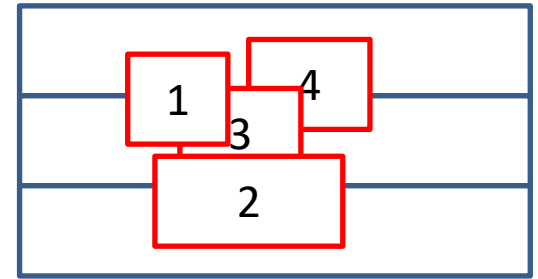


# Greedy Methods

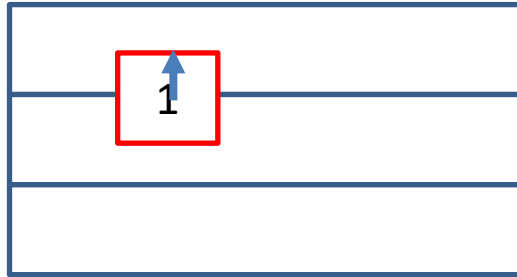
- Legalize one cell at a time
- **Tetris** [Hill, US Patent 6370673, 2002]
  - Sort cells in ascending x-coordinates
  - Pack cells to the left one at a time into a row so as to minimize the displacement
  - Already legalized cells are never moved again
  - Pros: Commonly used and extremely fast
  - Cons: May result in very uneven row length and may fail to pack all cells inside the placement region
- **Abacus** [Spindler, ISPD 2008]
  - Similar to Tetris
  - **Already legalized cells can be moved**
  - Get better results than Tetris

# Example for Tetris

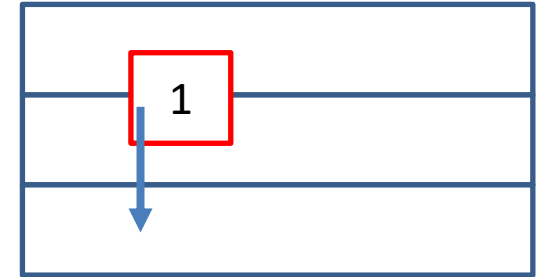
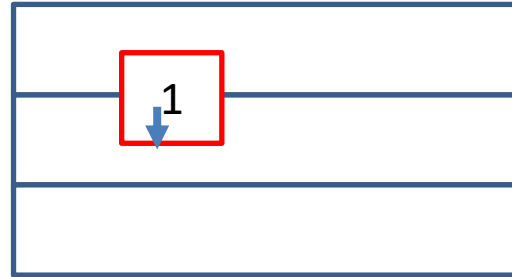
Global  
Placement



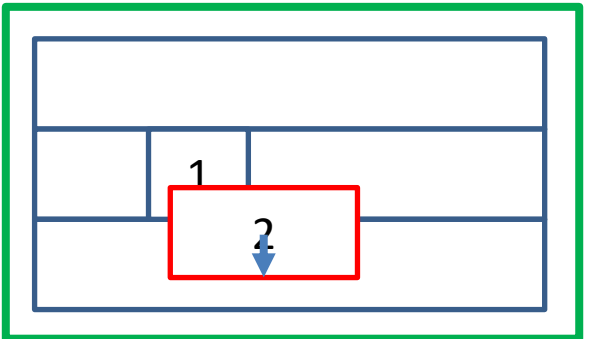
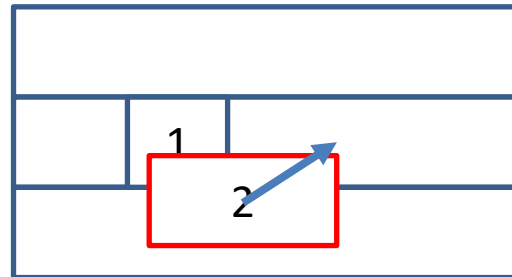
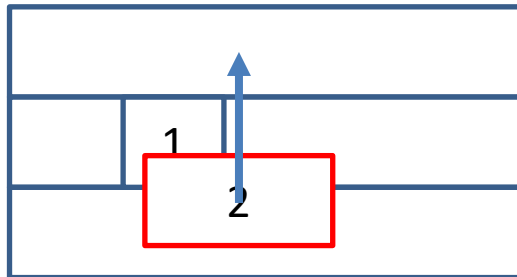
Legalize  
cell 1:



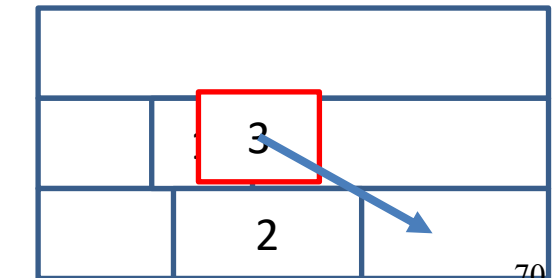
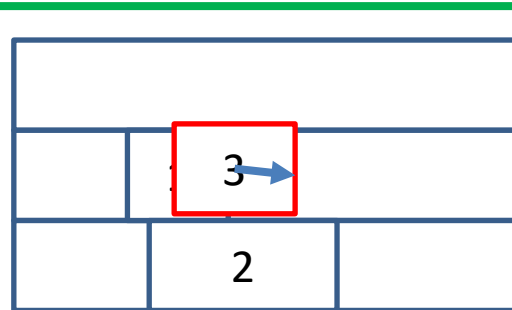
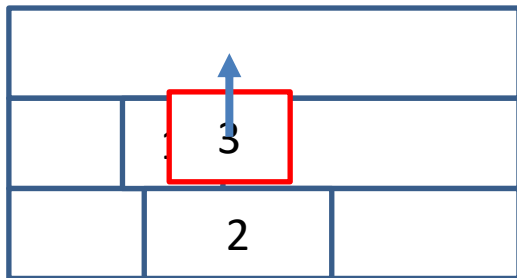
Pick Nearest Movement



Legalize  
cell 2:



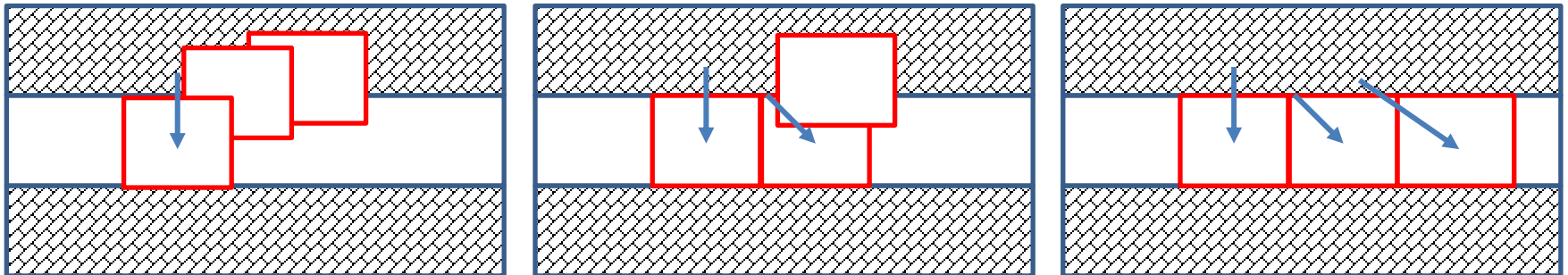
Legalize  
cell 3:



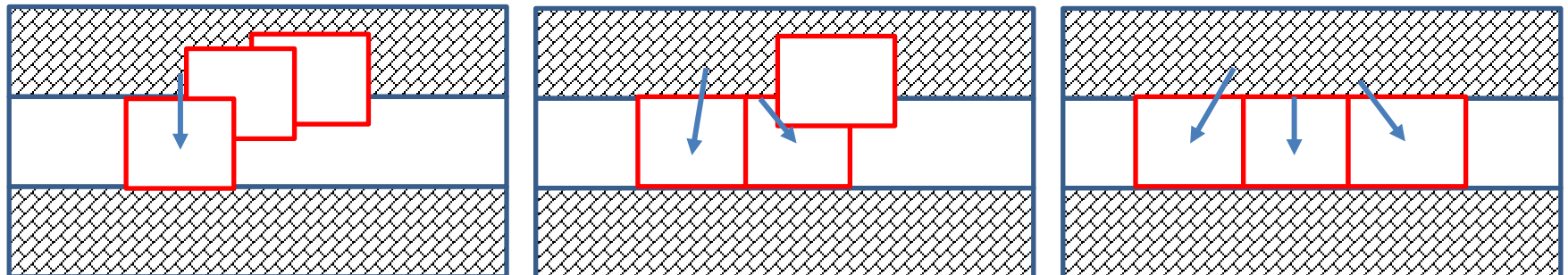
# Abacus

- Similar to Tetris: sort cells and legalize one cell at a time
- Legalization of one cell: move the cell over the rows, and place the cell to the best/nearest row
- **PlaceRow** (difference from Tetris): move already legalized cells within one row to minimize total movement

