Analytical Approach

- Write the placement objective and possibly placement constraints as *analytical* functions of module coordinates
- Therefore, formulate the placement problem as a mathematical program
- Many variations
  - Different ways to formulate the problem
  - Different ways to solve the resulting mathematical program

HPWL as Analytical Function

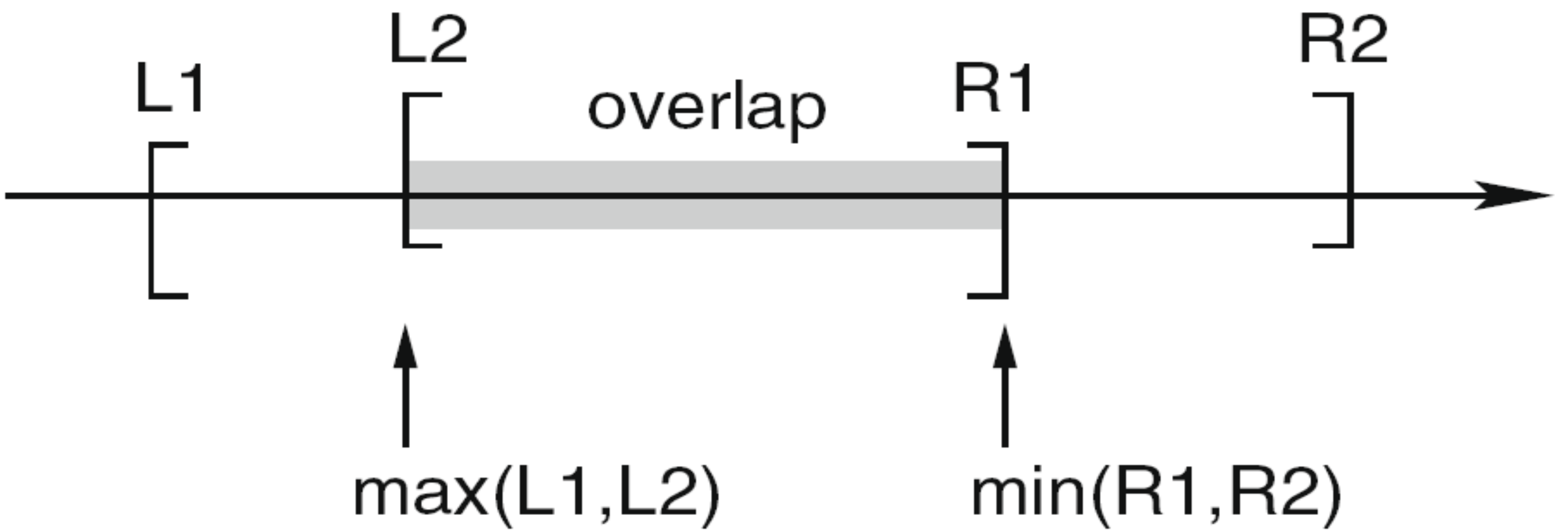
- HPWL of net e

$$
\begin{aligned}
&\mathrm{HPWL}_e(x_1, \ldots, x_n, y_1, \ldots, y_n) \\
&\quad = \left(\max_{i \in e} \{x_i\} - \min_{i \in e} \{x_i\} \right) \\
&\quad + \left(\max_{i \in e} \{y_i\} - \min_{i \in e} \{y_i\} \right)
\end{aligned}
$$

Overlapping Area as Analytical Function

- First, define:

$$\Theta([L1, R1], [L2, R2]) = [\min(R1, R2) - \max(L1, L2)]^{+}$$



Overlapping Area as Analytical Function (cont'd)

- Overlapping area between modules *i* & *j*:

$$\text{Overlap}_{ij}(x_i,\ y_i,\ x_j,\ y_j)$$

$$= \Theta\left(\left[x_i - \frac{\omega_i}{2},\ x_i + \frac{\omega_i}{2}\right], \left[x_j - \frac{\omega_j}{2},\ x_j + \frac{\omega_j}{2}\right]\right)$$

$$\Theta\left(\left[y_i - \frac{h_i}{2},\ y_i + \frac{h_i}{2}\right], \left[y_j - \frac{h_j}{2},\ y_j + \frac{h_j}{2}\right]\right)$$

