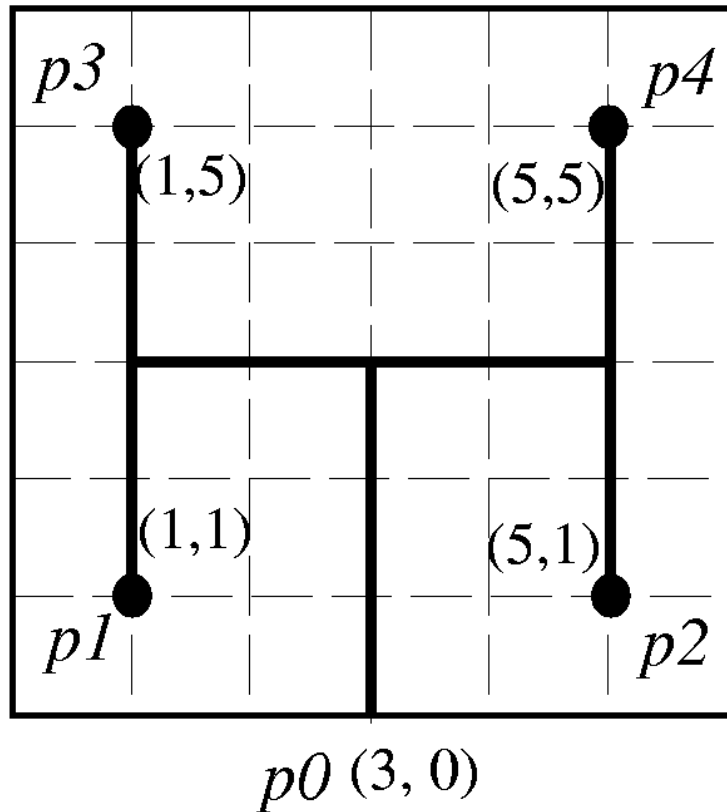
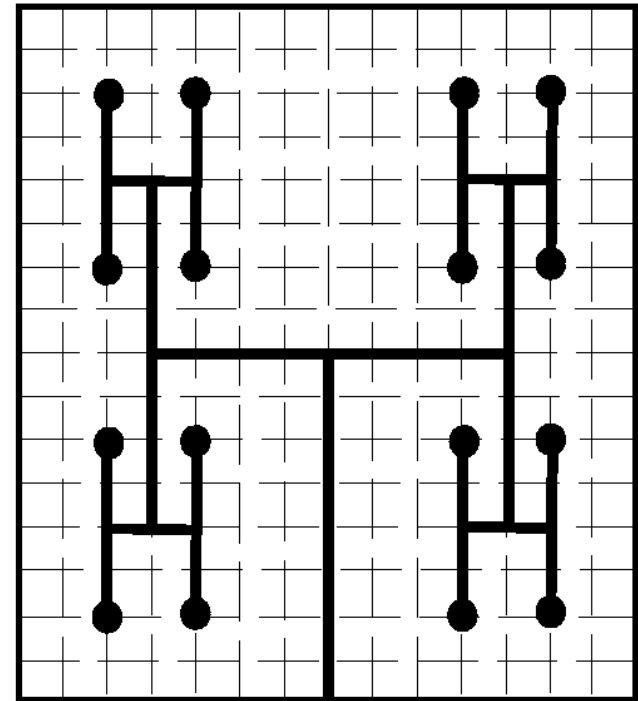


# H-Tree Based Algorithm

- H-tree: Dhar, Franklin, Wang, “Reduction of clock delays in VLSI structure,” *ICCD*, 1984.



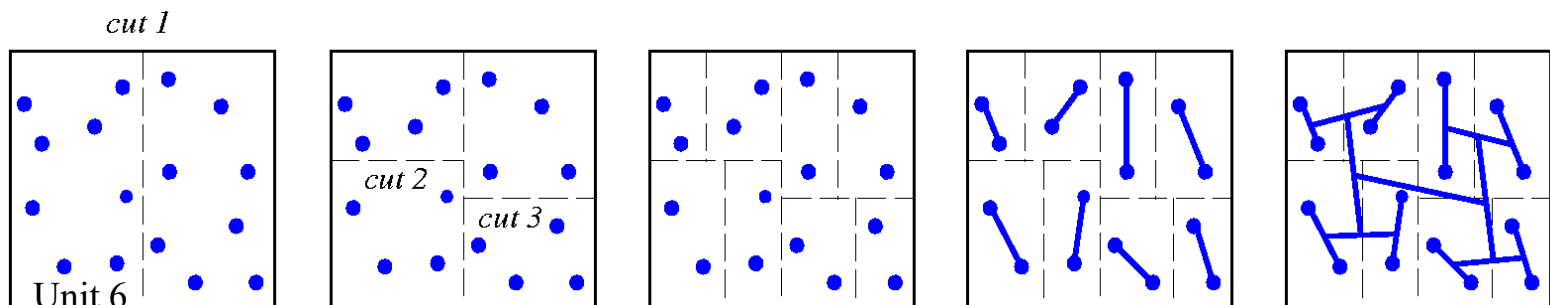
*H-tree over 4 points*



*H-tree over 16 points*

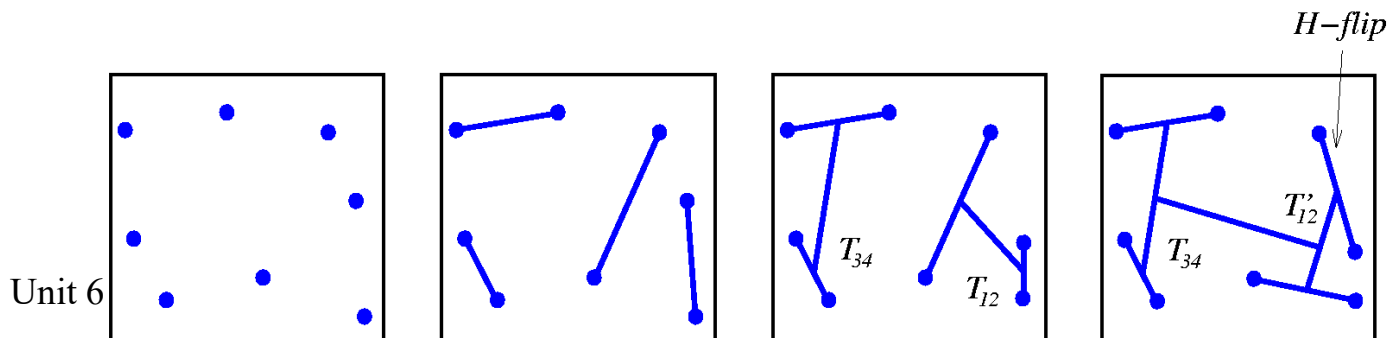
# The MMM Algorithm

- Jackson, Srinivasan, Kuh, “Clock routing for high-performance ICs,” *DAC*, 1990.
- Each clock pin is represented as a point in the region,  $S$ .
- The region is partitioned into two subregions,  $S_L$  and  $S_R$ .
- The center of mass is computed for each subregion.
- The center of mass of the region  $S$  is connected to each of the centers of mass of subregion  $S_L$  and  $S_R$ .
- The subregions  $S_L$  and  $S_R$  are then recursively split in  $Y$ -direction.
- The above steps are repeated with alternate splitting in  $X$ - and  $Y$ -direction.
- Time complexity:  $O(n \log n)$ .



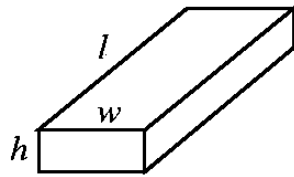
# The Geometric Matching Algorithm

- Kahng, Cong, Robins, “Matching based models for high-performance clock routing,” *IEEE TCAD*, 1993.
- Clock pins are represented as  $n$  nodes in the clock tree ( $n=2^k$ ).
- Each node is a tree itself with clock entry point being node itself.
- The minimum cost matching on  $n$  points yields  $n/2$  segments.
- The clock entry point in each subtree of two nodes is the point on the segment such that length of both sides is the same.
- The above steps are repeated for each segment.
- Apply  $H$ -flipping to further reduce clock skew (and to handle edges intersection).

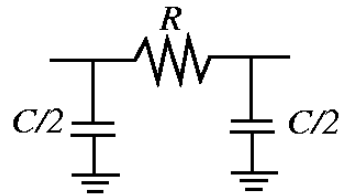


# Delay Calculation

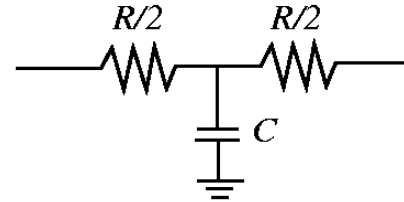
- Need to consider a more accurate delay model for general circuits.
- RC Delay:** The delay caused by wires is due to their capacitance and resistance.  
( $R \propto \frac{l}{wh}$ ;  $C \propto wl$ )
- Lumped circuit approximations for distributed RC lines:  $\pi$ -model,  $T$ -model,  $L$ -model.



