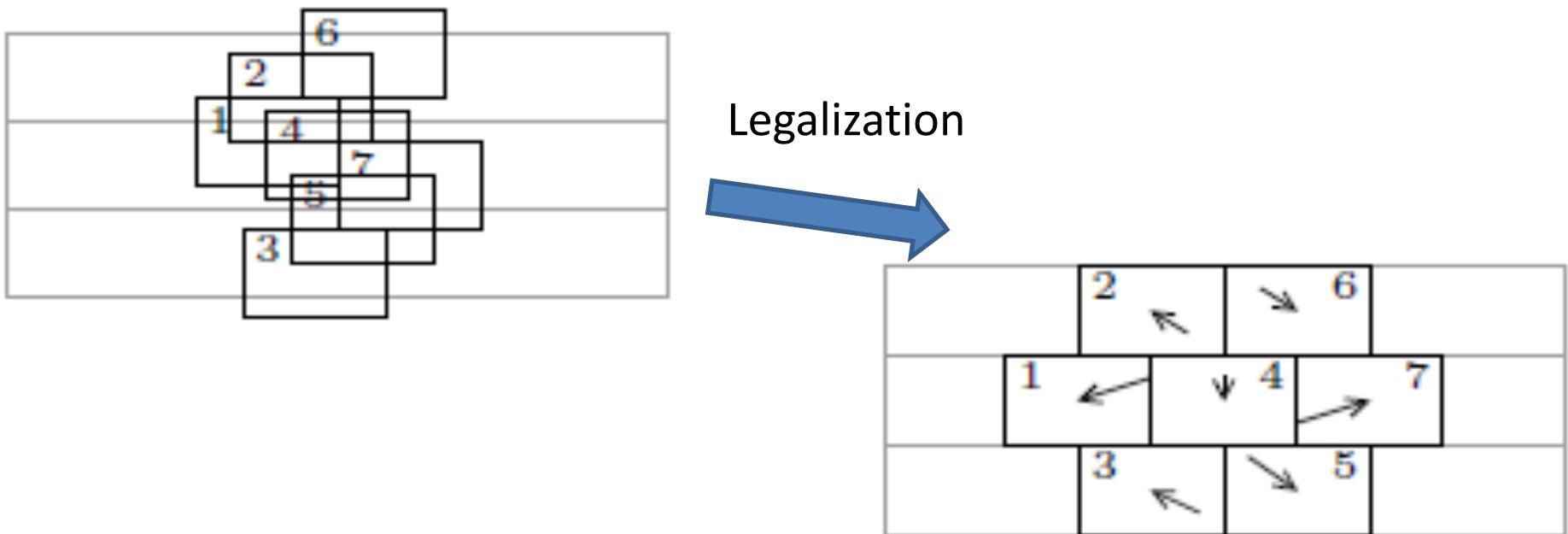


Legalization

- Legalization is a process to eliminate all overlaps (and assign cells to rows) by perturbing cells/modules as little as possible

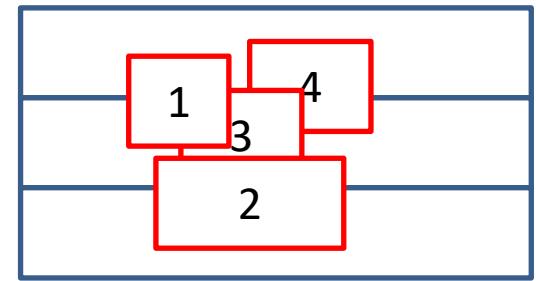


Greedy Methods

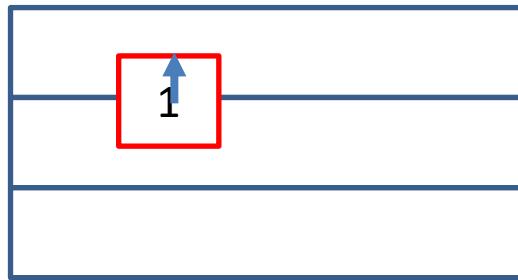
- Legalize one cell at a time
- **Tetris** [Hill, US Patent 6370673, 2002]
 - Sort cells in ascending x-coordinates
 - Pack cells to the left one at a time into a row so as to minimize the displacement
 - Already legalized cells are never moved again
 - Pros: Commonly used and extremely fast
 - Cons: May result in very uneven row length and may fail to pack all cells inside the placement region
- **Abacus** [Spindler, ISPD 2008]
 - Similar to Tetris
 - **Already legalized cells can be moved**
 - Get better results than Tetris

