Individual Assignment 3 (4 problems, 90 points)

1. (20 points) Suppose a monopolist firm sells a product with enough capacity. There are two markets. The demand function in Market 1 is $D_1(p) = 140 - p$; the demand function in Market 2 is $D_2(p) = 190 - 2p$.
   a. Which market has consumers who are more price sensitive (2 points)?

   b. If the firm sells the product under different prices in different markets. Find the prices that maximize the revenue and the total optimal revenue. (5 points)

   c. Suppose the firm cannot set two different prices due to arbitrage among the two markets, and hence it can only set one price. What is the optimal price to maximize the total revenue? What is the revenue loss compared with part (a)? (5 points)

   d. Suppose Markets 1 and 2 are in different countries. Due to the tariff and transportation costs, arbitrage between the two markets only happens when the price gap of the two markets is large. Specifically, the transportation cost per unit is 10, the tariffs of both countries are 10% of the import price. What is the optimal prices to maximize the total revenue and prevent arbitrage? What is the revenue loss compared with part (b)? (8 points)

2. (20 points) Canary Wharf (CW) Airlines operates a flight between London City Airport and Paris. There are two types of customers: business and leisure. The ticket price is £400 per business ticket and £150 per coach tickets. The plane has 200 seats.

   a. During weekdays, the business class demand is uniformly distributed between 31 and 60. What is the booking limit? (5 points)

   b. During weekends, the business class demand drops to uniformly distributed between 11 and 20. What is the booking limit now? (5 points)

   After some research, CW Airlines finds that during weekends, there are a new type of leisure travelers who have to work during the weekdays. Their willingness to pay is higher than the leisure travelers who can travel during either weekdays or weekends. Market research suggests this type of customers can afford £250 per ticket and the number is uniformly distributed between 20 and 55. This mid-valuation customers purchase their tickets earlier than the business customers, but later than the low-valuation flexible leisure travelers. Now to maximize the revenue during weekends, CW Airlines would like to introduce three prices and two booking limits: one for the £150 leisure customers and the other for all the leisure customers. That is, CW Airlines begins to sell £150 tickets; when the number of sold tickets reaches the first booking limit, CW Airlines increases ticket price from £150 tickets to £250; next when the number of total sold tickets reaches the second booking limit, CW Airlines further increase ticket price from £250 tickets to £400.

c. First we find the second booking limit. That is, use $f_f = 400$, $f_d = 250$, and the demand of business customers to find the second booking limit. (5 points)

d. Second we use $f_f = 250$, $f_d = 150$ and the demand of £250 leisure customers to find the first booking limit for the £150 leisure customers. (5 points)

3. (20 points) CW Airlines wants to adopt overbooking strategy. From history data, the manager estimates that for the flights between London to Madrid, the number of no-shows is uniformly distributed between 1 and 15. The fare is £200; denied boarding cost is estimated to be £800.
   a. How many tickets should CW Airlines oversell for each flight between London to Madrid? (5 points)

   b. What is the extra expected profit for CW Airlines to oversell compared with no overselling given the manager's estimation of the no-show distribution? (5 points)

A student from UCL Business Analytics program comes to the manager, saying that she has taken *Forecast Analytics* module and knows how to make better prediction for the no-show distribution. Based on his prediction, the number of no-show is uniformly distributed between 5 and 14. We assume this is the true distribution of the number of no-shows.

c. Based on the true distribution, what is the optimal overselling number now? (5 points)

d. What is the value of the more precise prediction? This is measured by the difference of the expected profits under the old overbooking decision and the new overbooking decision, calculated based on the true no-show distribution? (5 points)

4. (30 points) A seller received 1000 units of products to sell at the beginning of the sales season. The sales period is 100 days. The customers arrive in the system as a Poisson process with a constant rate 30 per day. Each customer has i.i.d. valuation. Assume no customer is strategic. That means, customers only purchase if the price is below their own valuation; otherwise, they leave the store and never come back.

   Assume customer valuation V is exponential distributed with mean 100. Assume the seller knows this distribution and sets a fixed price during the sales season.

   a. (3 points) Find the optimal price p^u the seller should set to optimize the expected revenue if there is no inventory constraint.

b. (2 points) Ignore the inventory constraint, based on the optimal fixed price you have obtained, what is the expected number of inventory you can sell within 100 days.

c. (2 points) Since the seller only have 1000 units of inventory. Calculate the fixed price p^c such that the expected sales per day is 1000.

d. (2 points) Which should be the best fixed price, p^u or p^c?

e. (7 points) Use the code on the Moodle to simulate the sales process. Modify the code to the current setting. Change the random number generator seed to your own student ID. Simulate the sales process with fixed price p^u, p^c respectively. Report your ending inventory and total revenue. Try other fixed prices. What is the optimal fixed price in your specific setting?

Assume the seller does not know the consumer valuation or the demand function. The seller has to experiment and find a proper price over the finite sales period. Use the code on the Moodle to simulate the sales process. Modify the code to the current setting. Change the random number generator seed to your own student ID.

f. (7 points) Currently the total inventory $I = 1000$, the sales period $T_{max} = 100$. Try different combinations of the learning period $\tau \in (0, T_{max})$ and number of experiment prices K. Find a good pair (τ, K) for your simulated revenue.

g. (7 points) Keep $T_{max} = 100$. Use the last five digits of your student ID as the initial inventory (set as I). Proportionally, set the daily customer arrival rate $Cb = \frac{I}{1000} * 30$. Try different combinations of the learning period $\tau \in (0, T_{max})$ and number of experiment prices K. Find a good pair (τ, K) for your simulated revenue.