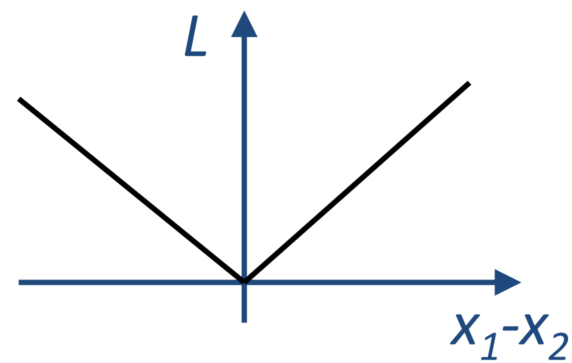
# Techniques for Analytical Placement

- **Quadratic** techniques
  - Transformed into a sequence of convex quadratic programs
    - convex quadratic program: a mathematical program with a convex and quadratic objective function and linear constraints
- **Non-quadratic** techniques
  - Transformed into a single general mathematical program

# Quadratic Wirelength

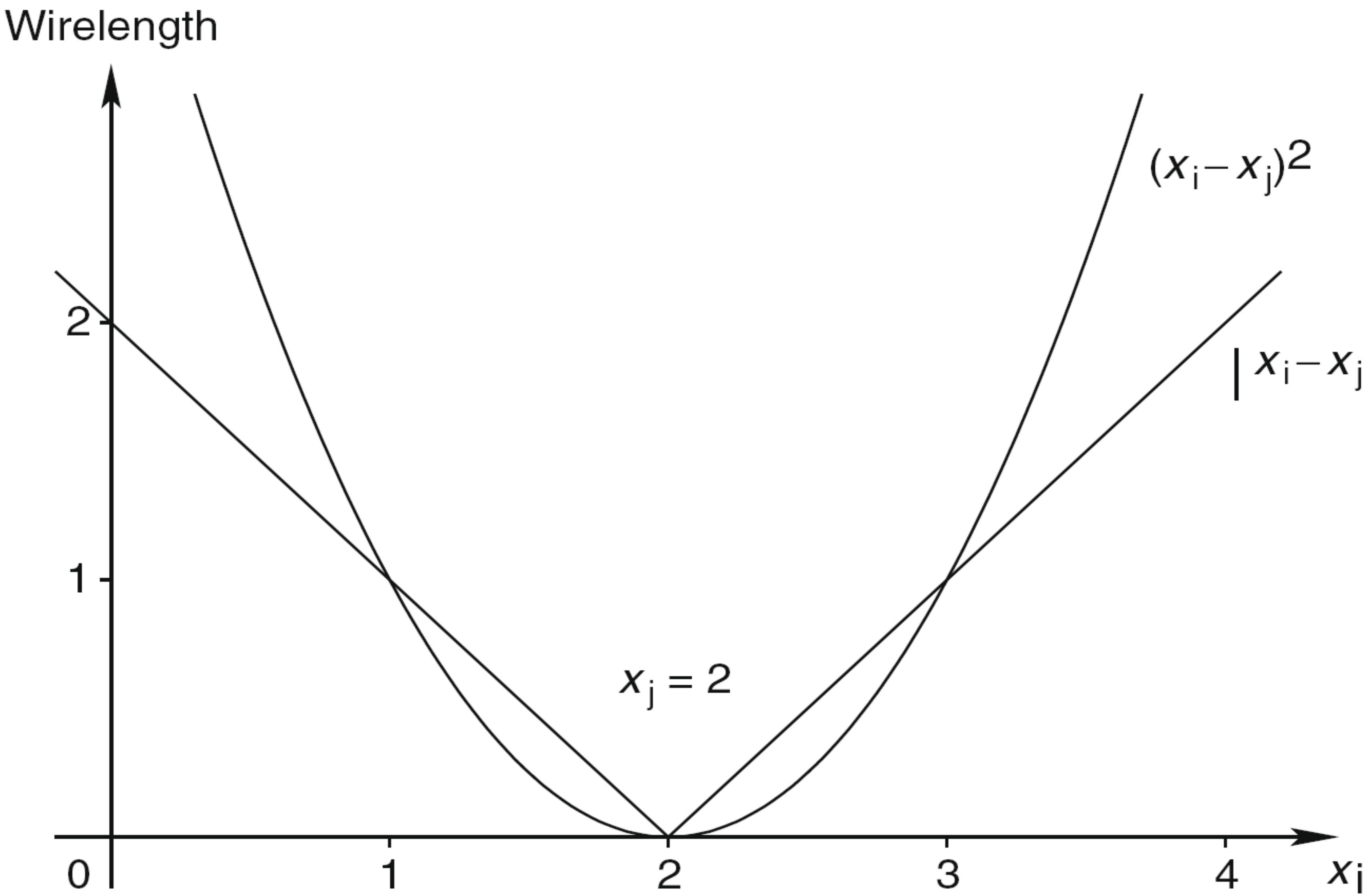
- WL (for 2-pin nets) can be written as a piece-wise linear function: $L = |x_1 - x_2|$ (in x direction)
- WL minimization can be written as a LP