An Exact (but Impractical) Formulation

Minimize $\sum_{e \in E} c_e \times \mathrm{HPWL}_e(x_1, \ldots, x_n, y_1, \ldots, y_n)$

Subject to $\mathrm{Overlap}_{ij}(x_i, y_i, x_j, y_j) = 0$ for all $i, j \in V$ s.t. $i \neq j$

$$0 \leq x_i - \frac{\omega_i}{2},\ x_i + \frac{\omega_i}{2} \leq W \text{ for all } i \in V$$

$$0 \leq y_i - \frac{h_i}{2},\ y_i + \frac{h_i}{2} \leq H \text{ for all } i \in V$$

Problems of the Exact Formulation

- The functions $Overlap_{ij}\left(x_i, y_i, x_j, y_j\right)$ for $i, j \in V$ are highly nonconvex and not differentiable
- The functions $HPWL_e\left(x_1, \ldots, x_n, y_1, \ldots, y_n\right)$ for $e \in E$ are not differentiable (although convex)
- There are $O(n^2)$ constraints
  - n is up to (tens of) millions in modern designs

Practical Analytical Placement Algorithms

- Global placement
  - Wirelength is approximated by some differentiable and convex functions
  - Nonoverlapping constraints are usually replaced by some simpler constraints to make the module distribution roughly even
- Legalization is performed to eliminate module overlaps
- Detailed placement is applied to refine the solution with a more accurate wirelength metric

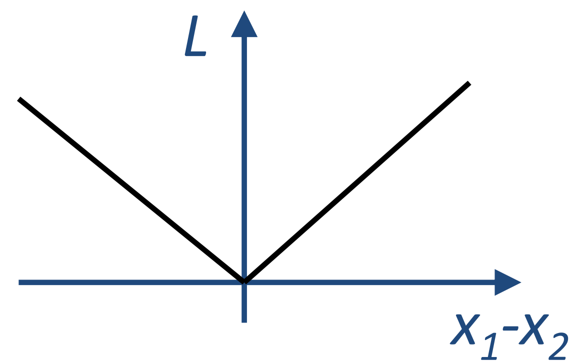
Techniques for Analytical Placement

- **Quadratic** techniques
  - Transformed into a sequence of convex quadratic programs
    - convex quadratic program: a mathematical program with a convex and quadratic objective function and linear constraints
- **Non-quadratic** techniques
  - Transformed into a single general mathematical program