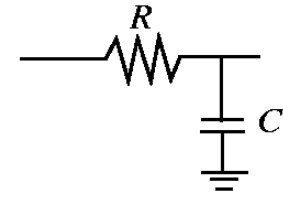
*a lumped wire*



$\pi$ -model

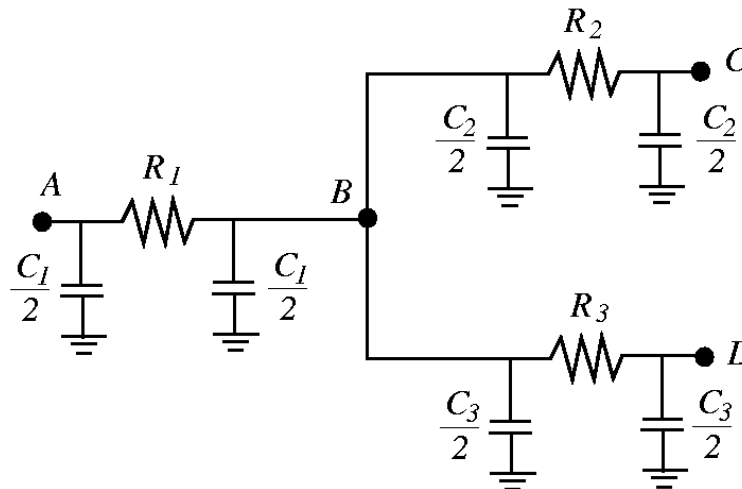


$T$ -model



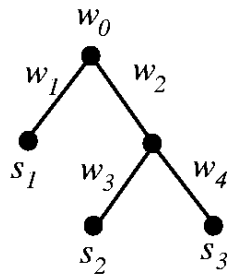
$L$ -model

- RC delay:  $D_{AB} = R_1(\frac{C_1}{2} + C_2 + C_3)$ ;  $B$  to  $C$ :  $D_{BC} = R_2(\frac{C_2}{2})$ ;  $B$  to  $D$ :  $D_{BD} = R_3(\frac{C_3}{2})$

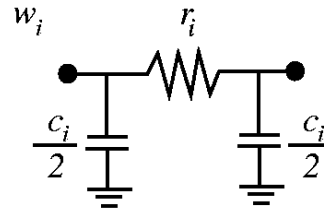


# Delay Calculation for a Clock Tree

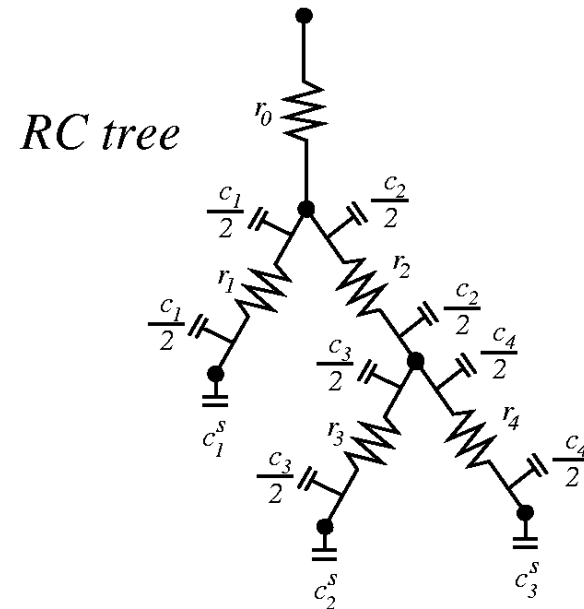
$$t_{03} = r_0(c_1 + c_2 + c_3 + c_4 + c_1^s + c_2^s + c_3^s) + r_2(\frac{c_2}{2} + c_3 + c_4 + c_2^s + c_3^s) + r_4(\frac{c_4}{2} + c_3^s).$$



*clock tree*



*delay model*



*RC tree*

# Exact Zero Skew Algorithm

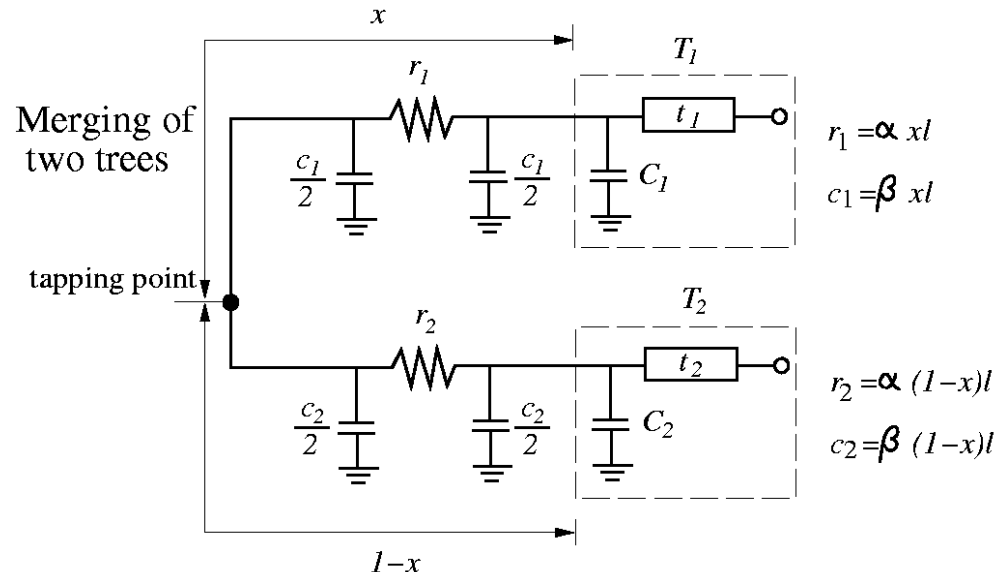
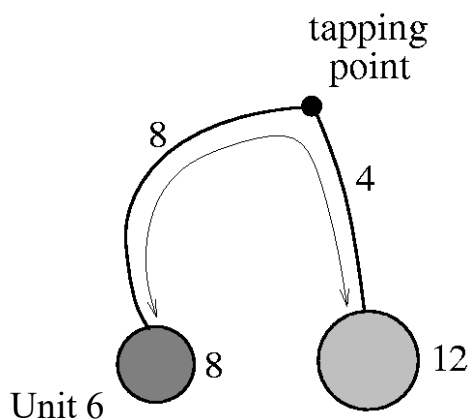
- Tsay, “Exact zero skew algorithm,” *ICCAD*, 1991.
- To ensure the delay from the **tapping point** to leaf nodes of subtrees  $T_1$  and  $T_2$  being equal, it requires that

$$r_1\left(\frac{c_1}{2} + C_1\right) + t_1 = r_2\left(\frac{c_2}{2} + C_2\right) + t_2$$

- Solving the above equation, we have

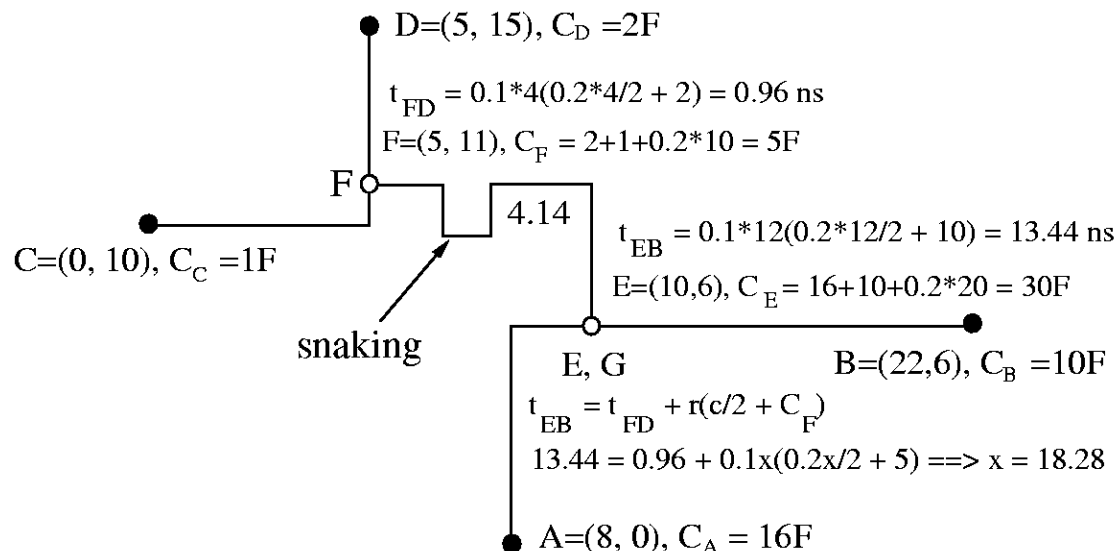
$$x = \frac{(t_2 - t_1) + \alpha l(C_2 + \frac{\beta l}{2})}{\alpha l(\beta l + C_1 + C_2)}$$

- $\alpha$  and  $\beta$  are the per unit values of resistance and capacitance,  $l$  the length of the interconnecting wire,  $r_1 = \alpha x l$ ,  $c_1 = \beta x l$ ,  $r_2 = \alpha (1-x) l$ ,  $c_2 = \beta (1-x) l$ .



