



Distributed signal processing in radio communication

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Bachelors's thesis presentation



Average consensus algorithm on the graph: motivation (1)

- Graph G is a set of edges and vertices
- An existing edge represents a possibility to exchange information
- Initialize i -th vertex with value i (e.g. temperature)

$$x_i(0) = i \quad (1)$$

- Distributed algorithm: communication is possible only between the neighbors in the graph
- Goal: obtain unique value in all vertices

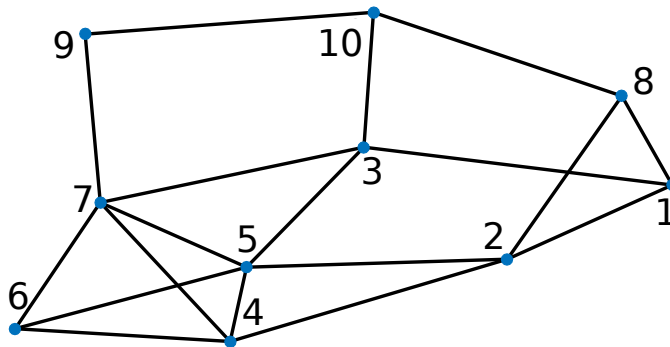


Figure 1 An example of a graph to present the problem



Average consensus algorithm on the graph: motivation (2)

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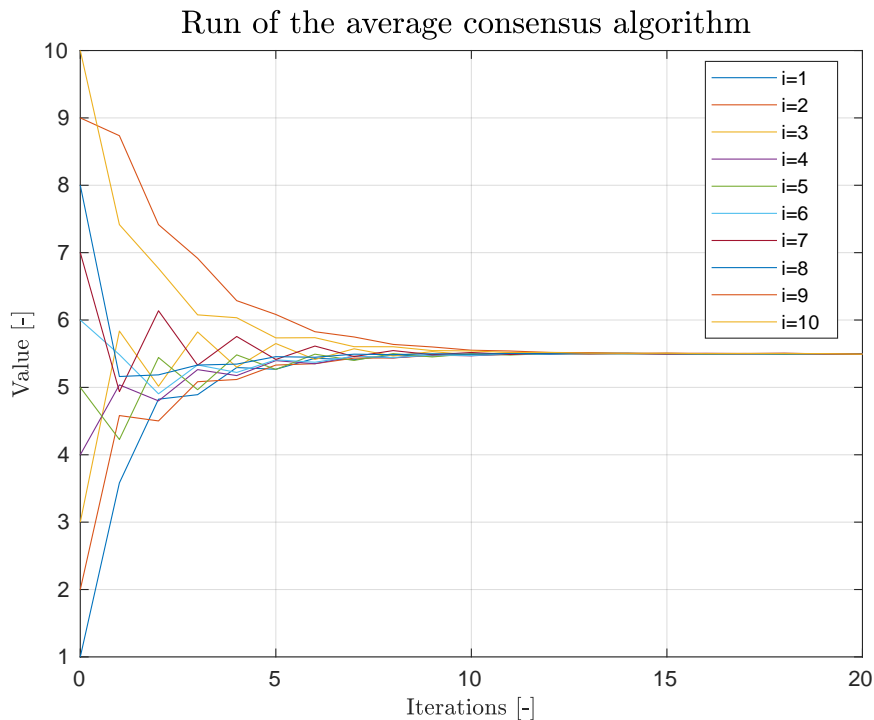


Figure 2 Run of the Average consensus algorithm on the graph



Graph theory

- Define basic terms with focus on matrix representation of the graph G
 - Number of nodes N
 - Adjacency matrix $\mathbf{A}(G) \in \mathbb{R}^{N \times N}$
 - Degree matrix $\mathbf{D}(G) \in \mathbb{R}^{N \times N}$
- Laplacian of the graph and its spectrum
 - Laplacian definition

$$\mathbf{L}(G) = \mathbf{D}(G) - \mathbf{A}(G) \quad (2)$$

- The second smallest eigenvalue λ_2 – Graph connectivity
- $\mathbf{L}(G)$ bears relevant information about the graph

$$G \Leftrightarrow \mathbf{L}(G) \quad (3)$$



Average consensus algorithm on the graph

- Linear update scheme

$$x_i(t+1) = x_i(t) + \sum_{j \in \text{Neighbors}} p_{ij}(x_j(t) - x_i(t)) \quad (4)$$