$$\begin{aligned} &\text{min. } L \\ &\text{s.t.} \quad x_1 - x_2 \le L \\ &\qquad\;\; x_2 - x_1 \le L \end{aligned}$$




- However, quadratic WL minimization is more common: $\widetilde{L} = (x_1 - x_2)^2$
  – Smooth function
  – Convex function → easy to minimize
  – Correlates well with linear WL
  – Often called **quadratic placement**

# Quadratic WL vs. Linear WL



# Cost Function of Quadratic Placement

Let $(x_i, y_i)$ = Coordinates of the center of cell $i$

$c_{ij}$ = Weight of the net between cell $i$ and cell $j$

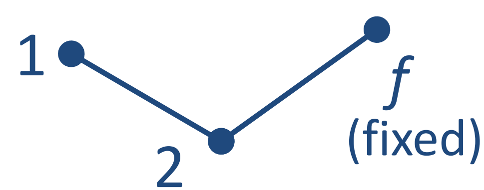
$\mathbf{x}, \mathbf{y}$ = Solution vectors

Cost of the net between cell $i$ and cell $j$

$$\tilde{L}_{\{i,j\}} = \frac{1}{2} c_{ij} \left( (x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

Total cost $\tilde{L} = \frac{1}{2} \mathbf{x}^T Q \mathbf{x} + \boldsymbol{d}_x{}^T \mathbf{x} + \frac{1}{2} \mathbf{y}^T Q \mathbf{y} + \boldsymbol{d}_y{}^T \mathbf{y} + \text{const}$

Horizontal cost




$$= \frac{1}{2} c_{12} (x_1 - x_2)^2 + \frac{1}{2} c_{2f} (x_2 - x_f)^2$$

$$= \frac{1}{2} \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} c_{12} & -c_{12} \\ -c_{12} & c_{12} + c_{2f} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -c_{2f} x_f \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{1}{2} c_{2f} x_f^2$$

# Notation

- Movable cells: 1,2, …, $r$
- Fixed cells: $r$+1, $r$+2, …, $n$
- $C = (c_{ij})_{r \times r}$: connectivity matrix among movable cells with $c_{ij} = c_{ji}$ for all $i, j$ in $\{1, 2, \ldots, r\}$
- $D = (d_{ij})_{r \times r}$: diagonal matrix with $d_{ii} = \sum_{j=1}^{n} c_{ij}$ for all $i$ in $\{1, 2, \ldots, r\}$
- $Q = D - C$
- $d_x = (d_{x_1}, d_{x_2}, \ldots, d_{x_r})^T$ with $d_{x_i} = -\sum_{j=r+1}^{n} c_{ij} x_j$
- $d_y = (d_{y_1}, d_{y_2}, \ldots, d_{y_r})^T$ with $d_{y_i} = -\sum_{j=r+1}^{n} c_{ij} y_j$