Example for Tetris

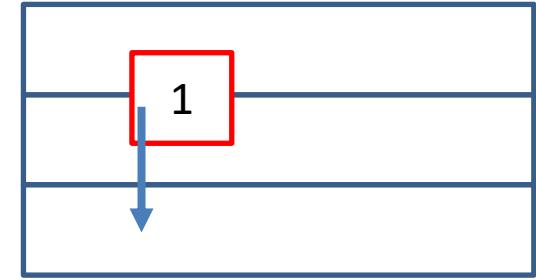
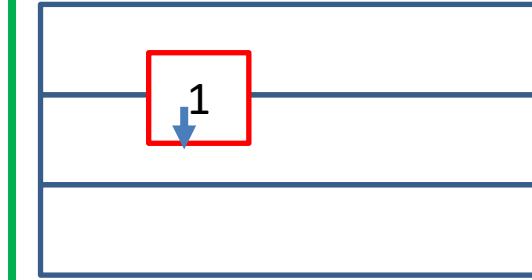
Global
Placement



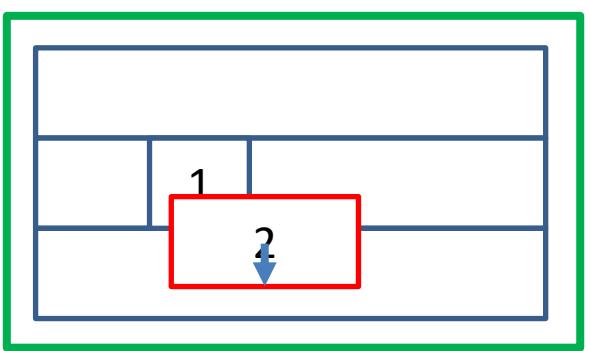
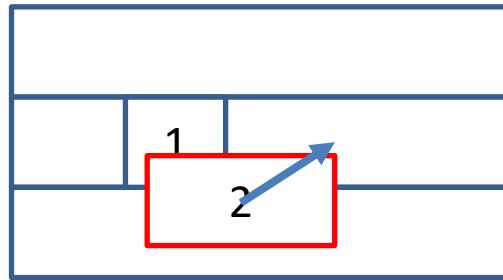
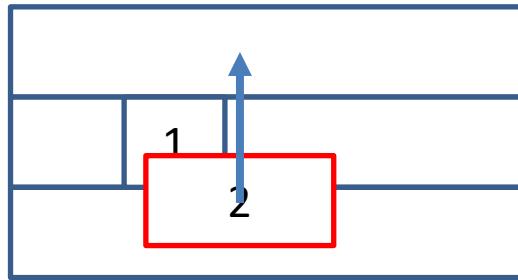
Legalize
cell 1:



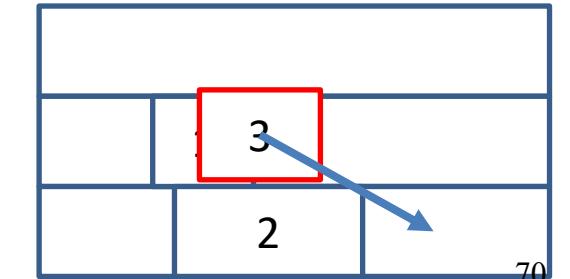
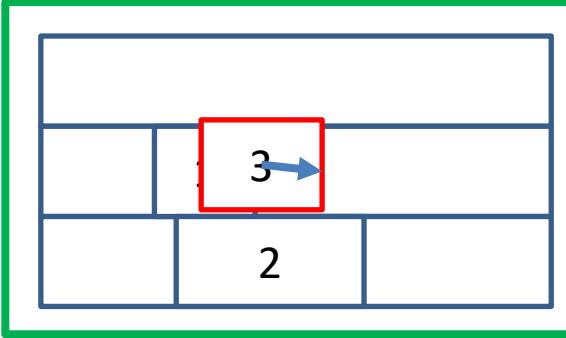
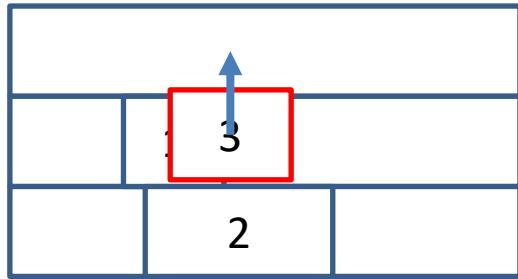
Pick Nearest Movement



Legalize
cell 2:



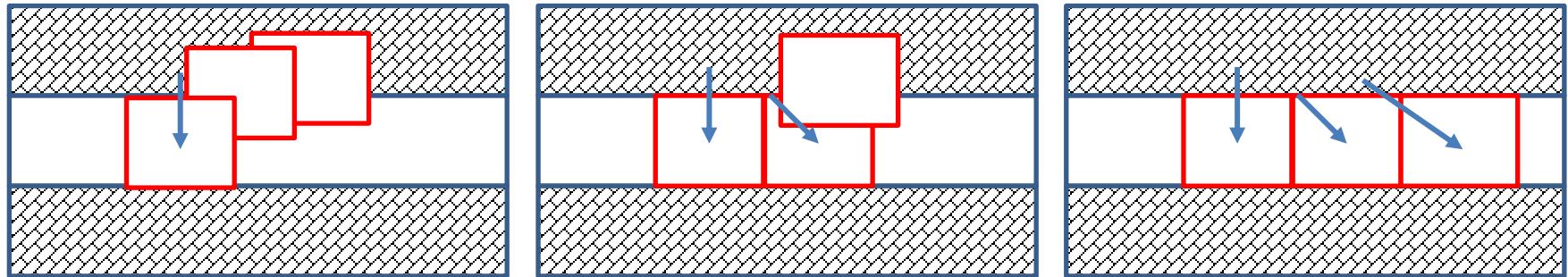
Legalize
cell 3:



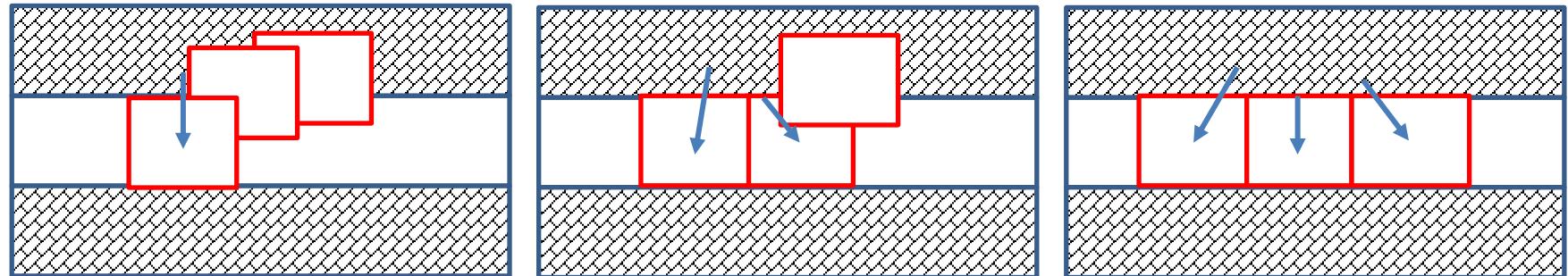
Abacus

- Similar to Tetris: sort cells and legalize one cell at a time
- Legalization of one cell: move the cell over the rows, and place the cell to the best/nearest row
- **PlaceRow** (difference from Tetris): move already legalized cells within one row to minimize total movement

Tetris:



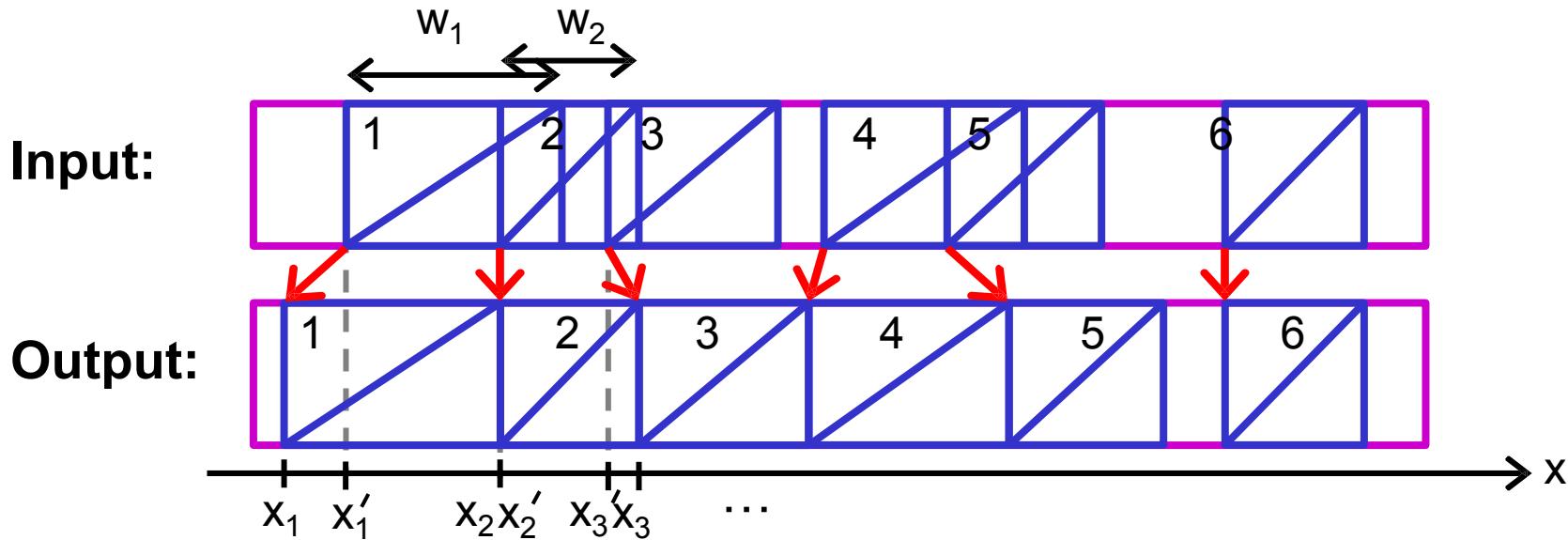
Abacus:



