Model Solution M.C.A. 2020 (Management Faculty) Optimization Techniques (2020 Pattern)- Set B

(MT-21) (Semester - 11)

(10)

Max. Marks : 50 Time: 2.5 Hours Instructions to the candidates: 1) All questions are compulsory. 2) Use of statistical table and non programmable calculator is allowed. 3) Figures to the right indicate full marks. Attempt the following MCQs (0.5 mark each) A minimization problem can be converted into a maximization problem by changing the Q1) 1 sign of coefficients in the _____ a) constraints b) Objective Function c) Both A and B d) None of the above The order in which machines are required for completing jobs is called as 2 a) Machine order b) Job order c) Processing order d) Working order Floats for critical activities will be always 3 a) one b) zero c) highest d) same as duration of the activity Problems based on the phenomenon of decision making under risk are referred to as 4 a) Numerical Problem b) Complex problem c) Probabilistic problems d) None of above The saddle point in a payoff matrix is always the _____. 5 a) largest number in the matrix b) Smallest number in its column and the smallest number in its row c) Smallest number in the Matrix d) largest number in its column and the smallest number in its row As simulation is not an analytical model, therefore the result of simulation must be viewed 6 a) Unrealistic b) Exact c) Approximation

If in a LPP, the solution of a variable can be made infinity large without violating the

d) Simplified

a) Infeasible b) Unbounded c) Alternative

constraints, the solution is ____

d) None of the above

8	to reduction it sawifest time ton a log popular to amounts of then that log has to black.
	a) right
	b) left
	e) centre d) None of the above
Q.	and the state of t
Y	a) tonative
	b) definite
	e) latest
	d) earliest
10	
(4)	a) a
	b) 1- a
	e) 1/a
	d) \alpha 2
11	in a mixed strategy, each player should optimize the
	a) maximum pavotis
	b) lower value of the game c) minimum loss.
	d) expected gain
13	A problem is classified as Markov chain provided a) There are finite number of possible states b) States are collectively exhaustive & mutually exclusive c) Long-run probabilities of being in a particular state will be constant over time d) All of the above In simplex method, if there is the between a decision variable and a slack (or surplus) variable for entering should be selected a) Slack variable
	b) Surplus variable
	c) Decision variable d) None of the above
14	To identify the outlined
1.4	To identify the optimal sequence, a 3 machine problem has to be converted into
	a) 1
	b) 2 c) M
	d) none of the above
15	CPM stands for
	a) Control Path Method
	b) Critical Path Method
	c) Control Path Management
	d) Crifical Plan Management
16	The minimin criterion is used when conso
	The minimin criterion is used when consequences are given in the form of
	o) table
	c) Opportunity loss
en a l	a) Pavoti
17	Each player should follow the same coates.
	Each player should follow the same strategy regardless of the other player's strategy in which of the following games?
	3) Constant operation

- b) Mixed strategy
- c) Pure strategy
- d) Dominance strategy
- 18 In Markov analysis, state probabilities must
 - a) Sum to one
 - b) Be less than one
 - c) Be greater than one
 - d) d. None of the above
- are the entities whose values are to be determined from the solution of the LPP.
 - a) Objective function
 - b) Decision Variables
 - e) Constraints
 - d) Opportunity costs
- The longest path in the network diagram is called _____path.
 - a) best
 - b) worst
 - e) sub-critical
 - d) critical
- Q 2) a) Solve the following LPP

Maximize $Z = -2x_1 - x_3$

subject to

 $x_1 + x_2 - x_3 \ge 5$

 $x_1 - 2x_2 + 4x_3 \ge 8$

 $x_1, x_2, x_3 \ge 0$

Solution:

PHASE-I

	Cj	0	0	0	0	0	-1	-1		
Basic variable s	Св	X ₁	X2	X3	Sı	S ₂	A ₁	A ₂	Solutio n values	Ratio
A_1	-1	1	1	-1	-1	0	1	0	5	
A_2	-1	1	-2	4	0	-1	0	1	8	2 -
	Z_{j}	-2	1	-3	1	1	-1	-1		
	C _j -	2	-1	3 ↑	-1	-1	0	0		