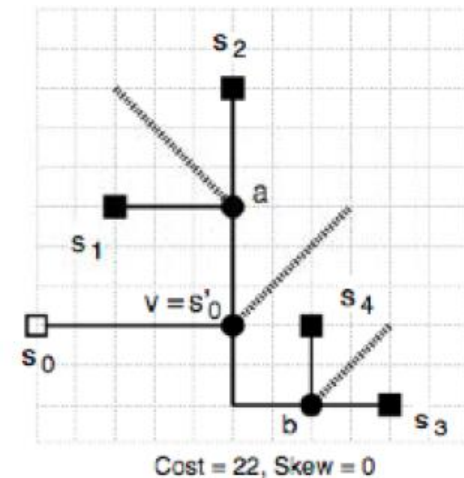
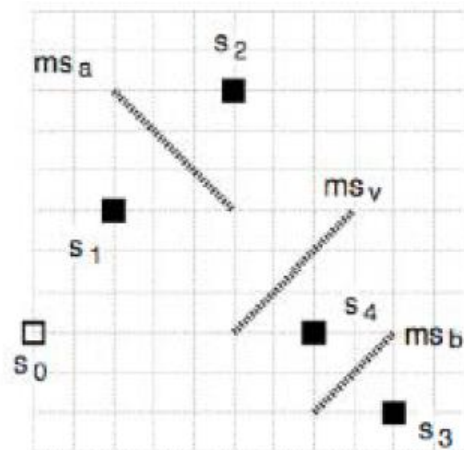
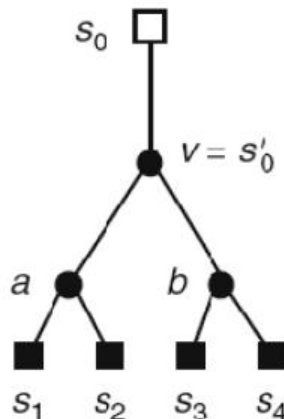
# Zero-Skew Computation

- **Balance delays:**  $r_1(\frac{c_1}{2} + C_1) + t_1 = r_2(\frac{c_2}{2} + C_2) + t_2$ .
- **Compute tapping points:**  $x = \frac{(t_2 - t_1) + \alpha l(C_2 + \frac{\beta l}{2})}{\alpha l(\beta l + C_1 + C_2)}$ ,  $\alpha(\beta)$ : per unit values of resistance (capacitance);  $l$ : length of the wire;  $r_1 = \alpha x l$ ,  $c_1 = \beta x l$ ;  $r_2 = \alpha(1-x)l$ ,  $c_2 = \beta(1-x)l$ .
- If  $x \notin [0,1]$ , we need **snaking** to find the tapping point.
- Exp:  $\alpha = 0.1 \Omega/unit$ ,  $\beta = 0.2 F/unit$ . (Find tapping points  $E$  for  $A$  and  $B$ ,  $F$  for  $C$  and  $D$ , and  $G$  for  $E$  and  $F$ .)



# Deferred Merge Embedding (DME)

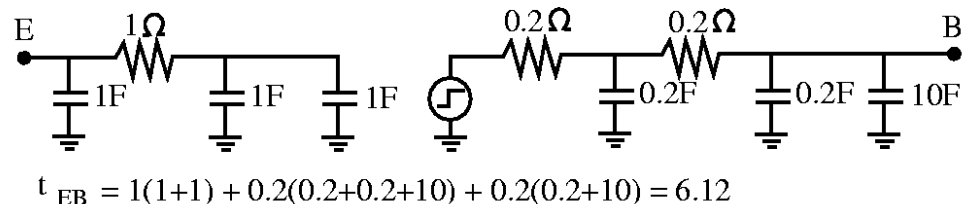
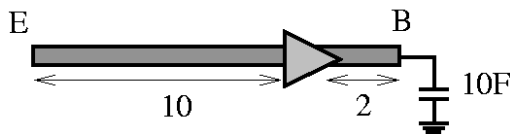
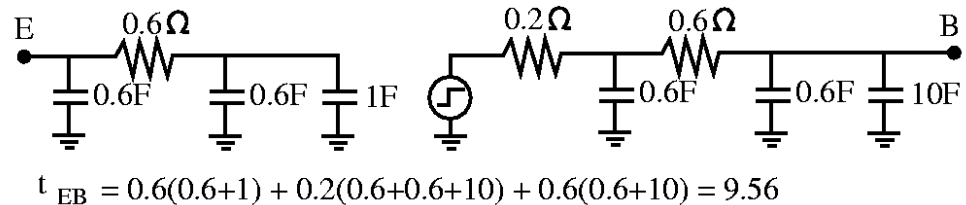
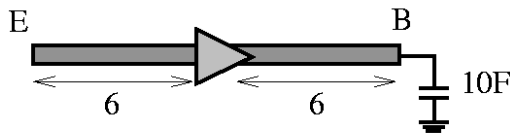
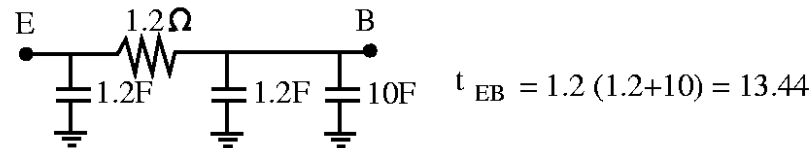
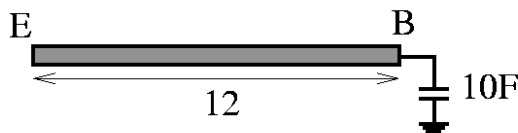
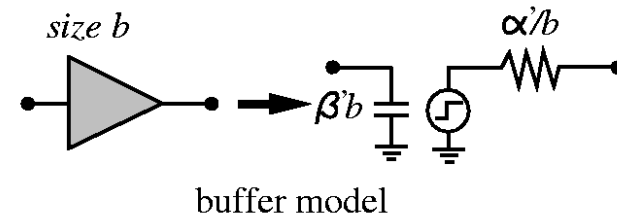
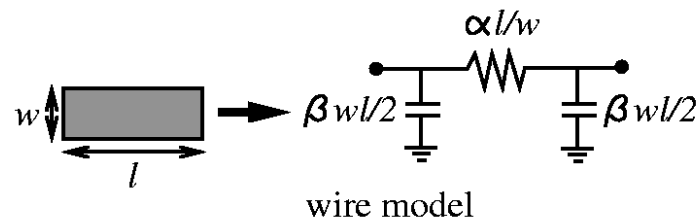
- Boese & Kahng, ASICON-92; Chao, Hsu, & Ho, DAC-92; Edahiro, NEC R&D, 1991
- Two stages
  - Bottom up: build a **segment** for potential tapping points
  - Top down: determine exact tapping points





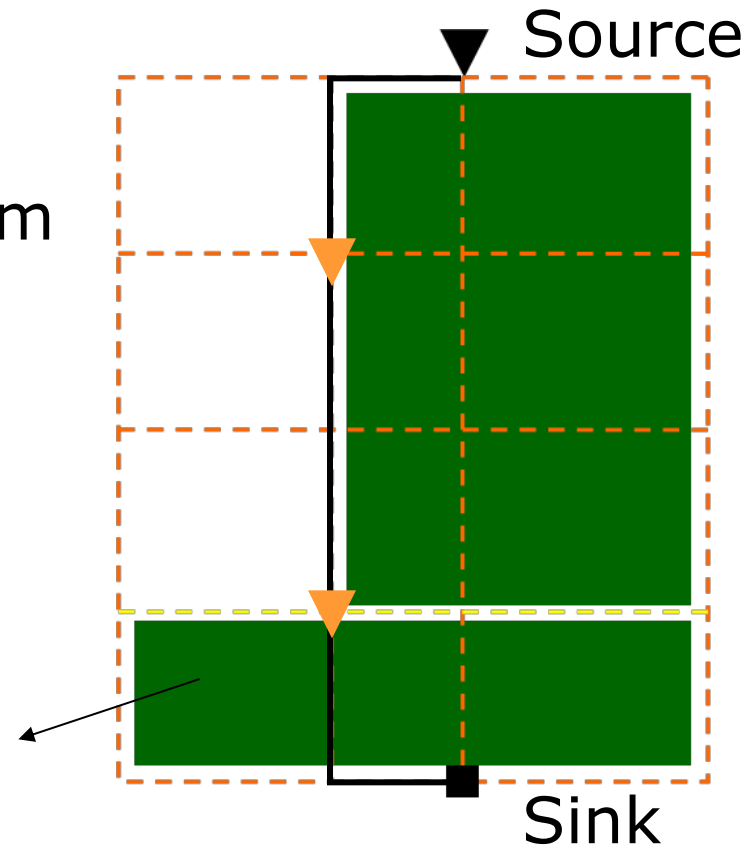
# Delay Computation for Buffered Wires

- Wire:  $\alpha=0.1\Omega$  /unit size,  $\beta=0.2F$ /unit size; buffer:  $\alpha'=0.2\Omega$ /unit size,  $\beta'=1F$ /unit size; unit-sized wire and buffer.