# Solution of Quadratic Placement

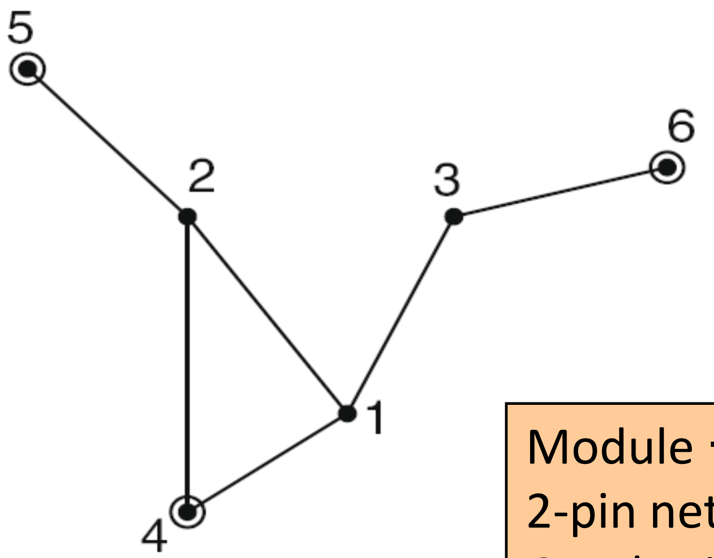
Total cost $\widetilde{L} = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \boldsymbol{d}_x{}^T\mathbf{x} + \frac{1}{2}\mathbf{y}^T Q\mathbf{y} + \boldsymbol{d}_y{}^T\mathbf{y} + \text{const}$

- The problems in x- and y-directions can be separated and solved independently
  - *Ignore non-overlapping and other constraints*
  - Minimize $\widetilde{L}_x = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \boldsymbol{d}_x{}^T\mathbf{x}$
  - Q can be proved to be positive and definite => the cost function is convex
  - Minimum solution can be found by setting derivatives to 0:

$$Q\mathbf{x} + \boldsymbol{d}_x{}^T = \mathbf{0}$$

# Force Interpretation of Quadratic WL

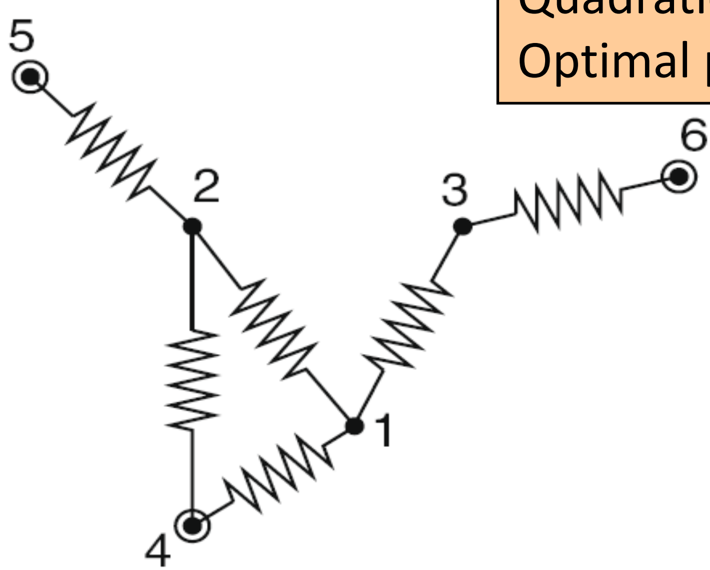
**Circuit**




- Fixed module
- Movable module

Module → Object
2-pin net → Spring
Quadratic WL → Spring potential energy
Optimal placement → Force equilibrium

**Spring system**




- Fixed object
- Movable object

# Force Calculation

- Hooke's Law:
  - Force = Spring Constant x Displacement
- Can consider forces in x- and y-direction separately