PlaceRow

Input: one row with N cells, x -pos of cells: global placement (x_i')

Output: new (legal) x -pos of cells (x_i) such that the overlap is removed and the total quadratic movement is minimized



QP:

$$\min \sum_{i=1}^N e_i (x_i - x_i')^2 \quad \text{s.t.} \quad \underbrace{x_i \geq x_{i-1} + w_{i-1}}_{\text{no overlap}} \leftarrow \text{width}$$

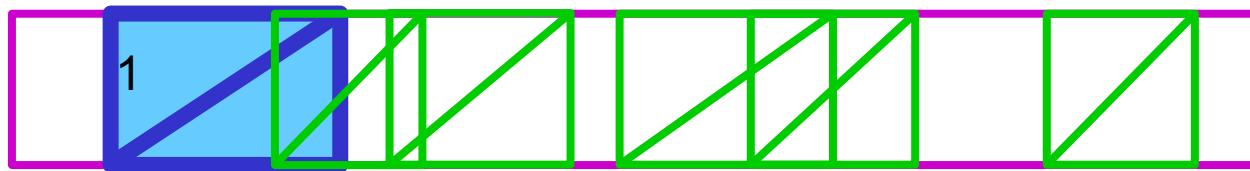
$\uparrow \text{weight}$ $\uparrow \text{global x-pos}$
 $\uparrow \text{legal x-pos}$

PlaceRow: Dynamic Programming

PlaceRow:

- Solve QP by dynamic programming approach:
solve sub problems optimally to obtain final solution
- Process cells from left to right

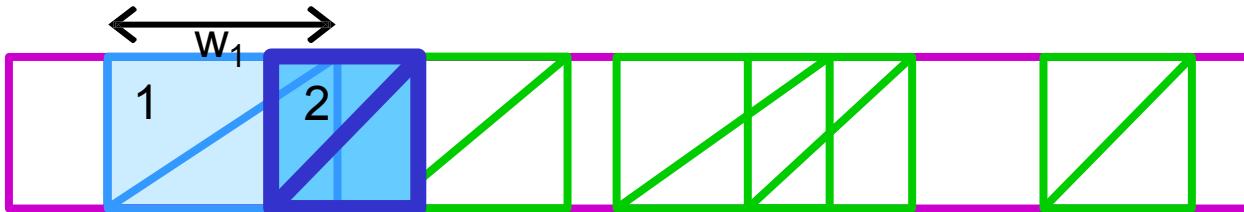
Cell 1:



first cell → do not move

PlaceRow: Cell 2

Cell 2:



overlap with previous cell? yes → **cluster** with previous cell

Clustering process:

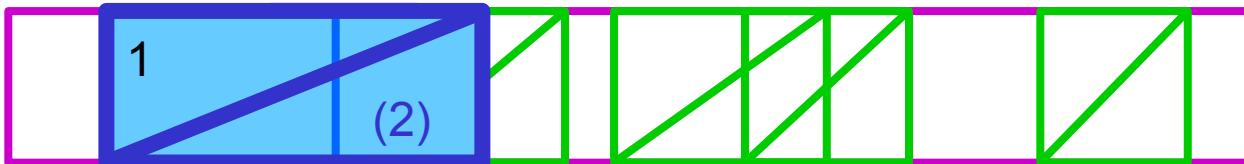
$$x_2 = x_1 + w_1$$

$$\min e_1(x_1 - x'_1)^2 + e_2(x_2 - x'_2)^2 \rightarrow \min \underbrace{(e_1 + e_2)}_{\text{new } e_1} \left(x_1 - \underbrace{\frac{e_1 x_1' + e_2 (x_2' - w_1)}{e_1 + e_2}}_{\text{new } x'_1} \right)^2$$

