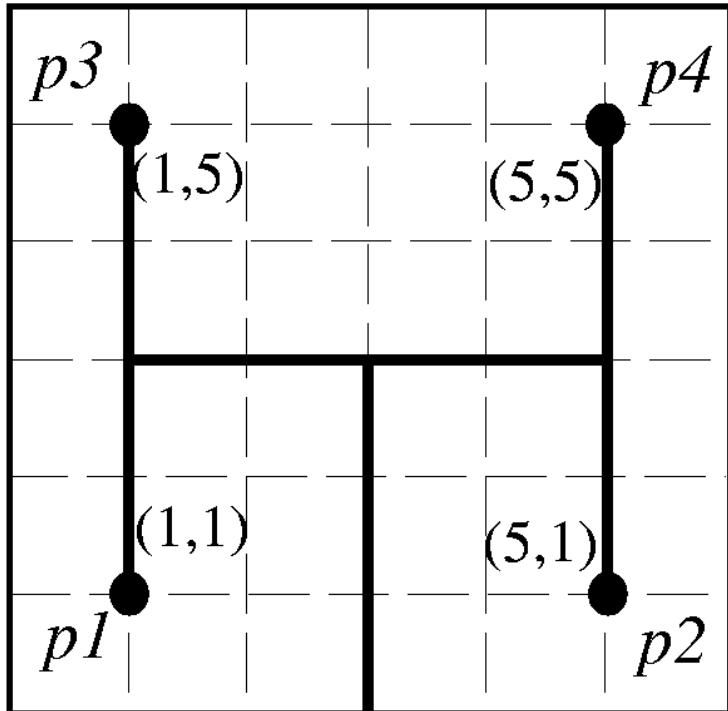
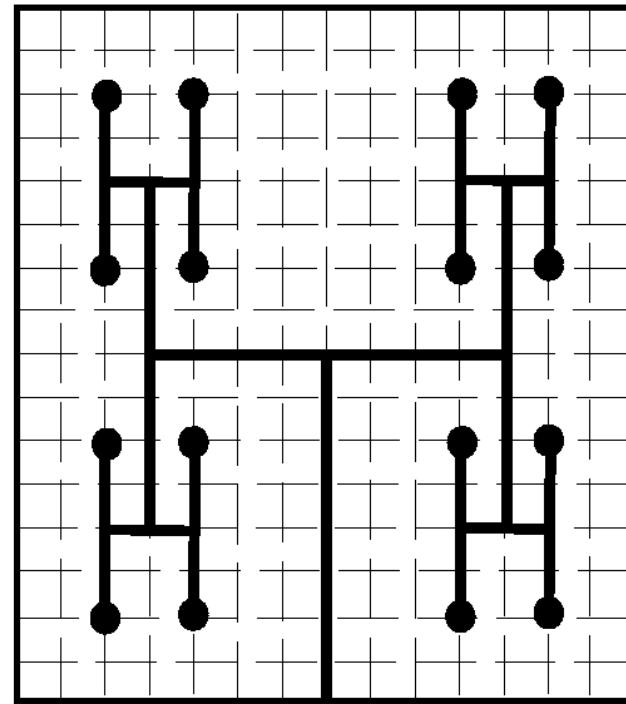


H-Tree Based Algorithm

- H-tree: Dhar, Franklin, Wang, “Reduction of clock delays in VLSI structure,” *ICCD*, 1984.



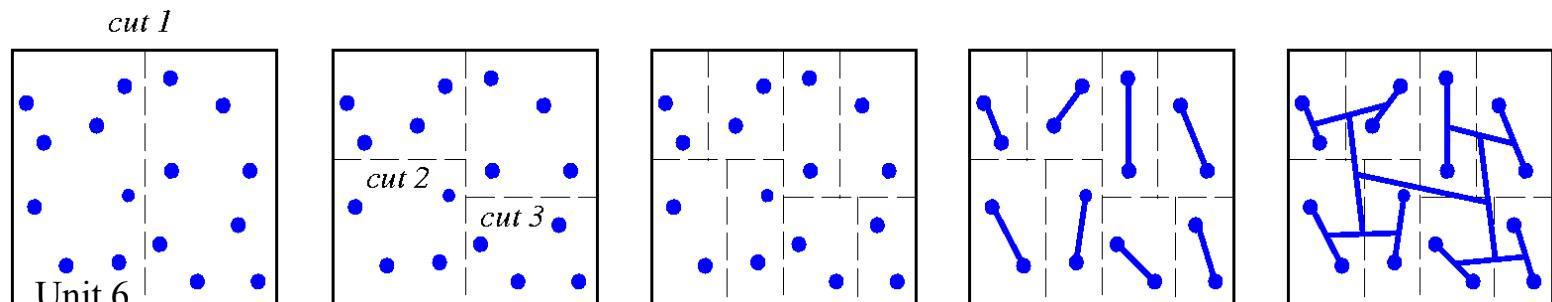
H-tree over 4 points



H-tree over 16 points

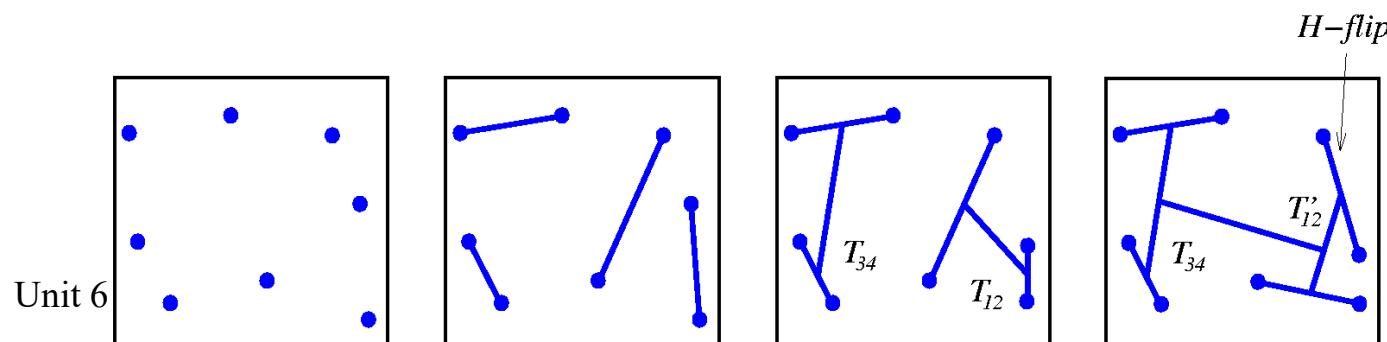
The MMM Algorithm

- Jackson, Srinivasan, Kuh, “Clock routing for high-performance ICs,” *DAC*, 1990.
- Each clock pin is represented as a point in the region, S .
- The region is partitioned into two subregions, S_L and S_R .
- The center of mass is computed for each subregion.
- The center of mass of the region S is connected to each of the centers of mass of subregion S_L and S_R .
- The subregions S_L and S_R are then recursively split in Y -direction.
- The above steps are repeated with alternate splitting in X - and Y -direction.
- Time complexity: $O(n \log n)$.



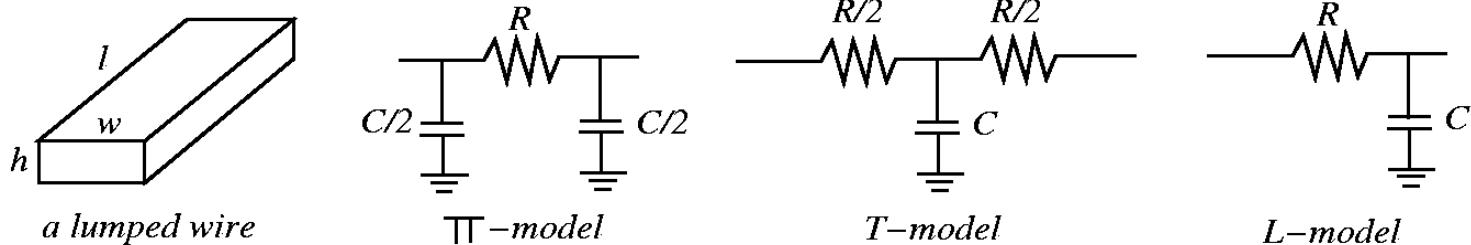
The Geometric Matching Algorithm

- Kahng, Cong, Robins, “Matching based models for high-performance clock routing,” *IEEE TCAD*, 1993.
- Clock pins are represented as n nodes in the clock tree ($n=2^k$).
- Each node is a tree itself with clock entry point being node itself.
- The minimum cost matching on n points yields $n/2$ segments.
- The clock entry point in each subtree of two nodes is the point on the segment such that length of both sides is the same.
- The above steps are repeated for each segment.
- Apply *H-flipping* to further reduce clock skew (and to handle edges intersection).