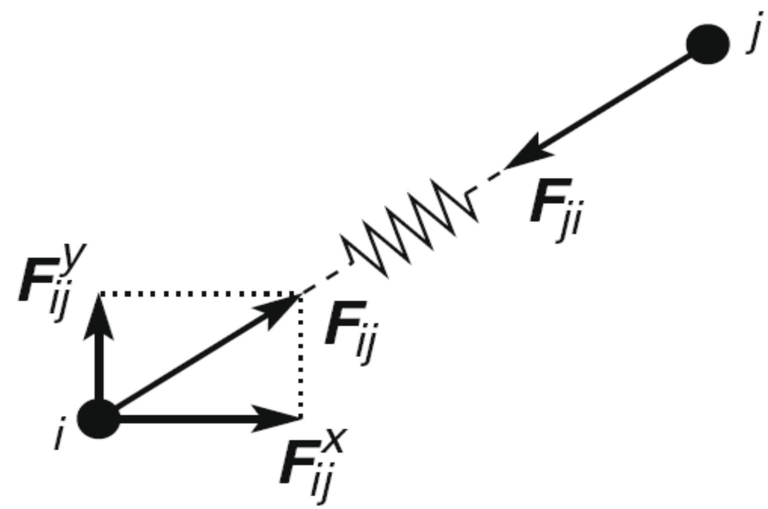
$$\text{Distance } d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

$$\text{Net Cost } c_{\{i,j\}}$$

$$|\mathbf{F_{ij}}| = c_{\{i,j\}} \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

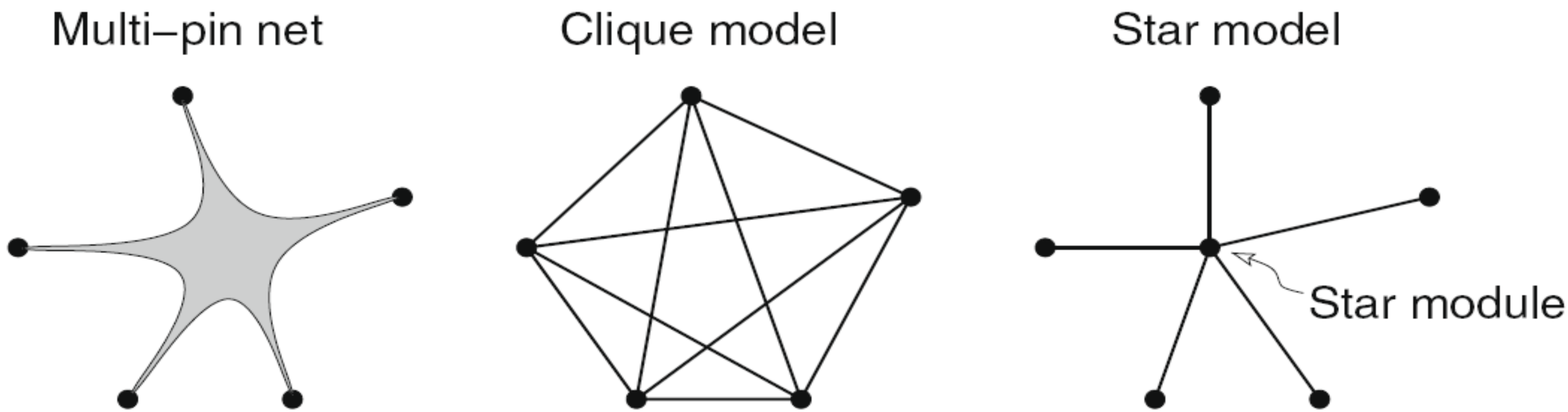
$$|\mathbf{F^x_{ij}}| = c_{\{i,j\}} \times |x_j - x_i|$$

$$|\mathbf{F^y_{ij}}| = c_{\{i,j\}} \times |y_j - y_i|$$



# Net Models for Multi-Pin Nets

- Multi-pin net is modeled as several 2-pin nets




Hybrid net model (best)

