The University of Texas at Arlington

Department of Mechanical and Aerospace Engineering
MAE 3303-001: Compressible Flow

Key Assignment:
Supersonic Airfoil Design

By:

Gayathri Kola

Submitted to:

Professor Zhen Xue Han, Ph.D.

27 April 2022

Table of Contents

Nomenclature

x_u = Upper x-distance from the Leading Edge
x_l = Lower x-distance from the Leading Edge
t_u = Upper y-distance to the airfoil peak
t_l = Lower y-distance to the airfoil peak
C_p = Pressure coefficient
C_n = Normal force coefficient
C_a = Axial force coefficient
C_l = Lift coefficient
C_d = Wave drag coefficient
C_{mLE} = Pitching moment coefficient about the leading edge
δ = Wedge angle of the airfoil.
M = Mach number
p = Static pressure
p_∞ = Static pressure for incoming flow
ρ_∞ = Fluid density for incoming flow
V_∞ = Fluid velocity for incoming flow
q_∞ = Dynamic pressure for incoming flow
α = Angle of attack
γ = Specific heat ratio of the fluid
c = Chord Length
L = Lift
D = Wave Drag

Problem Statement

The aim of this assignment is to maximize the lift-to-drag ratio (L/D) for straight edge supersonic airfoils. The analysis is performed using Shock-Expansion Wave theory to estimate the pressure coefficients which in turn can be used to calculate the lift, drag, and pitching moment coefficients. An 'Exhaustive Search' method is performed using MATLAB to produce a wide range of airfoils to choose from. A list of feasible designs are produced that meet three specific design constraints and the final airfoil with the maximum L/D is mapped.

Airfoil Geometric Model

Airfoils can be viewed as minimized versions of aircraft wings. According to the thin airfoil theory, an airfoil consists of the leading edge (LE) and trailing edge (TE) spanning over the length of the airfoil. The line connecting the leading edge to the trailing edge is the chord, c. The line diving the airfoil into half from the leading edge to the trailing edge is called the mean camber line. The distance between the chord line and the mean camber line spanning over the length of the airfoil is known as the camber. The thickness of the airfoil is the vertical distance from the maximum coordinate points on the top and bottom surfaces from the chord line.

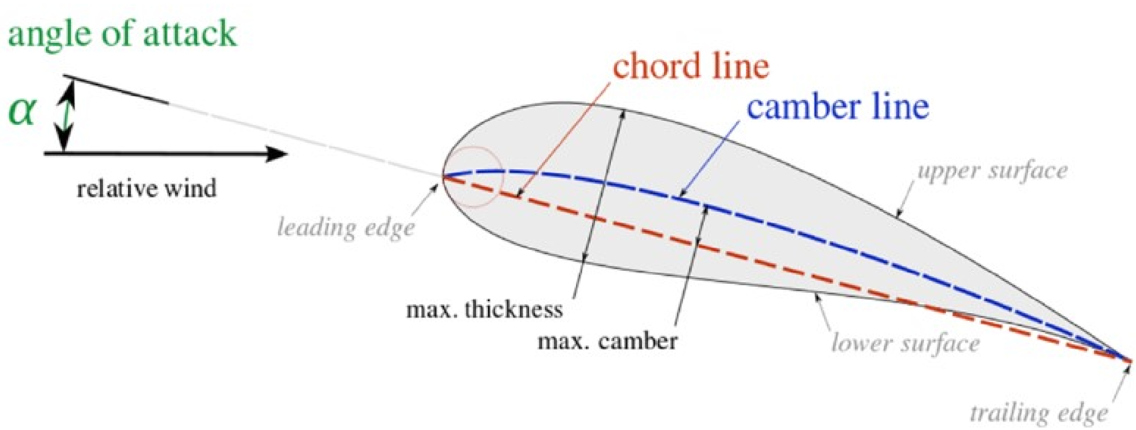

Figure 1. Standard Airfoil and Nomenclature [1].

In this assignment, straight edge, double wedge, and two-dimensional airfoil shapes are tested using the Shock-Expansion wave theory. A general airfoil coefficient calculation method includes experimental tests in a supersonic wind tunnel. Using this theory, we can estimate the important lift, drag, and pitching moment coefficients by calculating the pressure coefficients on each surface. This theory assumes a steady, inviscid flow that is supersonic throughout the surface with no interference. To simplify further, the flow is assumed to be tangent to the surface of the airfoil. A sharp wedge airfoil is similar to a thin airfoil. A key feature of the double wedge airfoil to be noted for this analysis is that the airfoil has a smooth surface with a very small thickness. The figure below labels the geometric parameters used in this analysis.