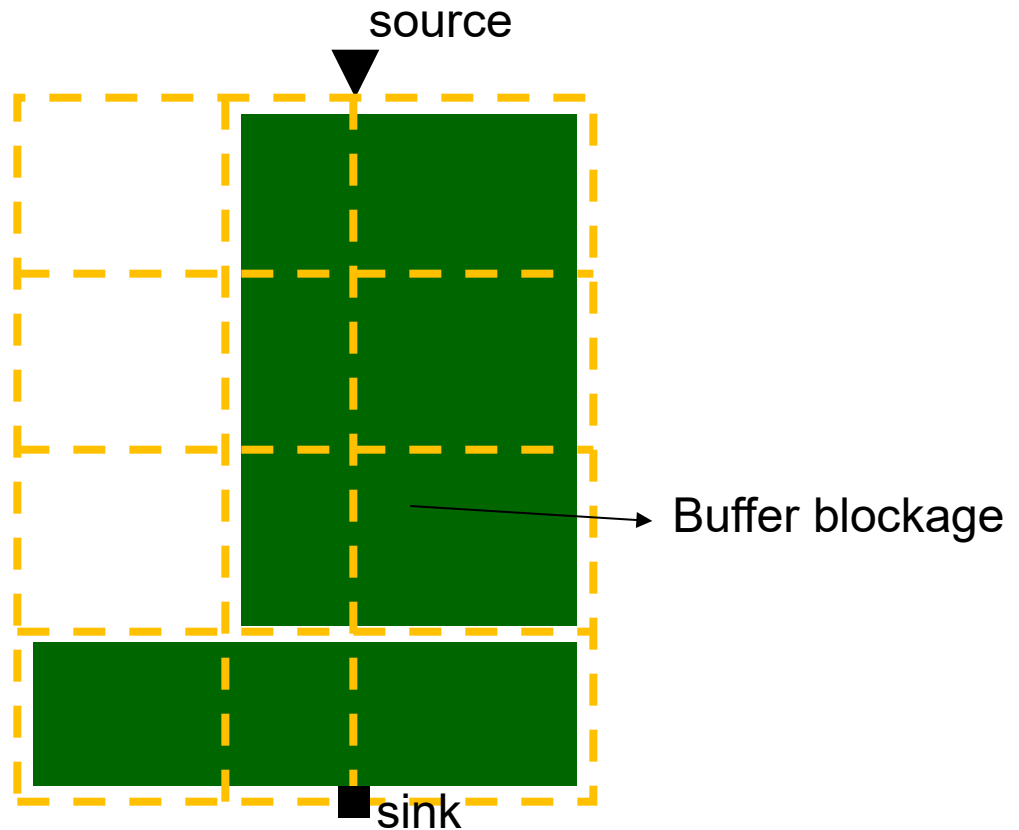
# Buffered Path Construction

Goal: Find a minimum-delay buffered path from source to sink with blockage avoidance