Quadratic Wirelength

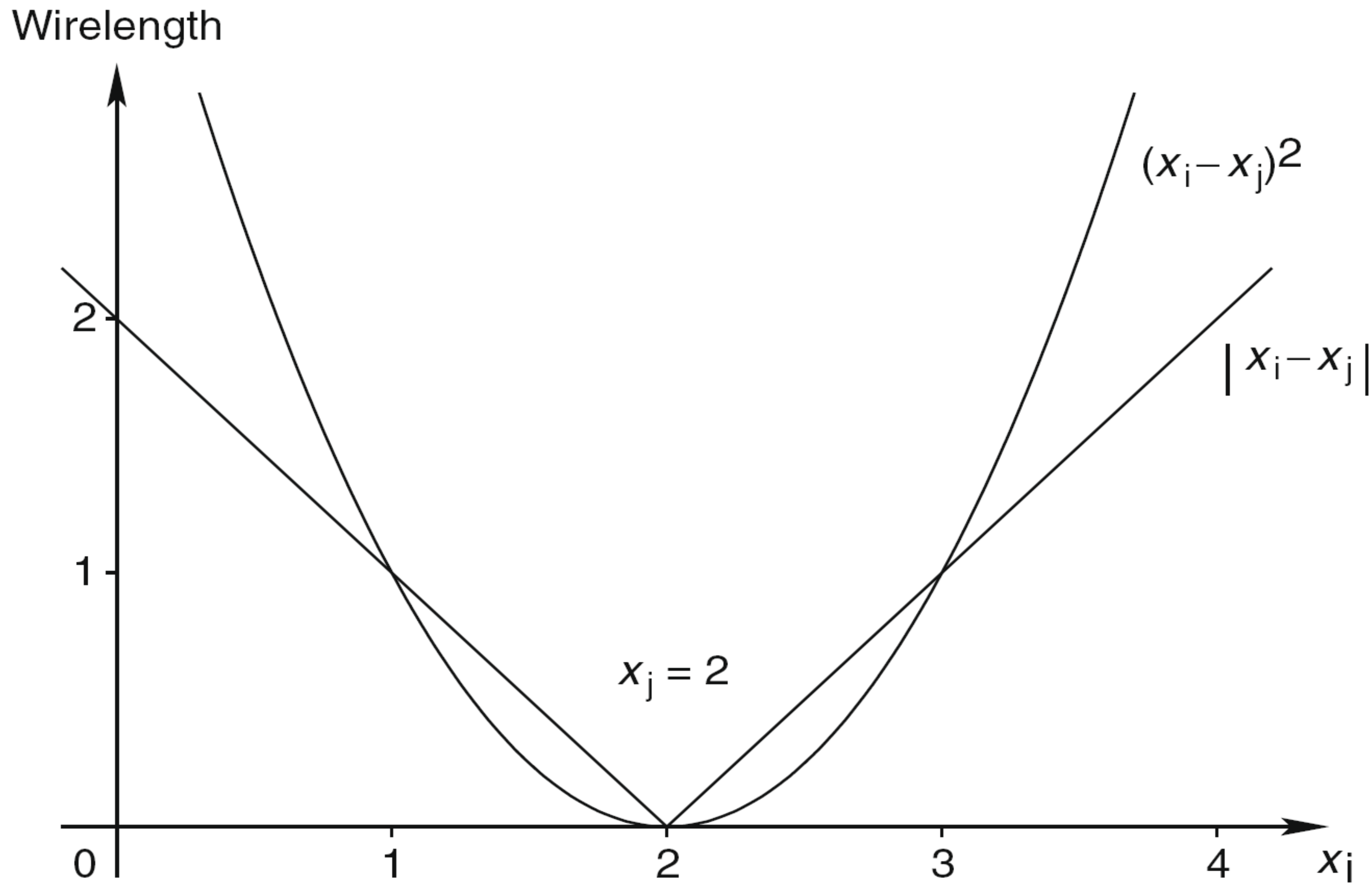
- WL (for 2-pin nets) can be written as a piece-wise linear function: $L = |x_1 - x_2|$ (in x direction)
- WL minimization can be written as a LP

$$\begin{aligned} &\min.\ L \\ &\text{s.t.} \quad x_1 - x_2 \le L \\ &\qquad\ \ x_2 - x_1 \le L \end{aligned}$$



- However, quadratic WL minimization is more common: $\tilde{L} = (x_1 - x_2)^2$
  - Smooth function
  - Convex function → easy to minimize
  - Correlates well with linear WL
  - Often called **quadratic placement**

Quadratic WL vs. Linear WL



Cost Function of Quadratic Placement

Let	(x_i, y_i)	=	Coordinates of the center of cell i
	c_{ij}	=	Weight of the net between cell i and cell j
	$\mathbf{x}, \mathbf{y}$	=	Solution vectors

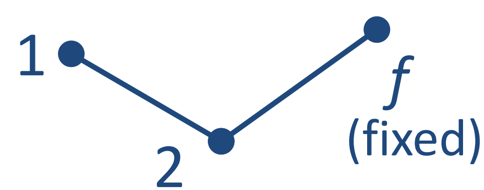
Cost of the net between cell i and cell j

$$\tilde{L}_{\{i,j\}} = \frac{1}{2} c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

Total cost $\tilde{L} = \frac{1}{2} \mathbf{x}^T Q \mathbf{x} + \boldsymbol{d}_x{}^T \mathbf{x} + \frac{1}{2} \mathbf{y}^T Q \mathbf{y} + \boldsymbol{d}_y{}^T \mathbf{y} + \text{const}$

Horizontal cost

$$= \frac{1}{2} c_{12} (x_1 - x_2)^2 + \frac{1}{2} c_{2f} (x_2 - x_f)^2$$



$$= \frac{1}{2} \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} c_{12} & -c_{12} \\ -c_{12} & c_{12} + c_{2f} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -c_{2f} x_f \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \frac{1}{2} c_{2f} x_f^2$$