Tetris:



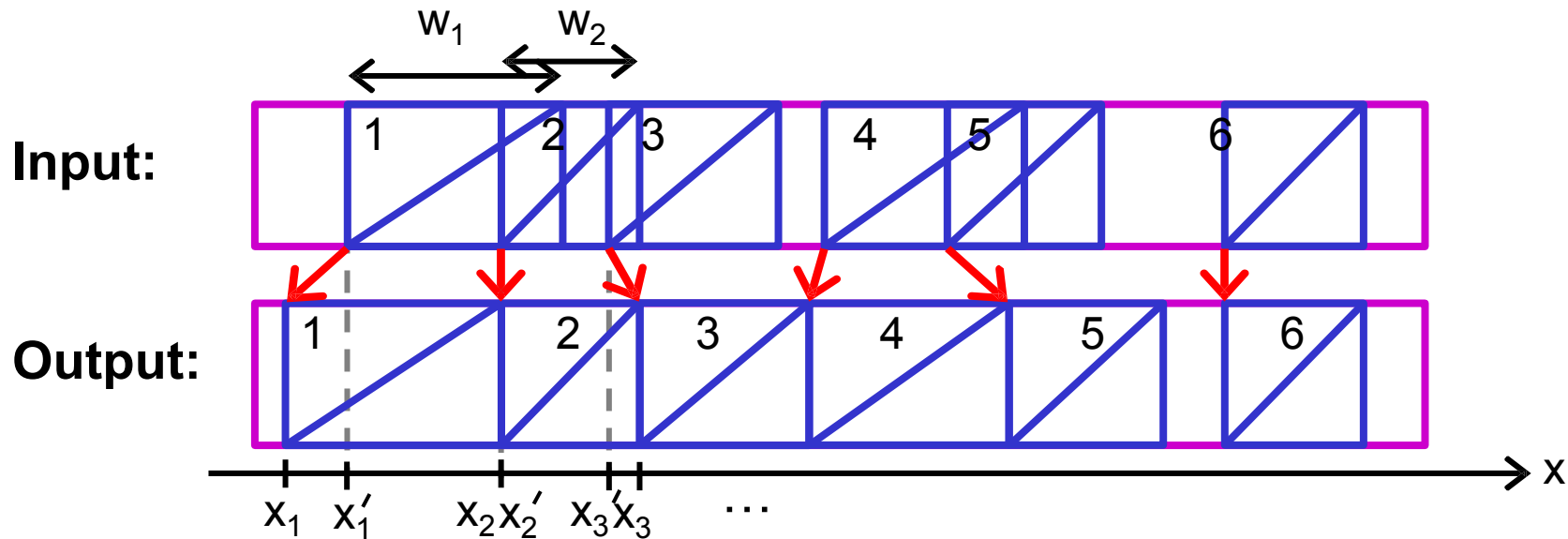
Abacus:



# PlaceRow

**Input:** one row with  $N$  cells, x-pos of cells: global placement ( $x_i'$ )

**Output:** new (legal) x-pos of cells ( $x_i$ ) such that the overlap is removed and the total quadratic movement is minimized



**QP:**

$$\min \sum_{i=1}^N e_i (x_i - x_i')^2 \quad \text{s.t.} \quad \underbrace{x_i \geq x_{i-1} + w_{i-1}}_{\text{no overlap}} \leftarrow \text{width}$$

