

# Package ‘ATEHonest’

June 26, 2020

**Title** Honest Inference for Treatment Effects under Unconfoundedness

**Version** 0.1.2

**Description** Construct matching estimators, and optimal linear estimators, along with confidence intervals for conditional and population average treatment effects under unconfoundedness that are valid in finite samples under the assumption that the regression function satisfies a Lipschitz constraint.

**Depends** R (>= 4.0.0)

**License** GPL-3

**Imports** Matrix,  
igraph,  
MASS,  
stats,  
methods

**Suggests** spelling,  
testthat,  
CVXR,  
lpSolve,  
knitr,  
rmarkdown

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 7.1.0

**VignetteBuilder** knitr

**Language** en-US

**BugReports** <https://github.com/kolesarm/ATEHonest/issues>

**URL** <https://github.com/kolesarm/ATEHonest>

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ATTEffBounds	<i>Efficiency bounds for confidence intervals</i>
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**Description**

Computes the asymptotic efficiency of two-sided fixed-length confidence intervals at smooth functions, as well as the efficiency of one-sided confidence intervals that optimize a given beta quantile of excess length, using the formula described in Appendix A of Armstrong and Kolesár (2018)

**Usage**

ATTEffBounds(op, sigma2, C = 1, beta = 0.8, alpha = 0.05)

**Arguments**

op	The output of ATTOptPath.
sigma2	Estimate of the conditional variance of the outcome, used to optimize the number of matches.
C	Lipschitz smoothness constant
beta	The quantile beta of excess length for determining performance of one-sided CIs.
alpha	Level of confidence interval, 1-alpha.

**Value**

A list with two elements, onesided and twosided, for one- and two-sided efficiency.

**References**

Armstrong, T. B., and M. Kolesár (2018): *Finite-Sample Optimal Estimation and Inference on Average Treatment Effects Under Unconfoundedness*, Unpublished manuscript

## Examples

```
Ahalf <- diag(c(0.15, 0.6, 2.5, 2.5, 2.5, 0.5, 0.5, 0.1, 0.1))
## Use NSW experimental subsample with 30 treated and untreated units
dt <- NSWexper[c(1:20, 426:445), ]
D0 <- distMat(dt[, 2:10], Ahalf, method="manhattan", dt$treated)
## Distance matrix for variance estimation
DM <- distMat(dt[, 2:10], Ahalf, method="manhattan")
sigma2 <- nnvar(DM, dt$treated, dt$re78, J=3)
## Compute the solution path, first 50 steps will be sufficient
op <- ATTOptPath(dt$re78, dt$treated, D0, maxsteps=50)
eb <- ATTEffBounds(op, mean(sigma2), C=1)
```

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ATTMatchEstimate

*Inference on the CATT using the matching estimator*


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## Description

Computes matching estimator and confidence intervals (CIs) for the CATT. If ATTMatchPath used a single M, the estimator and CIs are based on a matching estimator with this number of matches. Otherwise, optimize the number of matches from the set in M according to `opt.criterion`.

## Usage

```
ATTMatchEstimate(
  mp,
  sigma2,
  C = 1,
  sigma2final = sigma2,
  alpha = 0.05,
  beta = 0.8,
  opt.criterion = "RMSE"
)
```

## Arguments

<code>mp</code>	Output of ATTMatchPath
<code>sigma2</code>	Estimate of the conditional variance of the outcome, used to optimize the number of matches.
<code>C</code>	Lipschitz smoothness constant
<code>sigma2final</code>	vector of variance estimates with length n for determining the standard error of the optimal estimator. In contrast, <code>sigma2</code> is used only for determining the optimal tuning parameter.
<code>alpha</code>	Level of confidence interval, 1-alpha.
<code>beta</code>	The quantile beta of excess length for determining performance of one-sided CIs.
<code>opt.criterion</code>	One of "RMSE" (root mean squared error), "OCI" (one-sided confidence intervals), "FLCI" (fixed-length two-sided confidence intervals)

**Value**

Returns an object of class "ATTEstimate". An object of class "ATTEstimate" is a list containing the following components:

- e** Data frame with columns TODO
- k** weights TODO

**Examples**

```
Ahalf <- diag(c(0.15, 0.6, 2.5, 2.5, 2.5, 0.5, 0.5, 0.1, 0.1))
D0 <- distMat(NSWexper[, 2:10], Ahalf, method="manhattan", NSWexper$treated)
mp <- ATTMatchPath(NSWexper$re78, NSWexper$treated, D0, M=c(1, 2), tol=1e-12)
## Distance matrix for variance estimation
DM <- distMat(NSWexper[, 2:10], Ahalf, method="manhattan")
sigma2 <- nnvar(DM, NSWexper$treated, NSWexper$re78, J=3)
## Estimator based on a single match is better than with 2 matches for RMSE
ATTMatchEstimate(mp, mean(sigma2), C=1, sigma2final=sigma2)
```

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ATTMatchPath

---

*Compute the matching estimator for the ATT*


---

**Description**

Computes the matching estimator and the matching weights for a range of matches  $M$ . The output of this function is used as an input for [ATTMatchEstimate](#) for inference on the CATT.

**Usage**

```
ATTMatchPath(y, d, D0, M = 1:25, tol = 1e-12)
```

**Arguments**

- y** Outcome vector with length  $n$
- d** Vector of treatment indicators with length  $n$
- D0** matrix of distances with dimension  $[n_1 \ n_0]$  between untreated and treated units, where  $n_0$  is the number of untreated units and  $n_1$  is the number of treated units
- M** a vector of integers determining the number of matches. If Inf, then use the simple difference in means estimator.
- tol** numerical tolerance for determining nearest neighbors in constructing matches

**Value**

List with the following components

- ep** A data frame with columns  $M$ ,  $maxbias$ , and  $att$ , corresponding to the number of matches, the scaled worst-case bias, and the CATT estimate.
- K** A matrix where each row  $j$  corresponds to the linear weights  $k$  used to form the matching estimator with  $M[j]$  matches.
- d** Vector of treatment indicators, as supplied by  $d$

**Examples**

```
Ahalf <- diag(c(0.15, 0.6, 2.5, 2.5, 2.5, 0.5, 0.5, 0.1, 0.1))
D0 <- distMat(NSWexper[, 2:10], Ahalf, method="manhattan", NSWexper$treated)
mp <- ATMatchPath(NSWexper$re78, NSWexper$treated, D0, M=1:2, tol=1e-12)
```

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ATTOptEstimate

*Optimal estimation and inference for the CATT*


---

**Description**

Computes the estimator and confidence intervals (CIs) for the CATT. The tuning parameter is chosen to optimize `opt.criterion` criterion.

**Usage**

```
ATTOptEstimate(
  op,
  sigma2,
  C = 1,
  sigma2final = sigma2,
  alpha = 0.05,
  beta = 0.8,
  opt.criterion = "RMSE"
)
```

**Arguments**

<code>op</code>	Output of ATTOptPath.
<code>sigma2</code>	Estimate of the conditional variance of the outcome, used to optimize the number of matches.
<code>C</code>	Lipschitz smoothness constant
<code>sigma2final</code>	vector of variance estimates with length <code>n</code> for determining the standard error of the optimal estimator. In contrast, <code>sigma2</code> is used only for determining the optimal tuning parameter.
<code>alpha</code>	Level of confidence interval, $1-\alpha$ .
<code>beta</code>	The quantile <code>beta</code> of excess length for determining performance of one-sided CIs.
<code>opt.criterion</code>	One of "RMSE" (root mean squared error), "OCI" (one-sided confidence intervals), "FLCI" (fixed-length two-sided confidence intervals)

**Value**

Returns an object of class "ATTEstimate". An object of class "ATTEstimate" is a list containing the following components:

**e** Data frame with columns TODO

**k** weights TODO

### Examples

```
Ahalf <- diag(c(0.15, 0.6, 2.5, 2.5, 2.5, 0.5, 0.5, 0.1, 0.1))
## Use NSW experimental subsample with 30 treated and untreated units
dt <- NSWexper[c(1:20, 426:445), ]
D0 <- distMat(dt[, 2:10], Ahalf, method="manhattan", dt$treated)
## Distance matrix for variance estimation
DM <- distMat(dt[, 2:10], Ahalf, method="manhattan")
sigma2 <- nnvar(DM, dt$treated, dt$re78, J=3)
## Compute the solution path, first 50 steps will be sufficient
op <- ATTOptPath(dt$re78, dt$treated, D0, maxsteps=50)
ATTOptEstimate(op, mean(sigma2), C=1, sigma2final=sigma2,
               opt.criterion="RMSE")
ATTOptEstimate(op, mean(sigma2), C=1, sigma2final=sigma2,
               opt.criterion="FLCI")
```