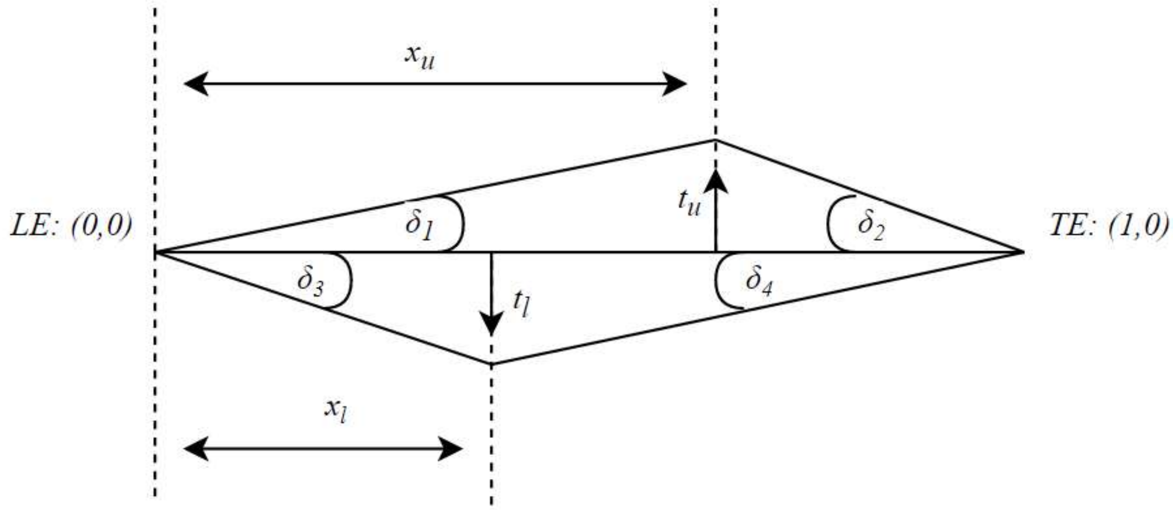

Figure 2. Geometric parameters for a double-wedge airfoil

Starting at the leading edge in the x-direction are upper and lower horizontal distances to the peaks on each side of the airfoil. The upper horizontal distance is labeled as x_u and the lower is labeled as x_l. The vertical distances to the wedge peaks are labeled as t_u and t_l Note, the thickness value for the lower surface is negative. The origin for the airfoil is conveniently placed at the leading edge. To make analysis for versatile, the chord length c is set to 1 making x_u and x_l non-dimensional parameters. By varying all four of the geometric parameters (x_u, x_l, t_u, t_l) from the leading edge (LE) to trailing edge (TE), different double wedge airfoil geometries can be constructed. Based on the values of each set of the four parameters, their corresponding wedge angles can be determined by using trigonometric relations. In this report, these wedge angles are denoted by the symbol δ to avoid confusion with the flow deflection angle θ.

Another external variable used is the angle of attack α. The angle of attack α and the flow deflection angle θ are used to analyze the type of shock waves occurring at the edges of the airfoil. Since the flow deflection angle is dependent on the angle of attack, only angle of attack is varied over a selected range along with geometric parameters. To summarize, five variables are being used for the performance analysis.

Aerodynamic Performance Analysis

As mentioned earlier, the performance analysis is done using Shockwave-Expansion wave theory. The coefficients of lift, wave drag, and pitching moment about leading edge can be calculated by analyzing which type of wave occurs at the edges when the flow direction deflects. Forces acting on an airfoil are a result of the pressure and shear stress distributions on the upper and lower surfaces. Pressure coefficient calculation can be done using shockwave-expansion wave theory for compressible flows. If an incompressible flow assumption was made, the drag coefficient cannot be computed since the inviscid theory does not predict the stall angle of attack. It is purely due to viscous forces as characterized by function of the Reynold's number. Another appropriate method in such a case would be the use of Shock-Expansion wave theory and account for wave drag coefficient and calculate the pressure coefficients on smooth edge surface to get a better approximation. The wave drag coefficient measures the drag due to shock waves in contact with the airfoil.

