Sensitivity analysis in moment condition models

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Contents

1 Introduction

The package `GMMSensitivity` implements estimators and confidence interval for sensitivity analysis in moment condition models considered in Armstrong and Kolesár [2018]. In this vignette, we demonstrate the implementation of these confidence intervals using the demand model for automobiles from Berry et al. [1995].

The package includes the dataset `blp`, which contains estimates of the Berry et al. [1995] model, as implemented by Andrews et al. [2017]. A description of the dataset can be obtained via the documentation in R, using `?blp`. For convenience, we give the description here:

`G` Matrix with 31 rows and 17 columns, estimate of derivative of the moment condition G

`H` Vector of length 17, estimate of derivative of average markup $h(\theta)$.

`W` Weight matrix used to obtain the estimate of theta.

`g_init` Average moment condition, evaluated at estimate $\hat{\theta}$ of θ from Berry et al. [1995].

`h_init` Estimate of the average markup, $h(\hat{\theta})$.

`names` Two lists, one for names of the moment conditions, and one for elements of theta

`ZZ` Gram matrix of the instruments, used to specify the set $\mathcal{C}$

`Sig` Variance of moment condition, estimate of Σ.

`sdZ` vector of standard deviations of the instruments

`perturb` scaling parameter to give interpretable meaning to violations of moment conditions. For demand-side moments, it corresponds to an estimate of $\delta_d = 0.01\overline{p}/(\overline{y}/\alpha)$, and for supply-side moments, it corresponds to an estimate of $\delta_s = -0.01\overline{p}/\overline{mc}$, where $\overline{p}$ is average price, $\overline{y}$ is average income, $\overline{mc}$ is average marginal cost, and α is a parameter in the utility function. With this scaling, if a given demand-side instrument enters the utility function with coefficient δ_d, a consumer is willing to pay 1% of the average car price for a unit increase in the instrument. If a given supply-side instrument enters the cost function with coefficient δ_s, increasing the instrument by one unit decreases the marginal cost by 1% of the average car price.

`n` Sample size, number of model/years.

2 Usage

The package implements estimators, confidence intervals, and efficiency calculations for the model (in the notation of Armstrong and Kolesár [2018])

$$g(\theta_0) = c, \quad c = B\gamma, \quad \|\gamma\|_p \leq M.$$

Suppose that we want to allow all excluded instruments in the Berry et al. [1995] application to be potentially invalid. Fix B to a scaling matrix so that if the jth supply-side instrument is invalid with $\gamma_{sj} = 1$, this means that changing the instrument by one standard deviation changes the marginal cost by γ_{sj}% of the average car price, and if the j the demand-side instrument is invalid with $\gamma_{dj} = 1$, then the consumer willingness to pay for one standard deviation change in the instrument is γ_{dj}% of the average 1980 car price. Let $p = 2$, and $M = \sqrt{\#I}$, where $\#I$ is the number of invalid instruments so that $\gamma = 1$ is included in the set (this is the same scaling as described in Armstrong and Kolesár [2018] in the paper). Then the confidence interval can be constructed as follows:

```
## Construct estimate of initial sensitivity
blp$k_init <- -drop(blp$H %*% solve(crossprod(blp$G, blp$W %*%
    blp$G), crossprod(blp$G, blp$W)))
## list collecting initial estimates of H, G, Sigma, n,
## g(thetahat), initial sensitivity k, and initial
## estimate of average markup h(thetahat)
eo <- list(H = blp$H, G = blp$G, Sig = blp$Sig, n = blp$n,
    g_init = blp$g_init, k_init = blp$k_init, h_init = blp$h_init)
## Rows corresponding to invalid instruments
I <- vector(mode = "logical", length = nrow(eo$G))
I[c(6:13, 20:31)] <- TRUE
## Matrix B, scaled as described in the paper
B0 <- blp$ZZ %*% diag(sqrt(blp$n) * abs(blp$perturb)/blp$sdZ)
## Value of M
M0 <- sqrt(sum(I))
## Select columns of B0 corresponding to invalid
## instruments
OptEstimator(eo, B0[, I], M = M0, p = 2, alpha = 0.05, opt.criterion = "FLCI")
```

```
#>
#>
#> Estimate    Max. bias    SE           CI
#> ----------  -----------  -----------  -----------------------
#> 0.5598805   0.06295884   0.02268748   (0.4596041, 0.6601569)
```

The efficiency κ_* for this confidence interval can be computed using the `EffBounds` function (which can also be used to compute efficiency of one-sided confidence intervals):

```
EffBounds(eo, B0[, I], M = M0, p = 2)$twosided
#> [1] 0.9704414
```

In contrast the CI based on the initial estimate is much wider:

```
OptEstimator(eo, B0[, I], M = M0, p = 2, alpha = 0.05, opt.criterion = "Valid")
#>
#>
#> Estimate    Max. bias   SE           CI
#> ----------  ----------  -----------  ------------------------
#> 0.3271788   0.1983659   0.01815665   (0.09894796, 0.5554097)
```

A specification test for whether the value $M = M0$ is too low, that is a test of the hypothesis $H_0\colon M \leq M0$, can be conducted using the `Jtest` function:

```
Jtest(eo, B0[, I], M = M0, p = 2, alpha = 0.05)
#> $J
#> [1] 404.6756
#>
#> $p0
#> [1] 0
#>
#> $pC
#> [1] 0.001361034
#>
#> $Mmin
#> [1] 4.836483
```

Here `J` is the J-statistic, `p0` is the p-value of the usual J-test (that assumes $M = 0$ under the null), `pC` is the p-value of the test, and `Mmin` is the smallest value of M that is not rejected.

If were only concerned about the validity of the demand-side instrument `"demand_firm_const"` (number of cars produced by the same firm), then since this is the sixth instrument, an analogous analysis could be conducted as:

```r
I <- vector(mode = "logical", length = nrow(eo$G))
I[6] <- TRUE
OptEstimator(eo, B0[, I, drop = FALSE], M = 1, p = 2, alpha = 0.05,
    opt.criterion = "FLCI")
#>
#>
#> Estimate    Max. bias     SE           CI
#> ----------  ------------  -----------  -----------------------
#> 0.3564058   0.002489521   0.01868668   (0.3194578, 0.3933538)
EffBounds(eo, B0[, I, drop = FALSE], M = 1, p = 2)$twosided
#> [1] 0.8593365
OptEstimator(eo, B0[, I, drop = FALSE], M = 1, p = 2, alpha = 0.05,
    opt.criterion = "Valid")
#>
#>
#> Estimate    Max. bias    SE           CI
#> ----------  -----------  -----------  -----------------------
#> 0.3271788   0.01070908   0.01815665   (0.2862084, 0.3681493)
Jtest(eo, B0[, I, drop = FALSE], M = 1, p = 2, alpha = 0.05)
#> $J
#> [1] 404.6756
#>
#> $p0
#> [1] 0
#>
#> $pC
#> [1] 0
#>
#> $Mmin
#> [1] 9.767053
```

References

Isaiah Andrews, Matthew Gentzkow, and Jesse M. Shapiro. Measuring the sensitivity of parameter estimates to sample statistics. *Quarterly Journal of Economics*, 132(4):1553–1592, November 2017.

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Steven T. Berry, James Levinsohn, and Ariel Pakes. Automobile prices in market equilibrium. *Econometrica*, 63(4):841–890, 1995.