	Cj	0	0	0	0	0	-1		
Basic variable s	Св	X ₁	X2	X ₃	Sı	S ₂	Aı	Solution values	Ratio
A_1	-1	5/4	1/2	0	-1	-1/4	-	7	28/5 -
X_3	0	1/4	-1/2	1	0	-1/4	0	2	8
	Z_j	-5/4	-1/2	0	1	1/4	-1		0
	C _j -	5/4	1/2	0	-1	-1/4	0		

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Basic variable	Сј	0° X1	0 X ₂	0 X 3	0 Sı	S ₂	Solution values
- <u>s</u> - X ₁	0	1	2/5	0	-4/5 1/5	-1/5 -1/5	28/5
X ₃	$\frac{0}{Z_{j}}$	0	-3/5 0	0	0	0	37.2
	C _j -	0	0	0	0	0	

Artificial variables are eliminated hence we proceed to Phase-II

		~~		
PH	A	SE	-	п

PHASE- II	Cj	-2	0	-1	0	0		
Basic variable s	Св	X ₁	X2	X3	Sı	S ₂	Solution values	Ratio
X_1	-2	1	2/5	0	-4/5	-1/5	28/5	14
<i>X</i> ₃	-1	0	-3/5	1	1/5	-1/5	3/5	
	Z_{j}	-2	-1/5	-1	7/5	3/5		
	C _j -	0	1/5	0	-7/5	-3/5		
	$\mathbf{Z}_{\mathbf{j}}$		I					

	Cj	-2	0	-1	0	0	
Basic variable s	Св	X ₁	X2	X3	Sı	S ₂	Solution values
X ₂	0	5/2	1	0	-2	-1/2	14
X ₃	-1	3/2	0	1	-1	-1/2	9
	$\mathbf{Z}_{\mathbf{j}}$	-3/2	0	-1	1	1/2	
	C_j - Z_j	-1/2	0	0	-1	-1/2	

Optimum Solution is arrived at with value of variables as : $X_1 = 0$, $X_2 = 14$, $X_3 = 9$ Maximum Z = -9; x1=0, x2=14, x3=9

$$X_1 = 0$$
, $X_2 = 14$, $X_3 = 9$

Solve the game for the given pay-off matrix: b)

-5	3	1	20
5	5	4	6
-4	-2	0	-5

Solution:

	B ₁	B_2	B ₃	B ₄	Row Minimun	
A_1	-5	3	1	20	-5	
A_2	5	5	4	6	4	Maximin = 4
A_3	-4	-2	0	-5	-5	
Column Max	5	5	4	20	-	
Saddla Daint - 1		Mini	imax = 4			

Saddle Point = 4

Oppinsal Strategy for B - B.

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Solve the following LPP: (12) 1

Manymore $z = 3x_1 + 2x_2$

Subject to the constraints:

$$x^4 - x^5 \le 5$$

$$0 \le c x_{cit}$$

Solution:

1000	C	3	2	0	0		
Basic variable	Св	Xi	N2	Sı	S2	Solution values	Ratio
Sı	0	1	1	1	0	4	4
S	0	1	-1	0	l	2	2
	2	0	0	0	0		
	Ci-Zi	3,	2	0	0	de la constitución de la constit	

	C_{j}	3	2	0	0		
Basic variable s	Св	Xı	X2	Sı	S2	Solution values	Ratio
Sı	0	0	2	1	-1	2	2 —
1/1	0	1		0	1	2	Reject
	Zi	3	-3	0	3		
	C-Z	0	\$	0	-3		

Secretary of Charles (Sec. 2019) Sec.	C_{i}	3	2	0	0	
Basic variable s	Св	Xı	N ₂	Sı	S ₂	Solution values
7.5	2	0	1	1/2	-1/2	1
71	3	1	0	1/2	1/2	3
And in the country the similar great to the country when the	Zj	3	2	5/2	1/2	
	$C_i - Z_i$	0	0	-5/2	-1/2	2

Optimum Solution is arrived at with value of variables as:

 $\frac{N_1=3}{N_2=2}$

Maximum Z = 13; $x_1=3$, $x_2=2$

Solution:		B's S0	rategy	Row Minimum
		\mathbf{B}_1	\mathbf{B}_2	
	Δı	9	=6	~ Ó
A's Strategy	Δ_2	-5	5	-5 ← Maximin
Column Maxima	1.	9	5	

Minimax

The maximin value is not equal to the minimax value, implying there is no saddle point in this problem. Hence it is a game of mixed strategies.