Generally, the lift, drag, and moment coefficients depend on the values of lift forces, incoming flow velocity and the dynamic pressure given by Eq. (1-3).

$$C_l = \frac{L}{\frac{1}{2}\rho_\infty {V_\infty}^2} \tag{1}$$

$$C_d = \frac{D}{\frac{1}{2}\rho_\infty {V_\infty}^2} \tag{2}$$

$$C_{m,LE} = \frac{M}{\frac{1}{2}\rho_\infty {V_\infty}^2} \tag{3}$$

However, using Shock-Expansion wave theory, the pressure coefficient on a surface can be estimated using the relation in Eq. (4).

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\frac{p}{p_\infty} - 1}{\frac{1}{2}\gamma M_\infty^2} \tag{4}$$

The ratio of pressures to the right most side of Eq. (4) is the value of static pressure in the surface of the airfoil and the incoming flow pressure. This indicates the pressure coefficient is dependent on the ratio of static pressures, specific heat ratio, and the incoming flow Mach number.

This theory gives the criteria for coefficient calculation based on the type of compression or expansion wave that occurs at the edges of the airfoil. Since the flow direction deflects at the sharp edges, the type of compression wave is characterized by the deflection angle θ. Following are the criteria based on the value of angle of attack α, half-wedge angle δ and deflection angle θ:

- Expansion waves: When $\alpha > \delta$

  Deflection angle, $\theta = \alpha - \delta$
- No waves: $\alpha = \delta$

  $$\alpha = \theta$$
- Oblique shock waves: When $\alpha < \delta$

  Deflection angle, $\theta = \delta - \alpha$

To account for the flow direction change at the bottom surface, the angle of attack needs to be added to the wedge angle on the bottom surface to get the correct angle of deflection.

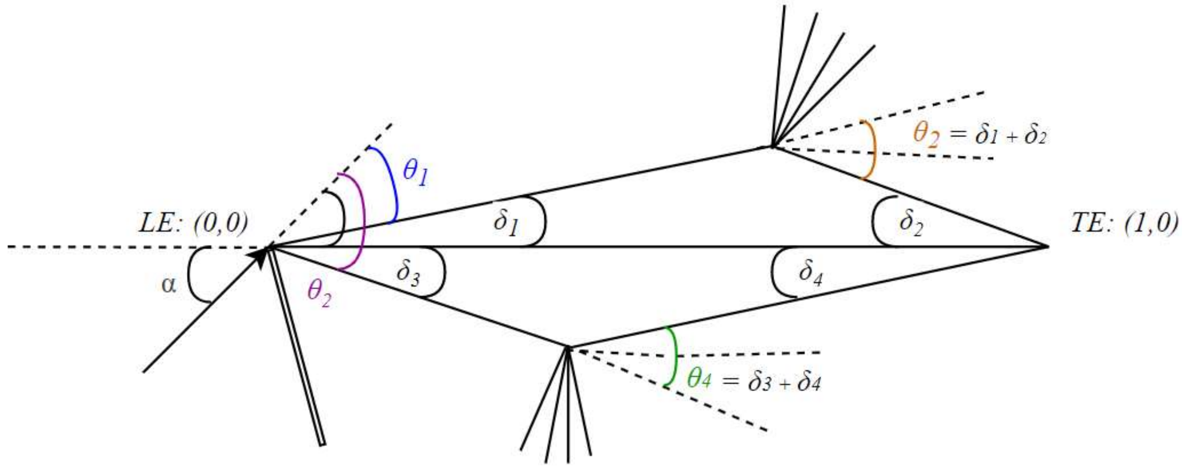

Figure 3 . Shock waves and expansion waves on the airfoil in supersonic flow.