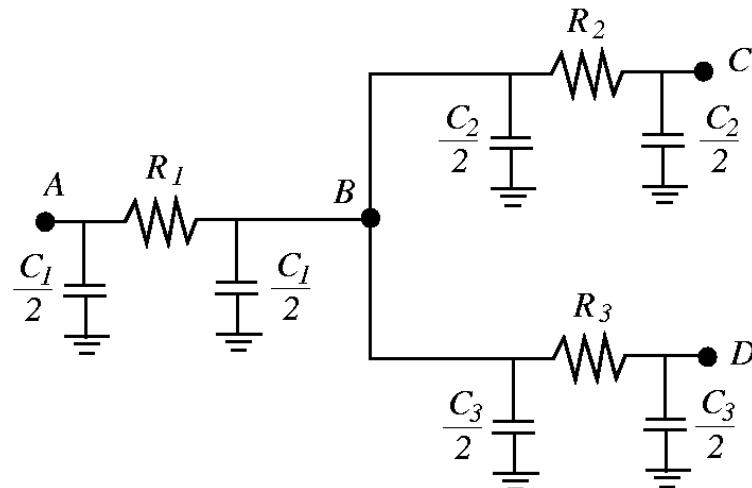


Delay Calculation

- Need to consider a more accurate delay model for general circuits.
- **RC Delay:** The delay caused by wires is due to their capacitance and resistance. $(R \propto \frac{l}{wh}; C \propto wl)$
- Lumped circuit approximations for distributed RC lines: π -model, T -model, L -model.

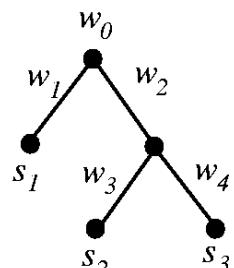


- RC delay: $D_{AB} = R_1 \left(\frac{C_1}{2} + C_2 + C_3 \right)$; B to C : $D_{BC} = R_2 \left(\frac{C_2}{2} \right)$; B to D : $D_{BD} = R_3 \left(\frac{C_3}{2} \right)$

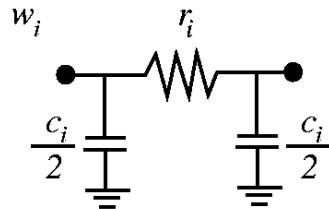


Delay Calculation for a Clock Tree

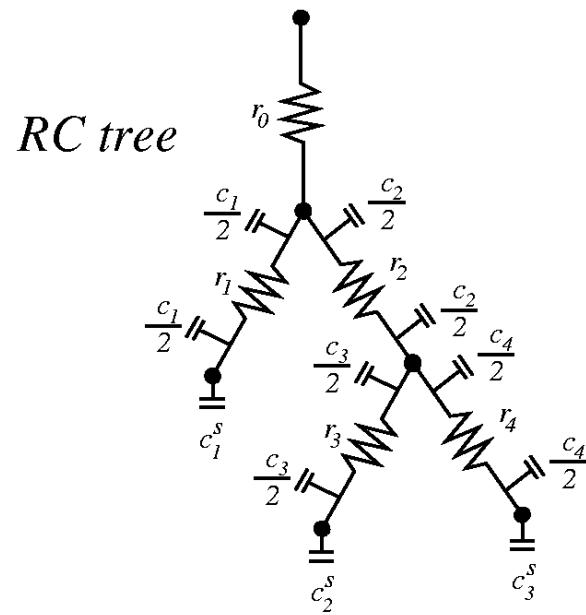
$$t_{03} = r_0(c_1 + c_2 + c_3 + c_4 + c_1^s + c_2^s + c_3^s) + r_2\left(\frac{c_2}{2} + c_3 + c_4 + c_2^s + c_3^s\right) + r_4\left(\frac{c_4}{2} + c_3^s\right).$$



clock tree



delay model



Exact Zero Skew Algorithm

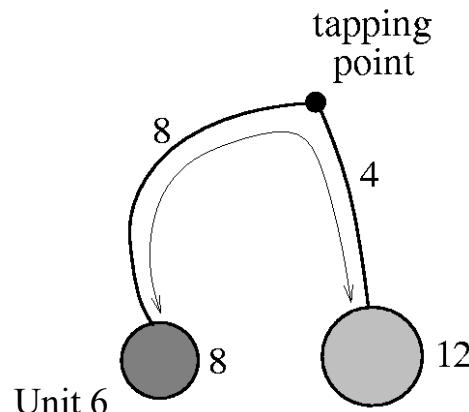
- Tsay, “Exact zero skew algorithm,” *ICCAD*, 1991.
- To ensure the delay from the **tapping point** to leaf nodes of subtrees T_1 and T_2 being equal, it requires that

$$r_1 \left(\frac{c_1}{2} + C_1 \right) + t_1 = r_2 \left(\frac{c_2}{2} + C_2 \right) + t_2$$

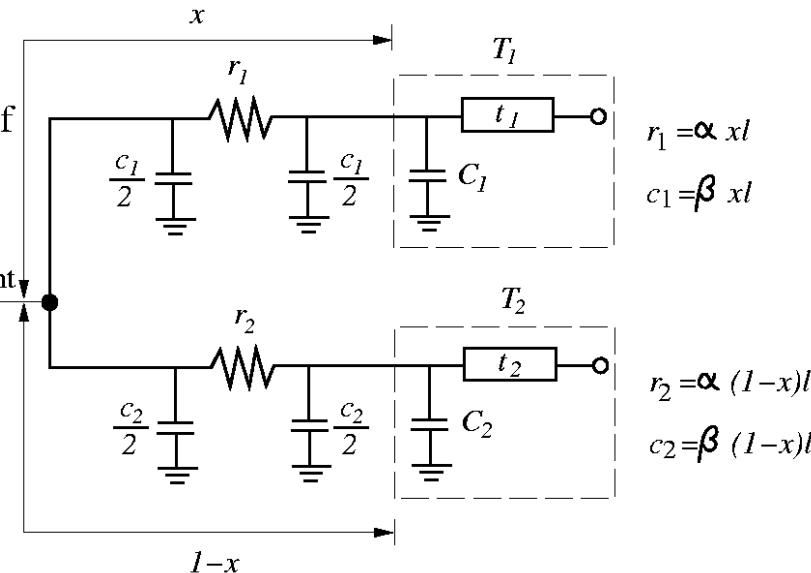
- Solving the above equation, we have

$$x = \frac{(t_2 - t_1) + \alpha l (C_2 + \frac{\beta l}{2})}{\alpha l (\beta l + C_1 + C_2)}$$

- α and β are the per unit values of resistance and capacitance, l the length of the interconnecting wire, $r_1 = \alpha xl$, $c_1 = \beta xl$, $r_2 = \alpha(1-x)l$, $c_2 = \beta(1-x)l$.