- Expressed as matrix multiplication

$$\mathbf{x}(t+1) = \mathbf{P}\mathbf{x}(t) \quad (5)$$

- Average and Convergence condition

$$\lim_{t \rightarrow \infty} \mathbf{P}^t = \frac{1}{N} \mathbf{1}\mathbf{1}^T \Leftrightarrow p_{ij} \rightarrow \frac{1}{N} \quad (6)$$

- Suits

$$\mathbf{P} = \mathbf{I} - \alpha \mathbf{L}, \quad \alpha = \frac{1}{\Delta} \in \mathbb{R} \quad (7)$$

- Δ is the greatest degree of the node in the graph



Noisy updates: problem

- Remains subject of research
- Updates affected by noise

$$\mathbf{x}(t+1) = \mathbf{P}(\mathbf{x}(t) + \mathbf{w}(t)) \quad (8)$$

Run of the algorithm with noisy updates

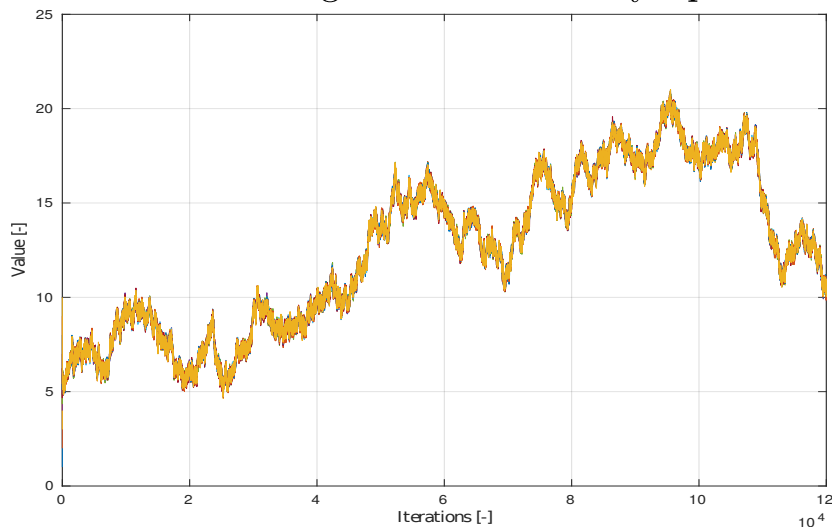


Figure 3 Updates affected by zero-mean additive noise



Noisy updates: solution

- Decreasing step size

$$\alpha \rightarrow \{\gamma(t) | \gamma(t+1) < \gamma(t)\}_{t=1}^{\infty} \quad (9)$$