Let p = Probability that player A uses strategy A₁
∴ 1-p = Probability that player A uses strategy A₂
q = Probability that player B uses strategy B₁
∴ 1-q = Probability that player B uses strategy B₂

for player A we have

$$p = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{5 - (-5)}{(5 + 9) - (-5 + (-6))}$$

$$\therefore p = \frac{10}{25} = \frac{2}{5} \quad and \quad 1 - p = 1 - \frac{2}{5} = \frac{3}{5}$$

Similarly, for player B we have,

$$q = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{5 - (-6)}{(5 + 9) - (-5 + (-6))}$$

$$\therefore q = \frac{11}{25} \quad and \ 1 - q = 1 - \frac{11}{25} = \frac{14}{25}$$

The maximum gain for player A will be given by the value of the game.

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{9 \times 5 - (-5)(-6)}{(9+5) - (-6-5)} = \frac{15}{25} = \frac{3}{5}$$

Hence the optimal strategy for player A is (p, 1- p) i.e $\left(\frac{2}{5}, \frac{3}{5}\right)$ and the optimal strategy for player B is (q, 1- q) i.e $\left(\frac{11}{25}, \frac{14}{25}\right)$

A should use strategy A_1 for $\frac{2}{5}$ th of the times and strategy A_2 for $\frac{3}{5}$ th of the times resulting in a gain of $\frac{3}{5}$ units to A.

Similarly, B should use strategy B_1 for $\frac{11}{25}$ of the times and strategy B_2 for $\frac{14}{25}$ of the times resulting in a loss of $\frac{3}{5}$ units to B,

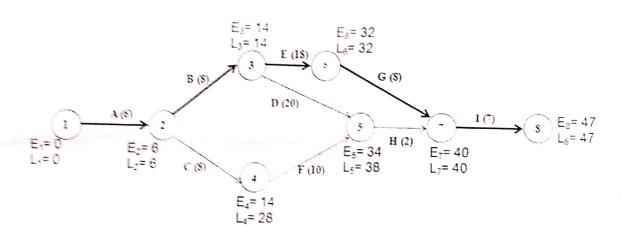
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	the state of the s	principles of the second secon	to the second se			
A			Time in day			
Activity		Most Optimistic	Most Likely	Most Pessimistic	Expected Time	Variance
15		-4	6	8	6	0.44
В	A	5	7	15	8	2.77
C	A	. 4	8	12	8	1.77
D	В	15	20	25	20	2.77
E	В	10	18	26	18	7.11
F	C	8	9	16	10	1.77
G	E	4	8	12	8	1.77
H	D.F ·	1	2	3	2	0.11
1	G.H	6	7	8	7	0.11

- i) Construct an arrow diagram for this problem
- ii) Determine the critical path and compute the expected completion time.
- iii) Determine the probability of completing the project in 55 days.

Solution:

The network diagram is shown below



ii) The critical path is 1-2-3-6-7-8 or A-B-E-G-I Expected Project completion time is 47 days

iii) Variance = 12.20;
$$\sigma$$
 = 3.492
P(X\le 55) = P(Z\le 2.29) = 0.9981

b) Consider the following profit table along with the given probabilities of each state.

	States					
Strategies	N_1	N ₂	N ₃			
Strategies		Probability	•			
	0.3	0.6	0.1			
Sı	20	18	-9			
S2	25	15	10			
S ₃	40	-10	12			

Calculate

- i. EMV
- ii. EVPI
- iii. VPI

Solutiona

$$EMV(S_1) = 0.3(20) + 0.6(18) + 0.1(-0) = 15.0$$

$$EMV(S_2) = 0.3(25) + 0.6(15) + 0.1(10) - 17.5$$

$$EMV(8.0 - 0.3(40) + 0.0(-10) + 0.1(12) - 7.2$$

Maximum EMV is 17.5.