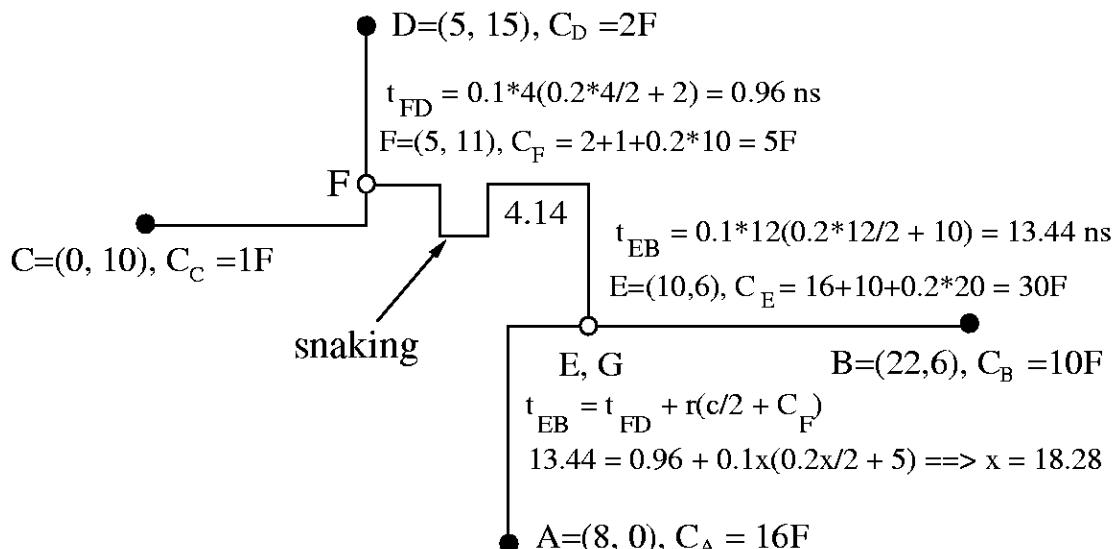


Merging of
two trees



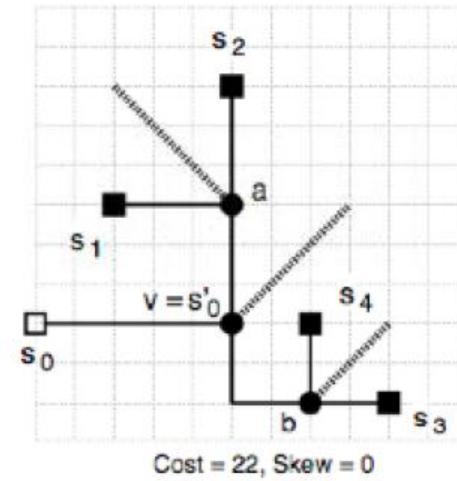
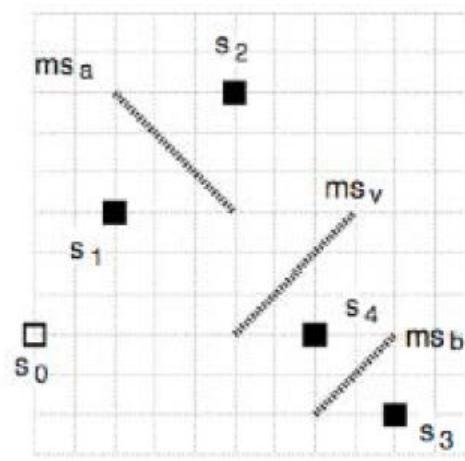
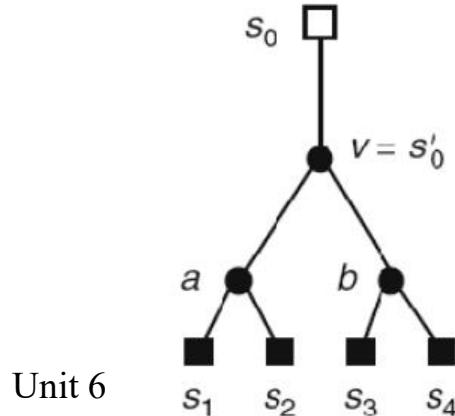
Zero-Skew Computation

- **Balance delays:** $r_1\left(\frac{c_1}{2} + C_1\right) + t_1 = r_2\left(\frac{c_2}{2} + C_2\right) + t_2$.
- **Compute tapping points:** $x = \frac{(t_2 - t_1) + \alpha l(C_2 + \frac{\beta l}{2})}{\alpha l(\beta l + C_1 + C_2)}$, $\alpha(\beta)$: per unit values of resistance (capacitance); l : length of the wire; $r_1 = \alpha xl$, $c_1 = \beta xl$; $r_2 = \alpha(1-x)l$, $c_2 = \beta(1-x)l$.
- If $x \notin [0,1]$, we need **snaking** to find the tapping point.
- Exp: $\alpha=0.1\Omega/unit$, $\beta=0.2F/unit$. (Find tapping points E for A and B , F for C and D , and G for E and F .)



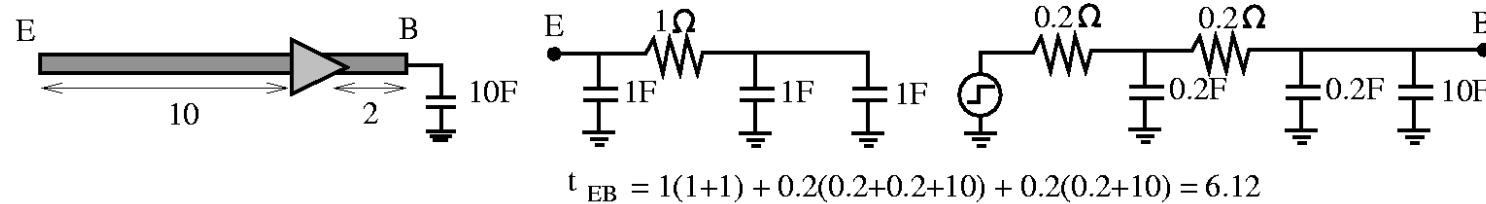
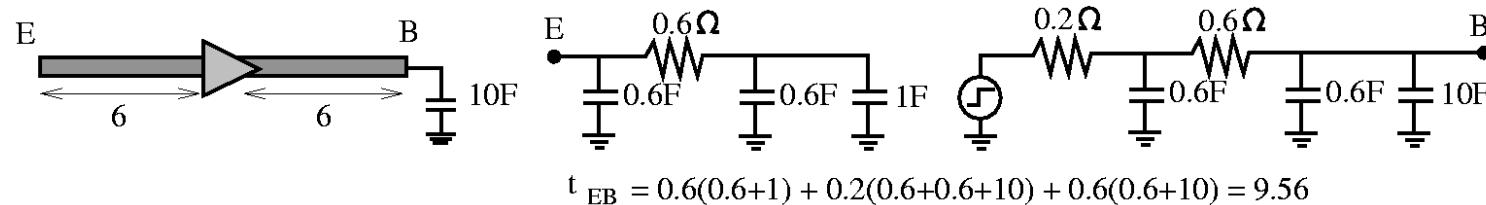
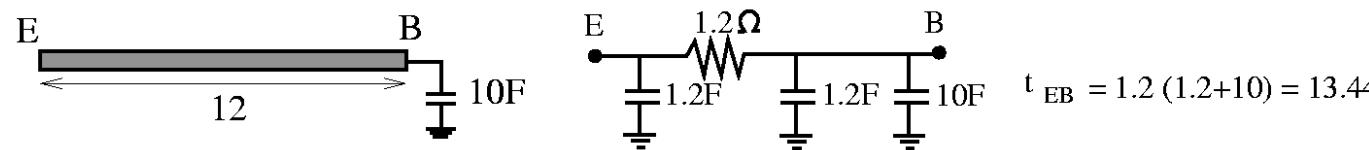
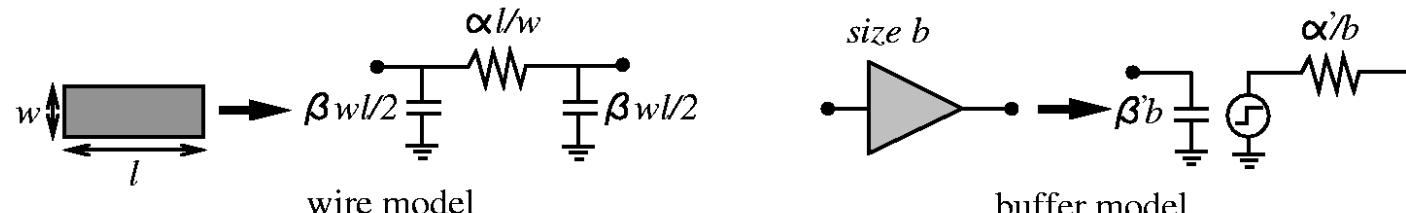
Deferred Merge Embedding (DME)

- Boese & Kahng, ASICON-92; Chao, Hsu, & Ho, DAC-92; Edahiro, NEC R&D, 1991
- Two stages
 - Bottom up: build a **segment** for potential tapping points
 - Top down: determine exact tapping points



Delay Computation for Buffered Wires

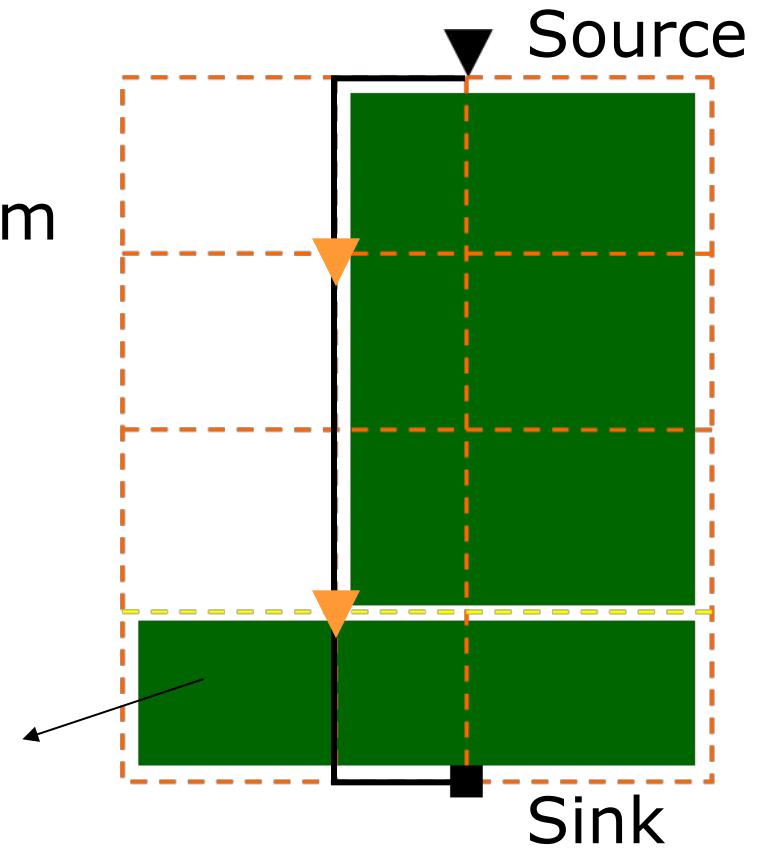
- Wire: $\alpha=0.1\Omega/\text{unit size}$, $\beta=0.2F/\text{unit size}$; buffer: $\alpha'=0.2\Omega/\text{unit size}$, $\beta'=1F/\text{unit size}$; unit-sized wire and buffer.



