Algorithmic Trading Bootcamp: Linear and Nonlinear Algorithms

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Course Outline

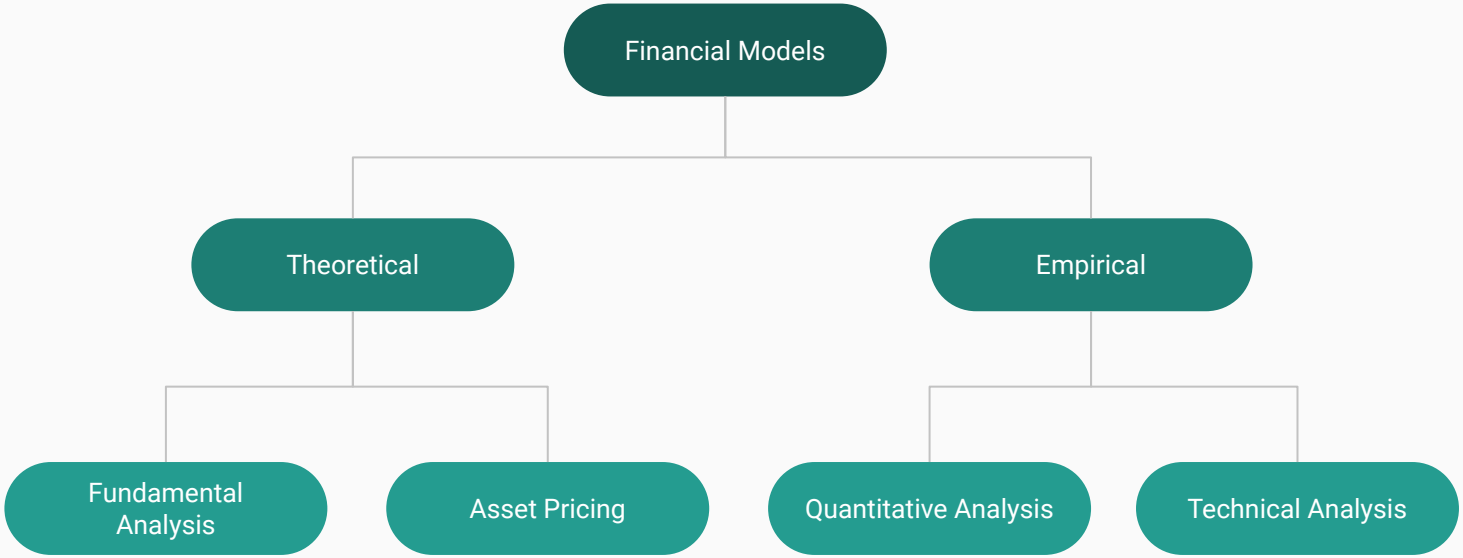
1. Linear Regression
2. Logistic Regression
3. Decision Trees
4. Random Forests and Gradient Boosted Machines

Linear Regression

Section 1

1. Ordinary least squares
2. Maximum likelihood estimates
3. Ridge regression
4. Lasso regression

Finance has been an early adopter of AI/ML



Linear Regression: The Market Model (MM)

$$(R - F) = a + b\,(M - F) + s$$

- Standard linear regression model where:
  - Y = (R - F) is the outcome variable or label, X = (M - F) is the predictor variable of feature
    - R is the realized return of a stock
    - F is the return on a risk-free asset (US treasury bill)
    - M is the realized return of a market portfolio (S&P 500)
    - **a** (alpha) is the expected stock-specific return
    - **b** (beta) is the level of systematic risk exposure to the market
    - **s** (sigma) is the unexpected stock-specific return

Model assumptions: Gauss-Markov theorem

- Linear regression makes the following assumptions:
  - Residuals are independent and identically distributed
  - Expected mean of the residuals is zero
  - Variance of the residuals is finite and constant
  - Residuals are normally distributed

Hands-on Exercise

- Simple linear regression with Statsmodels
  - Click here for Colab notebook

Linear regression algorithm

- Predicted response of a linear regression algorithm is a weighted sum of its input features:
- Y = w[0] + w[1]*x[1] + …..+ w[n]*x[n]
  - Y is predicted response
  - X[1]...x[n] are the input features
  - w[0]...w[n] are constant coefficients to be learned from training data
- Graphically it is a line in 2-dimensions, a plane in 3-dimensions and a hyperplane in n-dimension space

Performance measures for regression

- Error metrics are calculated based on the distance between predicted value and actual value
  - Mean squared error (MSE) : sensitive to outliers
  - Mean absolute errors (MAE): Not sensitive to outliers
  - Median absolute errors (MedAE): Not sensitive to outliers
  - R-squared: A standardized version of MSE

