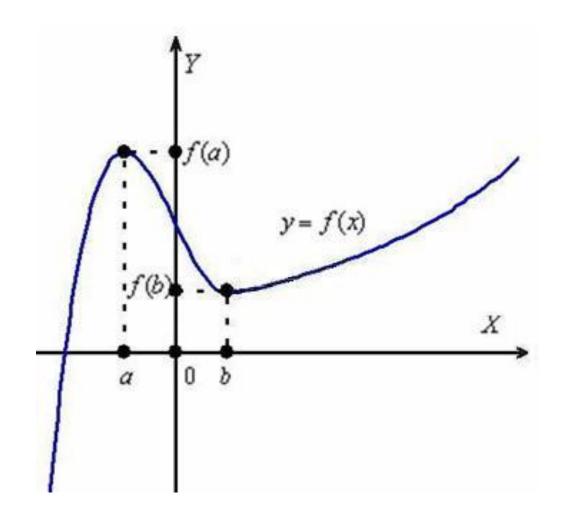
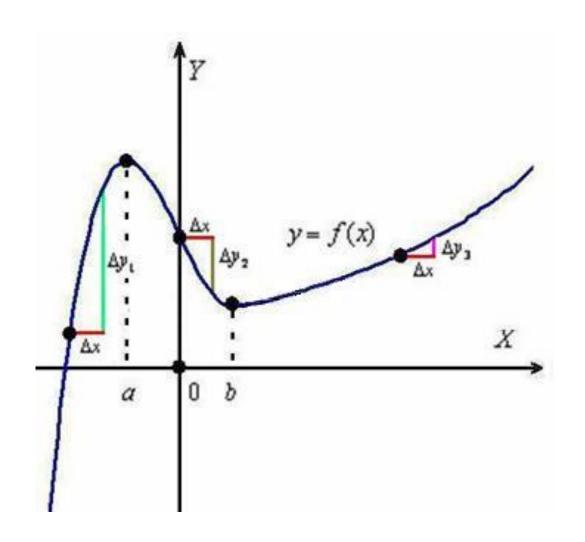
Производная функции. Градиентный спуск.

Производная функции – скорость изменения функции в данной точке

$$f'(x_0) = \lim_{x o x_0}rac{f(x)-f(x_0)}{x-x_0} = \lim_{\Delta x o 0}rac{f(x_0+\Delta x)-f(x_0)}{\Delta x} = \lim_{\Delta x o 0}rac{\Delta f(x)}{\Delta x}$$





Производные степенных функций	Производные тригонометрических функций	Производные обратных тригонометрических функций	Производные гиперболических функций
(c)'=0	$(\sin x)' = \cos x$	$(rcsin x)' = rac{1}{\sqrt{1-x^2}}$	$(\sinh x)' = \cosh x$
$(x^a)^\prime = ax^{a-1}$	$\left(\cos x ight)'=-\sin x$	$(rccos x)' = -rac{1}{\sqrt{1-x^2}}$	$(\cosh x)' = \sinh x$
$(a^x)' = a^x \ln a$	$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$	$(\operatorname{arctg} x)' = rac{1}{1+x^2}$	$(\tanh x)' = rac{1}{\cosh^2 x}$
$(\log_a x)' = \frac{1}{x \ln a}$	$(\operatorname{ctg} x)' = -rac{1}{\sin^2 x}$	$(\operatorname{arcctg} x)' = -rac{1}{1+x^2}$	$(\coth x)' = -rac{1}{\sinh^2 x}$

Правила дифференцирования. Дифференцирование произведения и частного.

$$egin{align} \left(kf(x)
ight)' = kf'(x) & \left(uv
ight)' = u'v + uv' \ & \left(f(x) + g(x)
ight)' = f'(x) + g'(x) & \left(rac{u}{v}
ight)' = rac{u'v - uv'}{v^2} \ & \left(rac{u}{v}
ight)' = rac{u'v - uv'}{v^2} \ & \left(rac{u}{v}
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ight)' = rac{u}{v} \ & \left(rac{u}{v}
ight)' = rac^2 \left(rac{u}{v} + u + uv' + u$$

Найти производную линейной функции **y=ax+b**, используя определение производной.

$$\Delta y=y\left(x+\Delta x
ight)-y\left(x
ight)=\left(a\left(x+\Delta x
ight)+b
ight)-\left(ax+b
ight)=\cancel{ax}+a\Delta x+\cancel{b}-\cancel{ax}-\cancel{b}=a\Delta x.$$

$$y'\left(x
ight) = \lim_{\Delta x o 0} rac{\Delta y}{\Delta x} = \lim_{\Delta x o 0} rac{a\cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x o 0} a = a$$

$$y=x^2$$
 .