Notation

- Movable cells: 1,2, …, r
- Fixed cells: r+1, r+2, …, n
- $C = (c_{ij})_{r \times r}$: connectivity matrix among movable cells with $c_{ij} = c_{ji}$ for all i, j in $\{1, 2, \ldots, r\}$
- $D = (d_{ij})_{r \times r}$: diagonal matrix with $d_{ii} = \sum_{j=1}^{n} c_{ij}$ for all i in $\{1, 2, \ldots, r\}$
- $Q = D - C$
- $d_x = (d_{x_1}, d_{x_2}, \ldots, d_{x_r})^T$ with $d_{x_i} = -\sum_{j=r+1}^{n} c_{ij} x_j$
- $d_y = (d_{y_1}, d_{y_2}, \ldots, d_{y_r})^T$ with $d_{y_i} = -\sum_{j=r+1}^{n} c_{ij} y_j$

Solution of Quadratic Placement

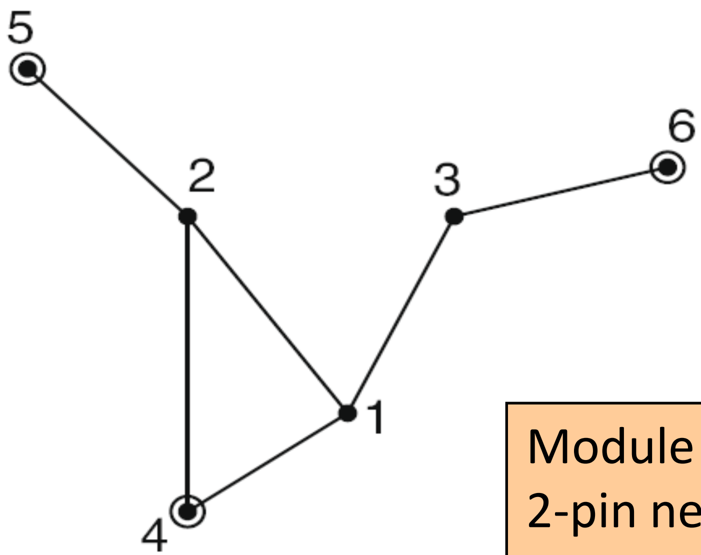
Total cost $\widetilde{L} = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \boldsymbol{d}_x{}^T\mathbf{x} + \frac{1}{2}\mathbf{y}^T Q\mathbf{y} + \boldsymbol{d}_y{}^T\mathbf{y} + \text{const}$

- The problems in x- and y-directions can be separated and solved independently
  - *Ignore non-overlapping and other constraints*
  - Minimize $\widetilde{L}_x = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \boldsymbol{d}_x{}^T\mathbf{x}$
  - Q can be proved to be positive and definite => the cost function is convex
  - Minimum solution can be found by setting derivatives to 0:

$$Q\mathbf{x} + \boldsymbol{d}_x{}^T = \mathbf{0}$$

Force Interpretation of Quadratic WL

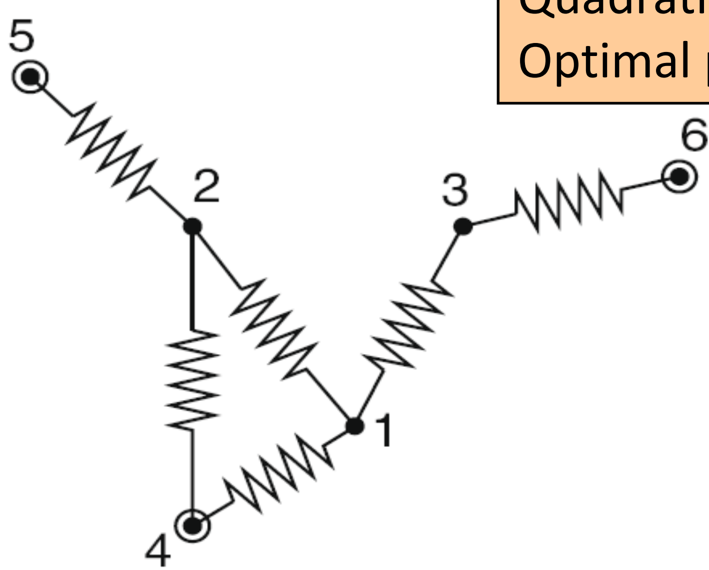
Circuit



- ◉ Fixed module
- • Movable module

Module → Object
2-pin net → Spring
Quadratic WL → Spring potential energy
Optimal placement → Force equilibrium