How a linear model learns

- Linear models learn the coefficient/weights from the training data by minimizing an error/performance metric:
  - Minimizes the sum of the squared distance between prediction and actual values
  - Assumes residual errors are normally distributed with constant variance
  - Ordinary least squares (OLS) regression algorithm is over 200 years old!
  - Same as maximum likelihood estimate (MLE) using Gauss-Markov theorem
- In higher dimensions, linear models are powerful baseline models
  - Easy to interpret and build
  - Fast to train and predict
  - Highly scalable on large datasets

Regularization reduces overfitting the data

- Financial data are abundant but have very low signal to noise ratio
  - Risk overfitting the data and learning from the noise instead of the signal
- Linear models that have highly correlated features tend to overfit data
  - Need methods to prevent overfitting of data
- Regularization uses two different types of shrinkage methods to:
  - Improve interpretation of predictions and inferences
  - Reduce generalization errors by reducing variance/overfitting

Lasso versus ridge regression models

- Two types of shrinkage methods that penalize complexity:
  - Lasso or L1 regularization: penalizes the sum of the absolute values of the coefficients
  - Ridge or L2 regularization: penalizes the sum of the coefficients squared
- In lasso regression, many of the coefficients are shrunk to zero
  - Used to eliminate features and increase interpretation of model
- In ridge regression, all coefficients are shrunk to near zero
  - Reduces the impact of any one feature
  - Coefficients with less variance are reduced more
- Features need to standardized prior to training

Hands-on Exercise

- Use linear regression models to forecast stock price returns
  - Click here for lasso and ridge regression models with Scikit-learn

Feature engineering is key to performance

- Features of a model enable good inference and prediction
  - Need systematic methods for selecting informative features
- Remove numerical features with low variances
  - Drop features that do not meet a variance threshold
- Drop some of the highly correlated features
  - Quantify the correlation among features
- L1/Lasso regularization can be used remove unimportant features
- Human expertise, judgement and experience are still important!

Logistic Regression Models

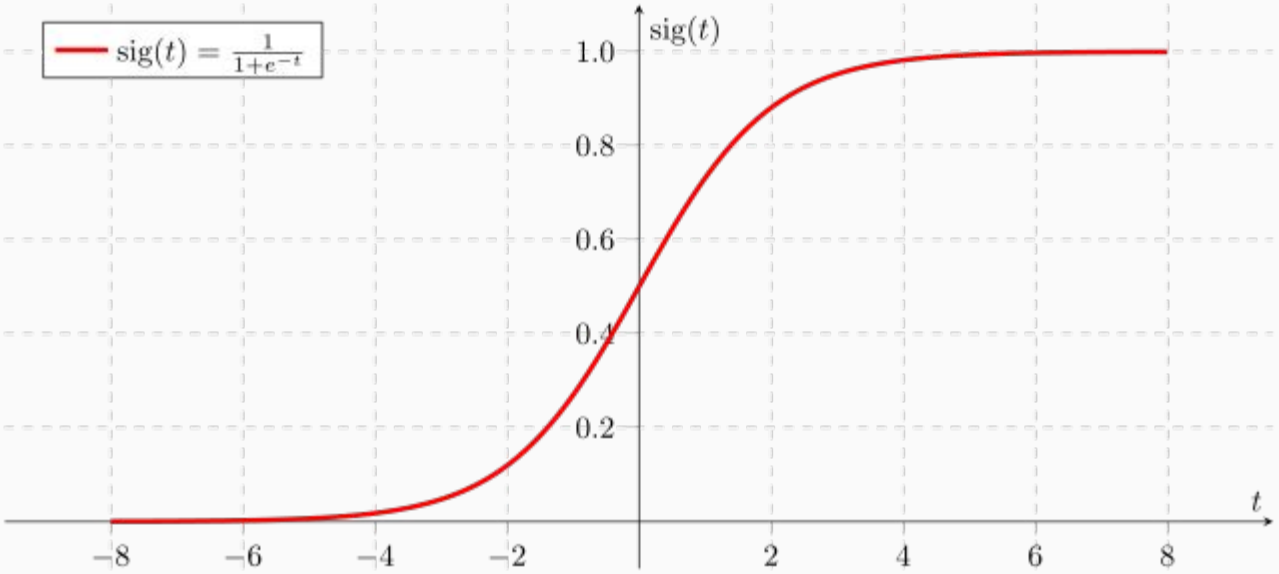
Section 3

1. Classification
2. Logistic algorithm
3. Hands-on exercise

Classification problems in finance

- For many problems in finance, you are interested in classifying an uncertain outcome as opposed to measuring its value:
  - Recession or growth
  - Bankruptcy or solvency
- Binary classes are easily extended to multiclass classifications
  - One versus rest method
- Also need to quantify the probability of belonging to a particular class
  - Probabilities need to add up to 1

