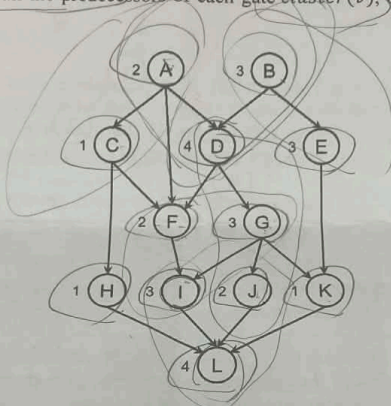# CS6135 VLSI Physical Design Automation
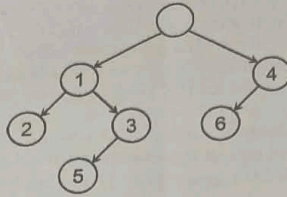## Final Exam: 15:30 - 18:30, May 27, 2025

1. (5 points) Vertex a and vertex b are two vertices in an edge-weighted complete graph, where the weight of each edge is a non-negative integer. Suppose that the vertex set of this graph is partitioned into two subsets, and vertices a and b are not in the same subset. Assume that the respective external and internal costs of vertex a are 18 and 12, and the respective external and internal costs of vertex b are 7 and 9. Can the cut cost be reduced by swapping vertices a and b? Justify your answer.

2. (15 points) Consider the circuit shown below. Assume the area of each gate is 1 unit, the area constraint of each cluster is 3 units, and the interconnection delay between two clusters is 4 units. The gate delay is given next to each gate. Show your work by applying the clustering algorithm discussed in class to the circuit below. You should provide the delay at each gate $l(v)$, show the optimal clustering of all the predecessors of each gate $cluster(v)$, and plot the final clustering result.
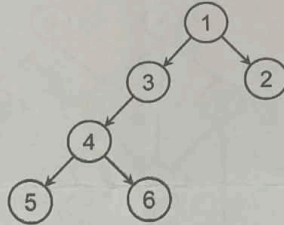

3. (5 points) Show your work by performing Edge Coarsening, which visits the gates $a, b, ..., h$ in the alphabetical order and breaks ties in the alphabetical order, on the following netlist: $n_1 = \{a, b, d, e\}$, $n_2 = \{b, d, f\}$, $n_3 = \{a, e, g\}$, $n_4 = \{c, e, f, h\}$, $n_5 = \{c, g\}$, $n_6 = \{a, d, f, h\}$, where the weight of each net n_i ($i = 1, 2, ..., 6$) is 1. You should provide the cluster results of each gate.

4. Consider a set of modules in the following table. **For each subproblem, you should plot the final floorplanning result.**

Module	Width	Height
1	1	5
2	2	1
3	4	2
4	3	3
5	5	4
6	2	3

(a) (7 points) Assume that each module cannot be rotated. Show your work to find the optimal packing for the sequence-pair (416235, 641352).

(b) (4 points) Assume that each module cannot be rotated. Show your work to find the placement for the following horizontal O-tree.

(c) (4 points) Assume that each module cannot be rotated. Show your work to find the placement for the following B*-tree.

(d) (10 points) Assume that each module can be rotated by 90-degree. Perform the Stockmeyer algorithm to find a minimum-area floorplan for the Polish expression 132VH4H5V6V.

5. An analytical placement approach usually models a k-pin net as one or more 2-pin nets, depending on the value of k. For each of the following net models, give the number of 2-pin nets and the number of extra x and y variables introduced for a k-pin net.
   (a) (2 points) Clique
   (b) (2 points) Star
   (c) (4 points) Hybrid
   (d) (2 points) BoundingBox