Buffered Path Construction

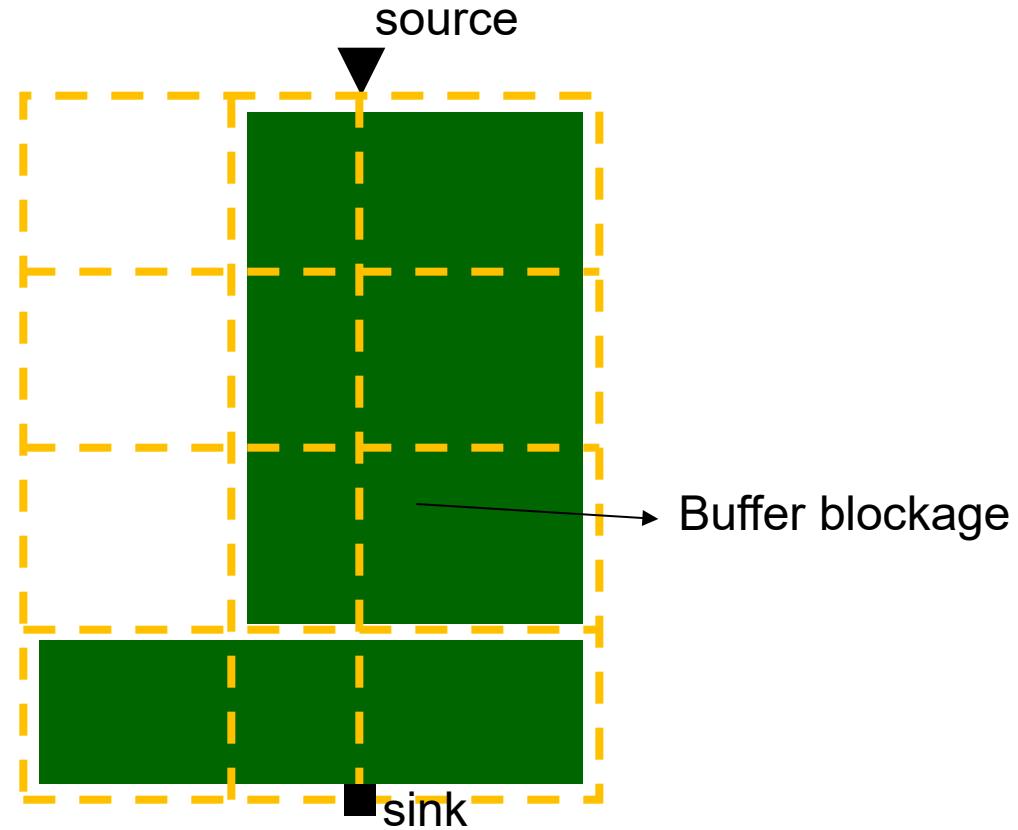
Goal: Find a minimum-delay buffered path from source to sink with blockage avoidance