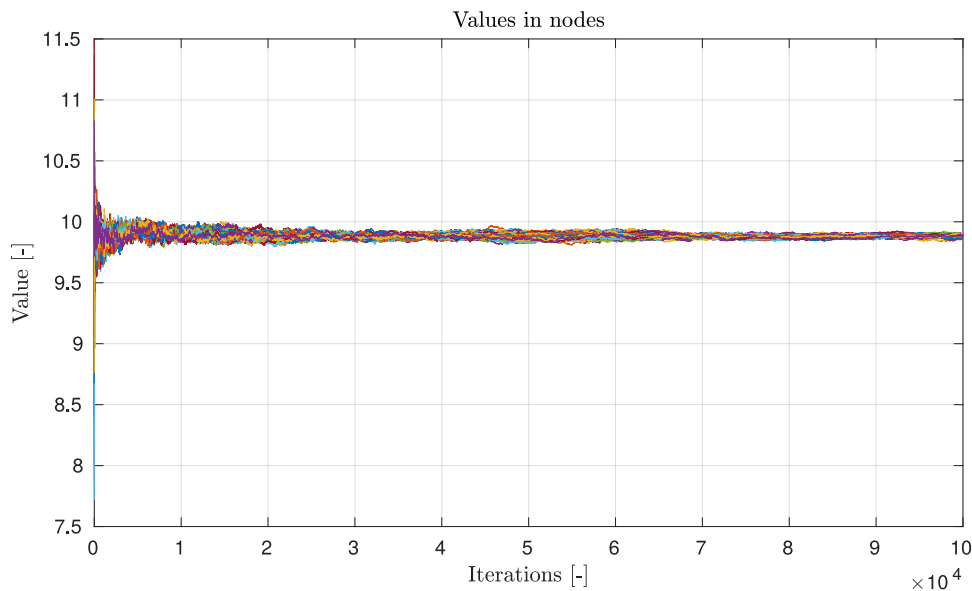


Figure 4 Run of the algorithm with decreasing step size



Noisy updates - solution (2)

- Converges in the sense of decreasing variance

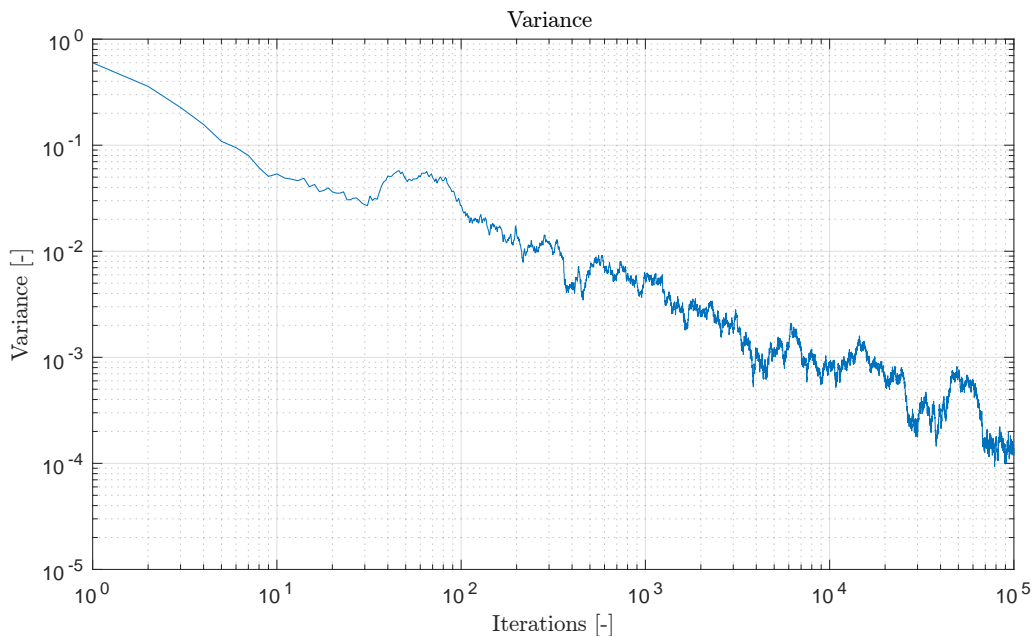


Figure 5 Decreasing variance of values in nodes from the previous example using decreasing step size



Example of application: Distributed time base synchronization (1)

- Motivation: we want to use TDMA
- Nodes need to have mutually synchronized time base