Logistic/Sigmoid function



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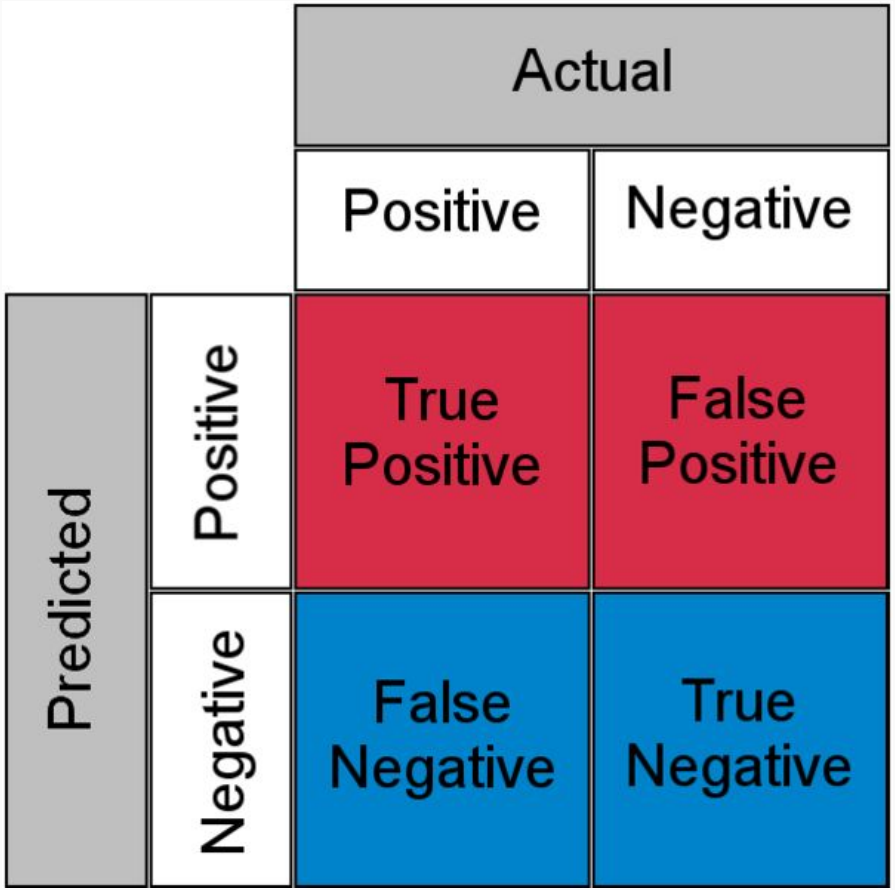
Logistic regression algorithm

- A linear model is extracted from a logistic/sigmoid function via a logarithmic transformation
  - Coefficients are linear in the feature vector X
- The logistic function constrains the value of the function's output to between 0 and 1
  - Can be interpreted as the probability of belonging to a particular class
- A logistic regressor is the simplest neural network

How a logistic regression model learns

- Logistic regression models learn the coefficient/weights from the training data by using maximum likelihood estimation methods
  - You can also use non-linear least squares method
- Lasso/L1 regularization reduces features and overfitting
- Ridge/L2 regularization reduces overfitting
- Features need to be rescaled or standardized

The Confusion Matrix



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Performance metrics for classifiers

- Accuracy = (TP + TN)/(TP + TN + FP + FN)
  - Misleading metric in imbalanced datasets
- Precision = TP/(TP + FP)
  - Focuses on the number of false positives
- Sensitivity or recall = TP/(TP + FN)
  - Focuses on the number of false negatives
- F-measure = 2 * (precision * sensitivity)/(precision + sensitivity)
  - Harmonic mean balances precision and sensitivity

Hands-on exercise

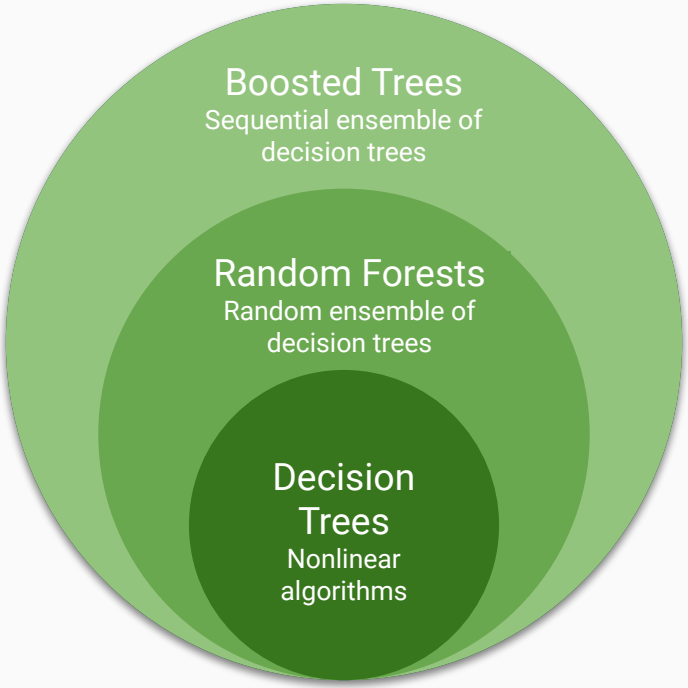
- Click here to use logistic regression models to forecast an economic recession