Buffer
blockage

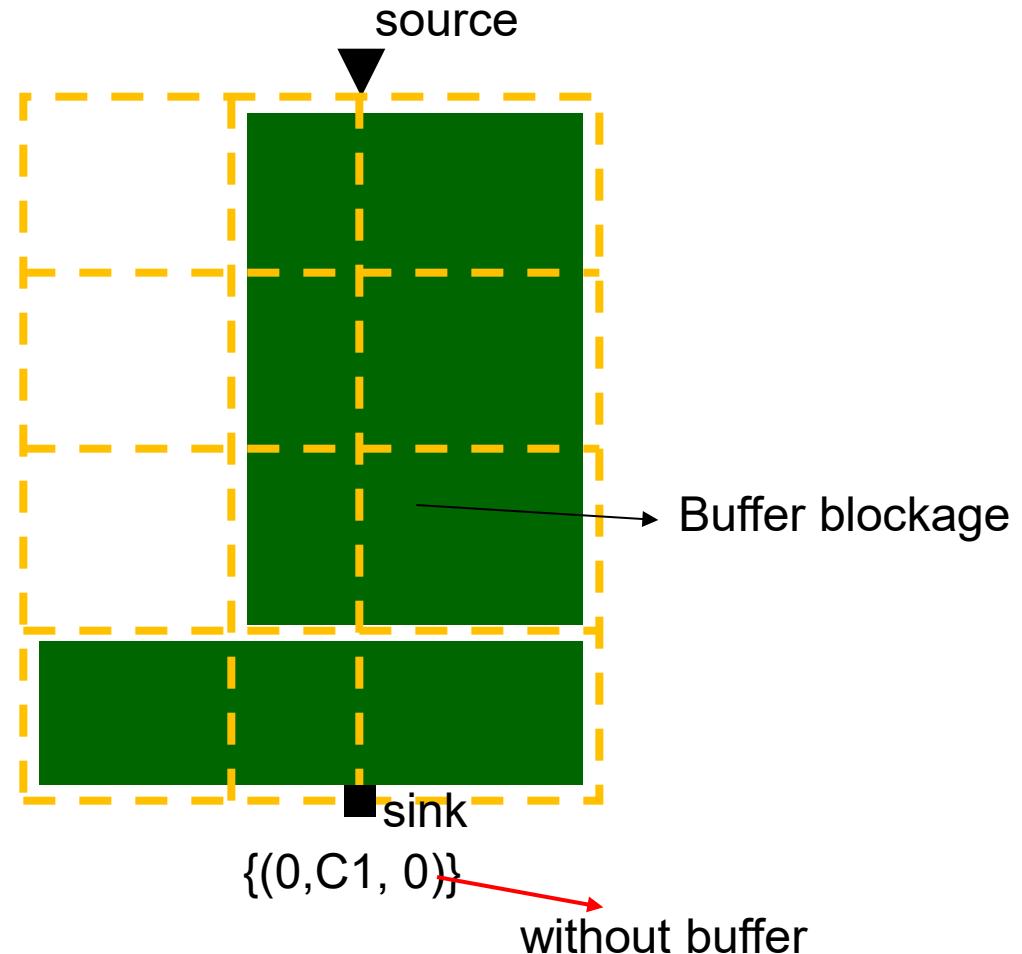


Fast Path Algorithm

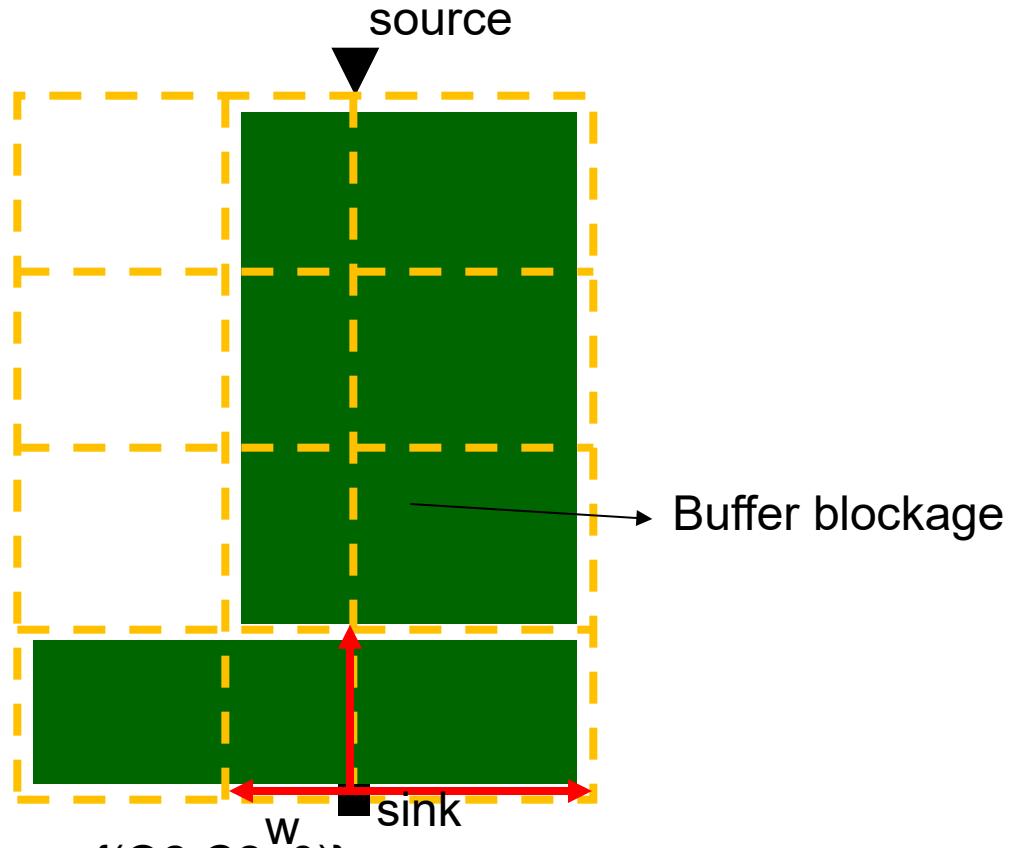
Proposed by
Zhou *et al.*, DAC
1999



Fast Path Algorithm

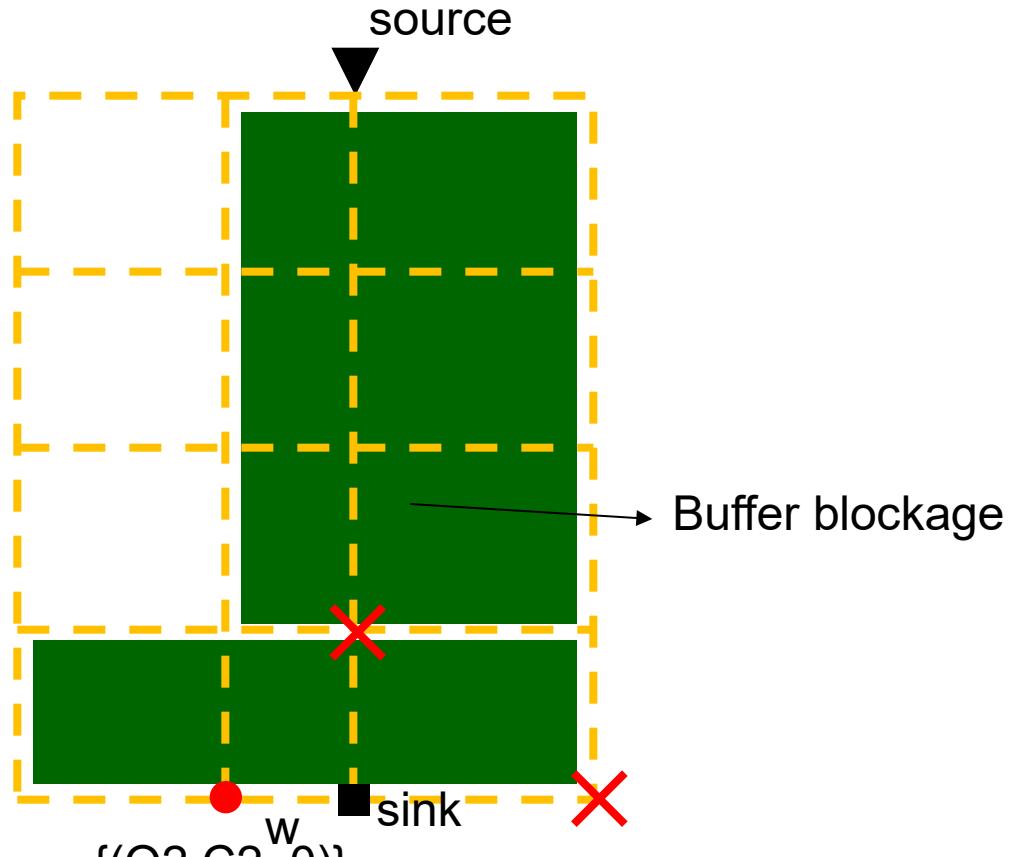


Fast Path Algorithm



C is capacitance of wire w,
Unit ⁶ R is resistance of w

Fast Path Algorithm

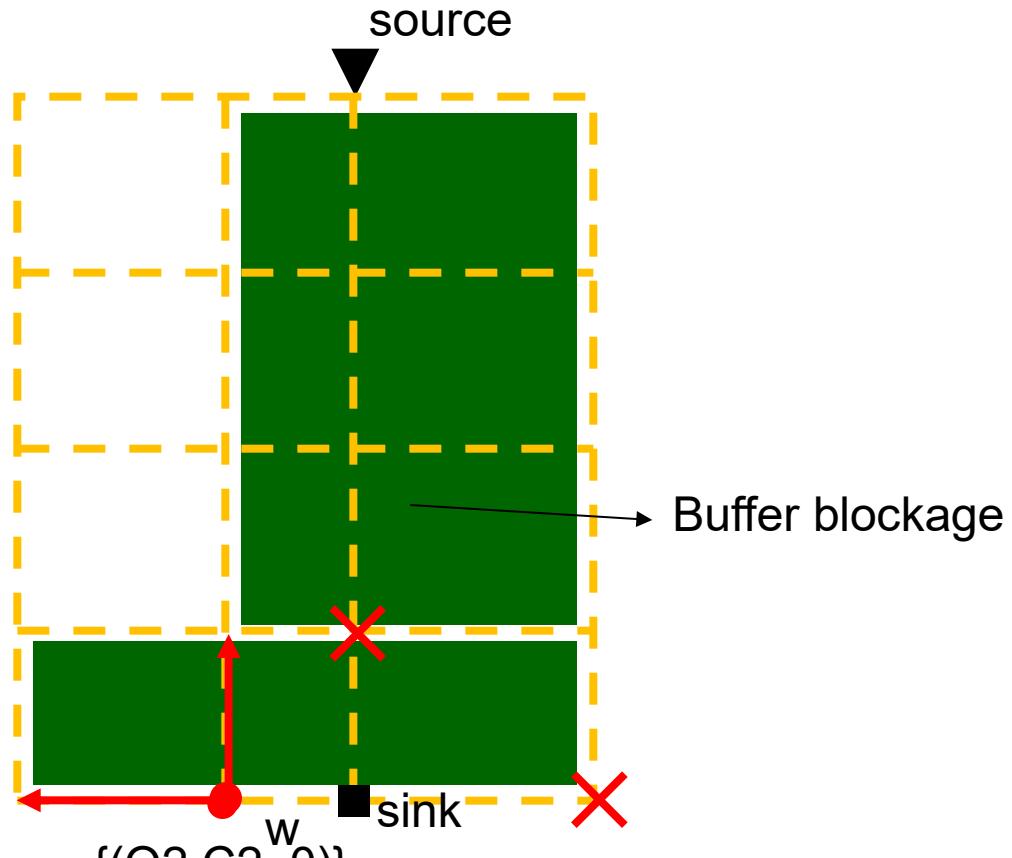