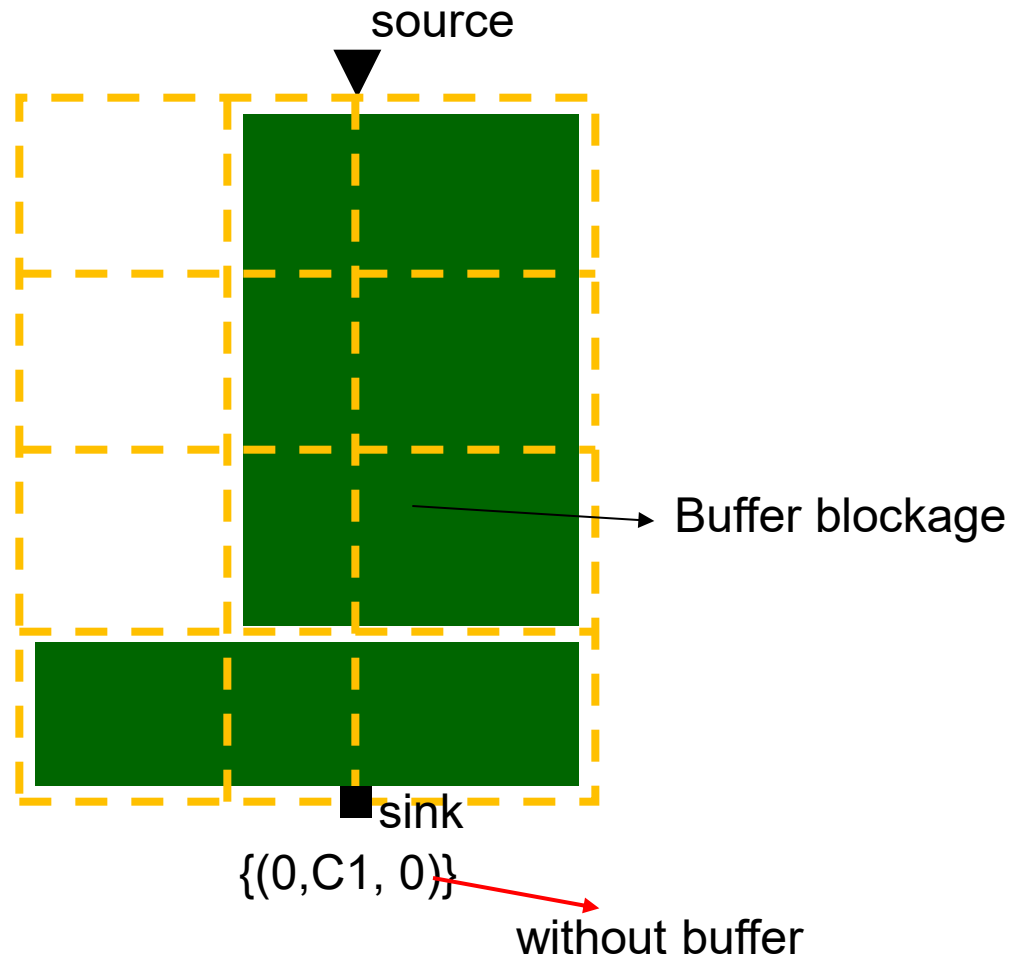


# Fast Path Algorithm

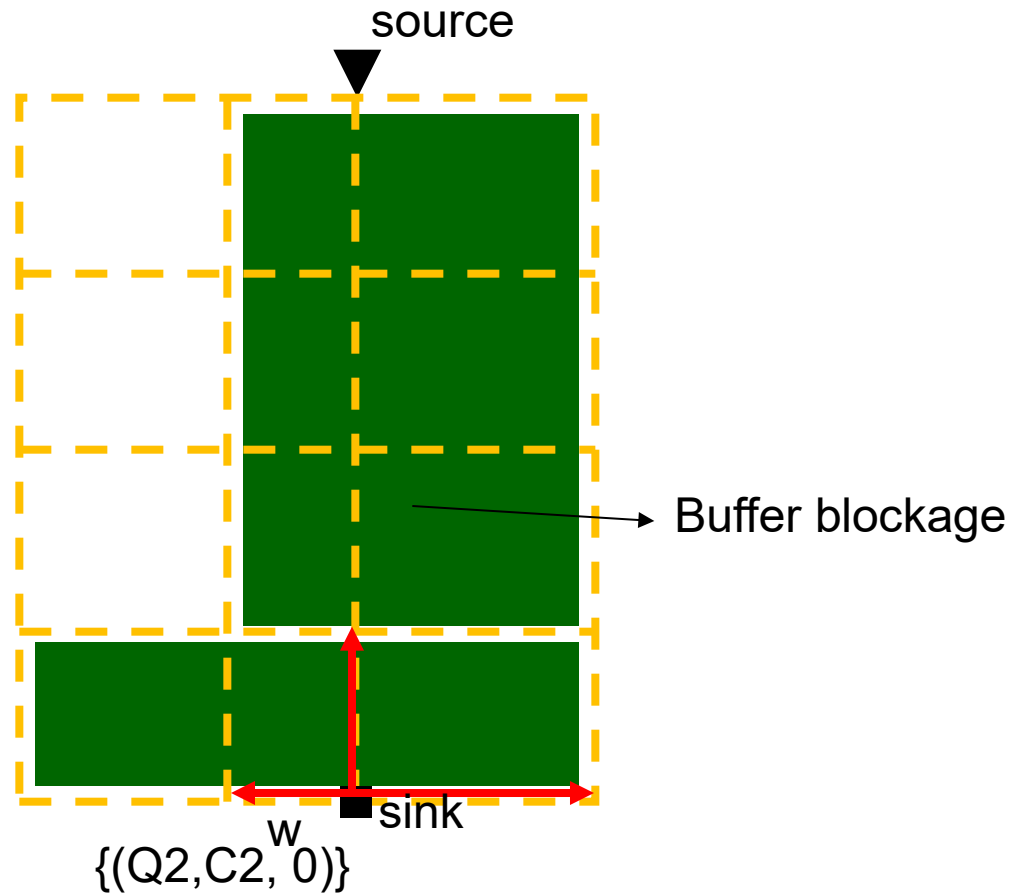
Proposed by  
Zhou *et al.*, DAC  
1999



# Fast Path Algorithm



# Fast Path Algorithm



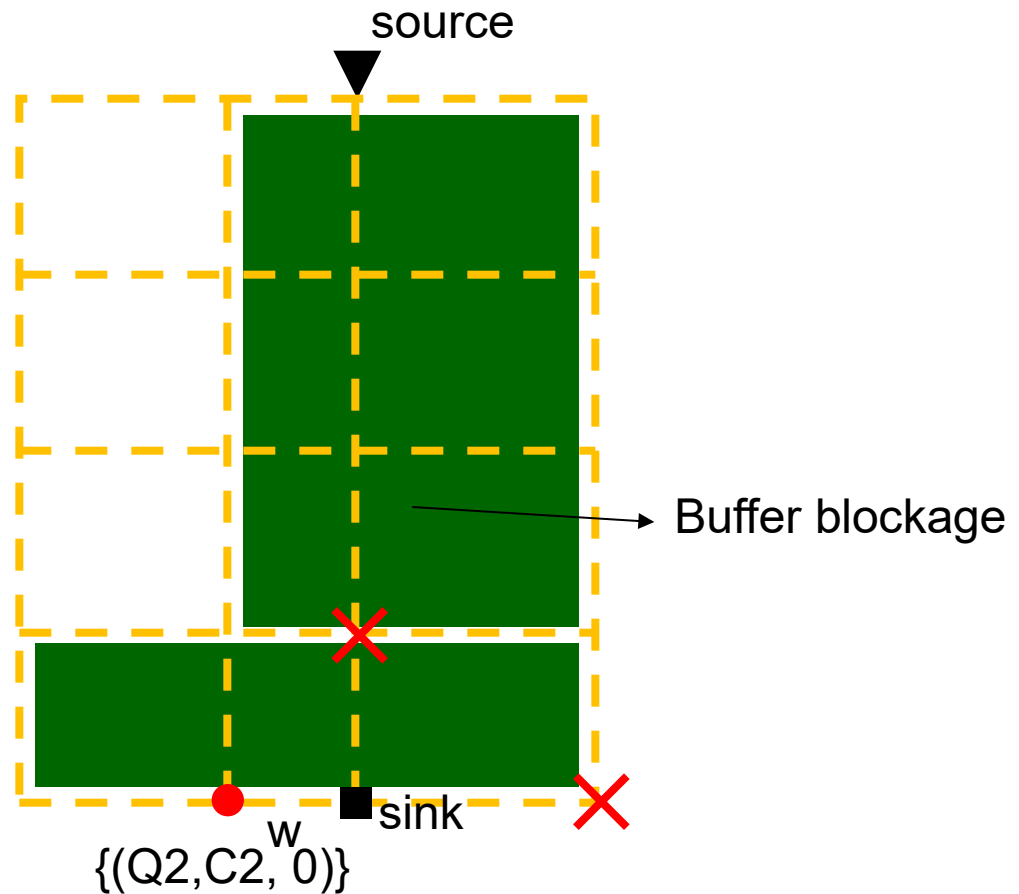
$$Q2 = Q1 + R (C/2 + C1)$$

$$C2 = C + C1$$

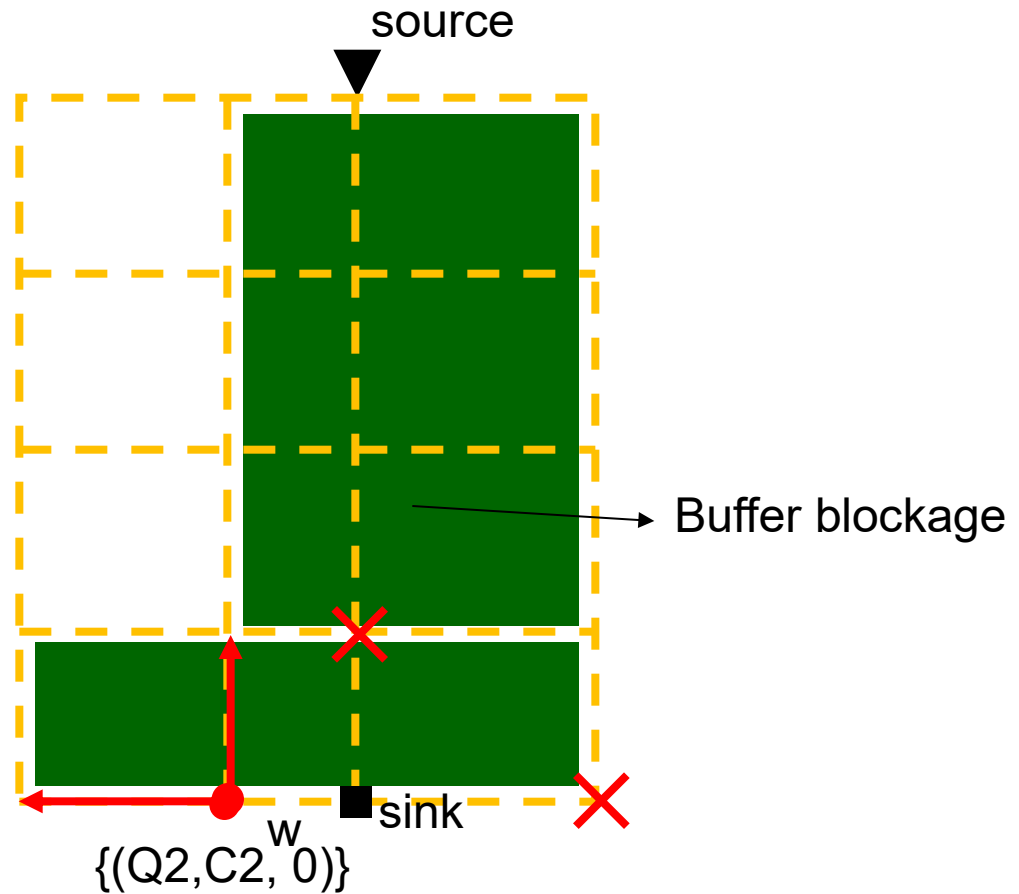
C is capacitance of wire w,