By using Eq. (4), the pressure coefficients can be obtained for each surface on the airfoil. The ratio of static pressures seen are relating to freestream static pressure and static pressure on the surface of the airfoil. Depending on the nature of the wave, pressure loss is obtained using the ratio in the Eq. (5) below.

$$\frac{p_0}{p} = (1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma - 1}} \tag{5}$$

The flow across a shock wave is not isentropic, so the total pressures need to be accounted for when finding the freestream-to-surface static pressure ratio. If the edge experiences an oblique shock wave, the total pressure drops across the wave and the flow velocity decreases in the flow direction. Alternatively, if the outgoing flow deflects away from the flow direction, the edge experiences an expansion wave and the outgoing flow velocity increases making the pressure decrease. After calculating the pressure coefficient on all four surfaces of the airfoil, the normal force, axial force, lift, wave drag, and pitching moment coefficients can be calculated using the following equations below:

$$C_n = \int_0^1 (C_{p,l} - C_{p,u}) d\bar{x} \tag{6}$$

$$C_a = \int_0^1 (C_{p,u} \frac{d\overline{y_u}}{d\bar{x}} - C_{p,l} \frac{d\overline{y_l}}{d\bar{x}}) d\bar{x} \tag{7}$$

$$C_l = C_n \cos\alpha - C_a \sin\alpha \tag{8}$$

$$C_d = C_n \sin\alpha + C_a \cos\alpha \tag{9}$$

$$C_{mLE} = \int_0^1 (C_{p,u} - C_{p,l})\bar{x}d\bar{x} + \int_0^1 \left(C_{p,u}\frac{d\overline{y_u}}{d\bar{x}}\right)\overline{y_u}d\bar{x} - \int_0^1 \left(C_{p,l}\frac{d\overline{y_l}}{d\bar{x}}\right)\overline{y_l}d\bar{x} \qquad (10)$$

Note, the equations above use average values of *x* and *y* coordinates. The slope of the lines in Eq. (7) and (10), can be substituted with the tangent of the wedge angles. The subscripts u and l are notated for upper and lower surfaces of the airfoil.

As mentioned before, the assumption is that the flow is adiabatic, the total temperature and total enthalpy of the flow remain constant. Note, the deflection angles θ_3, θ_4 in Fig. (3) are the sum of the wedge angles of respective side of the airfoil due to their geometry. Since the flow always deflects away near peaks, assuming the flow always moves tangent to the airfoil, only expansion waves occur. This simplifies further calculations.

Design Optimization

To complete analysis, there are three constraints imposed to get a list of feasible designs. The first constraint is of maximum thickness. Below in Eq. (11), is the combination of upper and lower thickness of the airfoil that equals to 5% of the chord length. The constraint is strictly equated to 5% because as the thickness of the airfoil increases, the drag coefficient increases. This defeats the purpose to achieve the highest lift-to-drag ratio.

$$\frac{t_u}{c} + \frac{t_l}{c} = 0.05 \qquad (11)$$

The second and third constraints are of the lift coefficient and pitching moment about the leading edge, mentioned below in Eq. (12-13)

$$C_l \geq 0.2 \qquad (12)$$

$$C_{m,LE} \leq 0.08 \qquad (13)$$

Since only few designs met the constraint requirements for the calculated lift coefficient, the values were rounded to the nearest significant figure to allow for more options. Hence, if the lift coefficients were approximately close to 0.2, such as 0.1795, they were rounded to 0.2 within the MATLAB code. The same approximation was made for pitching moment of coefficient about the leading edge.

A suitable range needs to be chosen for the angle of attack α. High angles of attack are not applicable for this type of analysis. For the design in MATLAB, four ranges were chosen for angle of attack α, x_u, x_l, and t_u. The lower thickness t_l is calculated for each geometry based on the constraint in Eq. (11). A range of -15 to 15 was selected for angle of attack α with a goal of having 30 equal divisions. Similarly, x_u ranged from 0.1 to 0.9 of the chord length. The lengths x_u and x_l

are normalized with the chord. The range for x_l was selected from the opposite side, going from 0.9 to 0.1. This allows for the double wedge to remain non-symmetrical.

The analysis was performed by a series of three loops. The first two loops were embedded into one another. This consisted of an inner loop that calculates the geometric parameters, namely, x_u, x_l, t_u and t_l. Secondly, an outer loop for the angle of attack α. Each loop has an indexing of 31. This means, for every single value of angle of attack α, 31 iterations are performed in the inner loop. The inner loop generates the same 31 geometries every time α changes. This brings the total number of values for geometry and angle of attack, $31 \times 31 = 961$ values.

The third loop finds the maximum value of lift-to-drag ratio out of all the feasible designs calculated and stored. If any of the constraints are not met, the embedded loops will automatically move to the next iteration. Finally, the best airfoil geometry and angle of attack α is displayed and automatically plotted as a result.

