Our goal here is to show that the subset of problems solved in polynomial time by WL is strictly included in the subset solved by our heuristic. We shall consider an extended version of WL (EWL) to facilitate the proof, where colors aren't redistributed among $[1; n]$ but are instead an injective function of the vertex's previous color and the multiset of its neighbour's colors into the set of colors (which is not anymore included in $[1; n]$). Two cases allow WL to reduce the number of possibilities and hence have a polynomial runtime : the first is when the colorings of G and G' are incompatible, and the second when there is a color shared by an unique vertex in each graph.

We must show three properties :

- When the graphs are isomorphic, WL, EWL and PN compute the isomorphism, and produce a negative certificate in the other case (correctness)
- When WL finds two incompatible coloring, then so does EWL, and PN also gives a negative certificate (polynomial negative certificate)
- When WL colors a vertex in an unique color in both graphs, then EWL does the same, and our algorithm also recognizes that those two vertices must be linked if the graphs are isomorphic (polynomial isomorphism)

The first is immediate, because the operations we use are invariant through permutations of the vertices, hence if the two graphs are isomorphic it might take exponential time but will surely end. The two other properties trivially hold for extended WL, and we must show that it implies that they hold for PN. However, there is an advantage of EWL over WL, because the coloring is unique for the two graphs (it being injective), so if there is exactly one vertex in each graph colored by C, then the isomorphism must assign one to the other (problems may arise in WL because the coloring is not necessarily injective). This gives an easy proof of the third depending on the second, because the fact that a vertex is colored uniquely in G and G' is implied by the fact that it is colored differently from every single other vertex. Hence we must only prove the second property.

We shall proceed by induction to prove that if EWL colors a vertice in G and another in G' in different colors, then PN also allows us to differentiate those two vertices. At the first run of the coloring, every vertex is colored by its degree so it is trivially true.

Now let us suppose that the property holds at run n, and show that it holds at run n+1 :

Consider two vertices V and W (one per graph) that had the same color for all previous runs, but which are colored differently at run n+1. Then their neighbourhoods are incompatible, which means that the multiset of colors is not the same, so there is a color c such that there is one more vertex of color c in N(V) than in N(W). However, those colors were attributed at run n, so it means that when comparing the PN lists of neighbours of V and W, those lists will be incompatible by hypothesis. Hence V and W can't be linked in PN, which ends the induction.