

Вектора и матрицы

Сумма матриц.

$$\begin{aligned} C + D &= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 2 & 4 & 6 & 8 \\ 1 & 3 & 5 & 7 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 0 & -4 \\ 3 & 0 & -4 & -2 \\ 0 & -4 & 2 & -9 \\ 3 & -4 & 0 & 1 \end{pmatrix} = \\ &= \begin{pmatrix} 1+2 & 2-1 & 3+0 & 4-4 \\ 4+3 & 5+0 & 6-4 & 7-2 \\ 2+0 & 4-4 & 6+2 & 8-9 \\ 1+3 & 3-4 & 5+0 & 7+1 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 3 & 0 \\ 7 & 5 & 2 & 5 \\ 2 & 0 & 8 & -1 \\ 4 & -1 & 5 & 8 \end{pmatrix}. \\ C + D &= \begin{pmatrix} 3 & 1 & 3 & 0 \\ 7 & 5 & 2 & 5 \\ 2 & 0 & 8 & -1 \\ 4 & -1 & 5 & 8 \end{pmatrix} \end{aligned}$$

Дистрибутивность, ассоциативность и коммутативность

$$A(B + C) = AB + AC$$

$$A(BC) = (AB)C$$

$$AB \neq BA$$

Свойства транспонирования

$$(AB)^T = B^T A^T$$

$$(A + B)^T = A^T + B^T$$

Скалярное произведение

$$\mathbf{a}^T = [a_1, a_2, \dots, a_n] \ , \quad \mathbf{b}^T = [b_1, b_2, \dots, b_n]$$

$$\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^T \mathbf{b} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

Скалярное произведение

$s^T = s$ transpose of a scalar s

$x^T y = y^T x$ inner product for vectors

Скалярное произведение. Единичный вектор.

$$\hat{x} = \frac{x}{||x||} , \quad ||x|| = \sqrt{\langle x, x \rangle} , \quad ||\hat{x}|| = 1$$

Скалярное произведение. Единичный вектор.

$$\hat{x} = \frac{x}{||x||} , \quad ||x|| = \sqrt{\langle x, x \rangle} , \quad ||\hat{x}|| = 1$$

Скалярное произведение.

$$\langle ax, y \rangle = a \langle x, y \rangle$$

$$\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$$

$$\langle Ax, y \rangle = \langle x, A^T y \rangle$$

Скалярное произведение.

$$\langle Ax, y \rangle = (Ax)^T y = x^T A^T y = \langle x, A^T y \rangle$$

By definition, $\langle x, y \rangle = x^T y$

Скалярное произведение.

$$\langle Ax, y \rangle = (Ax)^T y = x^T A^T y = \langle x, A^T y \rangle$$

By definition, $\langle x, y \rangle = x^T y$

Скалярное произведение.

$$\langle Ax, y \rangle = (Ax)^T y = x^T A^T y = \langle x, A^T y \rangle$$

By definition, $\langle x, y \rangle = x^T y$

Ещё об умножении.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 6 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 15 & 18 \end{pmatrix}$$

Ещё об умножении.

$$\begin{array}{c}
 \left(\begin{array}{c|c|c} \hline \boldsymbol{u}_1 & \cdots & \boldsymbol{u}_m \\ \hline \end{array} \right) \begin{array}{c} \left(\begin{array}{c|c} \hline \phantom{\boldsymbol{u}} & \boldsymbol{v}_1 \\ \hline \phantom{\boldsymbol{u}} & \vdots \\ \hline \phantom{\boldsymbol{u}} & \boldsymbol{v}_n \\ \hline \end{array} \right) & = & \left(\begin{array}{c|c} \hline \boldsymbol{u}_1 \\ \hline \end{array} \right) \left(\begin{array}{c|c} \hline \phantom{\boldsymbol{u}} & \boldsymbol{v}_1 \\ \hline \end{array} \right) + \dots + \left(\begin{array}{c|c} \hline \boldsymbol{u}_n \\ \hline \end{array} \right) \left(\begin{array}{c|c} \hline \phantom{\boldsymbol{u}} & \boldsymbol{v}_n \\ \hline \end{array} \right) \\
 \boldsymbol{A} & \boldsymbol{B} &
 \end{array}$$

