Below is the Cornish-Fisher series in two dimensions. Without loss of generality, consider the case with marginal standard deviations 1, and correlation ρ. Remember that we're trying to solve $P[X_1 \geq x_1] = \alpha_1$, $P[X_1 \geq x_1, X_2 \geq x_2] = \alpha_2$. Solution for x_1 in the first equation is given by the first expression on page 48 of Kolassa (2006). Mathematica code to calculate the second expansion is in `writecode.m`. The bivariate Edgeworth series is given by `bivariatetail` near the bottom. The one-dimensional series called `univariatetail` may be obtained by marginalizing the two dimensional series, and the standard Cornish-Fisher expansion is given by `r1` near the bottom. Just a couple of more lines of code substitutes in the one-dimensonal series into the two-dimensional Edgeworth series, and solves for x_2. The solution for x_2 is of form $x_2 = x_2^0 + x_2'(0)/\sqrt{n} + x_2''(0)/n$. Similarly, x_1^0 is the leading term for the first series. Then the lead terms satisfy

$$\alpha_2 = \bar{\Phi}(x_1^0, x_2^0, \rho)$$

and the term in $1/\sqrt{n}$ satisfies

$$x_2'(0) \to \frac{1}{6}(1-\rho^2)^{-2}\Big[\kappa_{222}\left(x_2^{0^2}-1\right)\left(\rho^2-1\right)^2 + \sqrt{1-\rho^2}m^{-1}\Big((\kappa_{111}x_1^0+\kappa_{222}x_2^0)\rho^3 + (\kappa_{222}x_1^0$$
$$+ \kappa_{111}x_2^0)\rho^2 - (2\kappa_{111}x_1^0 + 3\kappa_{122}x_1^0 + 3\kappa_{112}x_2^0 + 2\kappa_{222}x_2^0)\rho + 3(\kappa_{112}x_1^0+\kappa_{122}x_2^0)\Big)\Big]$$

Here $m = \bar{\Phi}((x_1^0 - \rho x_2^0)/\sqrt{1-\rho^2})/\phi((x_1^0-\rho x_2^0)/\sqrt{1-\rho^2})$. The term of size $1/n$ is pretty bad; it is below. There might be a cleaner representation, but I don't want to worry about that now. Mathematica has the facility for exporting results into various formats, but not R. The `Splice` command puts Mathematica results reformatted to Fortran into the file terms.mf, to create the file terms.m. This operation should be, but isn't, entirely automatic. Mathematica is erroneously adding some backslash characters at the end of lines. These needed to be removed.