Decision Tree Based Models

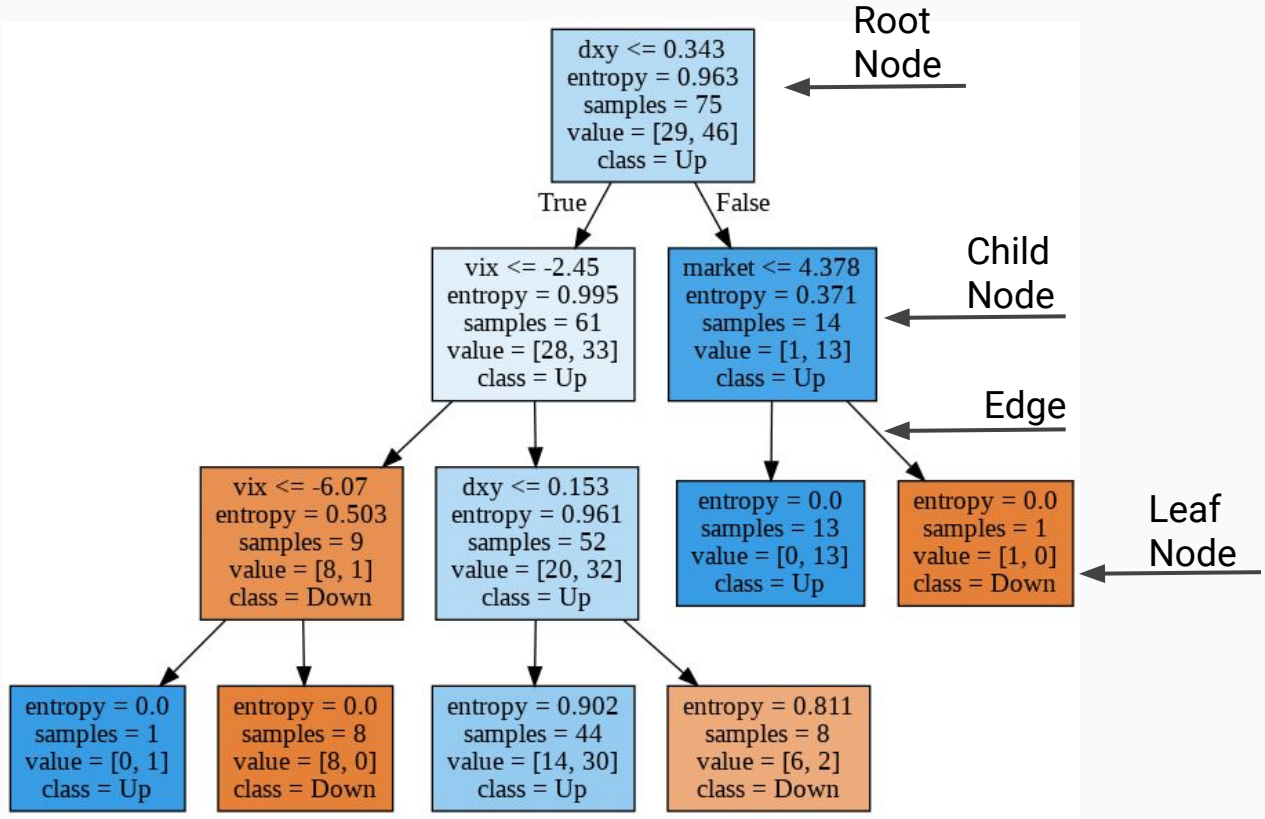
Section 3

1. Decision Trees
2. Hands-on Exercise
3. Regularizing Trees

Decision Tree (DT) based models use powerful nonlinear algorithms



A Decision Tree algorithm is a feature based series of true/false questions



Why use Decision Tree models

- One of the most powerful nonlinear base learners
  - Robust and scalable
- They are intuitive, white box models
  - Easy to interpret and visualize
- Not much data preprocessing is required
  - No need to standardize, normalize or scale data
- Quantifies uncertainty in predictions using probabilities
  - Only available in the classification algorithm