Results and Discussion

All the feasible designs that meet the design constraints are listed in Appendix A. The designs that meet the criteria range have angle of attack α ranging from -6° to 6°. The resulting lift-to-drag ratio ranges from 5.4 to 8.6 with the highest lift-to-drag being 8.6404.

The only airfoil reported to result in no waves at the leading edge has an angle of attack of α = 0°, with maximum thickness on the lower surface $t_l = -0.005$ at a distance of x_l = 0.96667. No detached shock waves were produced in this analysis because the angle of attack α range is small. Another reason is that the range for x_u and x_l does directly start from 0 or 1.

As mentioned earlier, the values for C_l and $C_{m,LE}$ were rounded to the nearest significant figure, giving more feasible designs with higher lift-to-drag ratios to choose from. The highest lift-to-drag ratio achieved is mentioned in Table. 1, along with the respective geometry, lift coefficient, drag coefficient and pitching moment about the leading edge. Among the two geometries, the first has the highest lift-to-drag ratio from the MATLAB analysis code. However, upon inspecting the feasible designs, another geometry (row 2) strictly satisfies the design constraints with the lift coefficient being C_l = 0.1997, very close to 0.2. However, for the purpose of this analysis, the geometry generated with the MATLAB code having the highest lift-to-drag ratio (row 1) was chosen as the best airfoil. Upon plotting, by visually observing, the airfoil looks fairly proportionate.

Table 1. Geometry and Orientation for the Best Airfoils.

Angle of Attack $\boldsymbol{\alpha}$	Upper *x* $\overline{X_u}$	Upper *y* $\overline{T_u}$	Lower *x* $\overline{X_l}$	Lower *y* $\overline{T_l}$	Lift Coefficient C_l	Drag Coefficient C_d	Pitching Coefficient C_{mLE}	Ratio $\frac{L}{D}$
-5	0.4467	0.0217	0.5533	-0.0283	0.1522	0.0716	0.0755	8.6406
5	0.1000	0.0000	0.9000	-0.0500	0.1997	0.0364	-0.0704	5.4929

Best Airfoil

After the aerodynamic performance analysis, and design optimization performed on MATLAB, and by manually inspecting the feasible designs, following in Fig. 4, is the best airfoil chosen. The design and geometric parameters are given in Table. 1 above. The maximum lift-to-drag ratio among all the feasible designs meeting design constraints. The lift-to-drag ratio for this airfoil is L/D = 8.6406.