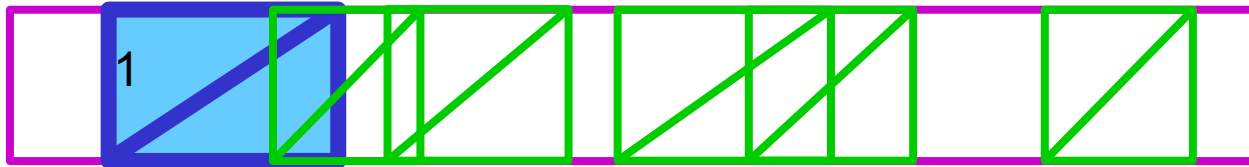
$\uparrow$  weight       $\uparrow$  legal x-pos       $\uparrow$  global x-pos

# PlaceRow: Dynamic Programming

## PlaceRow:

- Solve QP by dynamic programming approach:  
solve sub problems optimally to obtain final solution
- Process cells from left to right

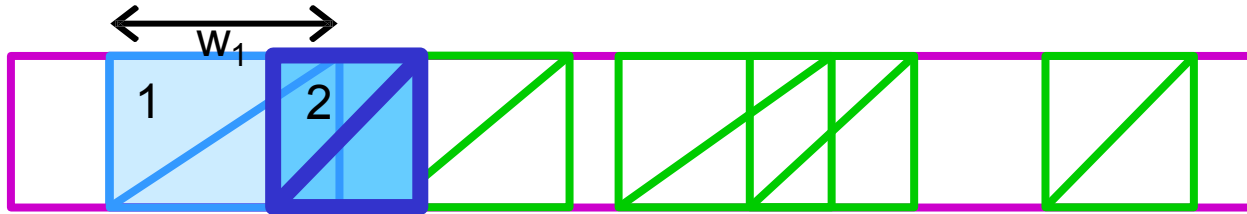
## Cell 1:



first cell → do not move

# PlaceRow: Cell 2

Cell 2:



overlap with previous cell? **yes** → **cluster** with previous cell

Clustering process:

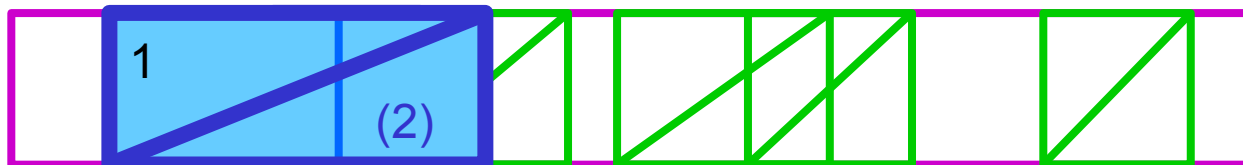
$$x_2 = x_1 + w_1$$

$$\min e_1 (x_1 - x'_1)^2 + e_2 (x_2 - x'_2)^2 \quad \Rightarrow \quad \min \underbrace{(e_1 + e_2)}_{\text{new } e_1} \left( x_1 - \underbrace{\frac{e_1 x'_1 + e_2 (x'_2 - w_1)}{e_1 + e_2}}_{\text{new } x'_1} \right)^2$$

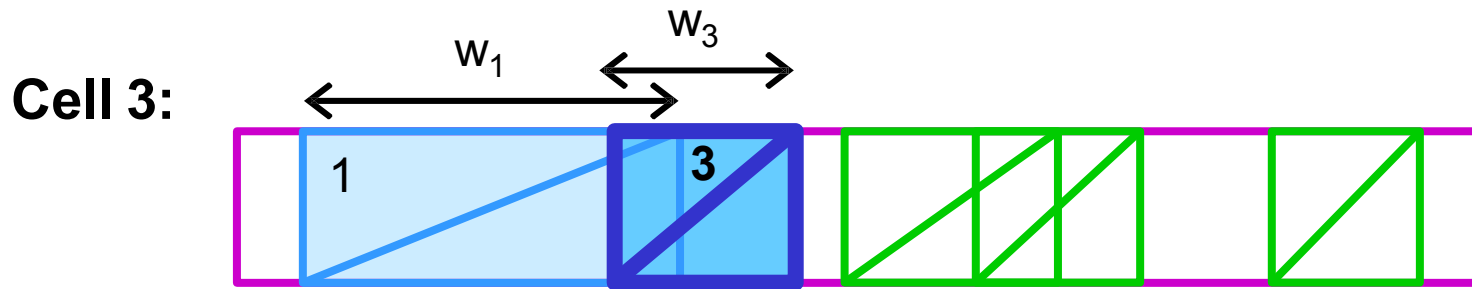
Update  $x'_1$ ,  $e_1$ , and  $w_1$ :

$$x'_1 \leftarrow \frac{e_1 x'_1 + e_2 (x'_2 - w_1)}{e_1 + e_2} \quad e_1 \leftarrow e_1 + e_2 \quad w_1 \leftarrow w_1 + w_2$$

