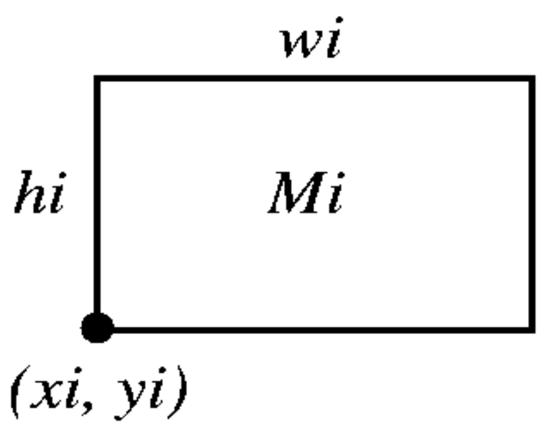
Non-slicing Floorplan Design by Mathematical Programming

- Sutanthavibul, Shragowitz, and Rosen, “An analytical approach to floorplan design and optimization,” DAC’90.
- Notation:
  - w_i, h_i: width and height of module M_i.
  - (x_i, y_i): coordinates of the lower left corner of module M_i.
  - $a_i \le w_i/h_i \le b_i$: aspect ratio w_i/h_i of module M_i. (Note: We defined aspect ratio as h_i/w_i before.)
- Goal: Find a **mixed integer linear programming (MILP)** formulation for the floorplan design.
  - **Linear** constraints? Objective function?



*Area = hi * wi*
Aspect ratio = wi / hi

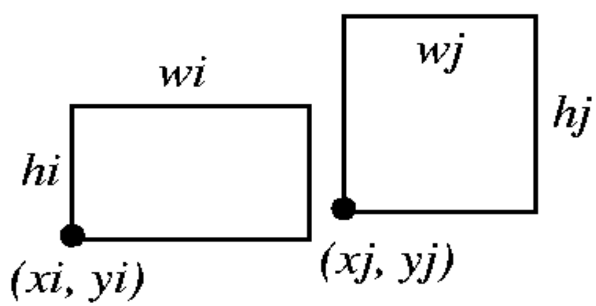
Non-overlap Constraints

- Two modules M_i and M_j do not overlap, if at least one of the following linear constraints is satisfied (cases encoded by p_{ij} and q_{ij}):

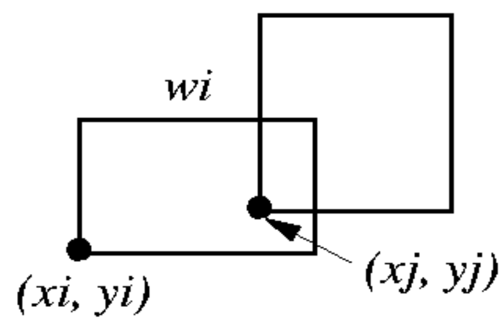
		p_{ij}	q_{ij}
M_i to the left of M_j:	$x_i + w_i \leq x_j$	0	0
M_i below M_j:	$y_i + h_i \leq y_j$	0	1
M_i to the right of M_j:	$x_i - w_j \geq x_j$	1	0
M_i above M_j:	$y_i - h_j \geq y_j$	1	1

- Let W, H be upper bounds on the floorplan width and height, respectively.
- Introduce two 0, 1 variables p_{ij} and q_{ij} to denote that one of the above inequalities is enforced; e. g., $p_{ij} = 0$, $q_{ij} = 1 \Rightarrow y_i + h_i \leq y_j$ is satisfied.

$$
\begin{aligned}
x_i + w_i &\leq x_j + W(p_{ij} + q_{ij}) \\
y_i + h_i &\leq y_j + H(1 + p_{ij} - q_{ij}) \\
x_i - w_j &\geq x_j - W(1 - p_{ij} + q_{ij}) \\
y_i - h_j &\geq y_j - H(2 - p_{ij} - q_{ij})
\end{aligned}
$$



xi + wi <= xj



xi + wi > xj

Cost Function & Constraints

- Minimize Area = xy, nonlinear! (x, y: width and height of the resulting floorplan)
- How to fix?

  \- Fix the width W and minimize the height y!
- Four types of constraints:
  1. No two modules overlap ($\forall i, j$: $1 \leq i < j \leq n$);
  2. Each module is enclosed within a rectangle of width W and height y ($x_i + w_i \leq W$, $y_i + h_i \leq y$, $1 \leq i \leq n$);
  3. $x_i \geq 0$, $y_i \geq 0$, $1 \leq i \leq n$;
  4. p_{ij}, $q_{ij} \in \{0, 1\}$.
- w_i, h_i are known.