$$x_2''(0) \to \Big(m\left(\rho^2-1\right)^2\kappa_{111}\phi(x_2^0,x_1^0,\rho)\left(x_1^{0^2}-1\right)$$
$$\times \Big(m\left(\rho^2-1\right)^2\left(\rho\kappa_{111}\left(x_1^{0^2}-1\right)-2\kappa_{222}\left(x_2^{0^2}-1\right)\right) - 2\sqrt{1-\rho^2}\left((\kappa_{111}x_1^0+\kappa_{222}x_2^0)\rho^3 + (\kappa_{222}x_1^0+\kappa_{111}x_2^0)\rho^2 - (2\kappa_{111}x_1^0 + \right.$$
$$-\phi(x_1^0,x_2^0,\rho)\Big(m^3x_2^0\left(2\kappa_{222}^2\left(2x_2^{0^2}-5\right)-3\kappa_{2222}\left(x_2^{0^2}-3\right)\right)\left(1-\rho^2\right)^{9/2}$$
$$+m\Big(\left(x_2^0\left(x_2^{0^2}+2\right)\kappa_{222}^2+2\kappa_{111}x_1^0\left(x_2^{0^2}+1\right)\kappa_{222}+\kappa_{111}^2x_1^{0^2}x_2^0\right)\rho^8+2\Big(x_1^0\left(-2x_1^{0^2}+x_2^{0^2}+2\right)\kappa_{111}^2+\kappa_{222}x_2^0\Big(-2x_1^{0^2}+x_2^{0^2}$$
$$+2\Big(2x_1^0\left(x_1^{0^2}-1\right)\kappa_{111}^2+\Big(3\kappa_{122}x_1^0\left(x_1^{0^2}-2x_2^{0^2}+1\right)-\left(3\kappa_{112}\left(x_1^{0^2}+1\right)-2\kappa_{222}\left(x_1^{0^2}-1\right)\right)x_2^0\Big)\kappa_{111}-3\Big(3x_1^0\Big(x_1^{0^2}+x_2^{0^2}$$
$$-\rho\left(\rho^2-1\right)\big((\kappa_{111}x_1^0+\kappa_{222}x_2^0)\rho^3+(\kappa_{222}x_1^0+\kappa_{111}x_2^0)\rho^2-(2\kappa_{111}x_1^0+3\kappa_{122}x_1^0+3\kappa_{112}x_2^0+2\kappa_{222}x_2^0)\rho+3(\kappa_{112}x_1^0+\kappa_{122}x_2^0)\big)$$
$$+m^2\Big(\Big(\left(x_1^{0^4}-7x_1^{0^2}+6\right)\kappa_{111}^2+4\kappa_{222}x_1^0x_2^0\kappa_{111}-\kappa_{222}^2\left(x_2^{0^2}-5\right)+3\left(\kappa_{1111}\left(x_1^{0^2}-2\right)+\kappa_{2222}\left(x_2^{0^2}-2\right)\right)\Big)\rho^9$$
$$-\Big(x_1^0\left(x_1^{0^2}+5\right)x_2^0\kappa_{111}^2+4\kappa_{222}\left(\left(x_2^{0^2}-1\right)x_1^{0^2}-2x_2^{0^2}+1\right)\kappa_{111}+x_1^0x_2^0\left(\kappa_{222}^2\left(x_2^{0^2}+1\right)-3(\kappa_{1111}+\kappa_{2222})\right)\Big)\rho^8$$
$$-\Big(-\kappa_{222}^2x_2^{0^4}-4\kappa_{222}^2x_2^{0^2}+\kappa_{222}^2x_1^{0^2}x_2^{0^2}-3\kappa_{1111}x_2^{0^2}+15\kappa_{2222}x_2^{0^2}+12\kappa_{122}\kappa_{222}x_1^0x_2^0+22\kappa_{222}^2+2\kappa_{222}^2x_1^{0^2}+15\kappa_{1111}x_1^{0^2}-3\kappa_{22}$$
$$+\Big(\kappa_{222}^2x_2^0x_1^{0^3}+6\kappa_{122}\kappa_{222}x_2^{0^2}x_1^{0^2}-6\kappa_{122}\kappa_{222}x_1^{0^2}-12\kappa_{1112}x_1^{0^2}+5\kappa_{222}^2x_2^{0^3}x_1^0+6\kappa_{112}\kappa_{222}x_2^{0^3}x_1^0+10\kappa_{222}^2x_2^0x_1^0+6\kappa_{112}\kappa_{222}x_2^0x$$
$$+\Big(\kappa_{222}^2x_1^{0^4}+27\kappa_{122}^2x_1^{0^2}+10\kappa_{222}^2x_1^{0^2}-\kappa_{222}^2x_2^{0^2}x_1^{0^2}+30\kappa_{1111}x_1^{0^2}+18\kappa_{1122}x_1^{0^2}-6\kappa_{2222}x_1^{0^2}-12\kappa_{122}\kappa_{222}x_2^{0^3}x_1^0+48\kappa_{122}\kappa_{222}x$$
$$-\Big(6\kappa_{122}\kappa_{222}x_1^{0^4}+9\kappa_{122}^2x_2^0x_1^{0^3}+5\kappa_{222}^2x_2^0x_1^{0^3}+12\kappa_{122}\kappa_{222}x_2^{0^2}x_1^{0^2}+24\kappa_{122}\kappa_{222}x_1^{0^2}-12\kappa_{1112}x_1^{0^2}+4\kappa_{222}^2x_2^{0^3}x_1^0+81\kappa_{122}^2x_2^0x_1^0$$
$$+\Big(9\kappa_{122}^2x_1^{0^4}+24\kappa_{122}\kappa_{222}x_2^0x_1^{0^3}-8\kappa_{222}^2x_1^{0^2}+27\kappa_{122}^2x_2^{0^2}x_1^{0^2}+8\kappa_{222}^2x_2^{0^2}x_1^{0^2}-27\kappa_{1111}x_1^{0^2}-36\kappa_{1122}x_1^{0^2}+3\kappa_{2222}x_1^{0^2}+24\kappa_{122}$$
$$-\Big(27\kappa_{112}^2x_2^0x_1^{0^3}+18\kappa_{112}\kappa_{222}x_2^0x_1^{0^3}-12\kappa_{1112}x_1^{0^2}+27\kappa_{112}^2x_2^{0^3}x_1^0+4\kappa_{222}^2x_2^{0^3}x_1^0+24\kappa_{112}\kappa_{222}x_2^{0^3}x_1^0-54\kappa_{112}^2x_2^0x_1^0-20\kappa_{222}^2x_2^0x$$
$$+\Big(9\kappa_{122}^2x_2^{0^4}+12\kappa_{122}\kappa_{222}x_1^0x_2^{0^3}-54\kappa_{122}^2x_2^{0^2}-12\kappa_{222}^2x_2^{0^2}+27\kappa_{122}^2x_1^{0^2}x_2^{0^2}+18\kappa_{1122}x_2^{0^2}+9\kappa_{2222}x_2^{0^2}-48\kappa_{122}\kappa_{222}x_1^0x_2^0+48\kappa$$