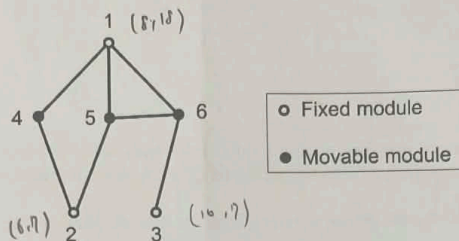6. The log-sum-exponential function of n numbers $z_1, z_2, ..., z_n$ is defined as:

$$LSE_\alpha(z_1, z_2, ..., z_n) = \alpha \times \left(\ln \left(\sum_{i=1}^{n} e^{\frac{z_i}{\alpha}} \right) \right),$$

where α is a user-specified parameter. $LSE_\alpha(z_1, z_2, ..., z_n)$ is an approximation of the maximum function $\max(z_1, z_2, ..., z_n)$ and α controls the accuracy of the approximation. The error function is defined as:

$$err_\alpha(z_1, z_2, ..., z_n) = LSE_\alpha(z_1, z_2, ..., z_n) - \max(z_1, z_2, ..., z_n).$$
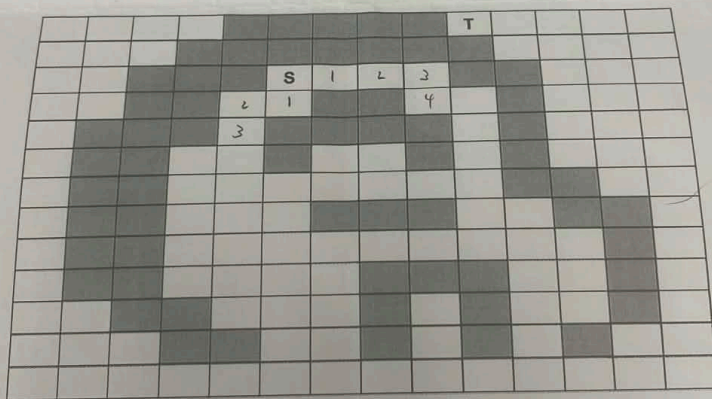
Assume that $\max(z_1, z_2, ..., z_n) = z_1$ and α is a non-negative real number. Prove that

(a) (5 points) $\text{err}_\alpha(z_1, z_2, ..., z_n) \geq 0$.

(b) (5 points) $\text{err}_\alpha(z_1, z_2, ..., z_n) \leq \alpha \times \ln n$.

7. (10 points) Consider a circuit with movable and fixed modules as represented by the graph below. In the graph, each vertex denotes a module, while each edge denotes a two-pin net and is associated with a weight 1.
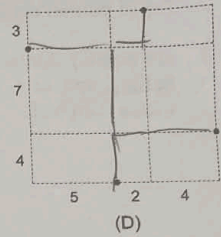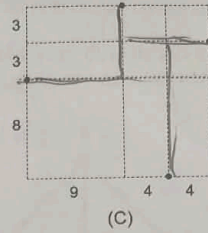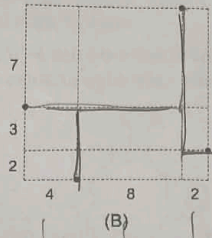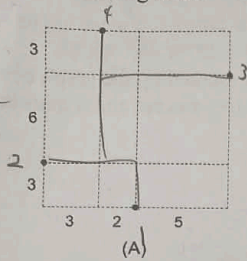

○ Fixed module

● Movable module

For the three fixed modules, module 1 is at $(8, 18)$, module 2 is at $(6, 7)$, and module 3 is at $(10, 7)$. Determine the locations of the three movable modules such that the total weighted quadratic wirelength of the circuit is minimized. **To answer this question, you only need to show your work for getting the y-coordinates of the three movable modules.**

8. Consider the following routing instance with shaded blockages.

(a) (5 points) Show your work for finding a shortest path from S to T using the Lee algorithm.

(b) (5 points) Show your work for finding a shortest path from S to T using the A*-search algorithm which defines the cost function of each grid cell x to be $f(x) = g(x) + h(x)$, where $g(x)$ is the label of x (i.e., the label given by the Lee algorithm) and h(x) is the Manhattan distance between x and T. Break the tie by picking the grid cell toward T.

9. Consider four nets shown below, each containing four pins. Use the FLUTE algorithm to complete the following tasks.

s_φ

$s_j = 4$

$\lceil 2$

$l_2 = 1$

$l_1 = 3$

(A) (B) (C) (D)

(a) (4 points) Write down the position sequence for each net.
(b) (2 points) Group the nets with the same position sequence. For instance, Group 1: (A), (B), (C); Group 2: (D).
(c) (4 points) Find the minimum wirelength of a rectilinear Steiner tree for each net.