Message Authentication Codes (MACs)

Tung Chou

Technische Universiteit Eindhoven, The Netherlands

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Outline

- Introduction
- HMAC
- Universal-hash based MACs
  - Poly1305
  - security issues
  - software implementation issues
- Diffie–Hellman key exchange

What are MACs?

- On Wikipedia:

  "a message authentication code (often MAC) is a short piece of information used to authenticate a message and to provide integrity and authenticity assurances on the message. Integrity assurances detect accidental and intentional message changes, while authenticity assurances affirm the message's origin"

Digital Signatures

- Construction:

$$\begin{tikzcd} \text{message } (m) \arrow[r] & \boxed{\text{hash}} \arrow[r] & h \arrow[rr, bend left, "sk"] & \boxed{\text{TP}} & \text{signature } (s) \arrow[ll, bend left, "pk"] \end{tikzcd}$$

- Usage:
  - S computes h and the $\text{SIGN}_{sk}(h)$.
  - S sends (m, s).
  - V gets (m', s').
  - V computes and check $\text{hash}(m') = \text{VERIFY}_{pk}(s')$.

- Security
  - attacker should not be able to forge a valid (m, s) pair
  - attacker might have collected many (m, s) pairs

Message Authentication Codes

- "Keyed hash function":

$$\text{message } (m) \longrightarrow \boxed{\text{MAC algorithm}} \longrightarrow \text{tag/authenticator } (t)$$

$$\uparrow$$

$$\text{shared secret key } (r)$$

- Usage:
  - S computes $t = \mathsf{MAC}_r(m)$ and sends (m, t).
  - R gets (m', t').
  - R computes and checks $\mathsf{MAC}_r(m') = t'$.

- Security
  - attacker should not be able to forge a valid (m, t) pair
  - attacker might have collected many (m, t) pairs

MACs vs Signatures

	MACs	Signatures
Integrity	yes	yes
Authenticity	yes	yes
Non-repudiation	no	yes
Key	secret-key	public-key

"Non-repudiation is about Alice showing to Bob a proof that some data really comes from Alice, such that not only Bob is convinced, but Bob also gets the assurance that he could show the same proof to Charlie, and Charlie would be convinced, too"

✍ secret-key crypto is "fast"

HMAC

- Build MAC from hash functions
- A naive construction:

$$t = H(r \mid\mid m)$$

- Merkle–Damgård construction based hashes (e.g., MD5, SHA1)

$$\begin{tikzcd} & m_1 \arrow[d] & m_2 \arrow[d] & & m_\ell \arrow[d] & \\ IV \arrow[r] & \boxed{f} \arrow[r] & \boxed{f} \arrow[r] & \cdots \arrow[r] & \boxed{f} \arrow[r] & h \end{tikzcd}$$

- Length extension attack: $h' = f(h, m_{\ell+1})$

HMAC (cont.)

- Another construction:

$$t = H(m \mid\mid r)$$

- HMAC:

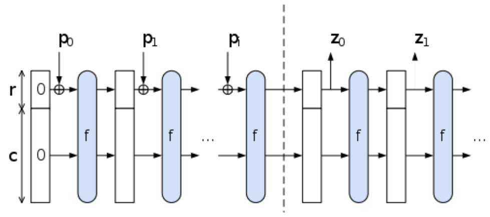
$$t = H\left((r \oplus p_o) || H((r \oplus p_i) || m)\right)$$

- HMAC-SHA1
  - widely used in Internet applications
  - 5.18 Sandy Bridge cycles/byte

✍ Reality: the most commonly used scheme might not be the best

SHA3

The "Sponge" construction:



`http://en.wikipedia.org/wiki/SHA-3`

The Wegman–Carter construction

- Why?
  - provides information theoretic security
  - usually involves field/ring arithmetic
  - better performance than HMAC
- Construction
  - "universal" hash function + one-time pad:

$$h_r(m_n) \oplus s_n$$

  - universal hash: low differential probability
  - one-time pad hides all information about the key

Poly1305

- Construction:

$$t = (((m_1 r^{\ell} + m_2 r^{\ell-1} + \cdots + m_{\ell} r) \mod 2^{130} - 5) + s) \mod 2^{128}$$

  - $2^{130} - 5$ is a prime
  - r, s are shared secret 128-bit values
  - $m_{i<\ell}$ is the ith 128-bit block of m padded by 1.
  - m_{ℓ} is the "remainder" of m padded by 1.