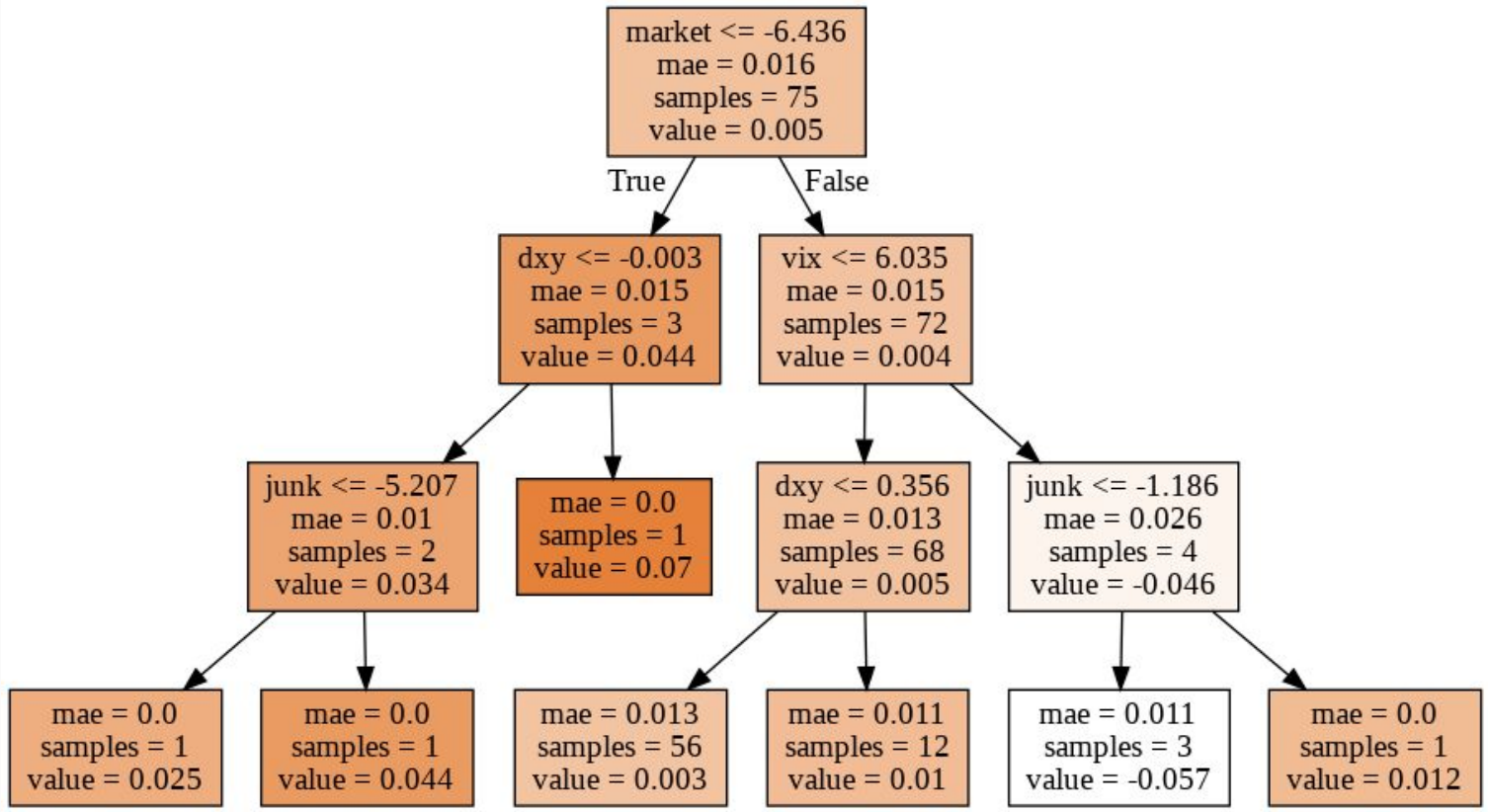
Scikit-Learn uses the CART algorithm

- Classification and Regression Tree (CART) algorithm
  - Recursively splits training set into two subsets based on a threshold value of a feature
  - Stops when it reaches its maximum depth or cannot split the data anymore
  - Produces a sequence of true/false questions leading to a most likely answer
  - Because questions have binary outcomes, non-leaf nodes will always have two children
- Solution produced by CART algorithm is not guaranteed to be optimal
  - Optimal tree is not solvable even for small datasets
- Uncertainty of classifications is quantified probabilistically
  - Simple ratio of each class divided by total sample size in the terminal node
  - Classification error rate is a similar ratio