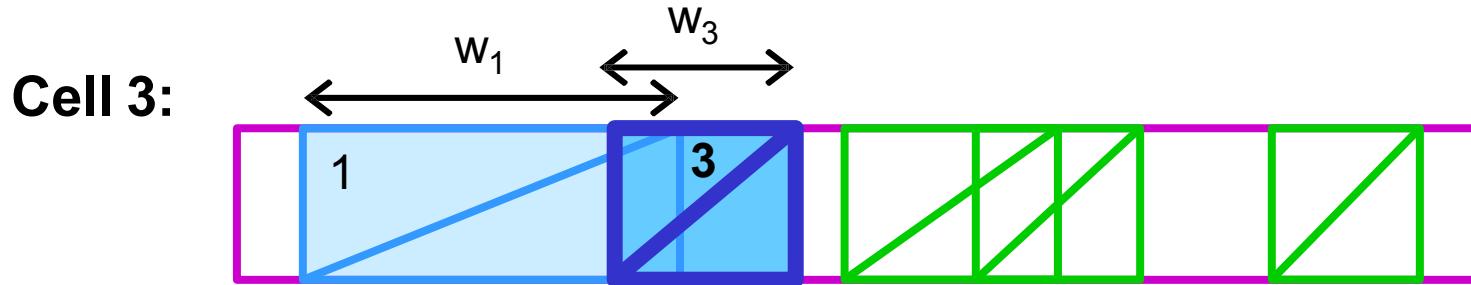
Update x'_1 , e_1 , and w_1 :

$$x'_1 \leftarrow \frac{e_1 x_1' + e_2 (x_2' - w_1)}{e_1 + e_2} \quad e_1 \leftarrow e_1 + e_2 \quad w_1 \leftarrow w_1 + w_2$$

Result: $\min e_1(x_1 - x'_1)^2 \rightarrow x_1 = x'_1$



PlaceRow: Cell 3



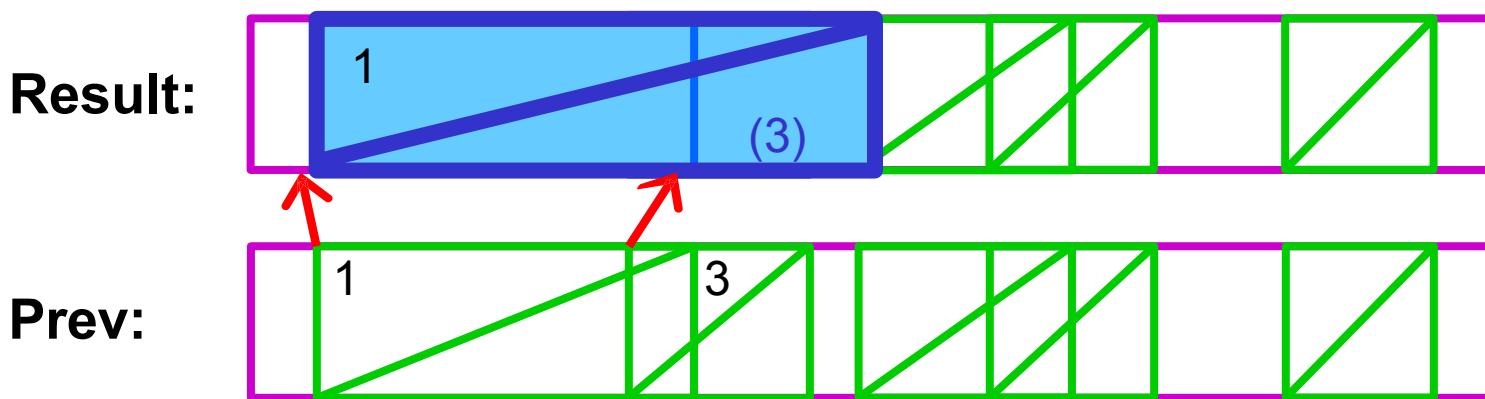
Overlap with previous cell? Yes → **cluster** with previous cell