**Result:**  $\min e_1 (x_1 - x'_1)^2 \rightarrow x_1 = x'_1$



# PlaceRow: Cell 3



Overlap with previous cell? **Yes** → **cluster** with previous cell

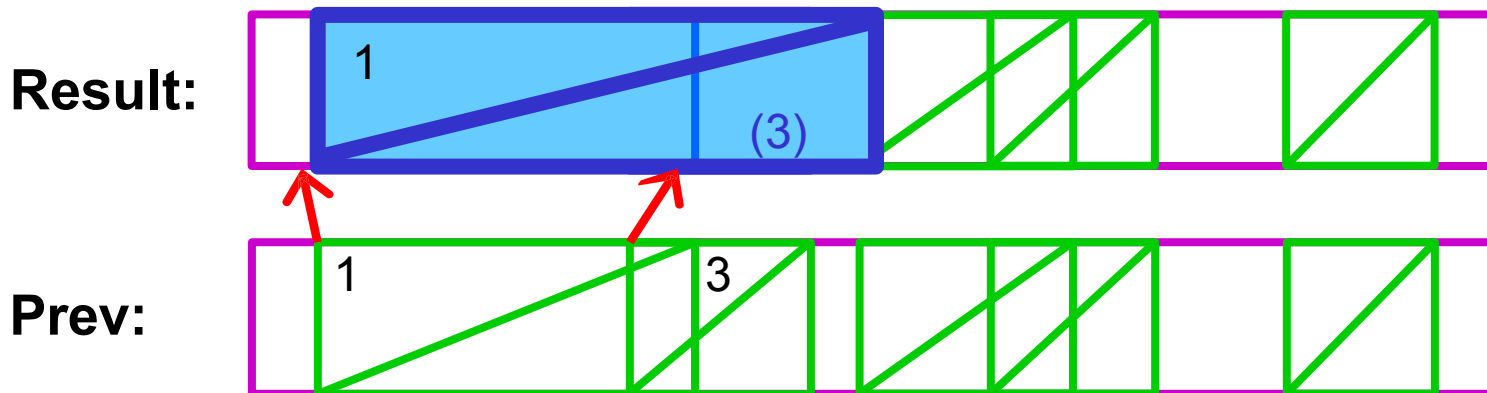
Update  $x_1'$ ,  $e_1$ , and  $w_1$ :

$$x_1' \leftarrow \frac{e_1 x_1' + e_3 (x_3' - w_1)}{e_1 + e_3}$$

$$w_1 \leftarrow w_1 + w_3$$

$$e_1 \leftarrow e_1 + e_3$$

Move cell 1:  $\min e_1 (x_1 - x_1')^2 \rightarrow x_1 = x_1'$



# PlaceRow: Cell 4

Cell 4:

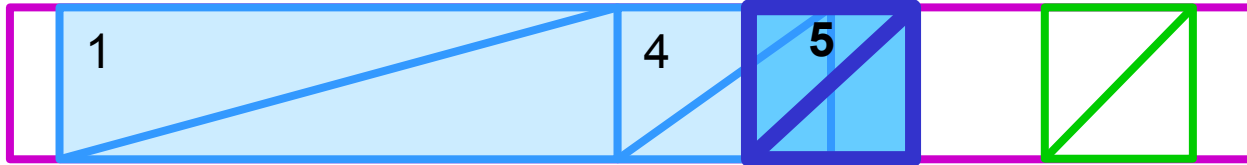


Overlap with previous cell? **No**  
→ **no clustering, no movement**



# PlaceRow: Cell 5

Cell 5:

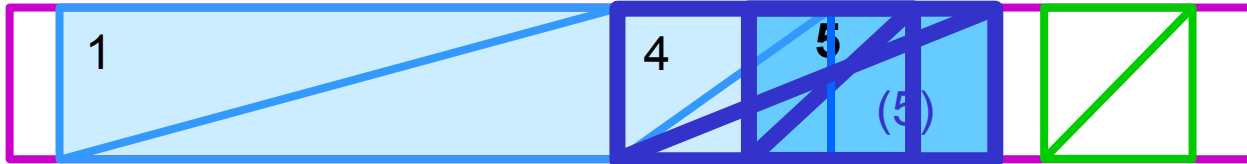


Overlap with previous cell? **yes** → **cluster** with previous cell 4

Update  $x_4, e_4$ , and  $w_4$ : 
$$x'_4 \leftarrow \frac{e_4 x'_4 + e_5 (x'_5 - w_4)}{e_4 + e_5} \quad e_4 \leftarrow e_4 + e_5$$
  

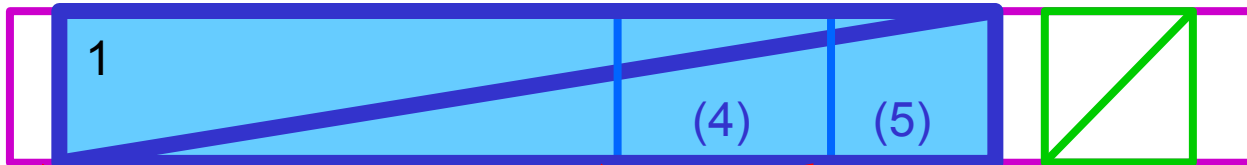
$$w_4 \leftarrow w_4 + w_5$$

Move cell 4:  $\min e_4 (x_4 - x'_4)^2 \rightarrow x_4 = x'_4$

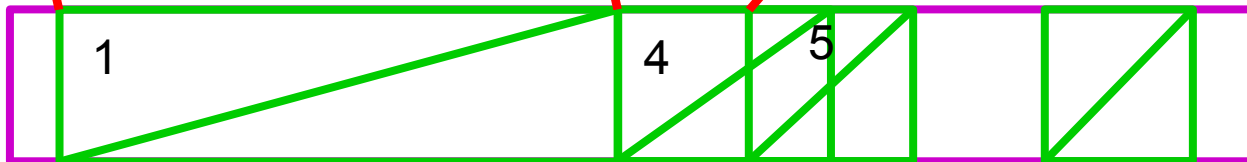


Overlap with previous cell? **yes** → **cluster** with previous cell 1

Result:

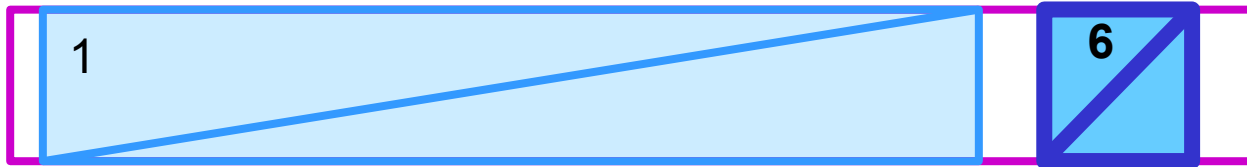


Prev:



# PlaceRow: Cell 6

Cell 6:



Overlap with previous cell? **No**  
→ **no clustering, no movement**

**Last cell** → done, PlaceRow finished

# PlaceRow: Summary

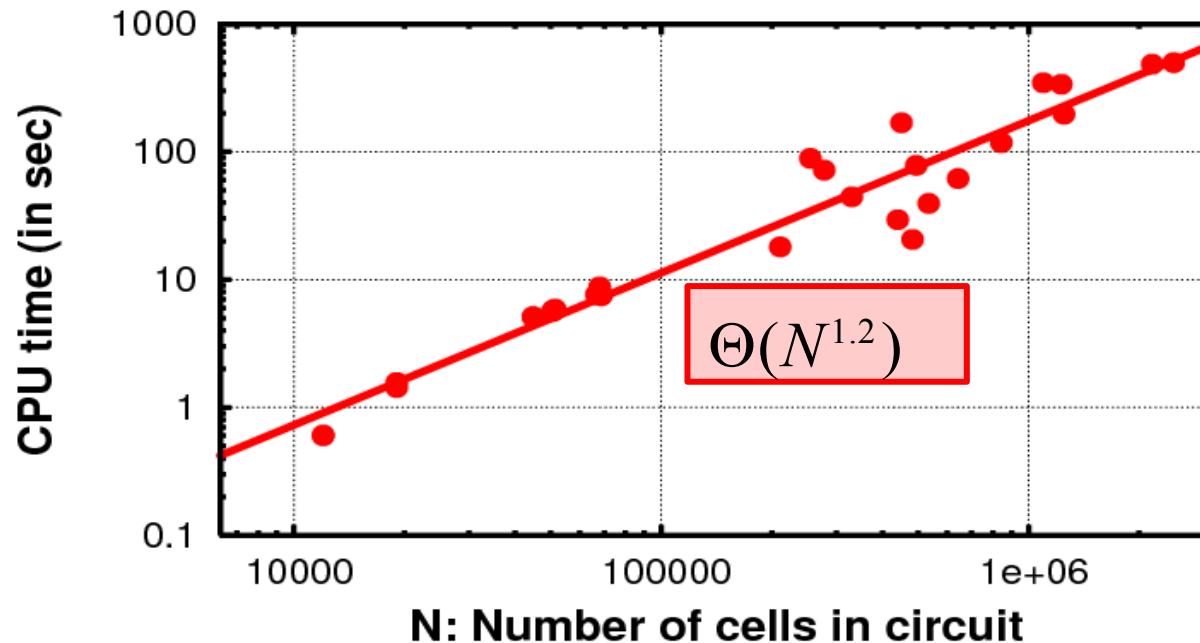
## PlaceRow:

- Called several times for legalizing one cell
- Places cells aligned to one row:  
minimize quadratic movement → quadratic program (QP)
- Solves QP by dynamic programming:
  - Process cells from left to right
  - If cell overlaps with previous cell:  
clustering → movement → further checks with left cells
  - Clustering: update width, weight, and global x-pos of cell  
constant execution time
- Linear worst-case complexity:  $O(N)$   $N$ : number of cells in the row  
At most  $N-1$  clustering operations for  $N$  cells

# Complexity

Worst-case for a complete circuit with N cells:  $O(N^2)$

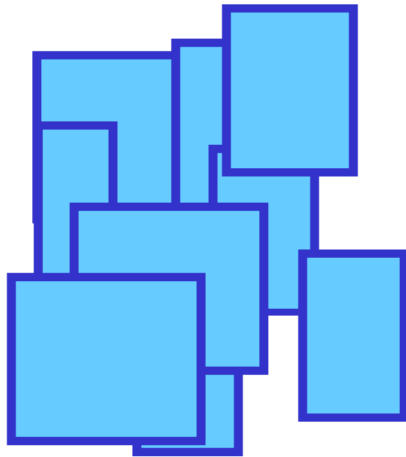
**Average-case (experimental results):**



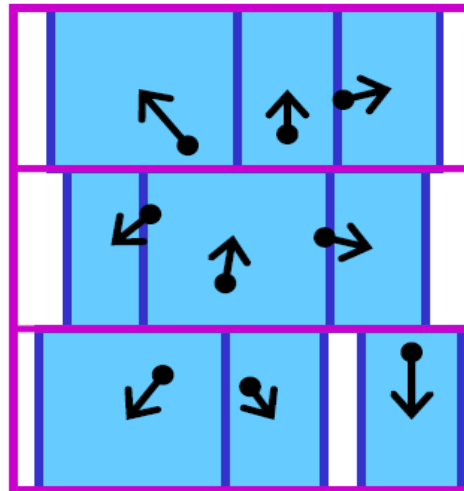
# Tetris vs. Abacus

- Abacus can get shorter total movement and shorter maximum movement than Tetris
- Abacus can avoid overflow in each row
- The runtime of Abacus is longer than Tetris, but both can complete in few seconds for large designs