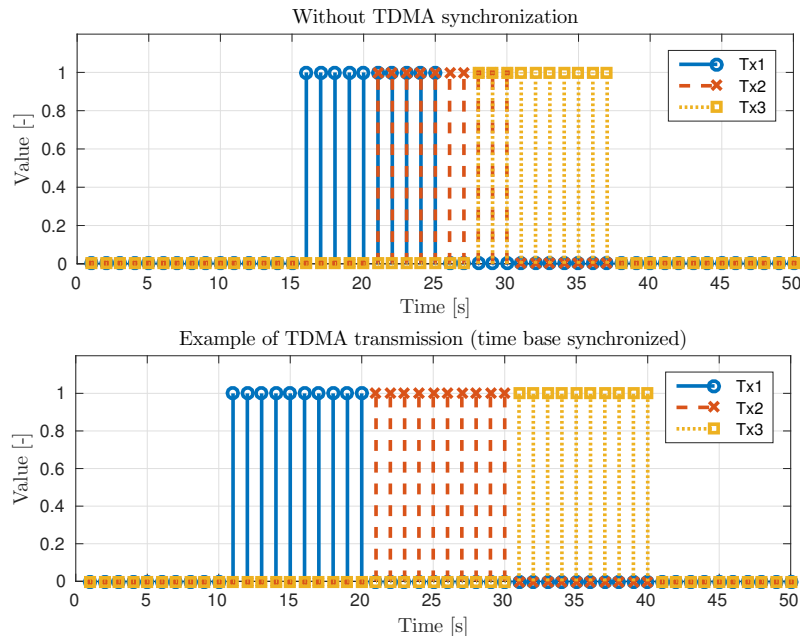


Figure 6 Pictures to explain Time Division Multiple Access



Example of application: Distributed time base synchronization (2)

- Each vertex transmits an impulse at defined moment (e.g. maximum of amplitude)
- We can apply the Average consensus algorithm on the offsets of impulses received from neighbors (see Figure below)

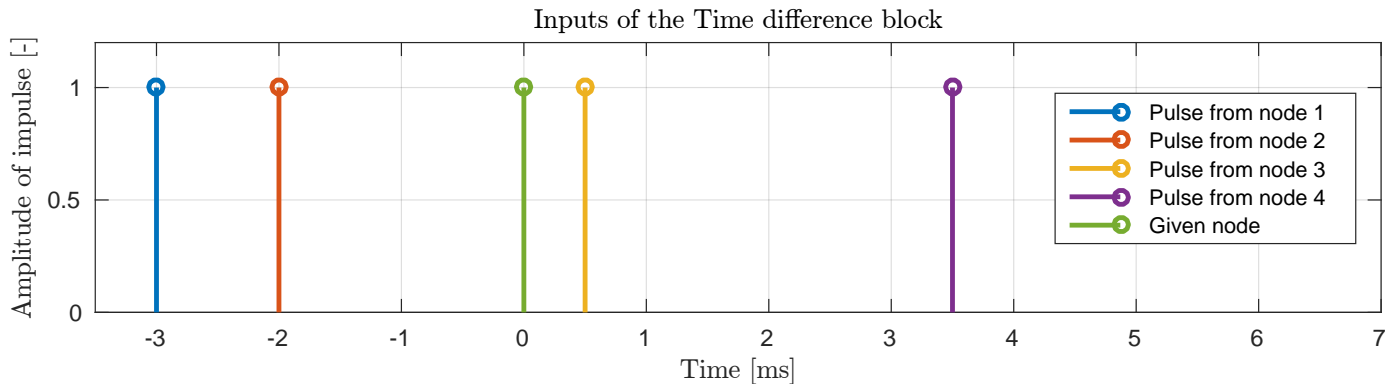


Figure 7 Figure to explain detection of time offsets



Example of application: Distributed time base synchronization (3)

- Modified update equation

$$t_i(n+1) = t_i(n) + T_i + \sum_{j \in \text{Neighbors}} p_{ij}(t_j(n) - t_i(n)), \quad (10)$$