Unit 6 R is resistance of w

# Fast Path Algorithm



# Fast Path Algorithm



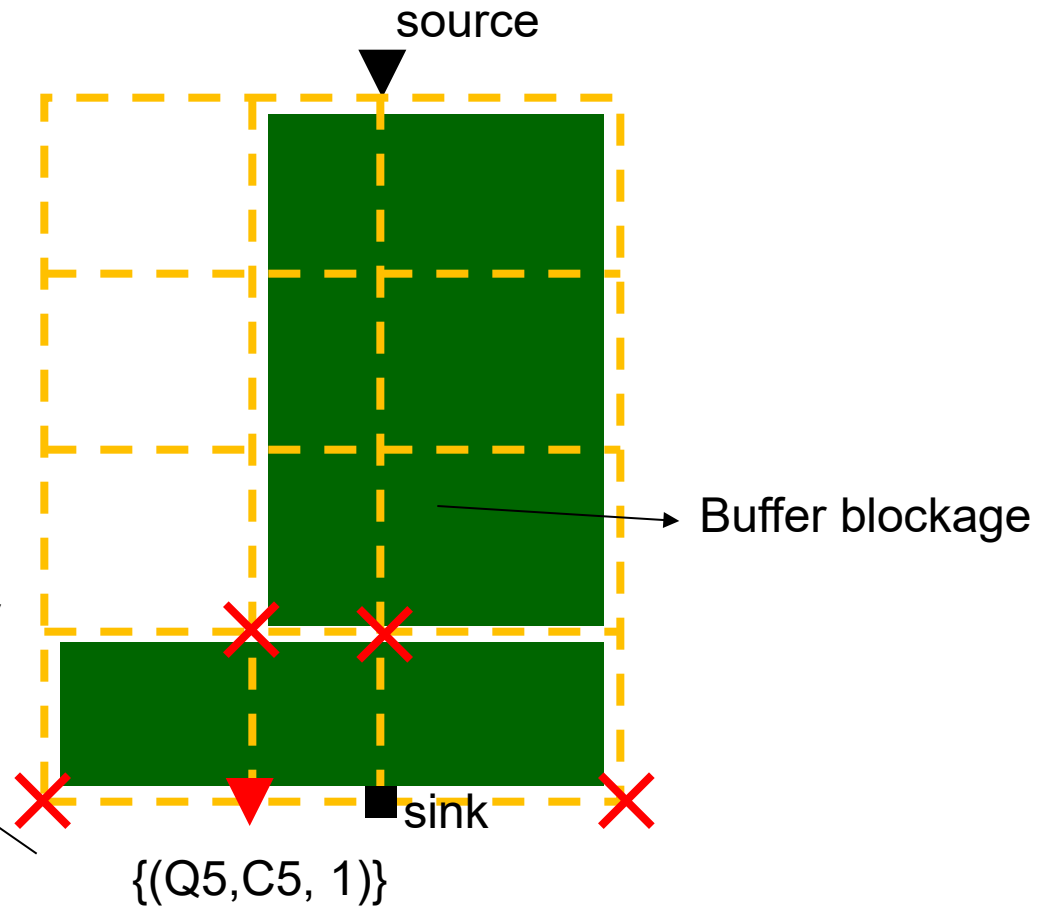
$$Q2 = Q1 + R (C/2 + C1)$$

$$C2 = C + C1$$

C is capacitance of wire w,

Unit 6 R is resistance of w

# Fast Path Algorithm


$$Q_5 = Q_2 + D_b + R_b \cdot C_2,$$

$$C_5 = C_b, \text{ where}$$

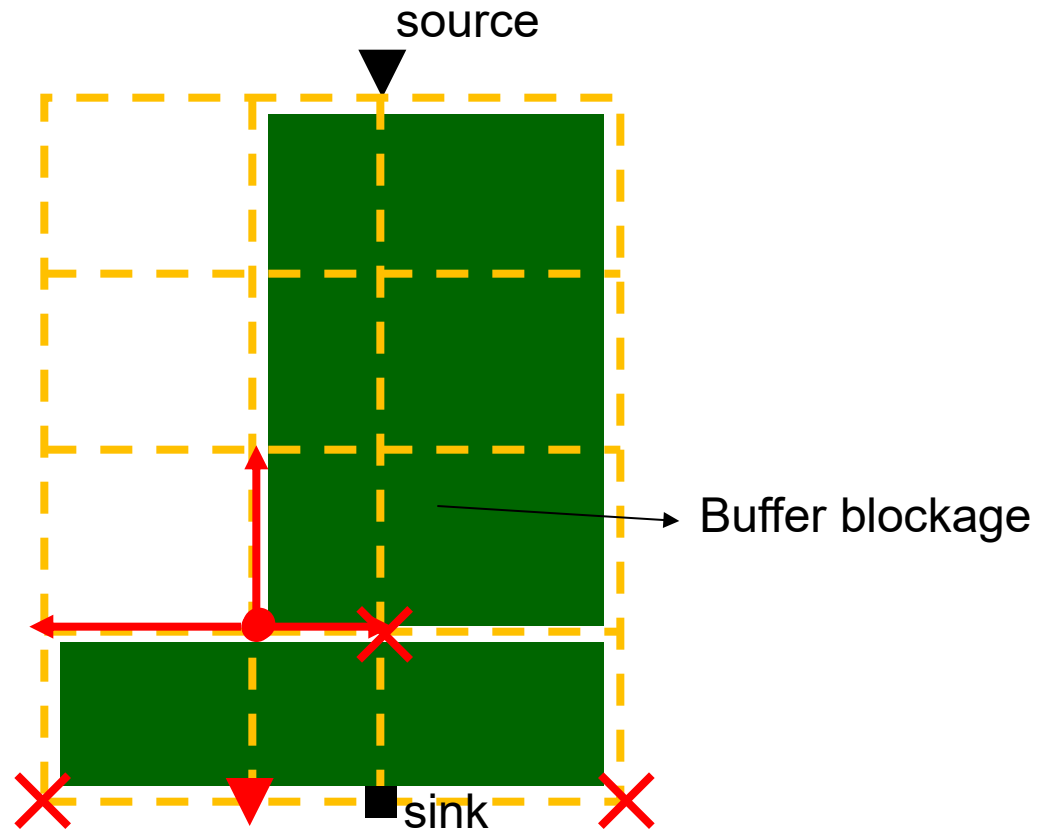