Ещё об умножении.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} (5 \ 6) + \begin{pmatrix} 2 \\ 4 \end{pmatrix} (7 \ 8)$$

Определитель матрицы.

$$\begin{aligned} |A| &= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} \square & \square & \square \\ \square & e & f \\ \square & h & i \end{vmatrix} - b \begin{vmatrix} \square & \square & \square \\ d & \square & f \\ g & \square & i \end{vmatrix} + c \begin{vmatrix} \square & \square & \square \\ d & e & \square \\ g & h & \square \end{vmatrix} \\ &= a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ &= aei + bfg + cdh - ceg - bdi - afh. \end{aligned}$$

Определитель матрицы.

$$\begin{aligned}|A| &= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} \square & \square & \square \\ \square & e & f \\ \square & h & i \end{vmatrix} - b \begin{vmatrix} \square & \square & \square \\ d & \square & f \\ g & \square & i \end{vmatrix} + c \begin{vmatrix} \square & \square & \square \\ d & e & \square \\ g & h & \square \end{vmatrix} \\ &= a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ &= aei + bfg + cdh - ceg - bdi - afh.\end{aligned}$$

Определитель матрицы.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$$\det \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = 45 + 84 + 96 - 105 - 48 - 72 = 0$$

The diagram illustrates the calculation of the determinant of a 3x3 matrix using the rule of Sarrus. The matrix is shown with its elements. The first two columns are repeated to the right, forming a 3x5 grid. The elements are: Row 1: 1, 2, 3, 1, 2; Row 2: 4, 5, 6, 4, 5; Row 3: 7, 8, 9, 7, 8. Three blue diagonal lines (downward) connect the top-left to the bottom-right: (1,1) to (3,3), (2,1) to (3,2), and (3,1) to (2,2). Three pink diagonal lines (upward) connect the top-right to the bottom-left: (1,3) to (3,2), (2,3) to (3,1), and (3,3) to (2,1). The products of the blue diagonals are 105, 48, and 72, with the first product (105) written above the first diagonal. The products of the pink diagonals are 45, 84, and 96, with the first product (45) written below the first diagonal. The final result is 0, highlighted in red.

$$\det(A) = \det(A^T)$$

$$\det(A) = \frac{1}{\det(A^{-1})}$$

$$\det(cA) = c^n \det(A) \quad \text{for } n \times n \text{ matrix}$$

$$\det(A^n) = (\det(A))^n$$

$$\det(AB) = \det(A) \det(B)$$

$$\det(A) = \lambda_1 \lambda_2 \dots \lambda_n \quad \det(A) = \text{product of all eigenvalues}$$

1	2	3
4	5	6
7	8	10

1	2	3
0	-3	-6
7	8	10

subtract row 2
by $4 \times$ row 1

...

1	2	3
0	-3	-6
0	0	1

$$\det(A) = 1 \times -3 \times 1$$

$\det(A)$ = product of diagonal elements
when all lower triangle elements
are transformed to 0

Обратная матрица.

$$\underline{A^{-1}} A = I$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \dots$$

Обратная матрица. Свойства.

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A^T)^{-1} = (A^{-1})^T$$

Обратная матрица. Свойства.

$$Ax = b$$
$$x = \underline{A^{-1}}b$$

matrix inverse

Обратная матрица. Свойства.

$$\begin{array}{ccc} \text{left inverse} & & \text{right inverse} \\ & \diagdown & / \\ & \mathbf{A}^{-1} \mathbf{A} & \mathbf{A} \mathbf{A}^{-1} \\ & \mathbf{A}^{-1} \mathbf{A} = \mathbf{A} \mathbf{A}^{-1} & \text{(if } \mathbf{A} \text{ is square)} \end{array}$$

Обратная матрица. Как считать?

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

cannot be zero

$$\operatorname{adj}(A) = C^T = \begin{pmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ + \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}.$$