- Without proper padding?
  - $m = \texttt{'FF'}$, $m' = \texttt{'FF'}, \texttt{'00'}$
  - zero-pad the message obtain a 128-bit block

$$m_1 = m'_1 = \texttt{'FF'},\ \texttt{'00'},\ \ldots,\ \texttt{'00'}$$

- Speed: 1.22 Sandy Bridge cycles/byte

Poly1305: avoiding security issue

- What is wrong with "real" polynomial evaluation?

$$t = m_1 r^{\ell-1} + m_2 r^{\ell-2} + \cdots + m_{\ell} + s$$

- The attacker forges a valid message–tag pair easily:

$$t + \Delta = m_1 r^{\ell-1} + m_2 r^{\ell-2} + \cdots + (m_{\ell} + \Delta) + s$$

- This does not provide low differential probability

Poly1305: avoiding security issue

- What is wrong with using the same pad twice?

$$t = m_1 r^\ell + m_2 r^{\ell-1} + \cdots + m_\ell r + s$$

$$t' = m'_1 r^\ell + m'_2 r^{\ell-1} + \cdots + m'_\ell r + s$$

- The attacker gets information of r by finding roots of

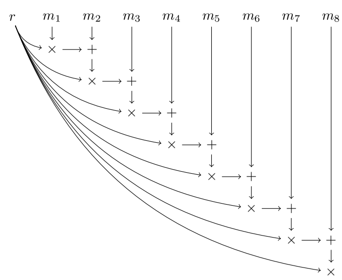
$$t - t' = (m_1 - m'_1) r^\ell + (m_2 - m'_2) r^{\ell-1} + \cdots + (m_\ell - m'_\ell) r$$

- "*nonce-misuse*" issue
  - In practice s is usually replaced by stream cipher output, e.g., AES$_k(n)$ for m_n
  - HMAC does not use nonce

Poly1305: polynomial evaluation

Consider $m_1 r^8 + m_2 r^7 + \cdots + m_8 r$

- *Horner's rule*:



- n multiplications (and $n-1$ additions)
- The issue of being *"on-line"*

GMAC

- The NIST-standard *authenticated encryption* scheme GCM
  - *Galois Counter Mode*
  - Special hardware support for AES-GCM in high-end CPUs

- Polynomial evaluation MAC:

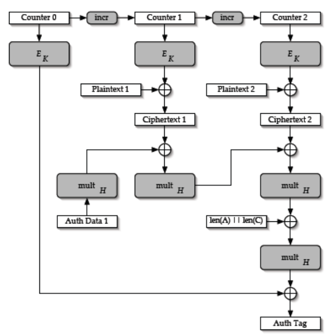
$$t = (m_1 r^\ell + m_2 r^{\ell-1} + \cdots + m_\ell r) + s$$

- Based on arithmetic in

$$\mathbb{F}_{2^{128}} = \mathbb{F}_2[x]/(x^{128} + x^7 + x^2 + x + 1)$$

✍ Binary fields: better in hardware

GCM



http://en.wikipedia.org/wiki/Galois/Counter_Mode

GMAC: speeds

reference	platform	PCLMUQDQ	cycles per byte
Käsper–Schwabe 2009	Core 2	no	14.40
	Sandy Bridge	no	13.10
Krovetz–Rogaway 2011	Westmere	yes	2.00
Gueron 2013	Sandy Bridge	yes	1.79
	Haswell	yes	0.40

Auth256*

- Construction
  - a *pseudo-dot-product* MAC:

    $$t = (m_1 + r_1)(m_2 + r_2) + (m_3 + r_3)(m_4 + r_4) + \cdots + s$$

  - base field $\mathbb{F}_{2^{256}} = \mathbb{F}_{2^8}[x]/(\phi)$. Tower field construction for $\mathbb{F}_{2^8}$.
- Compared to GMAC
  - higher security level
  - 0.5/1 multiplications per block
  - larger key size
  - very different field construction for low bit operation count

Wegman–Carter construction: security

- "δ-xor-universal hash": For all distinct (m, m') and Δ, we have

$$Pr\left(\mathsf{Hash}_r(m) = \mathsf{Hash}_r(m') \oplus \Delta\right) \le \delta$$

- The one-time pad hides all information about the key r.
- The best strategy for the attacker is to guess.

Auth256: Security Proof

Hash values:

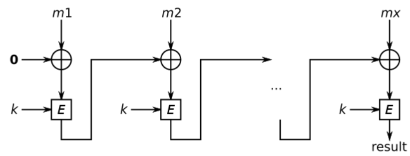
$$\begin{aligned} h &= (m_1 + r_1)(m_2 + r_2) + (m_3 + r_3)(m_4 + r_4) + \cdots + (m_{2\ell-1} + r_{2\ell-1})(m_{2\ell} + r_{2\ell}), \\ h' &= (m_1' + r_1)(m_2' + r_2) + (m_3' + r_3)(m_4' + r_4) + \cdots + (m_{2\ell-1}' + r_{2\ell-1})(m_{2\ell}' + r_{2\ell}). \end{aligned}$$

Then $h = h' + \Delta$ if and only if

$$\begin{aligned} &r_1(m_2 - m_2') + r_2(m_1 - m_1') + r_3(m_4 - m_4') + r_4(m_3 - m_3') + \cdots \\ &\quad = \Delta + m_1'm_2' - m_1m_2 + m_3'm_4' - m_3m_4 + \cdots . \end{aligned}$$

$m \neq m'$ implies that there are at most $|K|^{2\ell-1}$ solutions for r.

CBC-MAC



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