Spring system



- ◉ Fixed object
- • Movable object

Force Calculation

- Hooke's Law:
  - Force = Spring Constant x Displacement
- Can consider forces in x- and y-direction separately

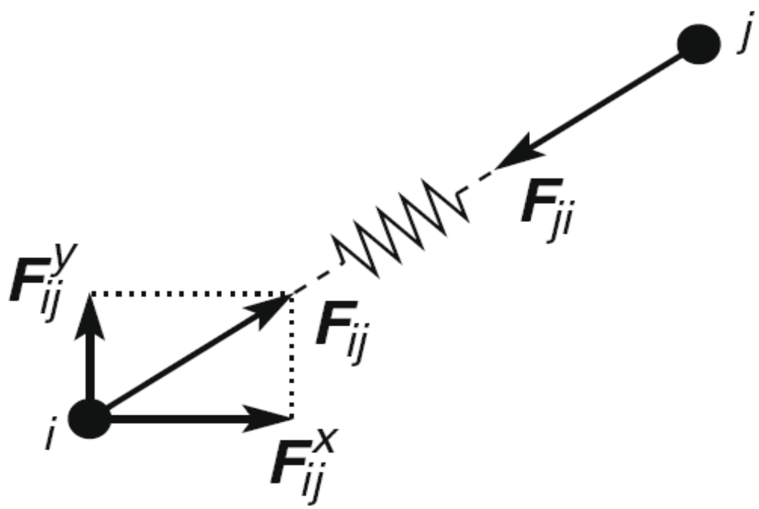
$$\text{Distance } d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

$$\text{Net Cost } c_{\{i,j\}}$$

$$\left|\mathbf{F_{ij}}\right| = c_{\{i,j\}} \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

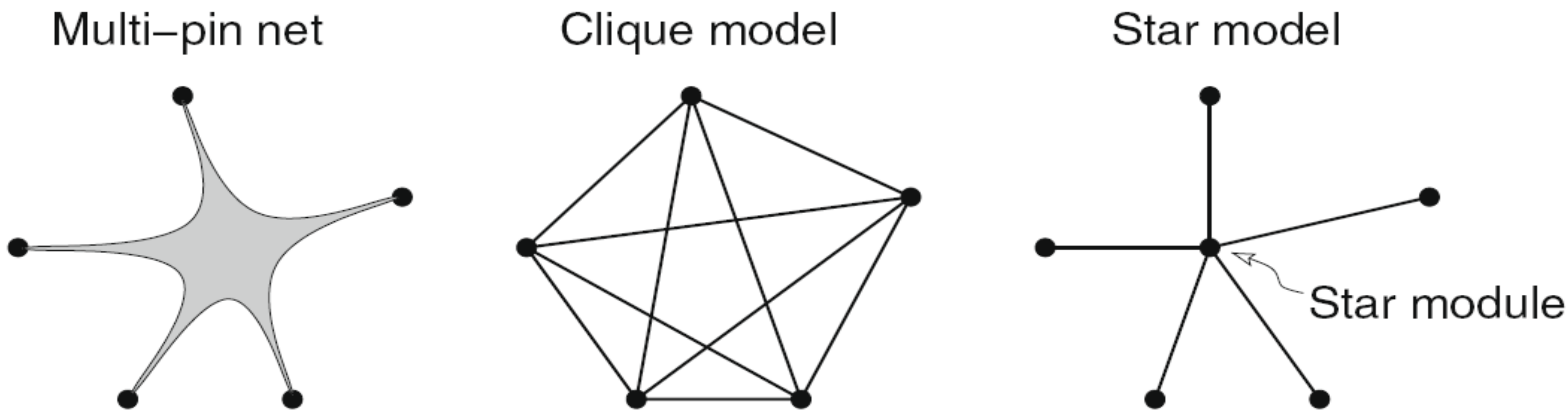
$$\left|\mathbf{F_{ij}^x}\right| = c_{\{i,j\}} \times \left|x_j - x_i\right|$$

$$\left|\mathbf{F_{ij}^y}\right| = c_{\{i,j\}} \times \left|y_j - y_i\right|$$



Net Models for Multi-Pin Nets

- Multi-pin net is modeled as several 2-pin nets



Hybrid net model (best)