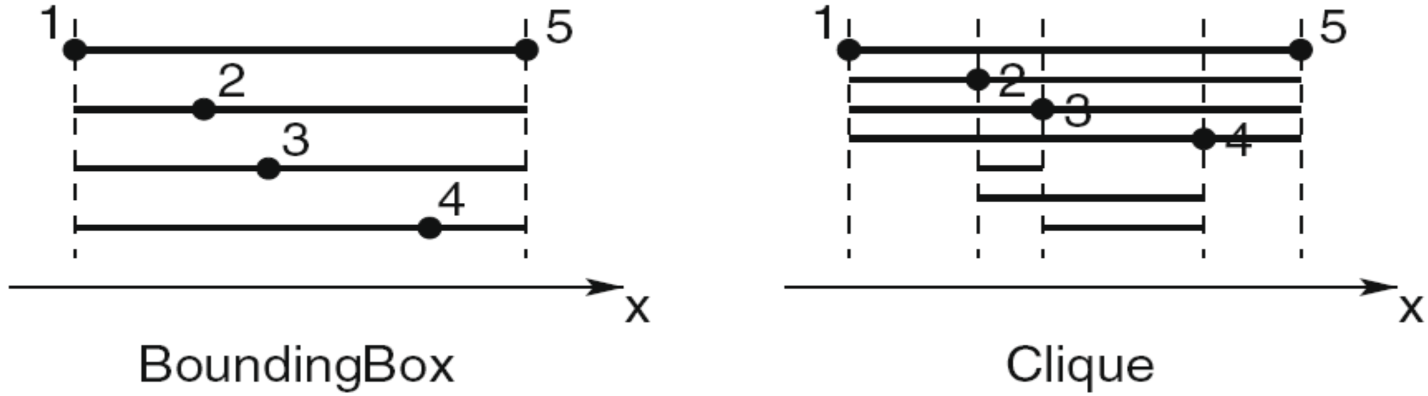
| # pins | Net Model |
|---|---|
| 2 | Clique |
| 3 | Clique |
| 4 | Star |
| 5 | Star |
| 6 | Star |
| … | … |

# Linearization Method in GORDIAN-L [DAC-91]

- Linear WL (star model): $$L^{star} = \sum_{e \in E} \sum_{i \in e} |x_i - x_e|$$
- Consider the function: $$\tilde{L}^{star} = \sum_{e \in E} \sum_{i \in e} \frac{(x_i - x_e)^2}{g_{ie}}$$
  - Exact if $g_{ie} = |x_i - x_e|$
  - Quadratic if $g_{ie}$'s are set to constant
  - $L^{star}$ can be optimized iteratively by setting $g_{ie}$ in current iteration according to the coordinates of previous iteration
  - In practice, set $g_{ie} = \sum_{i \in e} |x_i - x_e|$ for all $i \in e$

# BoundingBox Net Model in Kraftwerk [ICCAD-06]

- Make use of preceding linearization idea
- Accurately model HPWL
- For a *k*-pin net:

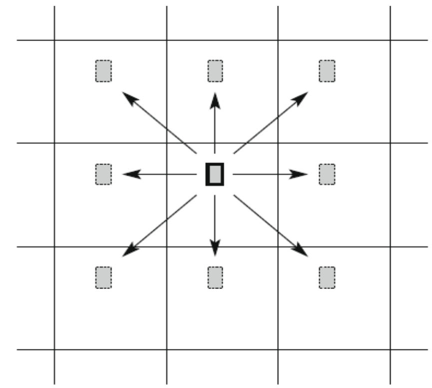



$$\tilde{L}^{BB} = \frac{1}{2} \sum_{\{i,j\} \in N} \omega_{\{i,j\}} \times (x_i - x_j)^2 \quad \text{where} \quad \omega_{\{i,j\}} = \frac{2}{k-1} \times \frac{1}{l_{\{i,j\}}}$$