$$Q2 = Q1 + R \left(\frac{C}{2} + C_1 \right)$$

$$C_2 = C + C_1$$

C is capacitance of wire w ,
Unit 6R is resistance of w

Fast Path Algorithm

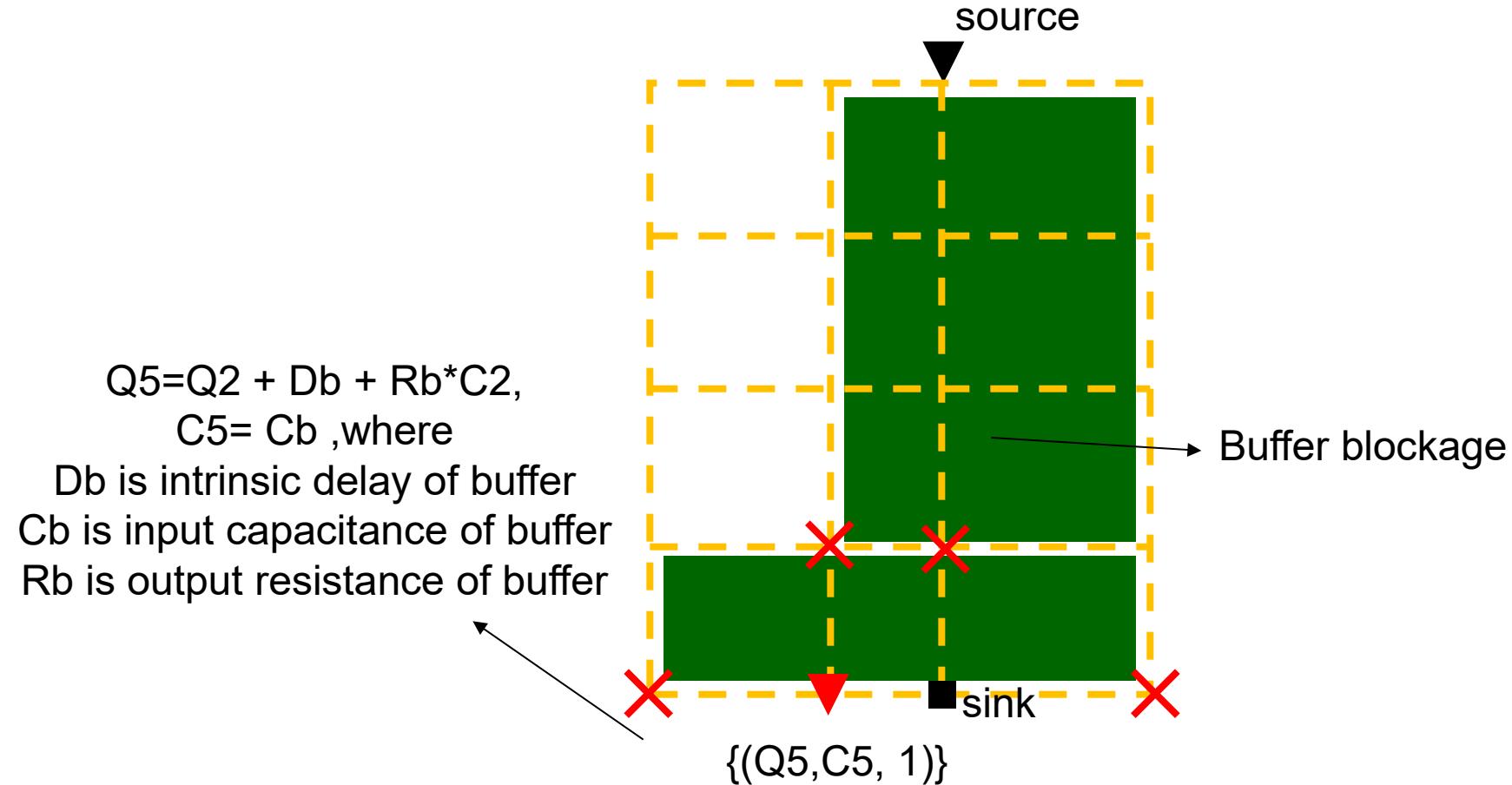


$$Q_2 = Q_1 + R \left(\frac{C}{2} + C_1 \right)$$

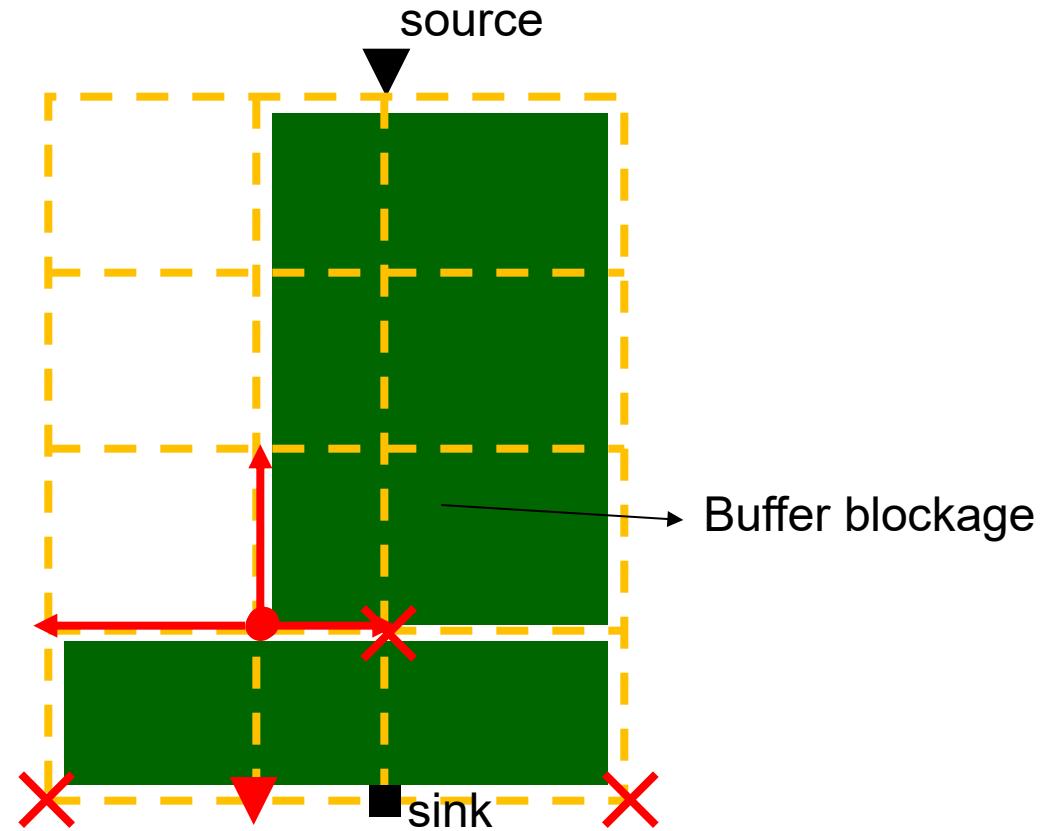
$$C_2 = C + C_1$$

C is capacitance of wire w ,