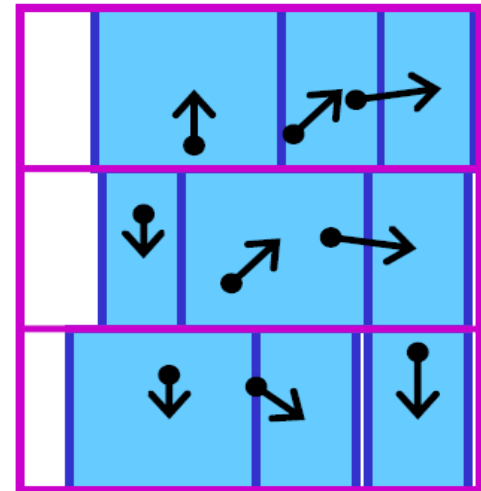
Global Placement



Abacus

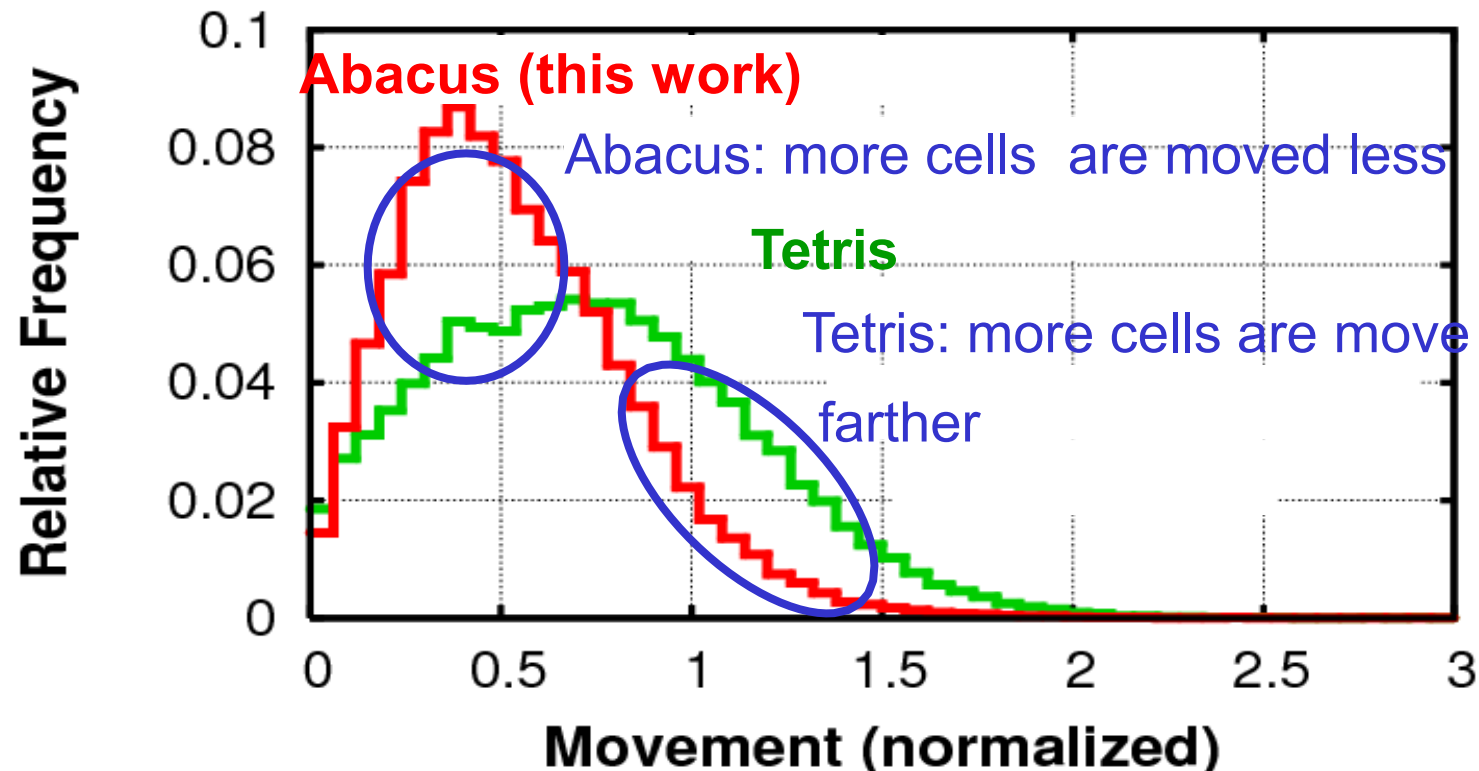


Tetris



# Comparison on Movement

Experimental results of one circuit:



→ Lower movement with Abacus