EMV_{max} =12.5 corresponds to Strategy S₂ Hence S₂ is optimal strategy

ii.
$$EVP1 = 0.3(40) + 0.0(18) + 0.1(12) - 24$$

iii.
$$VPI = EVPI = EMVmax$$

= 24 = 17.5
= 6.5

OR

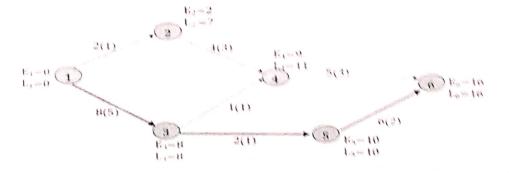
(O3) a)

. For the data given in the table below, draw the network. Crash systematically the activities and determine the optimal project duration and cost.

A	time ((chooks)	Cost in 182 (000)		
Activity	Normal	Crash		Create	
1 - 2	3	The second secon	10	13	
1 3	*	.8	13	31	
3 4	-1	1	30	24	
3 4	1		*	*	
3 5	3		N	13	
1 6	.5	1	10	16	
5 6	0	3	15	36	

- 1. Draw the project network.
- 2. Determine the critical path and the normal duration and the associated cost.
- Crash the activities so that the project completion time reduces to 9 weeks, with minimum additional cost.

Solution The network diagram is shown below



The normal duration of the project is to weeks.

The critical path is $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$ and the associated cost is:

Cost Slopes:

A spirites	THE RESERVE OF THE PARTY OF THE		A second second					
detivity	1 = 2	1 43	2-4	3-4	3-5	4-6	5-6	
Clist Stope	5000	2000	4000		7000	3000	6000	

First Crash:

Activity 1 - 3 has minimum costs slope. Crash activity 1 - 3 by 3 weeks.

Project duration = 13 weeks

Crashing cost = 2000x3 = Rs. 6,000

Second Crash:

Crash activity 5 - 6 by 2 weeks.

Project duration=11 weeks

Crashing cost = 6000x2 = Rs. 12,000

Third Crash:

Now there are three critical paths viz

1-3-5-6, 1-3-4-6 and 1-2-4-6 all with duration equal to 11 weeks.

Crash activity 4 -6 by 2 weeks and activity 5 - 6 by two weeks

Project duration=9 weeks

Crashing cost = 6000x2 + 3000x2

= Rs. 18.000

Thus, the project duration has been reduced from 16 weeks to 9 weeks.

Total additional cost = Rs. 6.000 + Rs. 12,000 + Rs. 18,000

= Rs. 36.000

b) A manufacturer of cycle has estimated the following distributing of demand for a

particular type of bicycle.

Section of the sectio	Commission of the Asset						
Demand	0	and later	2	3	4	5	6
Probability	0.14	0.27	0.27	0.18	0.09	0.04	0.01

Each cycle costs him Rs.7000 and he sells them for Rs.10000 each. Any cycles that are left unsold at the end of the season must be disposed off for Rs.6000 each. How many cycles should be in the stock so as to maximize his expected profit?

Solution:

Den	and	0	1	2	3	4	5	6	
Prob	abilit	0.14	0.27	0.27	0.18	0.09	0.04	0.01	EMV
	0	0	0	0	0	0	0	0	0
The same of the sa	1	-1000	3000	3000	3000	3000	3000	3000	2440
ies	2	-2000	2000	6000	6000	6000	6000	6000	3800
Strategi Stock	3	-3000	1000	5000	9000	9000	9000	9000	4080
Stra	4	-4000	0	4000	8000	12000	12000	12000	3640
	5	-5000	-1000	3000	7000	11000	15000	15000	2840
	6	-6000	-2000	2000	6000	10000	14000	18000	1880

As per EMV, Strategy S₃ is giving maximum profit of Rs. 4080, so it advised to have 3 cycles every day.

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(विकास) की विश्वभाग केलेंद्र का process five fields on three machines A, B and C. Processing times are स्थारण को कीलेंद्र को किलेंद्र का किलेंद्र केलेंद्र केलेंद्र

Hem	A_i	B_i	C_{L}
Ī	A	4	6
2	0)	3	9
3	H	3	11
4	0	2	18
3	3	6	7

1 While the sequence that minimizes the total elapsed time.

is a limit the wife times for all the machines

Sometimes there, $\min(A_i) = A_i - \min(C_i) = 0$, $\max(B_i) = 0$

So the condition $\max(B_i) \le \min(C_i)$ is satisfied.