If $l_{\{i,j\}}$ is set to $|x_i - x_j|$ for all $\{i, j\} \in N$, $\tilde{L}^{BB} = |x_1 - x_k|$

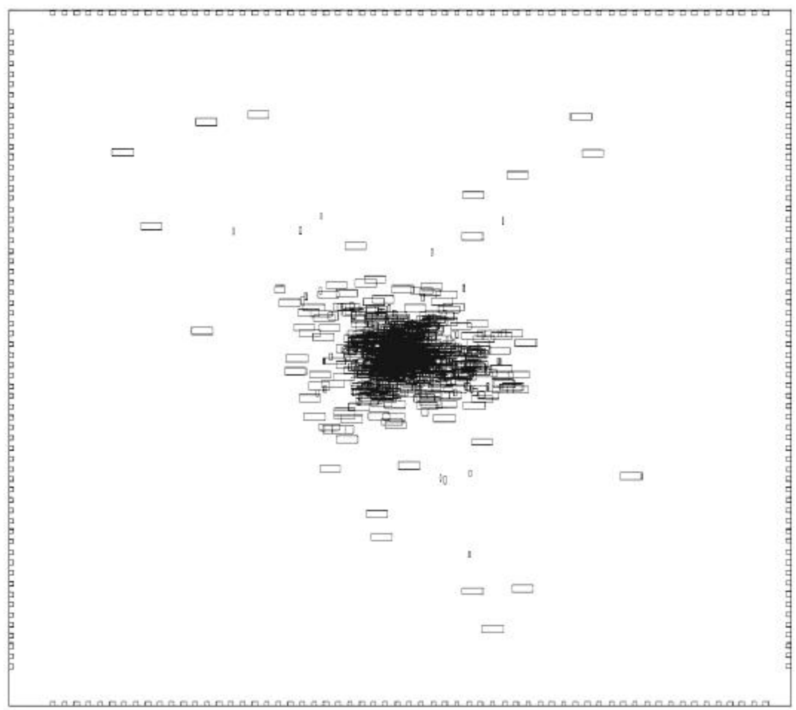
# Iterative Local Refinement (ILR) in FastPlace [ISPD-04]

- Mitigate the inaccuracy of quadratic WL by refining global placement with accurate WL metric
  - Divide the placement region into bins by a regular grid
  - Examine modules one by one
  - Tentatively move a module to its eight adjacent bins
  - Compute a score for each tentative move
    - HPWL reduction
    - Cell densities at the source and destination bins
  - Take the move with the highest positive score (no move if all scores are negative)
  - Repeat until there is no significant improvement



# Ignoring Nonoverlapping Constraints

- Consider WL minimization alone
  - If no fixed pins, a trivial solution is to place all modules at the same place
  - If there are fixed pins (e.g., I/O pins at boundary), it tends to get a lot of overlaps at the center of the placement

