

$m_{Pb} = 82 \cdot m_H + 127 m_n + W_0(Pb)$

$W_0(Pb) = -1632\ MeV$

36) $\frac{d\sigma}{d\Omega} = \frac{dN(\vartheta)}{t \cdot j_i N_j d\Omega} = \frac{dN(\vartheta)\, r^2}{t \cdot j_i \cdot N_j\, dS_1}$

$E_\gamma = 5{,}1\,MeV$

$\sigma_1 = 756\,fm^2$

$\sigma_2 = 670\,fm^2$

$\sigma_3 = 67{,}5\,fm^2$

$x = 3\,cm$

$\rho = 11{,}4\,\frac{g}{cm^3}$

$M_{Pb} = 207{,}2\,kg$

$j_0(\tilde{j}(x)) = ?$

$j(x) = j_0 \cdot \exp(-n \cdot x(\sigma_1 + \sigma_2 + \sigma_3))$

$\frac{j_0}{j(x)} = \frac{\cancel{j_0}}{\cancel{j_0} \exp\left(-\frac{N_A \cdot \rho}{M}(\sigma_t)\right)} = \exp\left(\frac{N_A \cdot \rho}{M}\sigma_t\right) = $

$=$

$\sigma_t = \sigma_1 + \sigma_2 + \sigma_3 = 1493{,}5\,fm^2$

$= \exp\left(\frac{6{,}1 \cdot 10^{28}}{m^3} \cdot \frac{11400\,kg \cdot 1493{,}5 \cdot 10^{-30}\,m^2 \cdot 0{,}03\,m}{207{,}2\,kg}\right) = 4{,}3$ ✓

---

37) $x = 0{,}23\,mm$

$\eta = 7{,}2\,\% = \frac{j(x)}{j} = 92{,}8\%$ $\boxed{n \cdot \sigma_t = k}$

$\sigma_t = ?$

$\rho_{Cu} = 8{,}65\,\frac{g}{cm^3} = \frac{0{,}00865\,kg}{cm^3} =$

$M = 112{,}4\,kg$

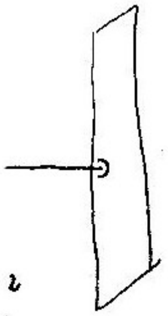
$\boxed{\sigma_t = \left(1 - \frac{j(x)}{j_0}\right) \frac{M}{N_A \cdot \rho \cdot x}} = (1 - 0{,}072) \cdot \frac{M}{N_A \cdot \rho \cdot x} = \underline{\underline{2{,}5 \cdot 10^{-25}\,m^2}}$ ✓ $\Rightarrow 9{,}38 \cdot 10^{-23}$...

$r_j = r_1 \cdot A^{1/3} = 1{,}1\,fm$

$S_j = \pi \cdot r_j^2 = \pi \cdot r_1^2 \cdot A^{\frac{2}{3}}$

$\frac{\sigma_t}{\pi \cdot r_1^2 \cdot A^{\frac{2}{3}}} = \underline{\underline{2800}}$ ✓

38) $\frac{j(x)}{j_0} = 0{,}2$



$\rho = 8{,}65 \frac{g}{cm^3}$

$M = 112{,}4\ kg$

$\sigma_t = 2{,}44 \cdot 10^5\ fm^2$

$(1\ fm)^2 = (1 \cdot 10^{-15})^2 =$

$1\ fm \ldots (1 \cdot 10^{-15}\ m$

$1\ fm^2 \ldots 1 \cdot 10^{-30}\ m$

$(2{,}44 \cdot 10^5) \ldots 2{,}44 \cdot 10^{-25}\ m$

$$\frac{j(x)}{j_0} = e^{-\sigma \cdot \frac{N_A \cdot \rho}{M} \cdot x} \quad / \ln$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\sigma \cdot \frac{N_A \cdot \rho}{M} \cdot x \Rightarrow \boxed{x = -\ln\left(\frac{j(x)}{j_0}\right) \frac{M}{\sigma \cdot N_A \cdot \rho}} =$$

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39) $x = 2\ mm$

$\frac{j(x)}{j_0} = 0{,}11$

$T = 1\ eV$

a) $\sigma_t = ?$

$\rho = 10{,}5 \frac{g}{cm^3}$

$M = 107{,}9\ kg$

b) $\frac{\sigma_t}{S}$

$$\frac{j(x)}{j_0} = \exp\left(-\sigma_t \cdot \frac{N_A \cdot \rho}{M} \cdot x\right) \quad / \ln$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\sigma_t \cdot \frac{N_A \cdot \rho}{M} \cdot x$$

$$\sigma_t = \frac{\ln\left(\frac{j_0}{j(x)}\right) \cdot M}{N_A \cdot \rho \cdot x} = \underline{\underline{1000\ fm^2}}$$

$$S_j = (1{,}1\ fm)^2 A^{\frac{2}{3}} = \underline{\underline{8{,}62 \cdot 10^{-29}}}\ fm^2$$

40) $^{55}Mn + n \longrightarrow {}^{56}Mn \longrightarrow$

$t = 1$ dan

$j = 10^{16}/m^2 s$

$A = 100/s$

$\tau = 2{,}6$ h

$\sigma_{Mn} = 1330$ fm^2

$m_{Mn} = ?$

1) $\frac{dN_1}{dt} = -j\sigma \cdot N_1$ 2) $\frac{dN_2}{dt} = j\sigma N_1 - \frac{N_2}{\tau}$

$$\frac{dN_2}{dt} = j\sigma \cdot N_{10} \cdot e^{-j\cdot\sigma\cdot t} - \frac{N_2}{\tau}$$