Convert the problem in 2- machine problem

The processing times for the new problem are given below:

ltem	$G_i = A_i + B_i$	$H_i = B_i + C_i$
١	8	10
2	14	14
3	11	14
4	8	10
3	9	13

Following Johnson's algorithm, the optimal sequence can be obtained as

Then the minimum clapsed time is obtained from the following table.

hem	Machine 4		Machine B			Machine C		
	Time	Time	Time in	Time out	ldle Time	Time in	Time out	Idle Time
1	0	4	4	8	4	8	14	8
4	4	10	10	12	2	14	22	-
3	10	13	13	19	1	22	29	•
3	13	21	21	24	2	29	40	-
2	21	30	30	35	6	40	49	-

Minimum Total Flapsed time= 49 hours

Table time for machine A = 49-30 = 19 hours.

kille time for machine B = 4 + 2 + 1 + 6 + (49 - 35) = 29 hours.

Adle time for machine C = 8 hours.

The number of units of an item that are withdrawn from the inventory on a day to day b) basis is a Markov Chain process in which the requirements for tomorrow depend on today's requirements. A one-day transition matrix is given below

		Comorrow				
		5 10 12				
	5	0.0	0.4	0.0		
Today	10	0.3	0.3	0.4		
	12	0.1	0.3	0.6		

- i) Develop a two day transition matrix.
- ii) Comment how a two-day transition matrix might be helpful to a manager who is responsible for the inventory management

Solution:

(i) Using the given one-day transition matrix, we get the two-day transition matrix (P)

$$P^{2} = P \times P = \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

By using row-column multiplication rule we have

$$P^{2} = \begin{bmatrix} 0.6(0.6) + 0.4(0.3) + 0.0(0.1) & 0.6(0.4) + 0.4(0.3) + 0.0(0.3) & 0.6(0.0) + 0.4(0.4) + 0.0(0.6) \\ 0.3(0.6) + 0.3(0.3) + 0.4(0.1) & 0.3(0.4) + 0.3(0.3) + 0.4(0.3) & 0.3(0.0) + 0.3(0.4) + 0.4(0.6) \\ 0.1(0.6) + 0.3(0.3) + 0.6(0.1) & 0.1(0.4) + 0.3(0.3) + 0.6(0.3) & 0.1(0.0) + 0.3(0.4) + 0.6(0.6) \\ \hline & 5 & 10 & 12 \\ & 5 & \begin{bmatrix} 0.48 & 0.36 & 0.16 \\ 0.31 & 0.33 & 0.36 \end{bmatrix} \\ & = 10 & 0.31 & 0.33 & 0.36 \end{bmatrix}$$

(ii) Consider that the inventory manager places order every day. If the material so ordered reaches him after two days (i.e. lead time), then the matrix P2 will be useful to him, for making the ordering decision.

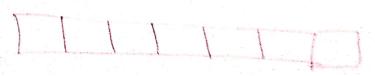
From the two-day transition matrix P3 we see that, if today's requirement of the item is 5, then two days later (when the ordered material will be received), the probability that 5 units will be required is 0.48, 10 units will be required is 0.36 and 12 units will be required is 0.16 and so on. Thus, these figures will guide him in deciding the quantity to be ordered today.

OR

Seven jobs are to be processed through 2 machines A and B. Processing times (in hours) Q(4)**a**) are given below:

Jobs	1	2	3	4	5	0	7
Machine A:	10	9	7	15	18	20	14
Machine B:	12	8	7	12	10	6	13

Find the clapsed time and idle times for Machines A and B.



Solution:

Optimal Sequent	:e:						
	1	7	4	5	2	3	6
	1	,			70.	1	

Elapsed Time:

Optimal	1	Machine: A	\	Machine:B		
Sequence	In	out	ldle	ln	Out	ldle
l	0	10	-	10	22	10
7	10	24	-	24	37	2
4	24	39	**	39	51	2
5	39	57	-	57	67	6
2	57	66	4	67	75	0
3	66	73	-	75	82	0
6	73	93	6	93	99	11

Total Elapsed Time = 99 Hours Idle time of Machine A = (99-93) = 6 hours Idle time of Machine B =(10+2+2+6+11)=31 hours

b) The present market shares of three brands of soft drinks are 60%, 30% and 10% respectively. The transition probability matrix is as below:

$$P = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{pmatrix}$$

- Find their expected market shares after two years.
- ii. Find their market shares in the long run.