How a DT classification model learns

- DT classification models learn feature threshold values recursively by minimizing a performance metric:
  - Gini Impurity = 1 - (R1^2 + …+ Rn^2)
    - where R is the ratio of the number of instances of a class to the training sample size of a node
  - Entropy = - {R1 * log(R1) + …. + Rn * log(Rn)}
    - where R is the ratio as defined above and log is the binary logarithm (base 2)
- Recursion stops automatically when the chosen metric equals zero
  - Gini impurity is the default metric and is faster to compute
  - Entropy produces slightly more balanced trees

Regression trees predict target values instead of classes at each node



How a DT regression model learns

- DT regression models learn feature threshold values recursively by minimizing a performance metric:
  - Mean squared error: {(A - Y1)^2 + …+(A - Yn)^2}/n
    - where A is the target value averaged over the entire sample size and Ys are target values of each training instance of the node
  - Mean absolute error: (|A -Y1| + ….+ |A - Yn|)/n ,
    - where A and Ys are the same as above
- Recursion stops automatically when the chosen metric equals zero
  - Mean squared error is sensitive to outlier values
  - Mean absolute error is not as sensitive to outlier values

Hands-on exercise

- Use built-in Colab document to use DT models to predict an economic recession and percentage loss in GDP
  - Click here for DT classification and regression models with Scikit-learn

Analyzing the importance of features in DT

- DT models can get overwhelming complex
  - Feature importance function helps summarize the decision-making process of a tree
- Quantifies how much a feature contributed in predicting the target value
  - Values are between 0 and 1 and the total sum of feature importances equals 1
- Nonlinear relationship between feature and target value
  - High importance value does not indicate a linear correlation with target value
  - Low importance value means that information in the feature is likely redundant

Regularization reduces overfitting the data

- Financial data are abundant but have very low signal to noise ratio
  - Risk is overfitting the data and learning its noisy structure
- Decision Trees can easily overfit training data with training scores of 100%
  - Need methods to prevent overfitting of data
- Regularization uses shrinkage methods to:
  - Improve interpretation of predictions and inferences
  - Reduce generalization errors by reducing variance/overfitting

Regularizing DTs with hyperparameters

- The following pre-pruning hyperparameters reduce overfitting of data
  - max_depth: reduces the complexity level of a tree
  - max_features: maximum number of features to be evaluated per node before a split
  - max_leaf_nodes: maximum number of leafs in a tree
  - min_samples_split: minimum sample size a node must have before a split
  - min_samples_leaf: minimum sample size required in a leaf node

Weaknesses of DT models

- DT models can get complex very fast even with a small set of features
  - Regression trees can get very deep
- Overfitting of data even with regularization
  - Poor out of sample performance, especially in regression models
- Highly sensitive to small variations in training data
  - Predictions of a single tree are unstable