Db is intrinsic delay of buffer

$C_b$  is input capacitance of buffer

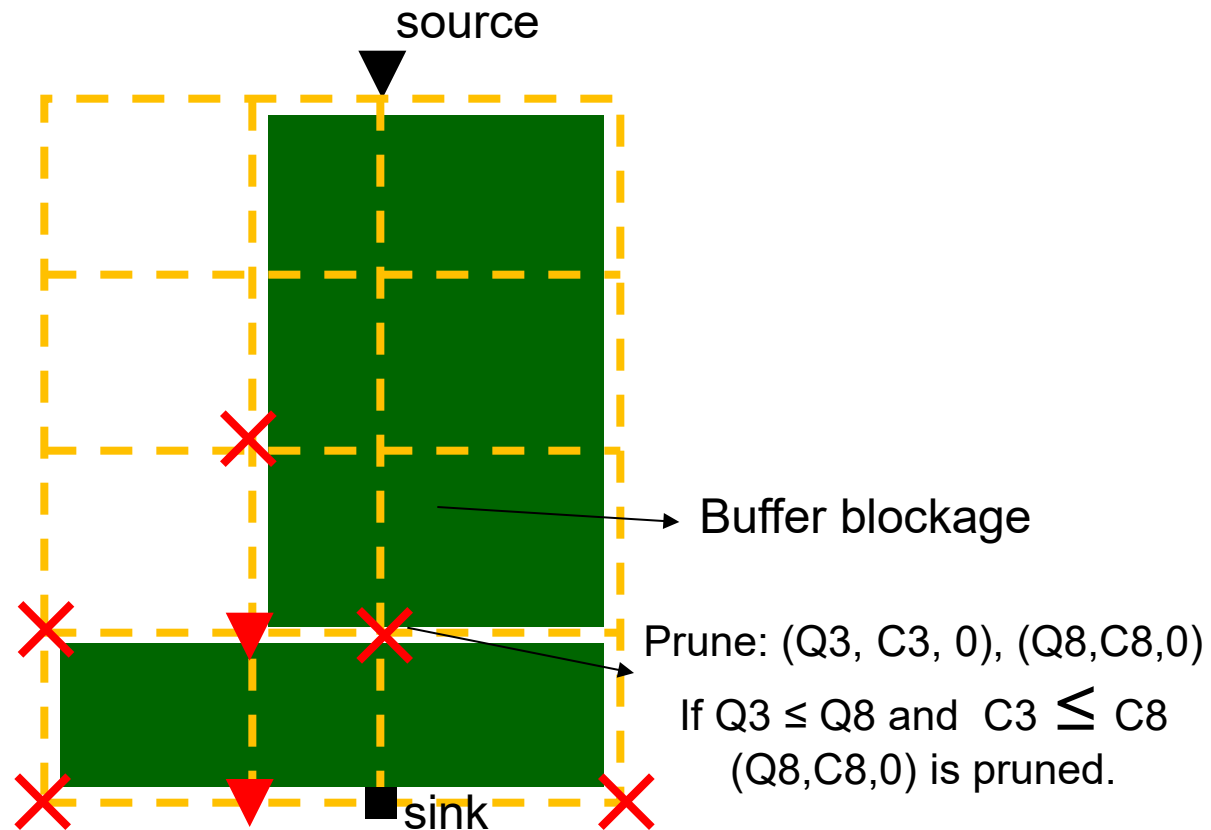
$R_b$  is output resistance of buffer



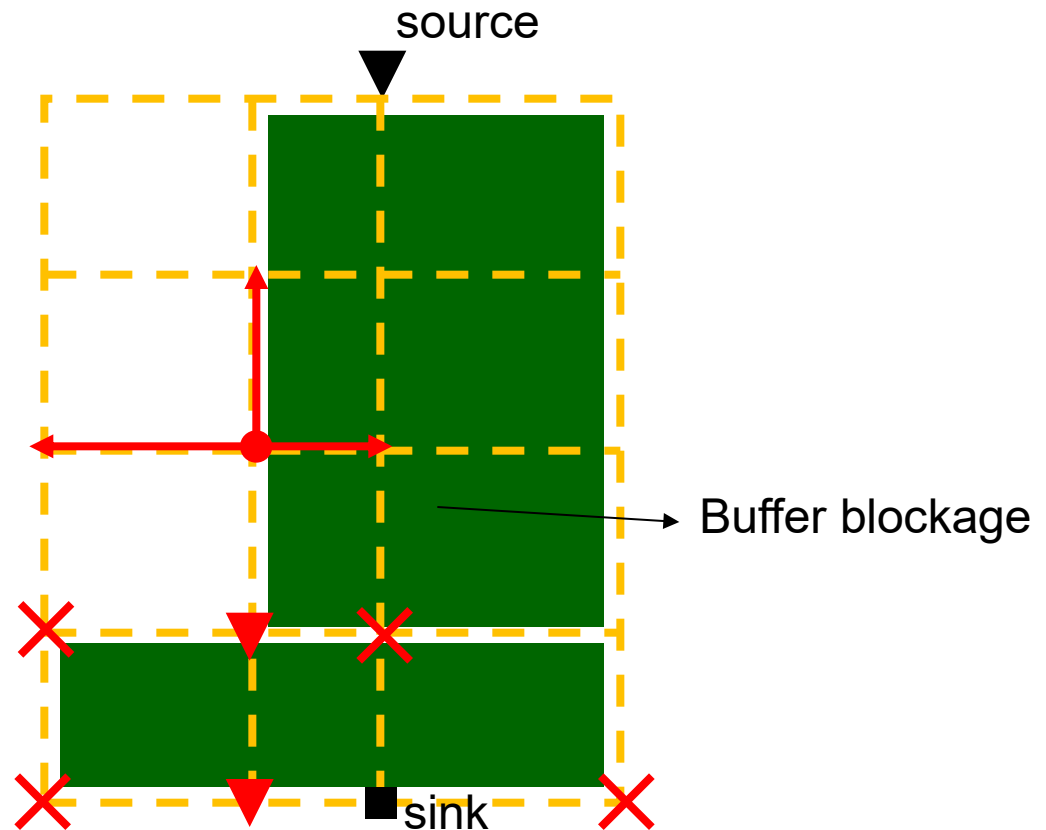
# Fast Path Algorithm



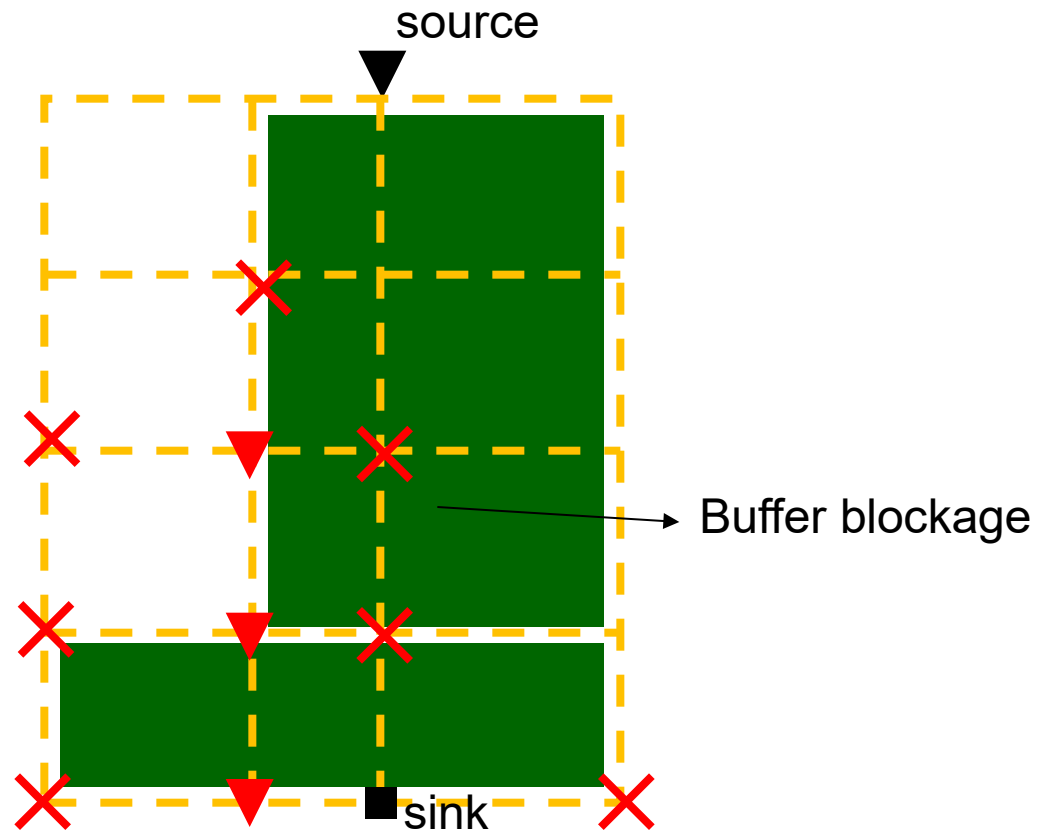
# Fast Path Algorithm



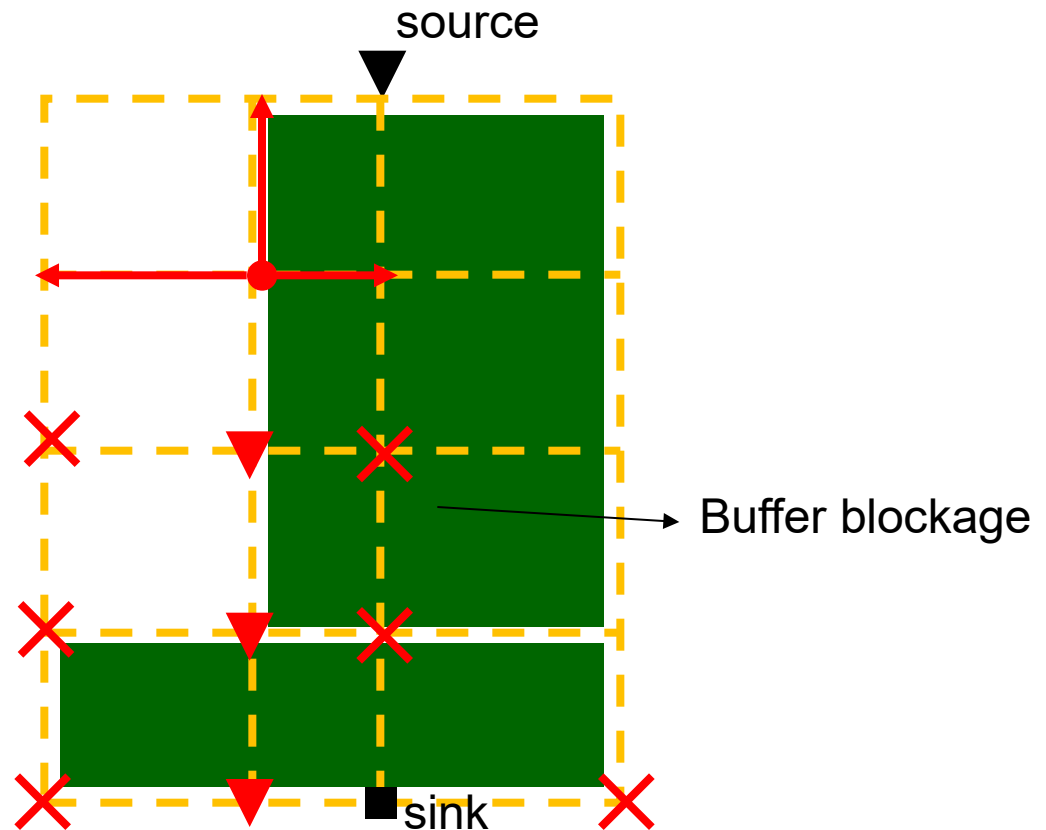