Update x'_1 , e_1 , and w_1 :

$$x'_1 \leftarrow \frac{e_1 x_1 + e_3 (x'_3 - w_1)}{e_1 + e_3} \quad w_1 \leftarrow w_1 + w_3$$

$$e_1 \leftarrow e_1 + e_3$$

Move cell 1: $\min e_1 (x_1 - x'_1)^2 \rightarrow x_1 = x'_1$



PlaceRow: Cell 4

Cell 4:

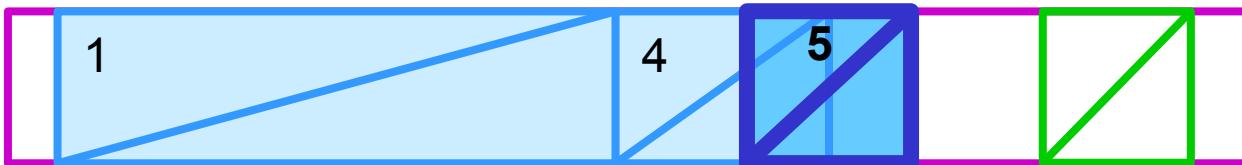


Overlap with previous cell? No

→ **no clustering, no movement**

PlaceRow: Cell 5

Cell 5:



Overlap with previous cell? **yes** → **cluster** with previous cell 4

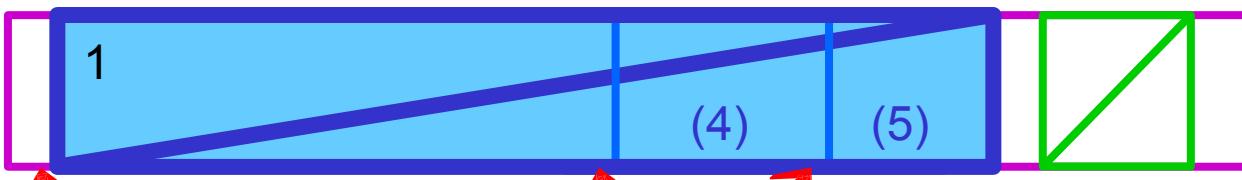
$$\text{Update } x_4', e_4, \text{ and } w_4: \quad x_4' \leftarrow \frac{e_4 x_4 + e_5 (x_5 - w_4)}{e_4 + e_5} \quad e_4 \leftarrow e_4 + e_5 \quad w_4 \leftarrow w_4 + w_5$$

Move cell 4: $\min e_4 (x_4 - x_4')^2 \rightarrow x_4 = x_4'$

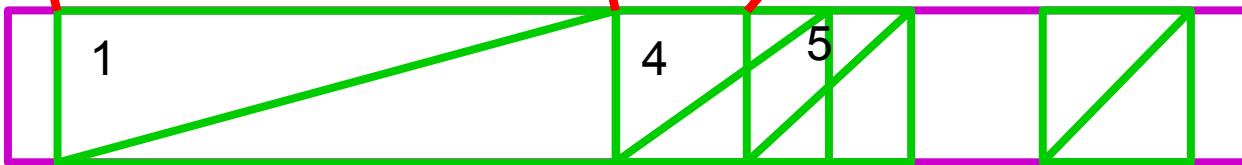


Overlap with previous cell? **yes** → **cluster** with previous cell 1

Result:

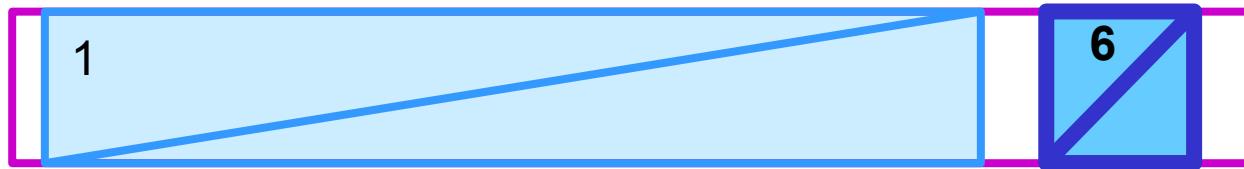


Prev:



PlaceRow: Cell 6

Cell 6:



Overlap with previous cell? **No**
→ **no clustering, no movement**

Last cell → done, PlaceRow finished

PlaceRow: Summary

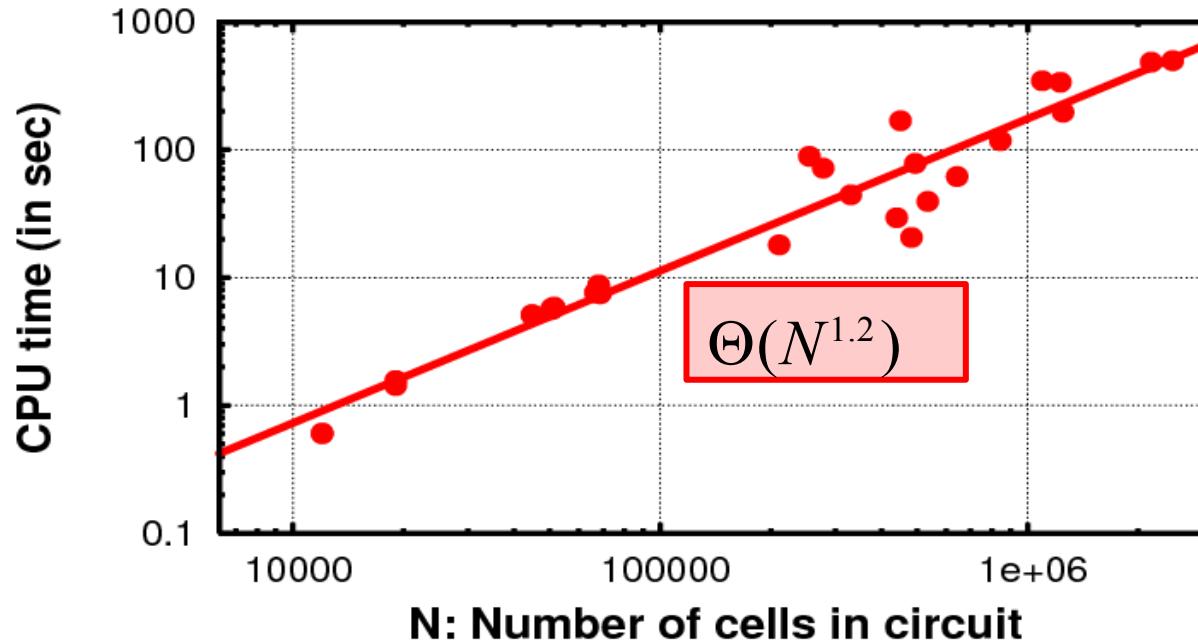
PlaceRow:

- Called several times for legalizing one cell
- Places cells aligned to one row:
minimize quadratic movement → quadratic program (QP)
- Solves QP by dynamic programming:
 - Process cells from left to right
 - If cell overlaps with previous cell:
clustering → movement → further checks with left cells
 - Clustering: update width, weight, and global x-pos of cell
constant execution time
- Linear worst-case complexity: $O(N)$ N : number of cells in the row
At most $N-1$ clustering operations for N cells

Complexity

Worst-case for a complete circuit with N cells: $O(N^2)$

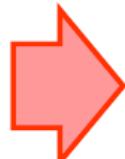
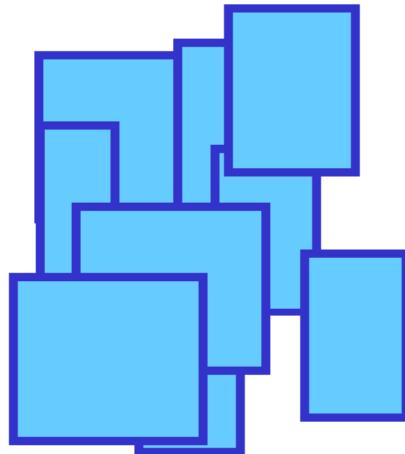
Average-case (experimental results):



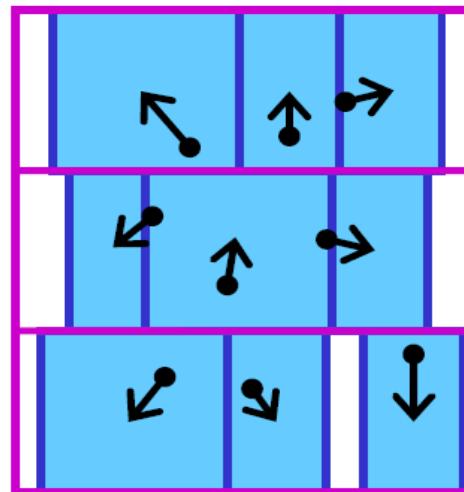
Tetris vs. Abacus

- Abacus can get shorter total movement and shorter maximum movement than Tetris
- Abacus can avoid overflow in each row
- The runtime of Abacus is longer than Tetris, but both can complete in few seconds for large designs