Nastavek: $N_2 = A \cdot e^{-j\sigma\cdot t} + B \cdot e^{-\frac{t}{\tau}} \quad / \frac{d}{dt}$

$$-j\sigma \cdot A\, \cancel{e^{-j\sigma \cdot t}} - \cancel{\frac{1}{\tau} B e^{-\frac{t}{\tau}}} = j\cdot\sigma\cdot N_{10} \cdot e^{-j\sigma\cdot t} - \frac{A}{\tau} e^{-j\sigma\cdot t} - \frac{B}{\tau} e^{-\frac{t}{\tau}}$$

$$-j\sigma\cdot A = j\cdot\sigma\cdot N_{10} - \frac{A}{\tau} \quad ; \quad \frac{1}{\tau_1} = j\cdot\sigma = 1{,}33\cdot 10^{-11}$$

$$A(1 - j\sigma) = j\sigma \cdot N_{10} \Rightarrow A = \frac{j\cdot\sigma\cdot N_{10}\cdot\tau}{1 - j\sigma\tau} = \frac{N_{10}\cdot\tau}{\tau_1\left(1 - \frac{\tau}{\tau_1}\right)}$$

$$\boxed{A = \frac{N_{10}\,\tau}{\tau_1 - \tau}}$$

Upoštevamo zač. pogoje $N_1(t=0)$, $N_2(t=0)$

$$N_2 = A + B = 0 \Rightarrow B = -A$$

$$N_2 = A\left(e^{-j\sigma t} - e^{-\frac{t}{\tau}}\right) = \frac{N_{10}\cdot\tau}{\tau_1 - \tau}\left(e^{-j\sigma\cdot t} - e^{-\frac{t}{\tau}}\right)$$

$$\frac{N_2}{\tau} = \frac{N_{10}}{\tau_1 - \tau}\left(e^{-j\sigma\cdot t} - e^{-\frac{t}{\tau}}\right) \Rightarrow \boxed{N_{10} = \frac{\left|\frac{dN_2}{dt}\right|_{t=1dan}(\tau_1 - \tau)}{\left(e^{-t/\tau_1} - e^{-t/\tau}\right)}}$$

$$\boxed{m_{Mn} = \frac{N_{10}\cdot M}{N_A}}$$

41) $^{59}_{27}Co^{N_1} + n \longrightarrow {}^{60}_{27}Co^{N_2}$ $\qquad$ $1\,m = \ldots \quad 1 \cdot 10^{-15}\,m = 1\,fm$

$1 \cdot 10^{-30}\,m^2 = 1\,fm^2$

$m_{Co} = 10\,g \qquad \sigma = 2000\,fm^2$

$t = 100\,h$

$j = 2 \cdot 10^{18}/m^2 s = 2 \cdot 10^{0}/$

$\rho_{Co} = 8{,}9\,\frac{g}{cm^3}$

$t_{\frac{1}{2}} = 5{,}2\,l \Rightarrow \tau = 7{,}5\,l$

$A = ?$

$N_1 = N_{10} \cdot e^{-\frac{t}{\tau}}$

$$\left|\frac{dN_1}{dt}\right| = -j \cdot \sigma \cdot N_1$$

$$\left|\frac{dN_2}{dt}\right| = j \cdot \sigma \cdot N_1 - \frac{N_2}{\tau} = j \cdot \sigma \cdot N_{10} \cdot e^{-j\sigma \cdot t} - \frac{N_2}{\tau}$$

Nastavek: $N_2 = A \cdot e^{-j\sigma \cdot t} + B \cdot e^{-\frac{t}{\tau}}$ lahko zanemarimo

$$-j\sigma \cdot A \cdot e^{-j\sigma \cdot t} - \cancel{\frac{B}{\tau} \cdot e^{-\frac{t}{\tau}}} = j\sigma N_{10}\, e^{-j\sigma \cdot t} - \frac{A}{\tau} \cdot e^{-j\sigma \cdot t} - \cancel{\frac{B}{\tau} e^{-\frac{t}{\tau}}}$$

$$-j\sigma \cdot A = j \cdot \sigma \cdot N_{10} - \frac{A}{\tau} \quad ; \quad j \cdot \sigma = \frac{1}{\tau_1} \Rightarrow \tau_1 = 29\,dni$$

$$A = \frac{j \cdot \sigma \cdot N_{10} \cdot \tau}{(1 - j\sigma\tau)} = \frac{\tau\, N_{10}}{\tau_1\left(1 - \frac{\tau}{\tau_1}\right)} = \left|\frac{\tau \cdot N_{10}}{\tau_1 - \tau}\right| = N_0''$$

lahko zanemarimo

$N_2(t=0) = 0$

$N_2(t_0) = A + B = 0 \Rightarrow A = -B$

$$N_2 = \left(\frac{\tau \cdot N_0}{\tau_1 - \tau}\right)\left(e^{-j\sigma t} - e^{-\frac{t}{\tau}}\right) = \frac{\tau \cdot N_0}{\tau_1 - \tau}\left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau}}\right)$$

$\tau_1 << \tau$

$N_2 = N_0 \cdot e^{-\frac{t}{\tau}} =$