Advanced Decision Trees

Section 4

1. Random Forest Algorithm
2. Hands-on Exercise
3. Gradient Boosted Machines
4. Hands-on Excercise

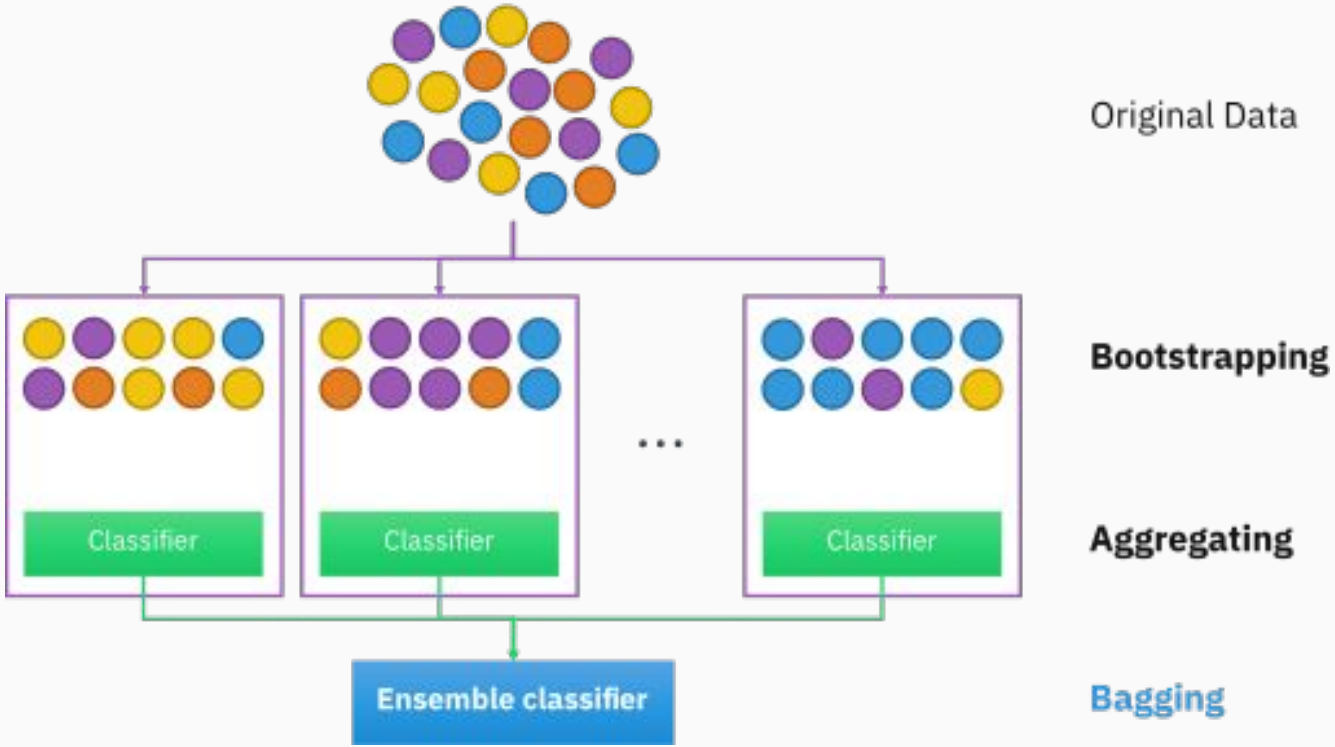
Why use Random Forest (RF) models

- One of the most powerful ensemble learning methods
  - Highly robust and scalable to very large datasets
- RF average has similar bias but lower variance than any single DT
  - Better generalization of predictions than DT models
- Like DT models, not much data preprocessing is required
  - No need to standardize, normalize or scale data
- Like DT models, quantifies uncertainty in predictions using probabilities
  - Only available in the classification algorithm

RF algorithm reduces variance of DTs

- Each DT is built uniquely using randomness:
  - Random sampling of training data with replacement (Bootstrapping)
  - Random subset of features
- DT algorithm is modified so it doesn't look for the best test for each node
  - It selects a random subset of features for each node
  - Selects the best possible test for one of the features in the subset above
- RF takes the prediction of each DT and averages them (Aggregating)
  - Each trained DT is still a weak learner but the ensemble is a much stronger learner
- Two different ways of aggregating classifier predictions
  - Soft voting averages predictions and hard voting takes the majority vote of classifiers

RF algorithm is trained using bagging (bootstrapping and aggregating)



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Using out-of-bag (OOB) data for validation

- RF trained using random sampling with replacement leaves out approximately 37% of the training data of size N:
  - Probability of not picking a training instance = (N-1)/N
  - Probability of not picking a training instance in N random draws = {(N-1)/N}^N
  - As N gets larger, {(N-1)/N}^N converges to exp(-1) = 0.368
- An OOB sample can be used as a validation dataset for a classification or regression tree not built using that dataset
  - The prediction average produces an unbiased estimate of the test error
- Cross validating large datasets can be costly
  - Validating using OOB is much faster and cheaper

Hands-on exercise

- Use built-in Colab to use Random Forest models to forecast stock price direction and returns

Analyzing the importance of features in RF

- RF models are black boxes and cannot be visualized
  - Feature importance function helps summarize factors driving decisions
  - Many more possible combinations are evaluated, so it's more reliable than DT
- Feature importance in RF models is a weighted average
  - The weight of each node that uses the feature to reduce impurity is equal to training sample size associated with the node
  - Feature importances of each tree in the forest are aggregated and normalized
- Nonlinear relationship between feature and target value still holds
  - High importance value does not indicate a linear correlation with target value
  - Low importance value means that information in the feature is likely redundant

Regularizing RFs with hyperparameters

- The following hyperparameters are key to reducing variance:
  - n_estimators: increase number of trees that populate the forest
  - max_features: reduce number of features included in the random subset
  - n_samples: increase bootstrap training sample size
- Use pre-pruning DT hyperparameters that reduce overfitting of data