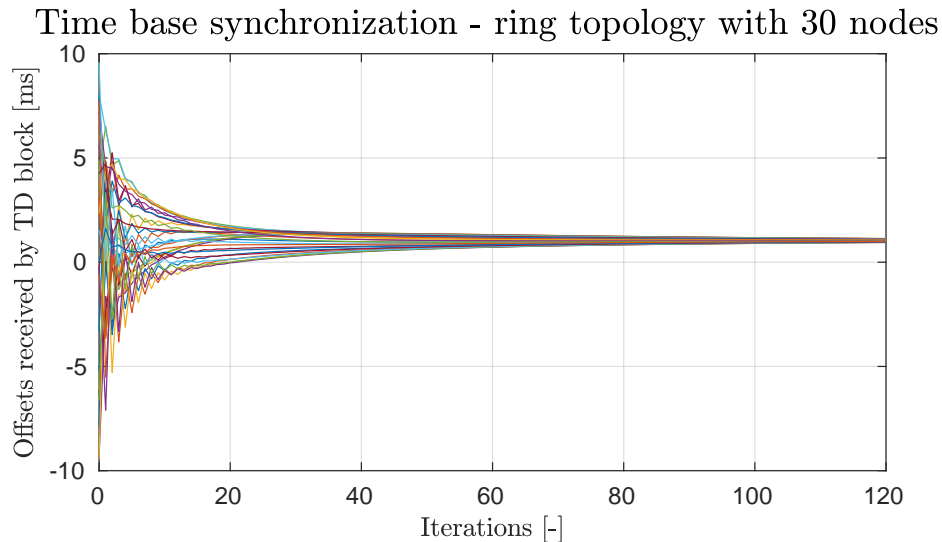


Figure 8 A run of the Time base synchronization example



Example of application: Distributed time base synchronization (4)

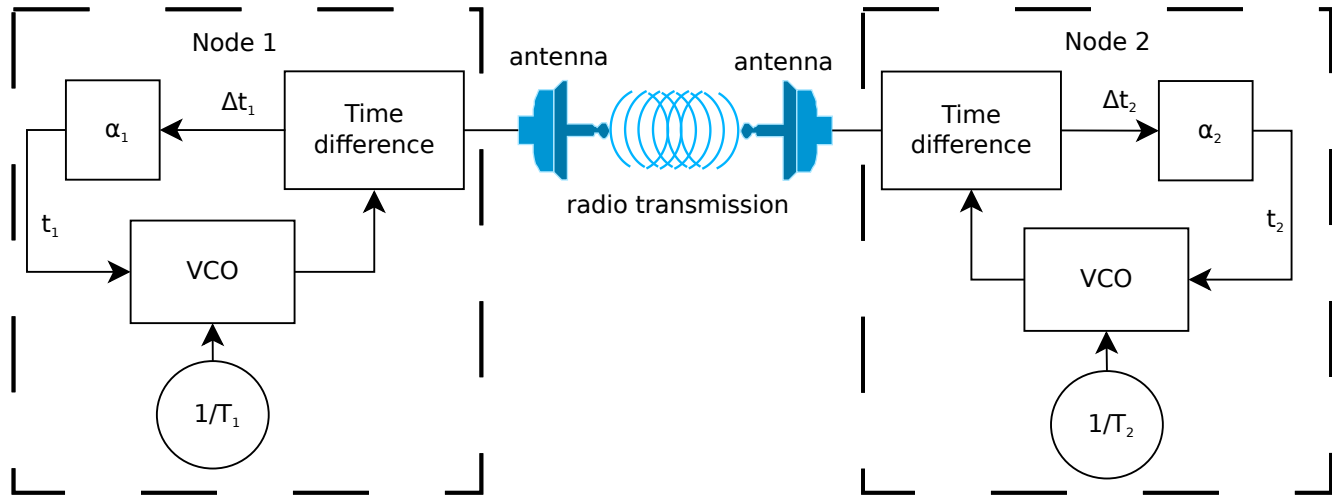


Figure 9 A block diagram representation of solution



Conclusion & My Contribution

I have

- studied the Linear average consensus algorithm on the graph
- implemented the algorithm in several examples
- observed, that the experimentally obtained outputs are consistent with the previous theoretical part



Thank you for your attention.



Otázka

- Je možné využít analytického aparátu pro zpracování signálu, viz [1], nad grafem při analýze chování distribuovaných algoritmů?
- Ano, ale zatím spíše jen velmi omezeně [1]:
 - Operace pro zpracování signálu nad grafy (harmonická analýza) mohou být zobecněny zavedením příslušných operátorů
 - Problém s nepravidelnostmi grafů
 - Jak graf sestavit v souladu s příslušnou transformací
 - Jak využít současné znalosti zpracování signálu
 - Výpočetně efektivní řešení
 - Nejednoznačný a heuristický přístup (normalizace Laplaceanu)
 - Analýza (často) vyžaduje určení vlastních vektorů Laplaceanu
- Jedná se o výzkumně otevřený problém [1]



References

- [1] David I. Shuman, Sunil K. Narang, Pascal Frossard, Antonio Ortega, and Pierre Vandergheynst. Signal Processing on Graphs: Extending High-Dimensional Data Analysis to Networks and Other Irregular Data Domains. CoRR. 2012, abs/1211.0053