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ATTOptPath

*Class of optimal linear estimators for the CATT*

---

### Description

Use a LASSO-like algorithm to compute the solution path  $\{\hat{L}_\delta : \delta > 0\}$  tracing out the class of optimal linear estimators that minimize variance subject to a bound on bias. The output of this function is used by [ATTOptEstimate](#) for optimal estimation and inference on the CATT.

### Usage

```
ATTOptPath(y, d, D0, maxsteps = 50, tol, path = NULL, check = FALSE)
```

### Arguments

y	Outcome vector with length n
d	Vector of treatment indicators with length n
D0	matrix of distances with dimension [n1 n0] between untreated and treated units, where n0 is the number of untreated units and n1 is the number of treated units
maxsteps	maximum number of steps in the solution path. If the full solution path is shorter than maxsteps, compute the whole path.
tol	numerical tolerance for rounding error when finding the nearest neighbors. All observations with effective distance within tol of the closest are considered to be active.
path	Optionally, supply previous output of ATTOptPath. If not provided, the path is started at the beginning (at $\delta = 0$ ). If provided, it starts at the step where the previous call to ATTOptPath ended.
check	check at each step that the solution matches that obtained by direct optimization using <a href="#">CVXR-package</a> (generic convex optimizer package).

**Value**

A list with the following elements:

**y** Output vector, as supplied by `y`

**d** Vector of treatment indicators, as supplied by `d`

**D0** Matrix of distances, as supplied by `D0`

**res** A matrix with rows corresponding to steps in the homotopy, so that the maximum number of rows is `maxsteps` (if homotopy started at the beginning), and columns corresponding to  $\delta$ ,  $m$ ,  $r$ ,  $\mu$ , and `drop`, an indicator if an observations has been dropped from an active set, or added.

**m0** A vector of length `n0` of corresponding to  $m$  at the last step.

**r0** A vector of length `n1` of corresponding to  $r$  at the last step.

**mu** A scalar corresponding to the Lagrange multiplier  $\mu$  at the last step.

**D** A matrix of effective distances with dimension `[n1 n0]` at the last step.

**Lam** A sparse matrix of Lagrange multipliers with dimension `[n1 n0]` at the last step.

**N0** A sparse matrix of effective nearest neighbors with dimension `[n1 n0]` at the last step.

**K** Matrix of weights  $k$  associated with the optimal estimator at each step

**ep** A data frame with columns `delta`, `omega`, `maxbias`, and `att`, corresponding to  $\delta$ ,  $\omega(\delta)$ , the scaled worst-case bias, and the CATT estimate.

**Examples**

```
x0 <- c(0, 1, 2, 3)
x1 <- c(1, 4, 5)
d <- c(rep(FALSE, length(x0)), rep(TRUE, length(x1)))
D0 <- distMat(c(x0, x1), d=d)
## Compute first three steps
p1 <- ATTOptPath(d, d, D0, maxsteps=3)
## Compute the remaining steps, checking them against CVX solution
p2 <- ATTOptPath(path=p1, maxsteps=4, check=TRUE)
```

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cv

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*Critical values for inference based on a biased Gaussian estimator.*


---

**Description**

Critical value  $cv_{1-\alpha}(B)$  such that the confidence interval  $X \pm cv_{1-\alpha}(B)$  will have coverage  $1 - \alpha$ , where  $X$  is normally distributed with variance equal to 1 and bias bounded by  $B$  in absolute value.

**Usage**

```
cv(B, alpha = 0.05)
```

**Arguments**

**B** Maximum bias, a non-negative vector.

**alpha** Scalar between 0 and 1 determining the confidence level,  $1-\alpha$

**Value**

Critical value  $cv_{1-\alpha}(B)$

**Examples**

```
# 90% critical value:
cv(B = 1, alpha = 0.1)
# 95% critical values
cv(B = c(0, 1, 3), alpha = 0.05)
```

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distMat	<i>Matrix of distances between observations</i>
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**Description**

Compute a matrix of distances between  $n$  observations using the distance measure in method.