Unit $^6 R$ is resistance of w

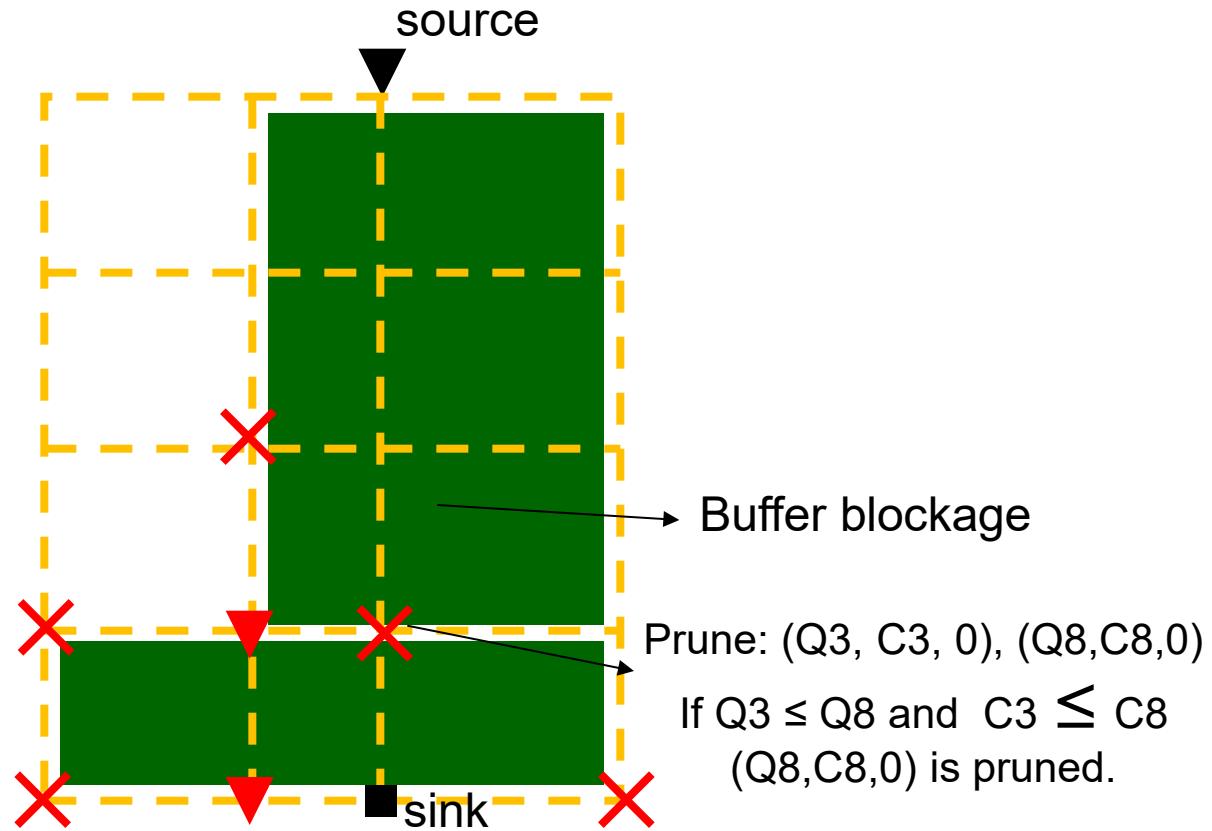
Fast Path Algorithm



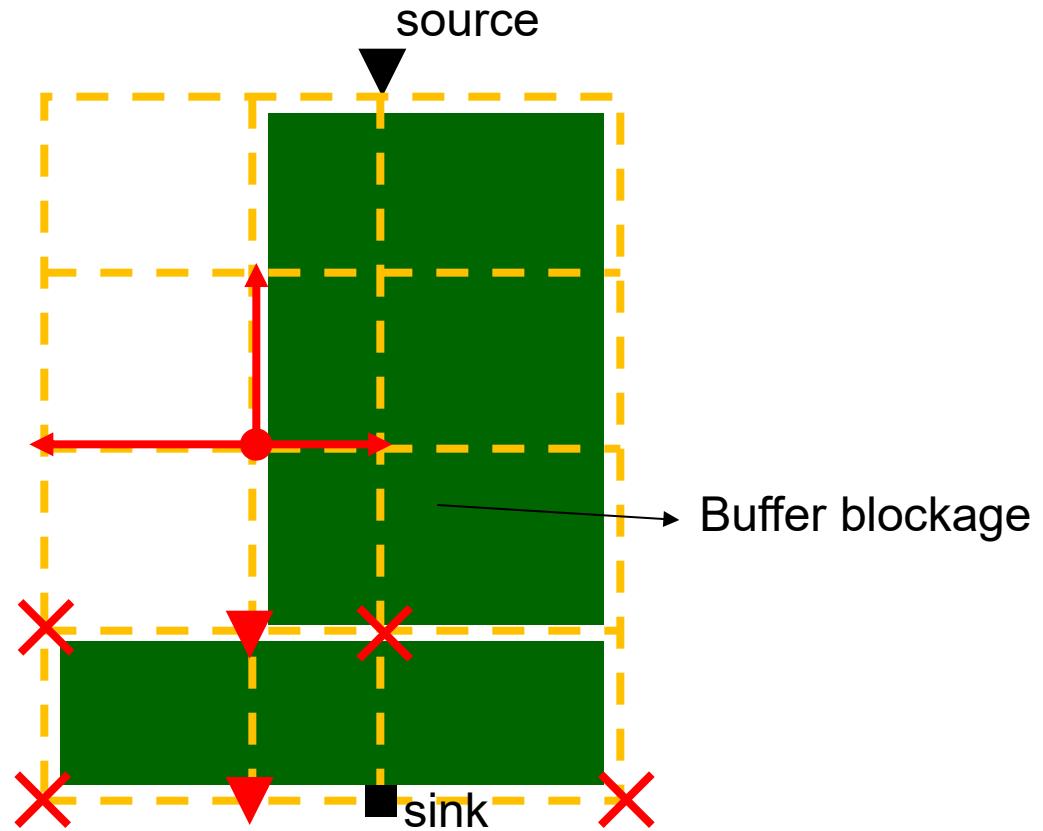
Fast Path Algorithm



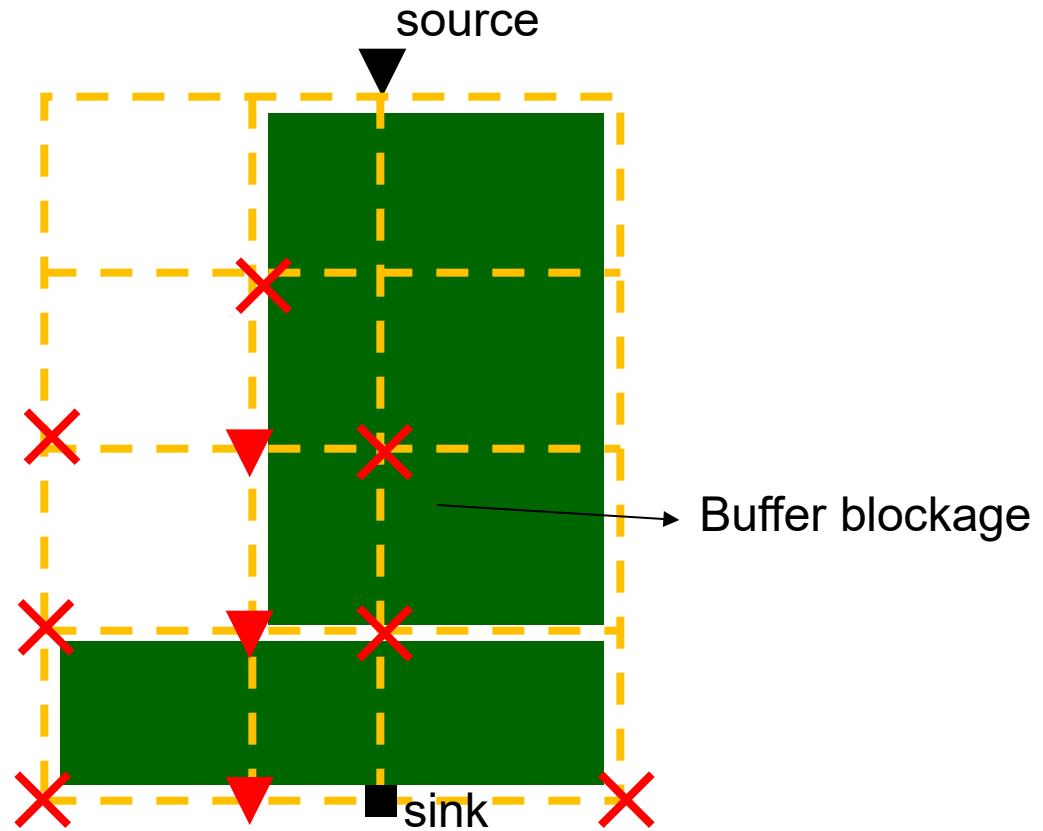
Fast Path Algorithm



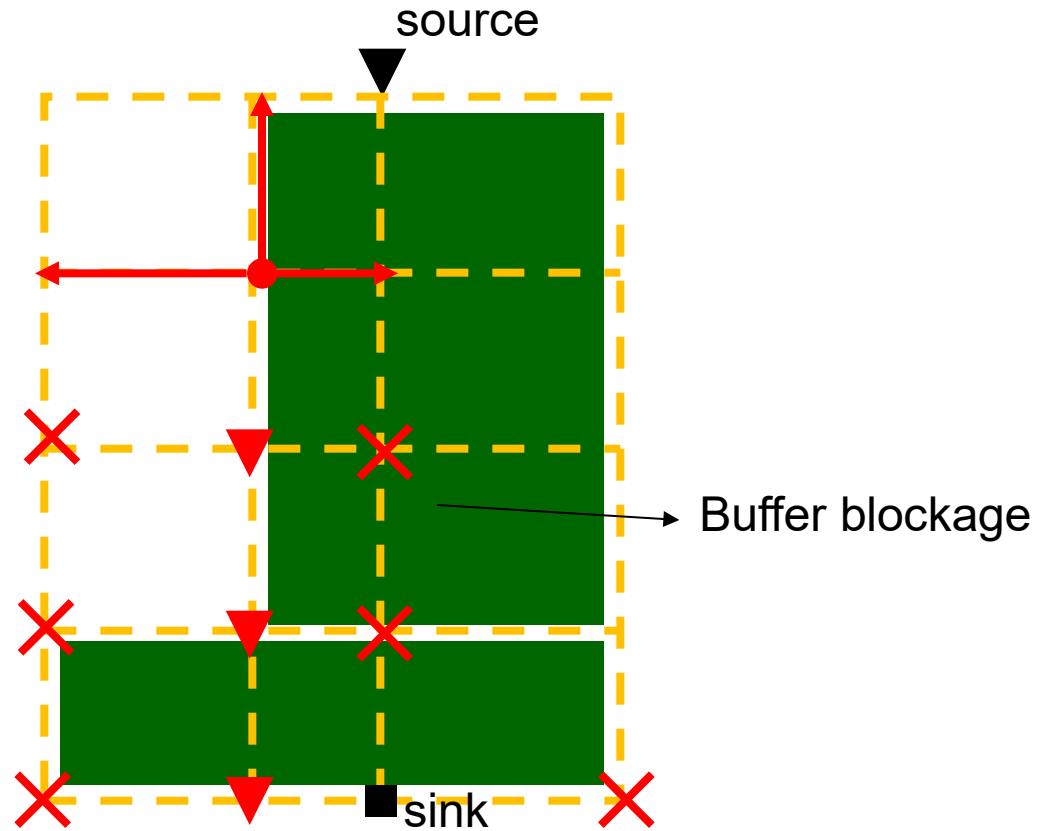
Fast Path Algorithm



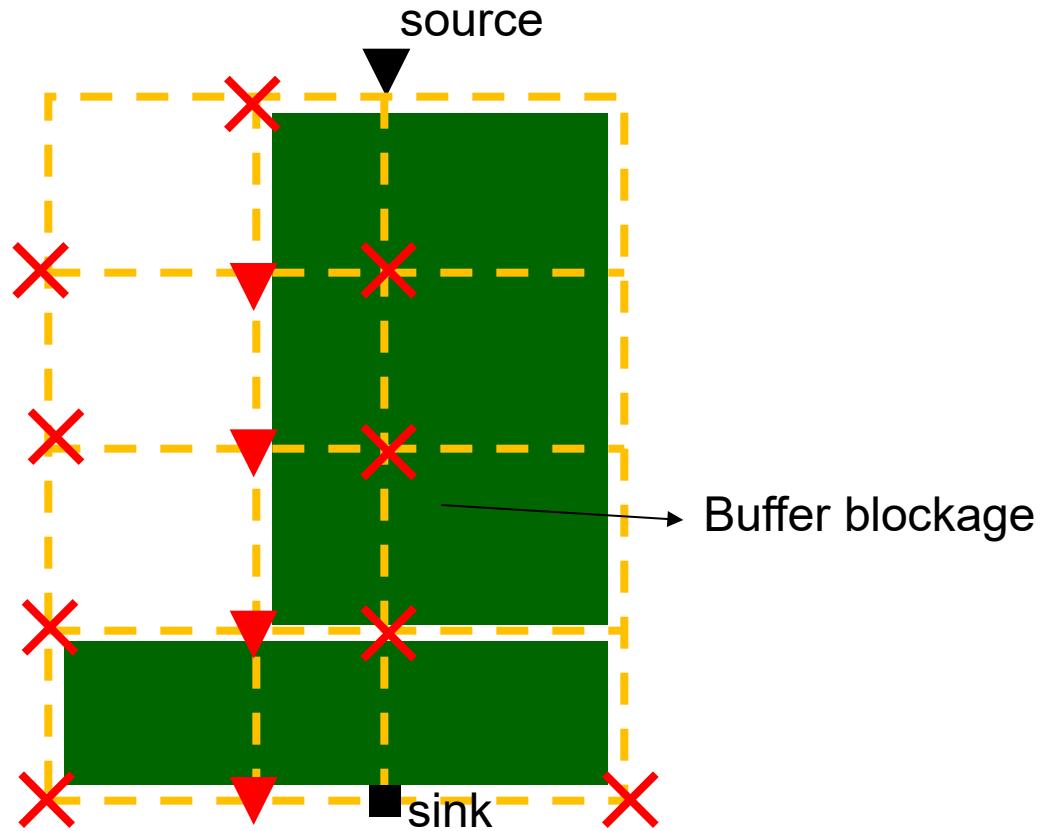