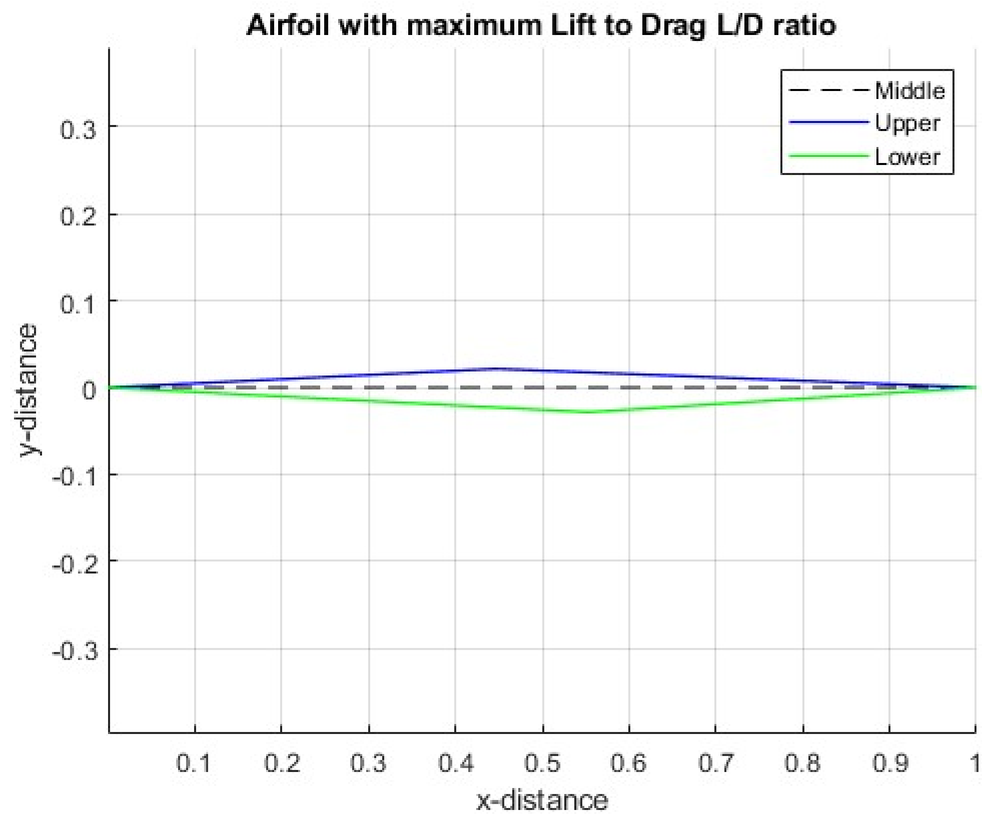

Figure 4. Final chosen airfoil

References:

[1] Cleynen, Olivier. (2011). *Airfoil Nomenclature* [JPEG]. Wikipedia.

[2] Anderson, John D. *Fundamentals of Aerodynamics*. Boston: McGraw-Hill, 2001. Print.

[3] MATLAB. (2021). *version 7.10.0 (R2021b)*. Natick, Massachusetts: The MathWorks Inc.

APPENDIX

Table 2. List of Feasible Designs and Respective Geometry.

Angle of Attack α	Upper x $\overline{X_u}$	Upper y $\overline{T_u}$	Lower x $\overline{X_l}$	Lower y $\overline{T_l}$	Lift Coefficient C_l	Drag Coefficient C_d	Pitching Coefficient C_{mLE}	Ratio $\frac{L}{D}$
-6	0.6333	0.0333	0.3667	-0.0167	0.1897	0.0261	0.0848	7.2674
-6	0.6600	0.0350	0.3400	-0.0150	0.1909	0.0268	0.0846	7.1314
-6	0.6867	0.0367	0.3133	-0.0133	0.1921	0.0275	0.0847	6.9793
-5	0.3933	0.0183	0.6067	-0.0317	0.1504	0.0175	0.0776	8.6112
-5	0.4200	0.0200	0.5800	-0.0300	0.1513	0.0175	0.0766	8.6388
-5	0.4467	0.0217	0.5533	-0.0283	0.1522	0.0176	0.0755	8.6406
-5	0.4733	0.0233	0.5267	-0.0267	0.1530	0.0178	0.0745	8.6178
-5	0.5000	0.0250	0.5000	-0.0250	0.1539	0.0180	0.0736	8.5711
-5	0.5267	0.0267	0.4733	-0.0233	0.1547	0.0182	0.0727	8.5012
-5	0.5533	0.0283	0.4467	-0.0217	0.1556	0.0185	0.0719	8.4088
-5	0.5800	0.0300	0.4200	-0.0200	0.1565	0.0189	0.0712	8.2943
-5	0.6067	0.0317	0.3933	-0.0183	0.1575	0.0193	0.0707	8.1582
-5	0.6333	0.0333	0.3667	-0.0167	0.1586	0.0198	0.0703	8.0008
-5	0.6600	0.0350	0.3400	-0.0150	0.1597	0.0204	0.0701	7.8226
-5	0.6867	0.0367	0.3133	-0.0133	0.1610	0.0211	0.0701	7.6239
-5	0.7133	0.0383	0.2867	-0.0117	0.1625	0.0219	0.0704	7.4055
-5	0.7400	0.0400	0.2600	-0.0100	0.1641	0.0229	0.0711	7.1682
-5	0.7667	0.0417	0.2333	-0.0083	0.1660	0.0240	0.0722	6.9136
-5	0.7933	0.0433	0.2067	-0.0067	0.1684	0.0253	0.0739	6.6440
-5	0.8200	0.0450	0.1800	-0.0050	0.1711	0.0269	0.0763	6.3636
-5	0.8467	0.0467	0.1533	-0.0033	0.1746	0.0287	0.0797	6.0794
-5	0.8733	0.0483	0.1267	-0.0017	0.1790	0.0309	0.0843	5.8033
-4	0.9000	0.0500	0.1000	0.0000	0.1531	0.0268	0.0748	5.7024
4	0.1000	0.0000	0.9000	-0.0500	0.1653	0.0295	-0.0540	5.6129
4	0.1267	0.0017	0.8733	-0.0483	0.1595	0.0270	-0.0517	5.9094
4	0.1533	0.0033	0.8467	-0.0467	0.1550	0.0248	-0.0506	6.2453
4	0.1800	0.0050	0.8200	-0.0450	0.1515	0.0230	-0.0503	6.5964
5	0.1000	0.0000	0.9000	-0.0500	0.1997	0.0364	-0.0704	5.4929
5	0.1267	0.0017	0.8733	-0.0483	0.1937	0.0336	-0.0680	5.7584
5	0.1533	0.0033	0.8467	-0.0467	0.1891	0.0313	-0.0668	6.0495
5	0.1800	0.0050	0.8200	-0.0450	0.1855	0.0292	-0.0664	6.3456
5	0.2067	0.0067	0.7933	-0.0433	0.1826	0.0275	-0.0665	6.6352
5	0.2333	0.0083	0.7667	-0.0417	0.1802	0.0261	-0.0669	6.9119
5	0.2600	0.0100	0.7400	-0.0400	0.1782	0.0248	-0.0676	7.1718
5	0.2867	0.0117	0.7133	-0.0383	0.1765	0.0238	-0.0684	7.4129
5	0.3133	0.0133	0.6867	-0.0367	0.1750	0.0229	-0.0693	7.6338