**Usage**

```
distMat(X, Ahalf = diag(NCOL(X)), method = "euclidean", d = NULL, p = 2)
```

**Arguments**

X	Design matrix of covariates with dimension $n$ by $p$ , or else a vector of length $n$ if there is a single covariate.
Ahalf	Weight matrix with dimension $p$ by $p$ so that the distances are computed between $Ahalf \%*\% X[i, ]$ .
method	the distance measure to be used. This must be one of "euclidean", "maximum", "manhattan", "canberra", "binary" or "minkowski". Any unambiguous substring can be given.
d	Vector of treatment indicators with length $n$ . If supplied, return the $n_1$ by $n_0$ sub-matrix corresponding to distances between treated and untreated observations. Otherwise return the full $n$ by $n$ matrix
p	The power of the Minkowski distance.

**Value**

Matrix of distances with dimension  $n$  by  $n$  or else  $n_1$  by  $n_0$

**Examples**

```
## 4 units, unit 1 and 3 are treated.
distMat(X=c(1, 2, 3, 4), d=c(TRUE, FALSE, TRUE, FALSE))
```



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nnvar	<i>Nearest-neighbor variance estimator</i>
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### Description

Calculate an n-vector of estimates of the variance of y using a nearest-neighbor estimator among observations with the same treatment status d.

### Usage

```
nnvar(DM, d, y, J = 3, tol = 0)
```

### Arguments

DM	distance matrix with dimension n by n.
d	Vector of treatment indicators with length n
y	Outcome vector with length n
J	number of nearest neighbors to average over
tol	numerical tolerance for determining nearest neighbors in constructing matches

### Value

An n-vector of estimates of the variance of y.

### Examples

```
X <- as.matrix(NSWexper[, 2:10])
DM <- distMat(X, chol(solve(cov(X))), method="euclidean")
sigma2 <- nnvar(DM, d=NSWexper$treated, y=NSWexper$re78, J=3)
```

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NSW	<i>Dataset from Dehejia and Wahba (1999)</i>
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### Description

Subset of National Supported Work and PSID data from Dehejia and Wahba (1999).

### Usage

```
NSW
```

**Format**

A data frame with 2,675 observations (2,490 controls from PSID, and 185 treated individuals from NSW) and 11 variables.

**treated** Treatment indicator

**age** Age in years

**education** Year of education

**black** Indicator for black

**hispanic** Indicator for Hispanic

**married** Indicator for married

**re74** Earnings in 1974 (in thousands of dollars)

**re75** Earnings in 1975 (in thousands of dollars)

**re78** Earnings in 1978 (in thousands of dollars)

**ue74** Indicator for zero earnings in 1974

**ue75** Indicator for zero earnings in 1975

**Source**

Rajeev Dehejia's website, <http://users.nber.org/~rdehejia/nswdata2.html>

**References**

Dehejia, R., and Wahba, S. (1999), "Causal Effects in Nonexperimental Studies: Reevaluating the Evaluation of Training Programs," *Journal of the American Statistical Association*, 94 (448), 1053-1062.

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NSWexper

*Experimental dataset from Dehejia and Wahba (1999)*

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**Description**

National Supported Work data from Dehejia and Wahba (1999).

**Usage**

NSWexper

**Format**

A data frame with 445 observations (185 treated and 260 controls) and 11 variables.

**treated** Treatment indicator

**age** Age in years

**education** Year of education

**black** Indicator for black

**hispanic** Indicator for Hispanic

**married** Indicator for married

**re74** Earnings in 1974 (in thousands of dollars)

**re75** Earnings in 1975 (in thousands of dollars)

**re78** Earnings in 1978 (in thousands of dollars)

**ue74** Indicator for zero earnings in 1974

**ue75** Indicator for zero earnings in 1975

**Source**

Rajeev Dehejia's website, <http://users.nber.org/~rdehejia/nswdata2.html>

**References**

Dehejia, R., and Wahba, S. (1999), "Causal Effects in Nonexperimental Studies: Reevaluating the Evaluation of Training Programs," *Journal of the American Statistical Association*, 94 (448), 1053-1062.

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