Solution: Here we have, for the present year (n = 0)

$$R_0 = \{0.6-0.3-0.1\} \\ \text{Hence, their market shares (state probabilities) after 1 year i.e. for n \times 1 is$$

$$R_{1} = R_{0} \times P$$

$$= [0.6 \ 0.3 \ 0.1] \begin{bmatrix} 0.7 \ 0.2 \ 0.1 \\ 0.2 \ 0.6 \ 0.2 \end{bmatrix}$$

$$= [(0.42 + 0.06 + 0.01) \ (0.12 + 0.18 * 0.01) \ (0.06 * 0.06 * 0.08)]$$

$$= [0.49 \ 0.31 \ 0.20]$$

Thus, the market shares of A, B and C in the next year will be 49%, 31% and 20% respectively. Similarly, market shares after two years i.e. for the period in = 2) are given by

$$R_2 = R_1 \times P$$

$$= \{0.49 \ 0.31 \ 0.21 | 0.7 \ 0.2 \ 0.1 \]$$

$$= \{0.425 \ 0.304 \ 0.271\}$$

Thus, the market shares will be 42 5%, 30 4% and 27 1% respectively

ii) Market shares in long run

$$\lambda = S_{\lambda} \times 100 = \frac{6}{19} \times 100 = 31.58\%$$

Market share of
$$B = 5_0 \times 100 = \frac{5}{10} \times 100 = 26.32\%$$
 and

Market share of
$$C = S_c \times 100 = \frac{8}{19} \times 100 = 42.10\%$$

Rainfall distribution in monsoon season is as follows:

	- A	1)	3	4	5
Rain in cm	0	1	2	0.05	0.03	0.02
Probability	0.50	0.25	0.15	0.05	0.03	0.02
AAGOROHIGA	V. D. W					

Simulate the rainfall for 10 days using following random numbers 67.63,39,55,29,78,70,6,78,76. Find the average rainfall.

Solution:

Generation of Random Number Intervals

Rain (in cm)	Probability	Cumulative Probability	Random Number Interval
0	0.5	0.5	0049
1	0.25	0.75	5074
5	0.15	0.9	7589
3	0.05	0.95	9094
4	0.03	0.98	9597
Š	0.02	1	9899

Simulation of Rainfall

Day	Random Number	Simulated Rainfall
1	67	1
2	63	l
3	39	0
4	35	1
3	29	0
6	78	2
7	70	1
8	6	0
9	78	2
10	76	2

Total Rainfall = 10 cm

Average Rainfall = 10/10= 1 cm

- b) Explain the following terms with examples:
 - i) Degeneracy
 - ii) Multiple Optimal Solution in LPP

OR

A bakery keeps stock of a popular brand of cake. Previous experience shows the daily demand pattern for the item with associated probabilities, as given below:

Daily demand (number)	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

Use the following sequence of random numbers to simulate the demand for next 10 days. Random numbers: 25, 39, 65. 76. 12, 05. 73, 89, 19, 49.

Also estimate the daily average demand for the cakes on the basis of the simulated data.

Solution:

Generation of Random Number Intervals

Daily Demand	Probability	Cumulative Probability	Random Number Interval
0	0.01	0.01	00
10	0.20	0.21	0120
20	0.25	0.36	2135
20	0.13	0.86	3685
30	0.30	0.98	8697
50	0.02	1	9899

Simulation of Daily demand

Day	Random Number	Simulated Daily Demand
The same of the sa	25	20
2	39	30
3	65	30
4	76	30
5	12	10
6	05	10
7	73	30
8	89	40
9	19	10
10	49	30

Total Daily Demand = 240 cakes Average Daily Demand = 240/10= 24 cakes Explain the following terms with examples: b)

- i) Dummy Activity
- ii) Optimistic Time