# Package 'ATEHonest'

June 25, 2020

Title Honest Inference for Treatment Effects under Unconfoundedness

Version 0.1.2

<b>Description</b> Construct matching estimators, and optimal linear estimators, along with confidence intervals for conditional and population average treatment effects under unconfoundedness that are valid in finite samples under the assumption that the regression function satisfies a Lipschitz constraint.
<b>Depends</b> R (>= 4.0.0)
License GPL-3
Imports Matrix, igraph, MASS, stats, methods
Suggests spelling, testthat, CVXR, lpSolve, knitr, rmarkdown
Encoding UTF-8
LazyData true
RoxygenNote 7.1.0
VignetteBuilder knitr
Language en-US
BugReports https://github.com/kolesarm/ATEHonest/issues
<pre>URL https://github.com/kolesarm/ATEHonest</pre>
R topics documented:
ATTEffBounds

2 ATTEffBounds

	ATTMatchEstimate	4
	ATTMatchPath	5
	ATTOptEstimate	6
	ATTOptPath	7
	cv	8
	distMat	8
	nnvar	9
	NSW	10
	NSWexper	11
Index		12

ATTEffBounds

Efficiency bounds for confidence intervals

## Description

Computes the asymptotic efficiency of two-sided fixed-length confidence intervals at smooth functions, as well as the efficiency of one-sided confidence intervals that optimize a given beta quantile of excess length, using the formula described in Appendix A of Armstrong and Kolesár (2018)

## Usage

```
ATTEffBounds(h, d, sigma2, C = 1, beta = 0.8, alpha = 0.05)
```

## Arguments

h	The output of ATTh.
d	Vector of treatment indicators with length n
sigma2	Estimate of the conditional variance of the outcome, used to optimize the number of matches.
С	Lipschitz smoothness constant
beta	The quantile beta of excess length for determining performance of one-sided CIs.
alpha	Level of confidence interval, 1-alpha.

#### References

Armstrong, T. B., and M. Kolesár (2018): Finite-Sample Optimal Estimation and Inference on Average Treatment Effects Under Unconfoundedness, Unpublished manuscript

ATTh 3

ATTh

Homotopy for average treatment effect for the treated

#### **Description**

Calculates optimal weights m and r for the control and treated observations as a function of  $\delta$ , or equivalently  $\mu$ , using the algorithm described in the appendix to Armstrong and Kolesár (2018)

#### Usage

```
ATTh(
   D0,
   h,
   maxiter = 50,
   check = FALSE,
   tol = .Machine$double.eps * ncol(D0) * nrow(D0)
)
```

#### **Arguments**

D0	matrix of distances with dimension $[n1 \ n0]$ between untreated and treated units, where $n0$ is the number of untreated units and $n1$ is the number of treated units
h	Optionally, supply previous output of ATTh. If not provided, the homotopy is started at the beginning. If provided, it starts at the step where the previous call to ATTh ended.
maxiter	maximum number of steps in the homotopy. If the homotopy has less steps than maxiter, returns the whole solution path.
check	check at each step that the solution matches that in the CVXR-package (generic convex optimizer package).
tol	numerical tolerance for rounding error when finding the nearest neighbor. All observations with effective distance within tol of the closest are considered to be active.

## Value

A list with two elements:

res A matrix with rows corresponding to steps in the homotopy, so that the maximum number of rows is maxiter (if homotopy started at the beginning), and columns corresponding to  $\delta$ , m, r,  $\mu$ , and drop, an indicator if an observations has been dropped from an active set, or added

 $\mathbf{m0}$  A vector of length  $\mathbf{n0}$  of corresponding to m at the last step.

 ${\bf r0}$  A vector of length n1 of corresponding to r at the last step.

**mu** A scalar corresponding to  $\mu$  at the last step.

**D** A matrix of effective distances with dimension [n1 n0] at the last step.

**Lam** A sparse matrix of Lagrange multipliers with dimension [n1 n0] at the last step.

NO A sparse matrix of nearest neighbors with dimension [n1 n0] at the last step.

4 ATTMatchEstimate

#### References

Armstrong, T. B., and M. Kolesár (2018): Finite-Sample Optimal Estimation and Inference on Average Treatment Effects Under Unconfoundedness, Unpublished manuscript

#### **Examples**

```
 x0 <- c(0, 1, 2, 3) \\ x1 <- c(1, 4, 5) \\ d <- c(rep(FALSE, length(x0)), rep(TRUE, length(x1))) \\ D0 <- distMat(c(x0, x1), d=d) \\ \# Compute first three steps \\ h <- ATTh(D0, maxiter=3) \\ \# Compute the remaining steps, checking them against CVX solution \\ h2 <- ATTh(D0, h=h, check=TRUE)
```

ATTMatchEstimate

Inference on the CATT using the matching estimator

#### Description

Computes matching estimator and confidence intervals (CIs) for the CATT. If ATTMatchPath used a single M, the estimator and CIs are based on a matching estimator with this number of matches. Otherwise, optimize the number of matches from the set in M according to opt.criterion.