5	0.3400	0.0150	0.6600	-0.0350	0.1737	0.0222	-0.0703	7.8339
5	0.3667	0.0167	0.6333	-0.0333	0.1725	0.0215	-0.0713	8.0125
5	0.3933	0.0183	0.6067	-0.0317	0.1715	0.0210	-0.0723	8.1695
5	0.4200	0.0200	0.5800	-0.0300	0.1704	0.0205	-0.0734	8.3045
5	0.4467	0.0217	0.5533	-0.0283	0.1695	0.0201	-0.0744	8.4171
5	0.4733	0.0233	0.5267	-0.0267	0.1686	0.0198	-0.0754	8.5072
5	0.5000	0.0250	0.5000	-0.0250	0.1677	0.0196	-0.0764	8.5740
5	0.5267	0.0267	0.4733	-0.0233	0.1668	0.0194	-0.0773	8.6173
5	0.5533	0.0283	0.4467	-0.0217	0.1660	0.0192	-0.0782	8.6361
5	0.5800	0.0300	0.4200	-0.0200	0.1651	0.0191	-0.0790	8.6298
5	0.6067	0.0317	0.3933	-0.0183	0.1641	0.0191	-0.0798	8.5973
5	0.6333	0.0333	0.3667	-0.0167	0.1632	0.0191	-0.0804	8.5373
5	0.6600	0.0350	0.3400	-0.0150	0.1622	0.0192	-0.0809	8.4487
5	0.6867	0.0367	0.3133	-0.0133	0.1610	0.0193	-0.0813	8.3297
5	0.7133	0.0383	0.2867	-0.0117	0.1598	0.0195	-0.0814	8.1787
5	0.7400	0.0400	0.2600	-0.0100	0.1584	0.0198	-0.0813	7.9939
5	0.7667	0.0417	0.2333	-0.0083	0.1569	0.0202	-0.0810	7.7735
5	0.7933	0.0433	0.2067	-0.0067	0.1550	0.0206	-0.0802	7.5160
5	0.8200	0.0450	0.1800	-0.0050	0.1528	0.0212	-0.0790	7.2208
5	0.8467	0.0467	0.1533	-0.0033	0.1502	0.0218	-0.0771	6.8893
6	0.1267	0.0017	0.8733	-0.0483	0.2280	0.0415	-0.0843	5.4883
6	0.1533	0.0033	0.8467	-0.0467	0.2232	0.0389	-0.0831	5.7333
6	0.1800	0.0050	0.8200	-0.0450	0.2195	0.0367	-0.0826	5.9772
6	0.2067	0.0067	0.7933	-0.0433	0.2166	0.0349	-0.0827	6.2113
6	0.2333	0.0083	0.7667	-0.0417	0.2142	0.0333	-0.0831	6.4311
6	0.2600	0.0100	0.7400	-0.0400	0.2121	0.0320	-0.0838	6.6348
6	0.2867	0.0117	0.7133	-0.0383	0.2104	0.0308	-0.0846	6.8215