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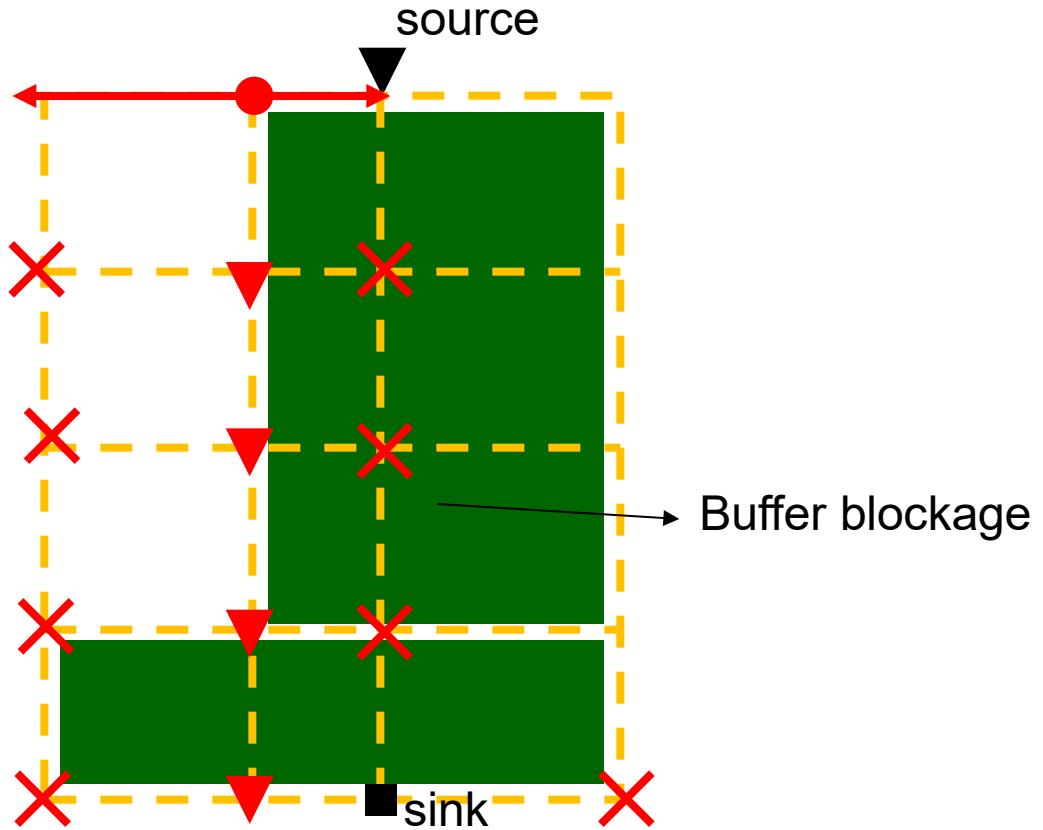
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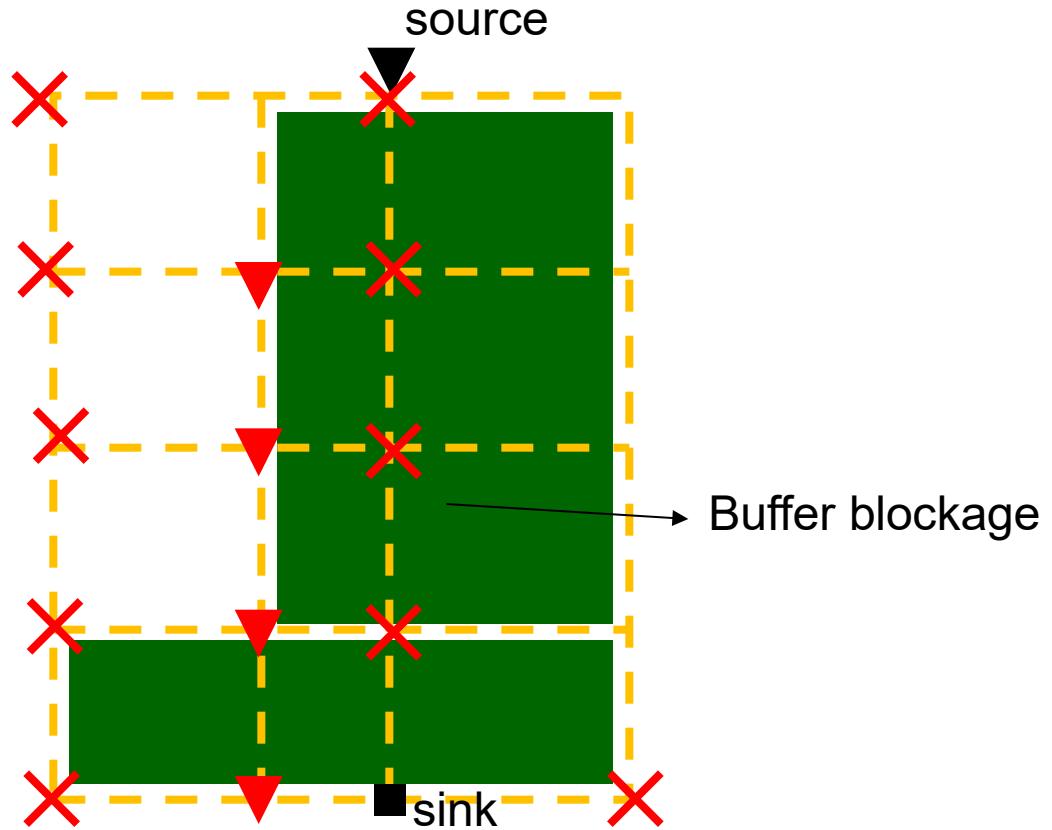
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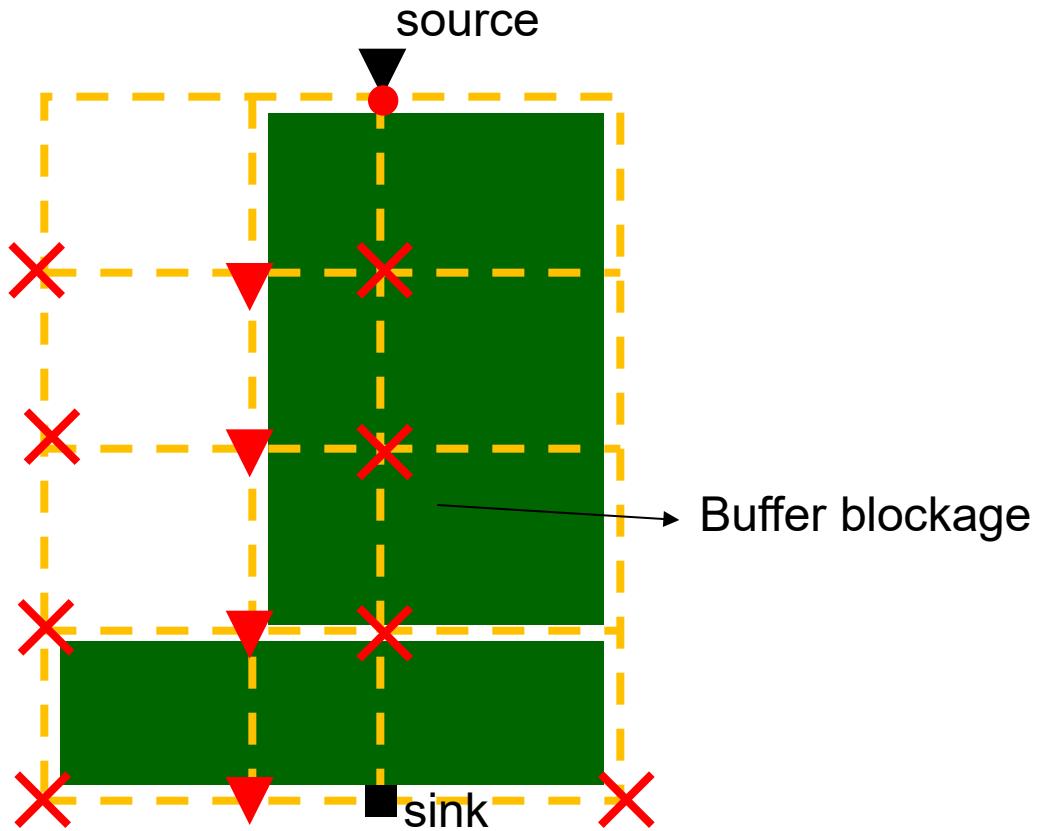
Fast Path Algorithm



Fast Path Algorithm



Fast Path Algorithm



Fast Path Algorithm