#### Usage

```
ATTMatchEstimate(
   mp,
   sigma2,
   C = 1,
   sigma2final = sigma2,
   alpha = 0.05,
   beta = 0.8,
   opt.criterion = "RMSE"
)
```

mp	Output of ATTMatchPath
sigma2	Estimate of the conditional variance of the outcome, used to optimize the number of matches.
С	Lipschitz smoothness constant
sigma2final	vector of variance estimates with length n for determining the standard error of the optimal estimator. In contrast, sigma2 is used only for determining the optimal tuning parameter.
alpha	Level of confidence interval, 1-alpha.

ATTMatchPath 5

beta	The quantile beta of excess length for determining performance of one-sided CIs.
opt.criterion	One of "RMSE" (root mean squared error), "OCI" (one-sided confidence intervals), "FLCI" (fixed-length two-sided confidence intervals)

#### Value

Returns an object of class "ATTEstimate". An object of class "ATTEstimate" is a list containing the following components:

e Data frame with columns TODO

k weights TODO

### **Examples**

```
Ahalf <- diag(c(0.15, 0.6, 2.5, 2.5, 2.5, 0.5, 0.5, 0.1, 0.1))

D0 <- distMat(NSWexper[, 2:10], Ahalf, method="manhattan", NSWexper$treated)

mp <- ATTMatchPath(NSWexper$re78, NSWexper$treated, D0, M=c(1, 2), tol=1e-12)

## Distance matrix for variance estimation

DM <- distMat(NSWexper[, 2:10], Ahalf, method="manhattan")

sigma2 <- nnvar(DM, NSWexper$treated, NSWexper$re78, J=3)

## Estimator based on a single match is better than with 2 matches for RMSE

ATTMatchEstimate(mp, mean(sigma2), C=1, sigma2final=sigma2)
```

ATTMatchPath

Compute the matching estimator for the ATT

## Description

Computes the matching estimator and the matching weights for a range of matches M. The output of this function is used as an input for ATTMatchEstimate for inference on the CATT.

#### Usage

```
ATTMatchPath(y, d, D0, M = 1:25, tol = 1e-12)
```

У	Outcome vector with length n
d	Vector of treatment indicators with length n
DØ	matrix of distances with dimension $[n1\ n0]$ between untreated and treated units, where $n0$ is the number of untreated units and $n1$ is the number of treated units
М	a vector of integers determining the number of matches. If Inf, then use the simple difference in means estimator.
tol	numerical tolerance for determining nearest neighbors in constructing matches

6 ATTOptEstimate

#### Value

List with the following components

**ep** A data frame with columns M, maxbias, and att, corresponding to the number of matches, the scaled worst-case bias, and the CATT estimate.

**K** A matrix where each row j corresponds to the linear weights k used to form the matching estimator with M[j] matches.

d Vector of treatment indicators, as supplied by d

#### **Examples**

```
Ahalf \leftarrow diag(c(0.15, 0.6, 2.5, 2.5, 2.5, 0.5, 0.5, 0.1, 0.1))\\ D0 \leftarrow distMat(NSWexper[, 2:10], Ahalf, method="manhattan", NSWexper$treated)\\ mp \leftarrow ATTMatchPath(NSWexper$re78, NSWexper$treated, D0, M=1:2, tol=1e-12)
```

ATTOptEstimate

Optimal estimation and inference for the CATT

## Description

Computes the estimator and confidence intervals (CIs) for the CATT. The tuning parameter is chosen to optimize opt.criterion criterion.

### Usage

```
ATTOptEstimate(
   op,
   sigma2,
   C = 1,
   sigma2final = sigma2,
   alpha = 0.05,
   beta = 0.8,
   opt.criterion = "RMSE"
)
```

ор	Output of ATTOptPath.
sigma2	Estimate of the conditional variance of the outcome, used to optimize the number of matches.
С	Lipschitz smoothness constant
sigma2final	vector of variance estimates with length n for determining the standard error of the optimal estimator. In contrast, sigma2 is used only for determining the optimal tuning parameter.
alpha	Level of confidence interval, 1-alpha.

ATTOptPath 7

The quantile beta of excess length for determining performance of one-sided

opt.criterion One of "RMSE" (root mean squared error), "OCI" (one-sided confidence inter-

vals), "FLCI" (fixed-length two-sided confidence intervals)

#### **Examples**

ATTOptPath

Compute the class of optimal linear estimators for the CATT

#### **Description**

Computes the class of optimal linear estimators that minimize variance subject to a bound on bias, and the optimal weights. The output of this function is used by ATTOptEstimate for optimal estimation and inference on the CATT.

## Usage

```
ATTOptPath(h, y, d)
```

h	The output of ATTh or	n which to base the estimate

- y Outcome vector with length n
- d Vector of treatment indicators with length n

8 distMat

cv

Critical values for inference based on a biased Gaussian estimator.