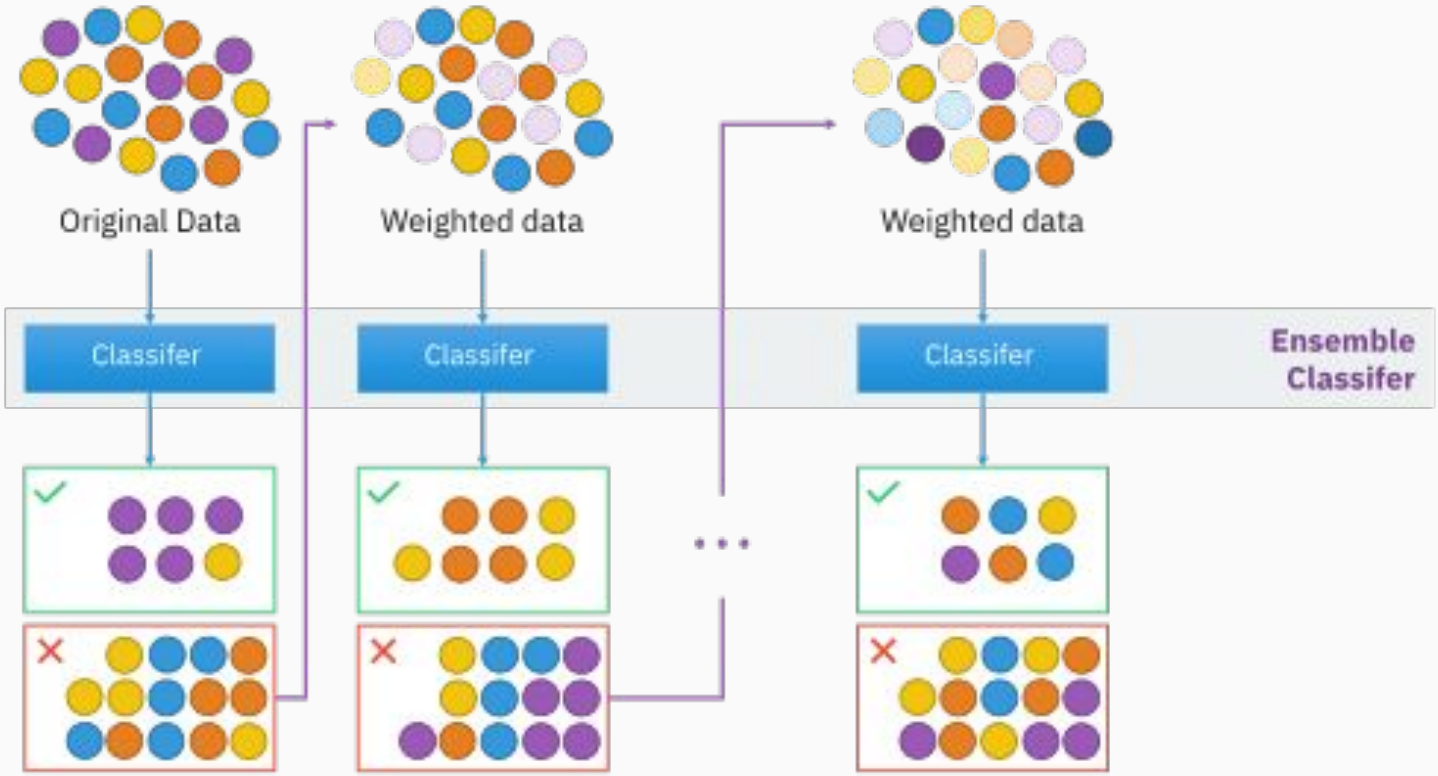
Weaknesses of RF models

- RF models are complex, black box models
  - Forests can be dense and trees can be deep
- More memory and time required to train RF models
  - Physical constraints on the complexity of RF models
- Poor performance in sparse, high dimensional datasets
  - For example, text data

Why use Gradient Boosting?

- Gradient boosted regression trees (GBRT), also known as Gradient boosting machines (GBM), are the most powerful nonlinear algorithms
  - Winning entries in most public contests use this algorithm
  - Used for regression and classification problems
- Gradient boosting enhances performance of RF models
  - Uses shallow trees that require less memory
  - Enables faster predictions of learning ensembles

How an adaptive boosting algorithm learns



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How gradient boosting algorithm learns

- Sequentially adds DTs to the ensemble, with each tree trying to correct the errors of the preceding ensemble
  - Unlike the RF algorithm, randomness is generally not used in building the ensemble
- At every iteration, algorithm tries to fit a new DT to the residual error of the previous ensemble
  - Unlike the adaptive boosting algorithm which modifies the weights of the training data
- Learning rate of the model controls the severity of the corrections
  - Higher learning rate leads to more complex GBM models and overfitting

Regularizing GBMs with hyperparameters

- The following hyperparameters are key to reducing variance:
  - n_estimators: reduce the number of trees that populate the ensemble
  - learning_rate: reduce the learning rate
- Use pre-pruning DT hyperparameters that reduce overfitting of data

Hands-on exercise

- Use built-in Colab to use GBM models to forecast stock price returns and probability of recession

Weaknesses of GBM models

- GBM models are complex, black box models
  - GBM ensembles can be dense
- Poor performance in data with a lot of statistical noise
  - For example, financial data
- Sensitive to outliers in the data set
  - Each new DT is required to fix the errors of its preceding ensemble