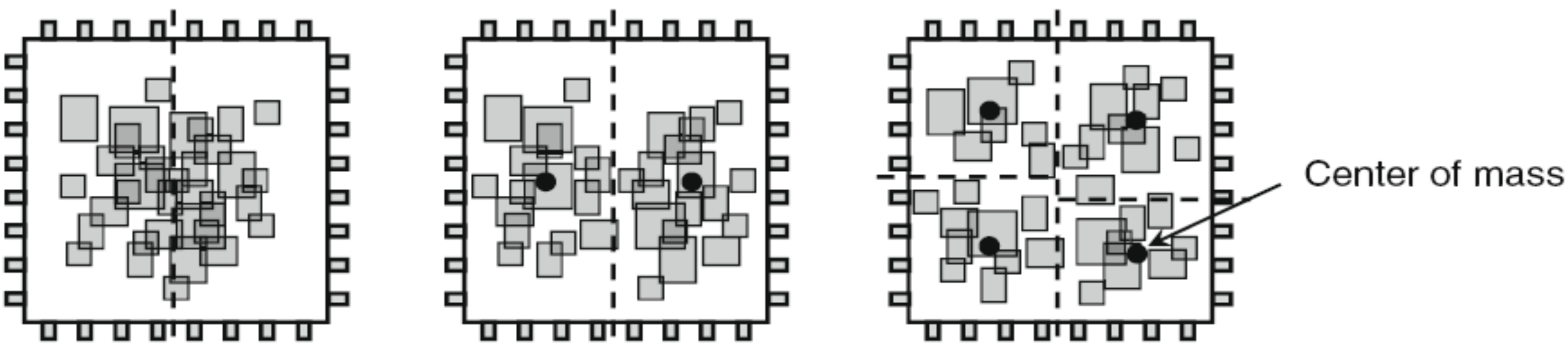


# Handling Nonoverlapping Constraints

- Ways to make module distribution more even in quadratic placement
  - Adding center-of-mass constraints
  - Adding forces to pull modules from dense regions to sparse regions
- Constraints/forces are added in **an iterative manner** to gradually spread out the modules
- Transformed into a sequence of convex quadratic programs

# Center-of-Mass Constraints in GORDIAN [TCAD-91]

- Given an uneven global placement solution
  - Find a good cut direction and position
    - Improve the cut value using FM
  - For each partition, add constraints that the center of gravity of cells should be in the center of region
    - The constraints are linear
  - Then perform quadratic placement again
    - Therefore, solving a single convex QP



# Density-based Force by Kraftwerk [ICCAD-06]

- Pull cells away from dense to sparse regions
- Definitions:
  - $\boldsymbol{x}'$= vector of **current** placement positions
  - $\boldsymbol{x}$= vector of **new** placement positions to be determined
  - $\hat{\boldsymbol{x}}$= vector of **target** placement positions
    - Based on module density $D(x,y)$

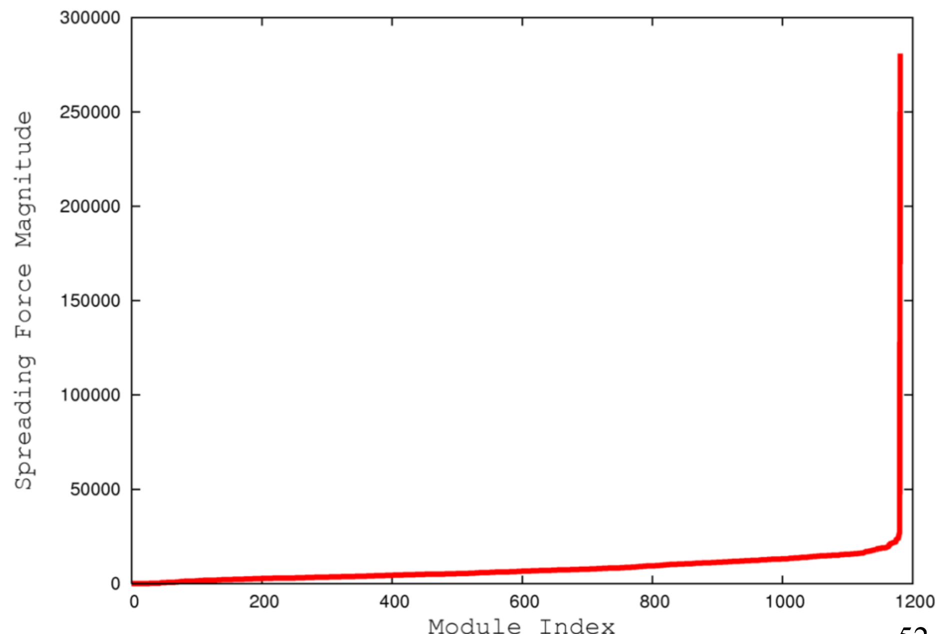
$$\Delta\phi = -D(x,\ y) \qquad \hat{x}_i = x'_i - \left|\frac{\partial\Phi}{\partial x}\right|_{(x'_i,\, y'_i)}$$