#### **Description**

Critical value  $cv_{1-\alpha}(B)$  such that the confidence interval  $X \pm cv_{1-\alpha}(B)$  will have coverage  $1-\alpha$ , where X is normally distributed with variance equal to 1 and bias bounded by B in absolute value.

#### Usage

```
cv(B, alpha = 0.05)
```

#### **Arguments**

B Maximum bias, a non-negative vector.

alpha Scalar between 0 and 1 determining the confidence level, 1-alpha

#### Value

```
Critical value cv_{1-\alpha}(B)
```

## Examples

```
# 90% critical value:
cv(B = 1, alpha = 0.1)
# 95% critical values
cv(B = c(0, 1, 3), alpha = 0.05)
```

distMat

Matrix of distances between observations

### **Description**

Compute a matrix of distances between n observations using the distance measure in method.

#### Usage

```
distMat(X, Ahalf = diag(NCOL(X)), method = "euclidean", d = NULL, p = 2)
```

nnvar 9

Arguments	
1 LI Sullicitus	

X	Design matrix of covariates with dimension n by p, or else a vector of length n if there is a single covariate.
Ahalf	Weight matrix with dimension p by p so that the distances are computed between Ahalf $**X[i,]$ .
method	the distance measure to be used. This must be one of "euclidean", "maximum", "manhattan", "canberra", "binary" or "minkowski". Any unambiguous substring can be given.
d	Vector of treatment indicators with length n. If supplied, return the n1 by n0 sub-matrix corresponding to distances between treated and untreated observations. Otherwise return the full n by n matrix
р	The power of the Minkowski distance.

#### Value

Matrix of distances with dimension n by n or else n1 by n0

#### **Examples**

```
## 4 units, unit 1 and 3 are treated.
distMat(X=c(1, 2, 3, 4), d=c(TRUE, FALSE, TRUE, FALSE))
```

nnvar

Nearest-neighbor variance estimator

## Description

Calculate an n-vector of estimates of the variance of y using a nearest-neighbor estimator among observations with the same treatment status d.

#### Usage

```
nnvar(DM, d, y, J = 3, tol = 0)
```

## Arguments

DM	distance matrix with dimension n by n.
d	Vector of treatment indicators with length n
У	Outcome vector with length n
J	number of nearest neighbors to average over
tol	numerical tolerance for determining nearest neighbors in constructing matches

## Value

An n-vector of estimates of the variance of y.

10 NSW

#### **Examples**

```
X <- as.matrix(NSWexper[, 2:10])
DM <- distMat(X, chol(solve(cov(X))), method="euclidean")
sigma2 <- nnvar(DM, d=NSWexper$treated, y=NSWexper$re78, J=3)</pre>
```

NSW

Dataset from Dehejia and Wahba (1999)

#### **Description**

Subset of National Supported Work and PSID data from Dehejia and Wahba (1999).

#### Usage

NSW

#### **Format**

A data frame with 2,675 observations (2,490 controls from PSID, and 185 treated individuals from NSW) and 11 variables.

treated Treatment indicator

age Age in years

education Year of education

black Indicator for black

hispanic Indicator for Hispanic

married Indicator for married

re74 Earnings in 1974 (in thousands of dollars)

re75 Earnings in 1975 (in thousands of dollars)

re78 Earnings in 1978 (in thousands of dollars)

ue74 Indicator for zero earnings in 1974

ue75 Indicator for zero earnings in 1975

#### **Source**

Rajeev Dehejia's website, http://users.nber.org/~rdehejia/nswdata2.html

## References

Dehejia, R., and Wahba, S. (1999), "Causal Effects in Nonexperimental Studies: Reevaluating the Evaluation of Training Programs," Journal of the American Statistical Association, 94 (448), 1053-1062.

NSWexper 11

**NSWexper** 

Experimental dataset from Dehejia and Wahba (1999)

#### Description

National Supported Work data from Dehejia and Wahba (1999).

#### Usage

NSWexper

#### **Format**

A data frame with 445 observations (185 treated and 260 controls) and 11 variables.

treated Treatment indicator

age Age in years

education Year of education

black Indicator for black

hispanic Indicator for Hispanic

married Indicator for married

re74 Earnings in 1974 (in thousands of dollars)

re75 Earnings in 1975 (in thousands of dollars)

re78 Earnings in 1978 (in thousands of dollars)

ue74 Indicator for zero earnings in 1974

ue75 Indicator for zero earnings in 1975

#### **Source**

Rajeev Dehejia's website, http://users.nber.org/~rdehejia/nswdata2.html

#### References

Dehejia, R., and Wahba, S. (1999), "Causal Effects in Nonexperimental Studies: Reevaluating the Evaluation of Training Programs," Journal of the American Statistical Association, 94 (448), 1053-1062.

## **Index**

```
* datasets
NSW, 10
NSWexper, 11

ATTEffBounds, 2
ATTh, 3
ATTMatchEstimate, 4, 5
ATTOptEstimate, 6, 7
ATTOptPath, 7

cv, 8

distMat, 8

nnvar, 9
NSW, 10
NSWexper, 11
```