$$\Delta y=y\left(x+\Delta x
ight) -y\left(x
ight) =\left(x+\Delta x
ight) ^{2}-x^{2},$$

$$\Delta y = (x+\Delta x)^2 - x^2 = \cancel{x}^2 + 2x\Delta x + (\Delta x)^2 - \cancel{x}^2 = (2x+\Delta x)\,\Delta x$$

$$y'\left(x
ight)=\lim_{\Delta x
ightarrow0}rac{\Delta y}{\Delta x}=\lim_{\Delta x
ightarrow0}rac{\left(2x+\Delta x
ight)\cancel{\Delta x}}{\cancel{\Delta x}}=\lim_{\Delta x
ightarrow0}\left(2x+\Delta x
ight)=2x$$

$$y = ax^2 + bx + c$$

$$egin{aligned} \Delta y &= y \left(x + \Delta x
ight) - y \left(x
ight) = \left[a (x + \Delta x)^2 + b \left(x + \Delta x
ight) + c
ight] - \left[a x^2 + b x + c
ight] \ &= \left[a x^2 + 2 a x \Delta x + a (\Delta x)^2 + b x + b \Delta x + c
ight] - \left[a x^2 + b x + c
ight] \ &= \left[a x^2 + 2 a x \Delta x + a (\Delta x)^2 + b x + b \Delta x + c - c x^2 - b x - c \right] = 2 a x \Delta x + a (\Delta x)^2 + b \Delta x \ &= \left(2 a x + b + a \Delta x \right) \Delta x. \end{aligned}$$

$$y'\left(x
ight)=\lim_{\Delta x o0}rac{\Delta y}{\Delta x}=\lim_{\Delta x o0}rac{\left(2ax+b+a\Delta x
ight)\cancel{\Delta x}}{\cancel{\Delta x}}=\lim_{\Delta x o0}\left(2ax+b+a\Delta x
ight)=2ax+b$$

$$y=rac{1}{x}$$

$$y'\left(x
ight) = \lim_{\Delta x o 0} rac{y\left(x + \Delta x
ight) - y\left(x
ight)}{\Delta x} = \lim_{\Delta x o 0} rac{rac{1}{x + \Delta x} - rac{1}{x}}{\Delta x} = \lim_{\Delta x o 0} rac{rac{x - (x + \Delta x)}{(x + \Delta x)x}}{\Delta x} = \lim_{\Delta x o 0} rac{rac{-\Delta x}{(x + \Delta x)x}}{\Delta x} = \lim_{\Delta x o 0} rac{-\Delta x}{\Delta x} = \lim_{\Delta x o 0} rac{-\Delta x}{(x + \Delta x)x}$$

Частная производная – производная по одной переменной в случае, если функция имеет несколько переменных.

$$z = x^5 + y^5 - 5x^3y^3$$

$$z'_{x} = (x^{5} + y^{5} - 5x^{3}y^{3})'_{x} = 5x^{4} - 15x^{2}y^{3}$$
$$z'_{y} = (x^{5} + y^{5} - 5x^{3}y^{3})'_{y} = 5y^{4} - 15x^{3}y^{2}$$

$$z = \frac{xy}{\sqrt{x^2 + y^2}}$$

$$z'_{x} = \left(\frac{xy}{\sqrt{x^2 + y^2}}\right)'_{x} = \frac{y\sqrt{x^2 + y^2} - xy\frac{2x}{2\sqrt{x^2 + y^2}}}{x^2 + y^2} = \frac{y(x^2 + y^2) - x^2y}{(x^2 + y^2)\sqrt{x^2 + y^2}} = \frac{y(x^2 + y^2)\sqrt{x^2 + y^2}}{(x^2 + y^2)\sqrt{x^2 + y^2}} = \frac{y(x^2 + y^2)\sqrt{x^2 + y^2}}{(x^2 + y^2)\sqrt{x^2 + y^2}} = \frac{y(x^2 + y^2)\sqrt{x^2 + y^2}}{(x^2 + y^2)\sqrt{x^2 + y^2}} = \frac{y(x^2 + y^2)\sqrt{x^2 + y^2}}{(x^2 + y^2)\sqrt{x^2 + y^2}} = \frac{y(x^2 + y^2)\sqrt{x^2 + y^2}}{(x^2 + y^2)\sqrt{x^2 + y^2}}$$

$$=\frac{y^3}{(x^2+y^2)\sqrt{x^2+y^2}};$$

$$z = \frac{xy}{\sqrt{x^2 + y^2}}$$

$$z'_{y} = \left(\frac{xy}{\sqrt{x^{2} + y^{2}}}\right)'_{y} = \frac{x\sqrt{x^{2} + y^{2}} - xy\frac{2y}{2\sqrt{x^{2} + y^{2}}}}{x^{2} + y^{2}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y^{2}) - xy^{2}}{(x^{2} + y^{2})\sqrt{x^{2} + y^{2}}} = \frac{x(x^{2} + y$$

$$=\frac{x^3}{(x^2+y^2)\sqrt{x^2+y^2}};$$