- **Hold force**: $\boldsymbol{F}_x^{hold} = -(\boldsymbol{Q}\boldsymbol{x}' + \boldsymbol{d}_x)$
- **Move force**: $\boldsymbol{F}_x^{move} = \hat{\boldsymbol{Q}}(\boldsymbol{x} - \hat{\boldsymbol{x}})$, here $\hat{\boldsymbol{Q}} = \mathrm{diag}(\hat{c}_i)$
- **Force equilibrium**:

$$(\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{d}_x) - (\boldsymbol{Q}\boldsymbol{x}' + \boldsymbol{d}_x) + \hat{\boldsymbol{Q}}(\boldsymbol{x} - \hat{\boldsymbol{x}}) = 0$$

# Force-Vector Modulation in RQL [DAC-07]

- Some additional spreading forces are huge
  - A module is pulled far away from its natural position
  - Causes significant increase in WL
  - Only a few percent of all additional forces are huge
- Idea: Nullifies the huge forces before next QP
  - Correcting mistakes made during spreading
  - Significant WL reduction
  - Minor effect in spreading



# Non-Quadratic Techniques

- Formulate the placement problem as a single non-linear & non-quadratic program

$$\text{Minimize} \sum_{e \in E} c_e \times \mathrm{WL}_e(\boldsymbol{x}, \boldsymbol{y})$$

$$\text{Subject to } D_b(\boldsymbol{x}, \boldsymbol{y}) <= T_b \text{ for all bin } b$$

  - $\mathrm{WL}_e()$ is continuously differentiable and more accurate in approximating HPWL than quadratic WL
  - Placement region is divided into **bins** such that non-overlapping constraints are replaced by **bin density constraints**

# Choices of Wirelength Functions

HPWL
$$\max_{v_i, v_j \in e, i<j} |x_i - x_j| + \max_{v_i, v_j \in e, i<j} |y_i - y_j|$$

Quadratic
$$\sum_{e \in E} \Big( \sum_{v_i, v_j \in e, i<j} w_{ij} |x_i - x_j|^2 + \sum_{v_i, v_j \in e, i<j} w_{ij} |y_i - y_j|^2 \Big)$$

Log-Sum-Exp (LSE)
$$\eta \sum_{e \in E} \Big( \log \sum_{v_k \in e} \exp(x_k/\eta) + \log \sum_{v_k \in e} \exp(-x_k/\eta) + \log \sum_{v_k \in e} \exp(y_k/\eta) + \log \sum_{v_k \in e} \exp(-y_k/\eta) \Big)$$

$L_p$-norm
$$\sum_{e \in E} \Big( \big(\sum_{v_k \in e} x_k^p\big)^{\frac{1}{p}} - \big(\sum_{v_k \in e} x_k^{-p}\big)^{-\frac{1}{p}} + \big(\sum_{v_k \in e} y_k^p\big)^{\frac{1}{p}} - \big(\sum_{v_k \in e} y_k^{-p}\big)^{-\frac{1}{p}} \Big)$$

# Choices of Wirelength Functions (cont'd)

**CHKS**

$$CHKS(x_1, x_2) = \frac{\sqrt{(x_1 - x_2)^2 + t^2} + x_1 + x_2}{2},$$

**Weighted-average** (WA)

$$\sum_{e \in E} \left( \frac{\sum_{v_i \in e} x_i \exp(x_i/\gamma)}{\sum_{v_i \in e} \exp(x_i/\gamma)} - \frac{\sum_{v_i \in e} x_i \exp(-x_i/\gamma)}{\sum_{v_i \in e} \exp(-x_i/\gamma)} + \frac{\sum_{v_i \in e} y_i \exp(y_i/\gamma)}{\sum_{v_i \in e} \exp(y_i/\gamma)} - \frac{\sum_{v_i \in e} y_i \exp(-y_i/\gamma)}{\sum_{v_i \in e} \exp(-y_i/\gamma)} \right).$$