Mixed ILP for Floorplanning

Mixed ILP for the floorplanning problem with rigid, fixed modules.

$$
\begin{array}{rll}
 & \text{min} \quad y & \\
\textit{subject to} & & \\
x_i + w_i \le W, & 1 \le i \le n & (1) \\
y_i + h_i \le y, & 1 \le i \le n & (2) \\
x_i + w_i \le x_j + W(p_{ij} + q_{ij}), & 1 \le i < j \le n & (3) \\
y_i + h_i \le y_j + H(1 + p_{ij} - q_{ij}), & 1 \le i < j \le n & (4) \\
x_i - w_j \ge x_j - W(1 - p_{ij} + q_{ij}), & 1 \le i < j \le n & (5) \\
y_i - h_j \ge y_j - H(2 - p_{ij} - q_{ij}), & 1 \le i < j \le n & (6) \\
x_i, y_i \ge 0, & 1 \le i \le n & (7) \\
p_{ij}, q_{ij} \in \{0, 1\}, & 1 \le i < j \le n & (8)
\end{array}
$$

- Size of the mixed ILP: for n modules,
  - # continuous variables: $O(n)$; # integer variable: $O(n^2)$; # linear constraints: $O(n^2)$.
  - Unacceptably huge program for a large n!
- Popular LP software: LINDO, lp_solve, CPLEX, GUROBI, etc.

Mixed ILP for Floorplanning (cont'd)

$$
\begin{array}{rll}
\text{min} & y & \\
\textit{subject to} & & \\
x_i + r_i h_i + (1 - r_i) w_i \le W, & 1 \le i \le n & (9) \\
y_i + r_i w_i + (1 - r_i) h_i \le y, & 1 \le i \le n & (10) \\
x_i + r_i h_i + (1 - r_i) w_i \le x_j + M(p_{ij} + q_{ij}), & 1 \le i < j \le n & (11) \\
y_i + r_i w_i + (1 - r_i) h_i \le y_j + M(1 + p_{ij} - q_{ij}), & 1 \le i < j \le n & (12) \\
x_i - r_j h_j - (1 - r_j) w_j \ge x_j - M(1 - p_{ij} + q_{ij}), & 1 \le i < j \le n & (13) \\
y_i - r_j w_j - (1 - r_j) h_j \ge y_j - M(2 - p_{ij} - q_{ij}), & 1 \le i < j \le n & (14) \\
x_i, y_i \ge 0, & 1 \le i \le n & (15) \\
p_{ij}, q_{ij} \in \{0, 1\}, & 1 \le i < j \le n & (16)
\end{array}
$$

- For each module i with free orientation, associate a 0-1 variable r_i;
- $r_i = 0$: 0° rotation for module i.
- $r_i = 1$: 90 ° rotation for module i.
- $M = \max\{W, H\}$.

Soft Modules

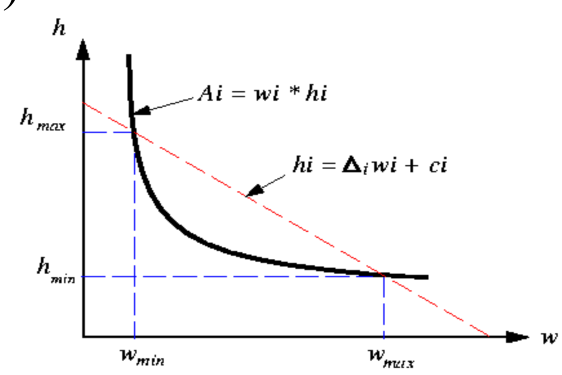
- Assumptions: w_i, h_i are unknown; area lower bound: A_i.
- Module size constraints: $w_i h_i \geq A_i$; $a_i \leq \frac{w_i}{h_i} \leq b_i$.
- Hence, $w_{min} = \sqrt{A_i a_i}$, $w_{max} = \sqrt{A_i b_i}$, $h_{min} = \sqrt{\frac{A_i}{b_i}}$, $h_{max} = \sqrt{\frac{A_i}{a_i}}$
- $w_i h_i \geq A_i$ nonlinear! How to fix? (The following fixing scheme is different from the one given in the paper.)
  - Can apply a first-order approximation of the equation: a line passing through (w_{min}, h_{max}) and (w_{max}, h_{min}).

$$h_i = \Delta_i w_i + c_i \qquad /^* \; y = mx + c \; ^*/$$

$$\Delta_i = \frac{h_{\max} - h_{\min}}{w_{\min} - w_{\max}} \qquad /^* \; slope \; ^*/$$