# Fast Path Algorithm



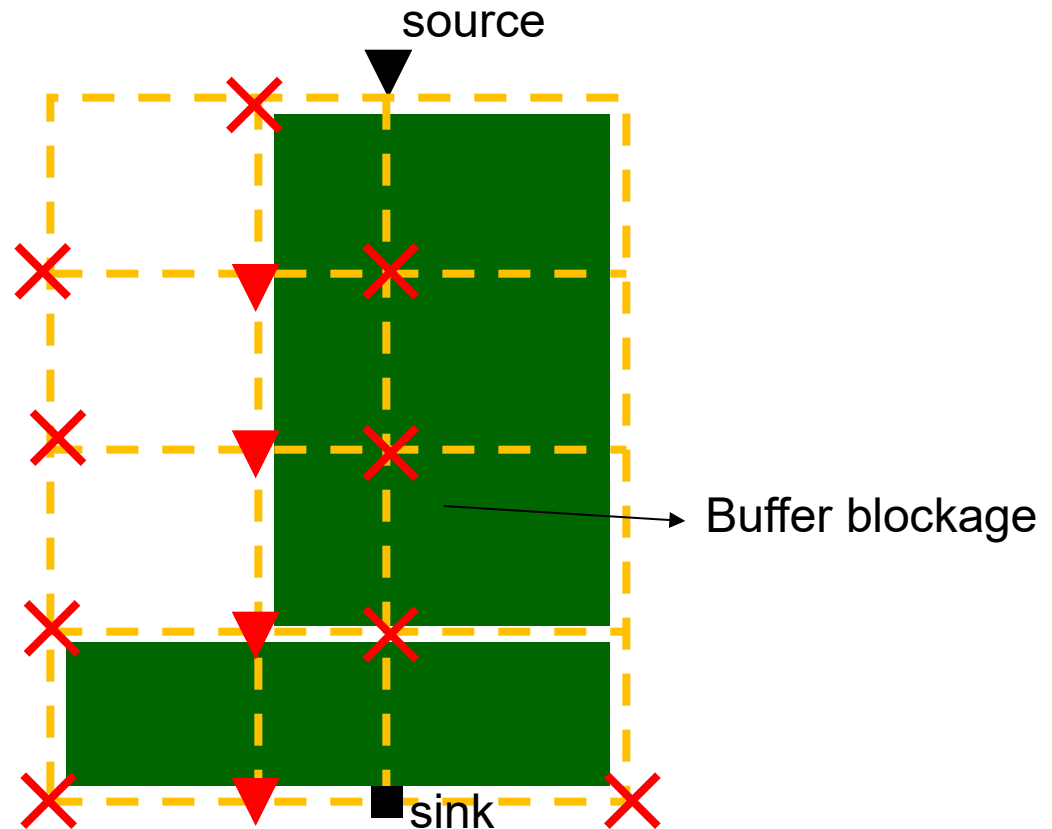
# Fast Path Algorithm



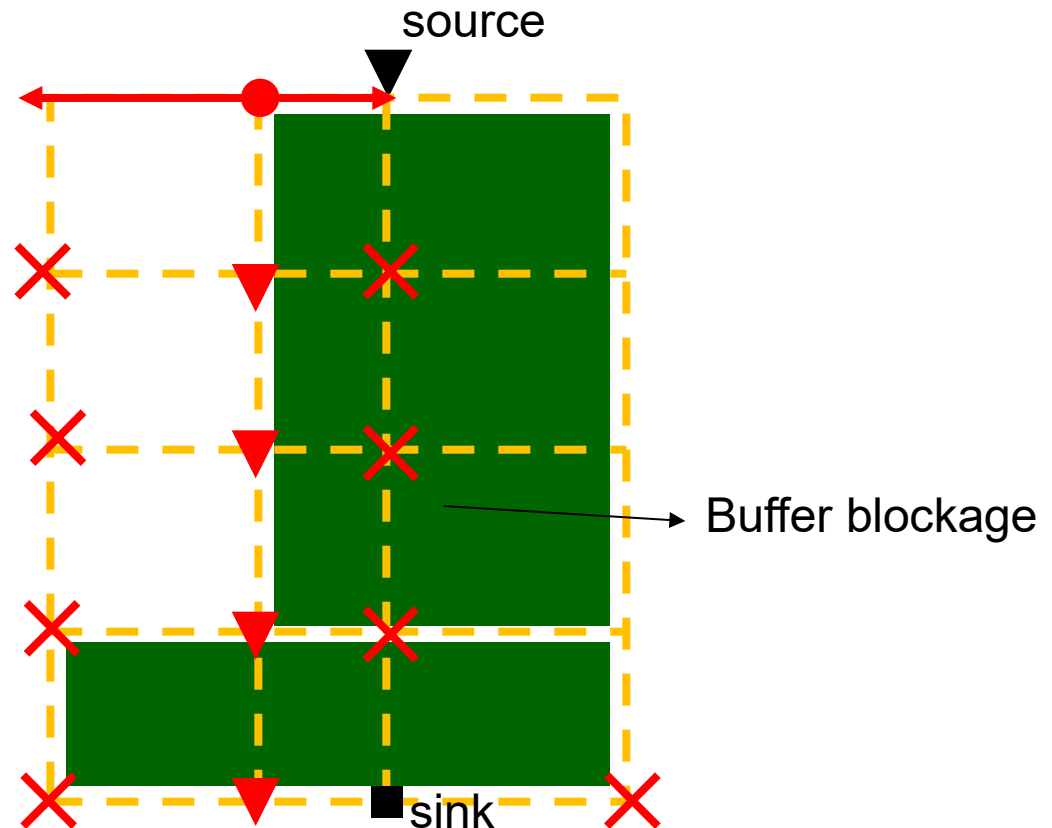
# Fast Path Algorithm



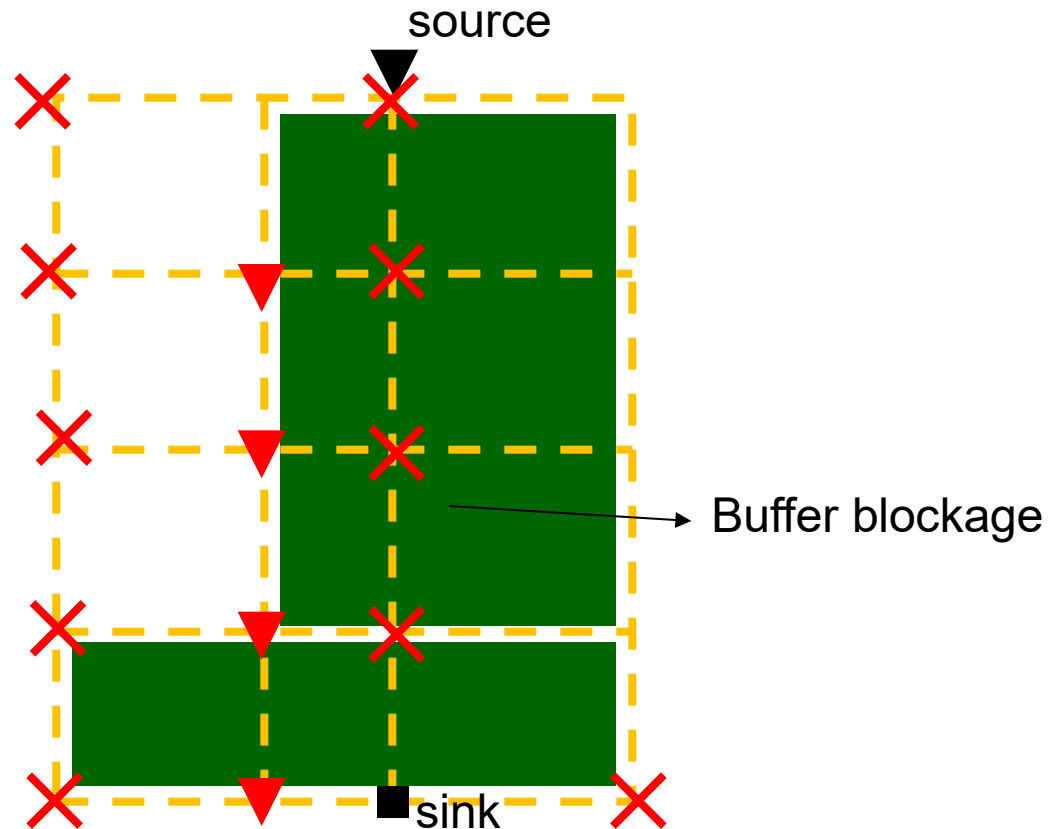
# Fast Path Algorithm



# Fast Path Algorithm

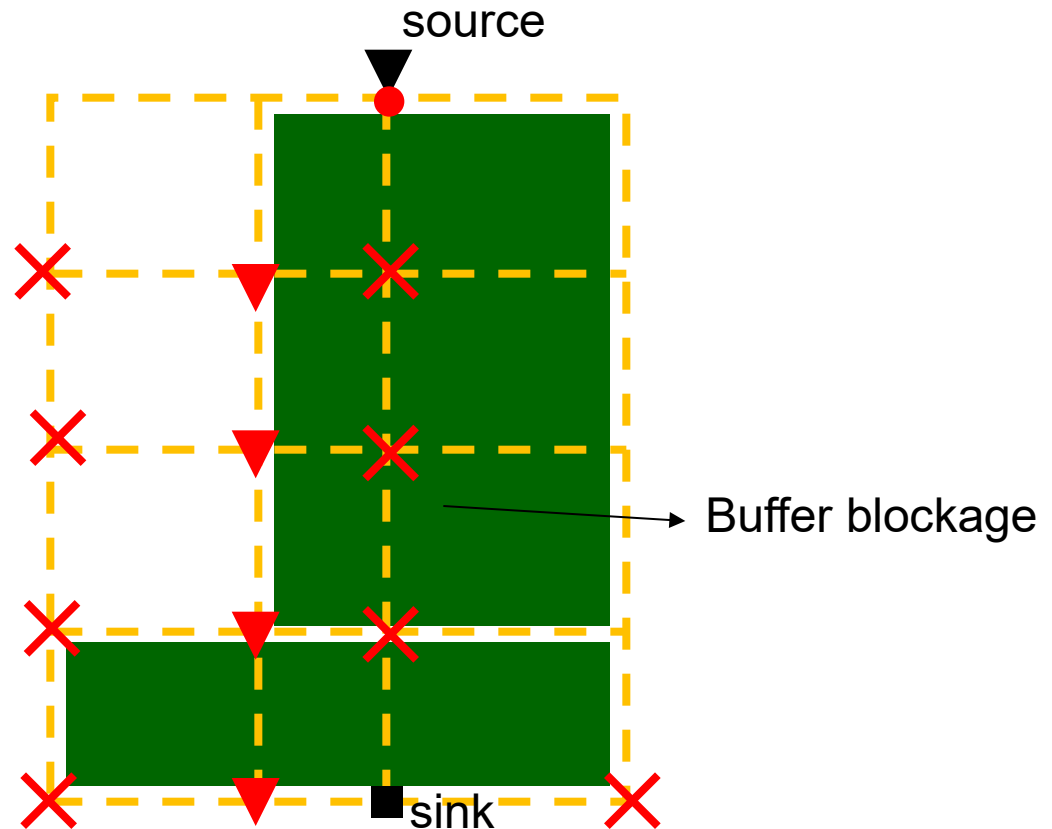


# Fast Path Algorithm





# Fast Path Algorithm



# Fast Path Algorithm