# pins	Net Model
2	Clique
3	Clique
4	Star
5	Star
6	Star
…	…

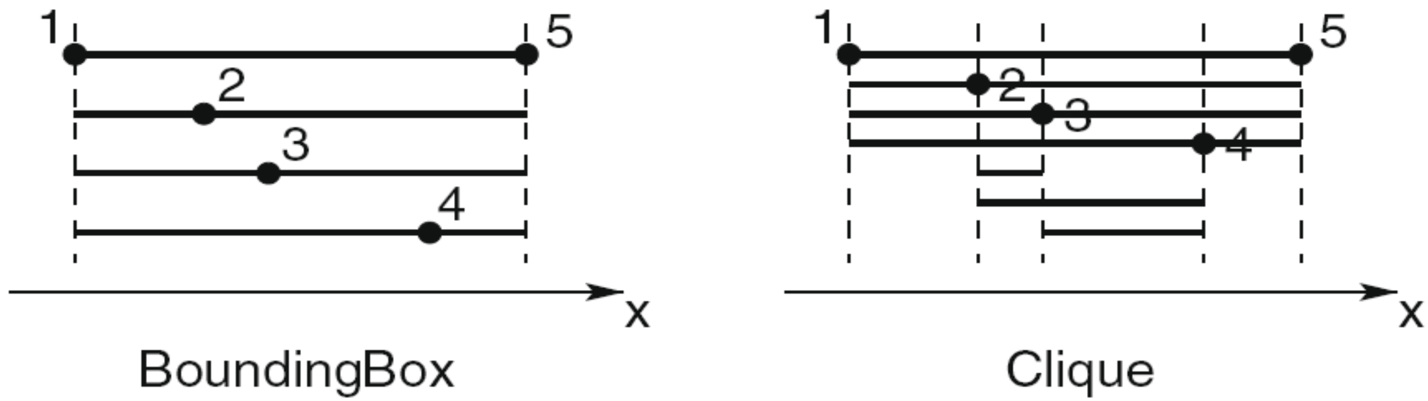
Linearization Method in GORDIAN-L [DAC-91]

- Linear WL (star model): $L^{star} = \sum_{e \in E} \sum_{i \in e} |x_i - x_e|$

- Consider the function: $\tilde{L}^{star} = \sum_{e \in E} \sum_{i \in e} \frac{(x_i - x_e)^2}{g_{ie}}$
  - Exact if $g_{ie} = |x_i - x_e|$
  - Quadratic if g_{ie}'s are set to constant
  - L^{star} can be optimized iteratively by setting g_{ie} in current iteration according to the coordinates of previous iteration
  - In practice, set $g_{ie} = \sum_{i \in e} |x_i - x_e|$ for all $i \in e$

BoundingBox Net Model in Kraftwerk [ICCAD-06]

- Make use of preceding linearization idea
- Accurately model HPWL
- For a *k*-pin net:

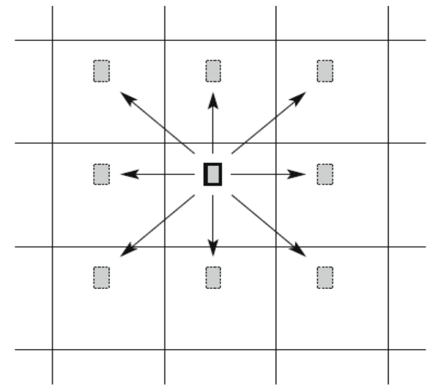


$$\tilde{L}^{BB} = \frac{1}{2} \sum_{\{i,j\} \in N} \omega_{\{i,j\}} \times (x_i - x_j)^2 \quad \text{where} \quad \omega_{\{i,j\}} = \frac{2}{k-1} \times \frac{1}{l_{\{i,j\}}}$$

If $l_{\{i,j\}}$ is set to $|x_i - x_j|$ for all $\{i,j\} \in N$, $\tilde{L}^{BB} = |x_1 - x_k|$

Iterative Local Refinement (ILR) in FastPlace [ISPD-04]

- Mitigate the inaccuracy of quadratic WL by refining global placement with accurate WL metric
  - Divide the placement region into bins by a regular grid
  - Examine modules one by one
  - Tentatively move a module to its eight adjacent bins



  - Compute a score for each tentative move
    - HPWL reduction
    - Cell densities at the source and destination bins
  - Take the move with the highest positive score (no move if all scores are negative)
  - Repeat until there is no significant improvement