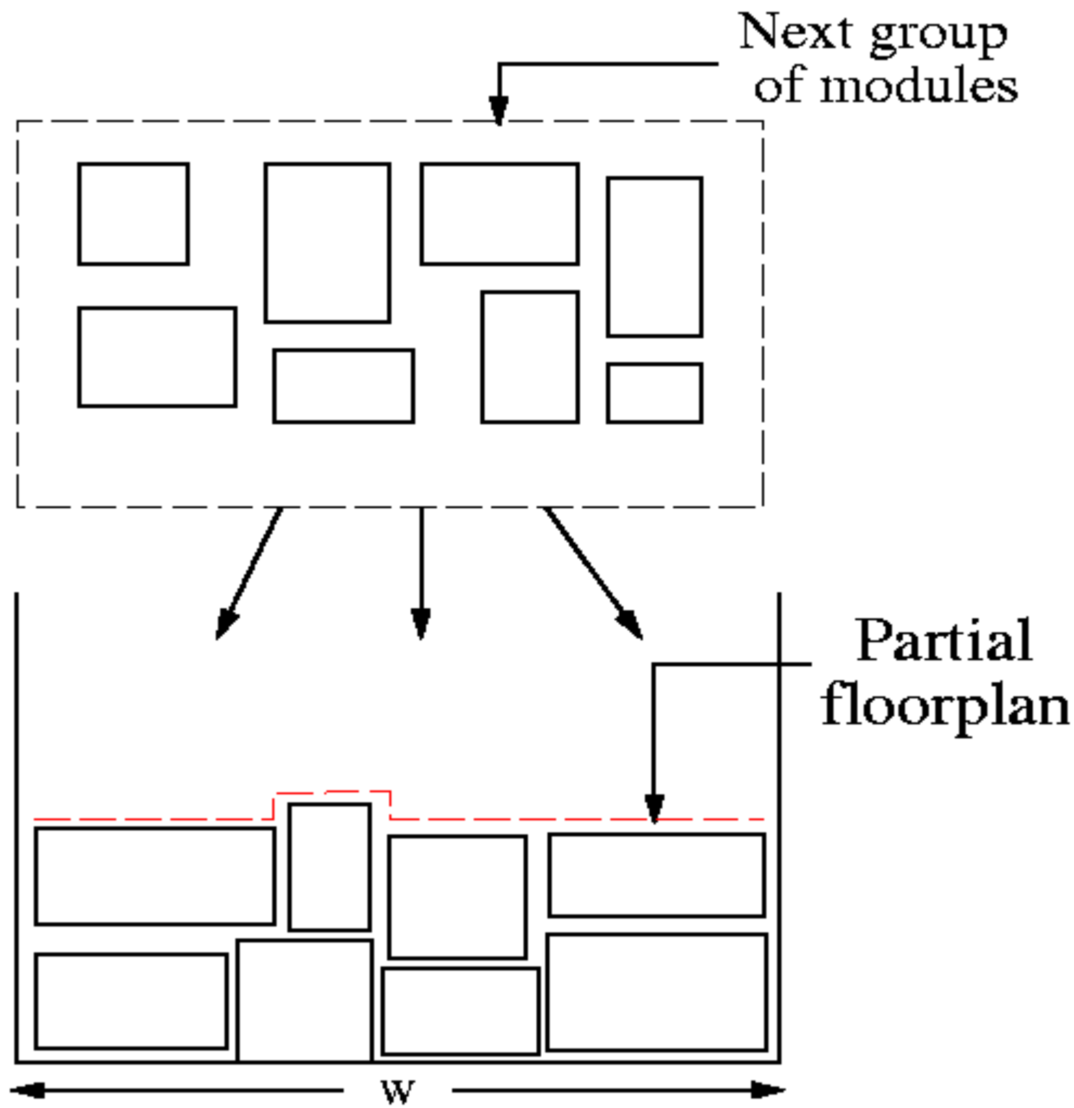
$$c_i = h_{max} - \Delta_i w_{min} \qquad /^* \; c = y_0 - mx_0 \; ^*/$$

  - Substitute $\Delta_i w_i + c_i$ for h_i to form linear constraints (x_i, y_i, w_i are unknown; Δ_i , c_i, h_i can be computed as above).



Reducing the Size of the Mixed ILP

- Time complexity of a mixed ILP: exponential!
- Recall the large size of the mixed ILP: # variables, # constraints: $O(n^2)$.
  - How to fix it?
- Key: solve a partial problem at each step (successive augmentation)
- Questions:
  - How to select next subgroup of modules? $\Rightarrow$linear ordering based on connectivity.
  - How to minimize # of required variables?


Next group
of modules
Partial
floorplan
W

Reducing the Size of the Mixed ILP (cont'd)

- Size of each successive mixed ILP depends on (1) # of modules in the next group; (2) "size" of the partially constructed floorplan.
- Keys to deal with (2):
  - Minimize the problem size of the partial floorplan.
  - Replace the already placed modules by a set of covering rectangles.
  - # rectangles is usually much smaller than # placed modules.