MSING0096 Homework 1
Due Oct 11st 3pm

1. (16pt) Assume that $X \sim N(10, 3^2)$, Compute following probabilities

   (a) (4pt) $p(X > 13)$

   (b) (4pt) $p(|X| > 9)$

   (c) (4pt) $p(X = 10)$

   (d) (4pt) Find z, such that $p(|X| > z) = 0.75$.

2. (15pt) A manufacturer of booklets packages them in boxes of 100. It is known that on average, the booklets weigh one ounce, with a standard deviation of 0.05 ounce. The manufactures is interested in calculating

   $$P(\text{100 bookelts weigh more than 100.4 ounces})$$

   a number that would help detect whether too many booklets are being put into a box. Explain how you would calculate an approximate value of this probability. Mention any relevant theorems or assumptions needed.

3. (10pt) By tracking how users are playing a newly launched game app, we random selected 15 players and record how many minutes they spent on this game in one day.

   $$8.6; 9.4; 7.9; 6.8; 8.3; 7.3; 9.2; 9.6; 8.7; 11.4; 10.3; 5.4; 8.1; 5.5; 6.9$$

   Please construct a 95% confidence interval of average time consumers spend with this game daily.

4. (15pt) Please donwload the dataset 1 from moodle, which contains **hourly** PM2.5 level in Beijing from Jan 1st, 2014 - Jan 31st, 2014.

   Please find variance of the daily median PM2.5 level (median of 24 PM2.5 levels observed in a day) in December 2014. Construct the 95% confidence interval.

5. (32pt) Please download the dataset 2 from moodle, which contains 100 random number drawn from $N(\mu, \sigma^2)$. μ and σ are unknown.

   (a) (2pt) Please compute the sample average and standard deviation.

   (b) (8pt) A student constructs a confidence interval of μ, which is $[4.688, 6.013]$. What is the confidence level of this confidence interval?

   (c) (5pt) Please conduct a test $H_0 : \mu = 5, H_1 : \mu \neq 5$ at 5% level.

   (d) (5pt) If you are told that $\sigma = 2.5$, please conduct a test $H_0 : \mu = 5, H_1 : \mu \neq 5$ at 5% level.

   (e) (12pt) If the random sample actually is drawn from $N(5.2, 2.5^2)$, please compute the probability of Type II error of the hypothesis test in (d).

6. (12pt)25 new mothers were investigated about their sleeping hours per day before and after having a baby. Researchers want to know if new mothers' sleeping hours have been significantly reduced.

Individual	Before	After	D=Before-After
1	9	6	3
2	8	6.5	1.5
3	7.5	5.5	2
...	...	...	...
24	10	7	3
25	8	7	1

The mean and sample standard deviation of D (before - after) are compuated

$$\bar{X}_D = 2.5, s_D = 2.5$$

Please test this hypothesis at 5% level and calculate the p-value.