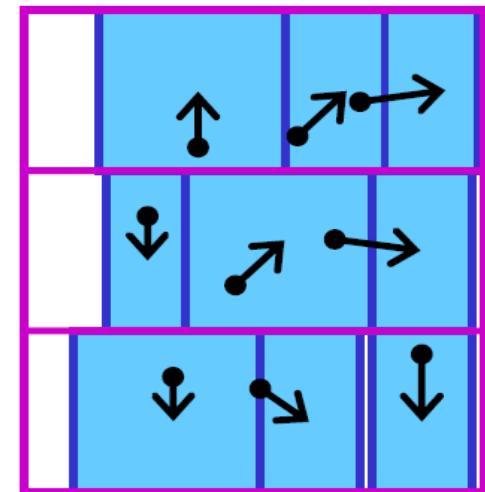
Global Placement



Abacus

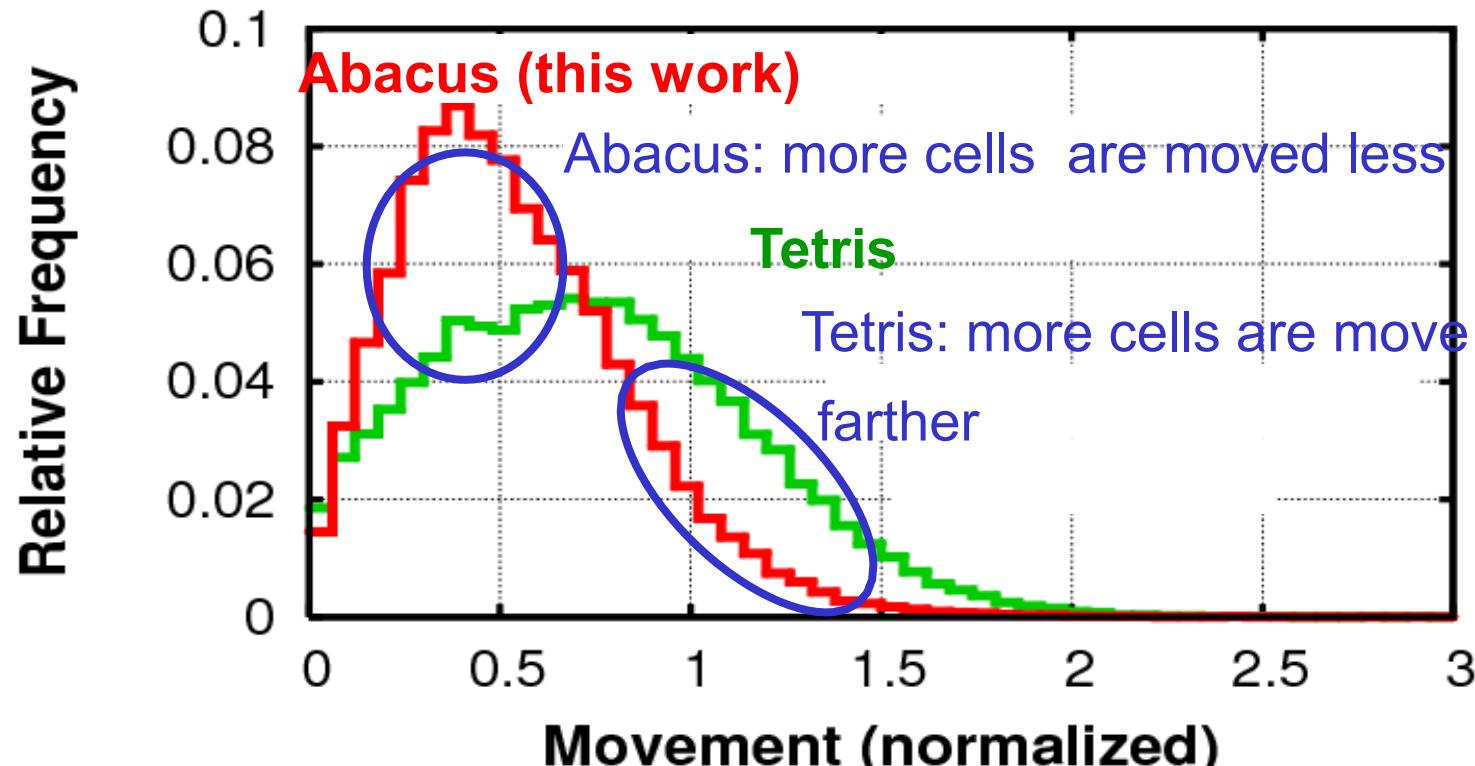


Tetris



Comparison on Movement

Experimental results of one circuit:



→ Lower movement with Abacus