Czech Technical University in Prague

F3 Faculty of Electrical Engineering
Department of Radioelectronics

Distributed signal processing in radio communication

Jakub Kolář

Bachelors's thesis presentation

Average consensus algorithm on the graph: motivation (1)

- Graph G is a set of edges and vertices
- An existing edge represents a possibility to exchange information
- Initialize $i-$th vertex with value i (e.g. temperature)

$$x_i(0) = i \tag{1}$$

- Distributed algorithm: communication is possible only between the neighbors in the graph
- Goal: obtain unique value in all vertices

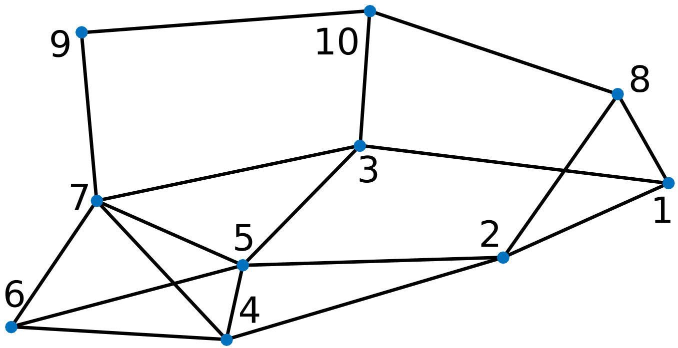

Figure 1 An example of a graph to present the problem

Average consensus algorithm on the graph: motivation (2)

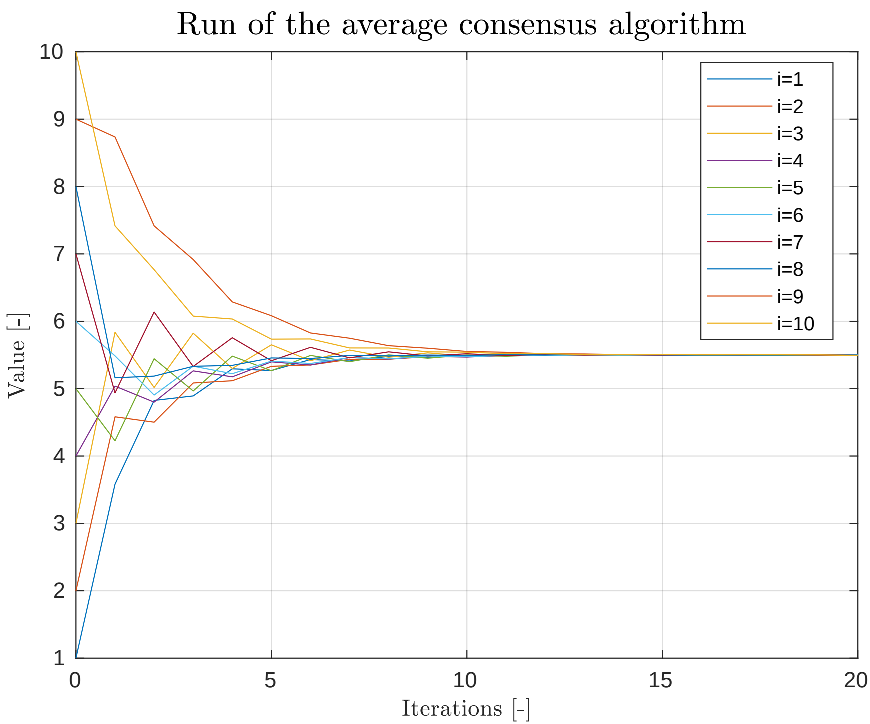

Figure 2 Run of the Average consensus algorithm on the graph

Graph theory

- Define basic terms with focus on matrix representation of the graph G
  - Number of nodes N
  - Adjacency matrix $\boldsymbol{A}(G) \in \mathbb{R}^{N \times N}$
  - Degree matrix $\boldsymbol{D}(G) \in \mathbb{R}^{N \times N}$
- Laplacian of the graph and its spectrum
  - Laplacian definition

$$\boldsymbol{L}(G) = \boldsymbol{D}(G) - \boldsymbol{A}(G) \tag{2}$$

  - The second smallest eigenvalue λ_2 – Graph connectivity
- $\boldsymbol{L}(G)$ bears relevant information about the graph

$$G \Leftrightarrow \boldsymbol{L}(G) \tag{3}$$

Average consensus algorithm on the graph

- Linear update scheme

$$x_i(t+1) = x_i(t) + \sum_{j \,\in\, Neighbors} p_{ij}(x_j(t) - x_i(t)) \tag{4}$$

- Expressed as matrix multiplication

$$\boldsymbol{x}(t+1) = \boldsymbol{P}\boldsymbol{x}(t) \tag{5}$$

- Average and Convergence condition

$$lim_{t\to\infty}\boldsymbol{P}^t = \frac{1}{N}\boldsymbol{1}\boldsymbol{1}^T \Leftrightarrow p_{ij} \to \frac{1}{N} \tag{6}$$

- Suits

$$\boldsymbol{P} = \boldsymbol{I} - \alpha\boldsymbol{L},\ \alpha = \frac{1}{\Delta} \in \mathbb{R} \tag{7}$$

  - Δ is the greatest degree of the node in the graph

Noisy updates: problem

- Remains subject of research
- Updates affected by noise

$$\boldsymbol{x}(t+1) = \boldsymbol{P}(\boldsymbol{x}(t) + \boldsymbol{w}(t)) \tag{8}$$

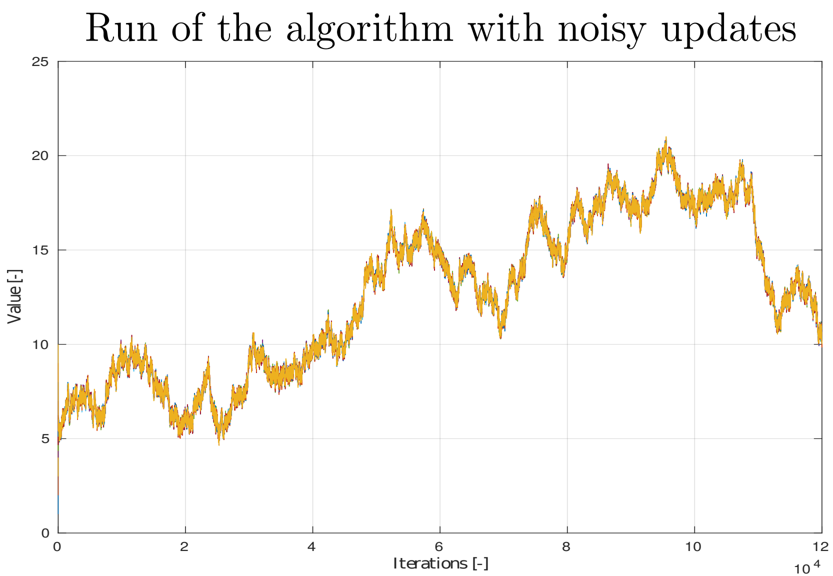

Figure 3 Updates affected by zero-mean additive noise

Noisy updates: solution

- Decreasing step size

$$\alpha \rightarrow \{\gamma(t) | \gamma(t+1) < \gamma(t)\}_{t=1}^{\infty} \tag{9}$$

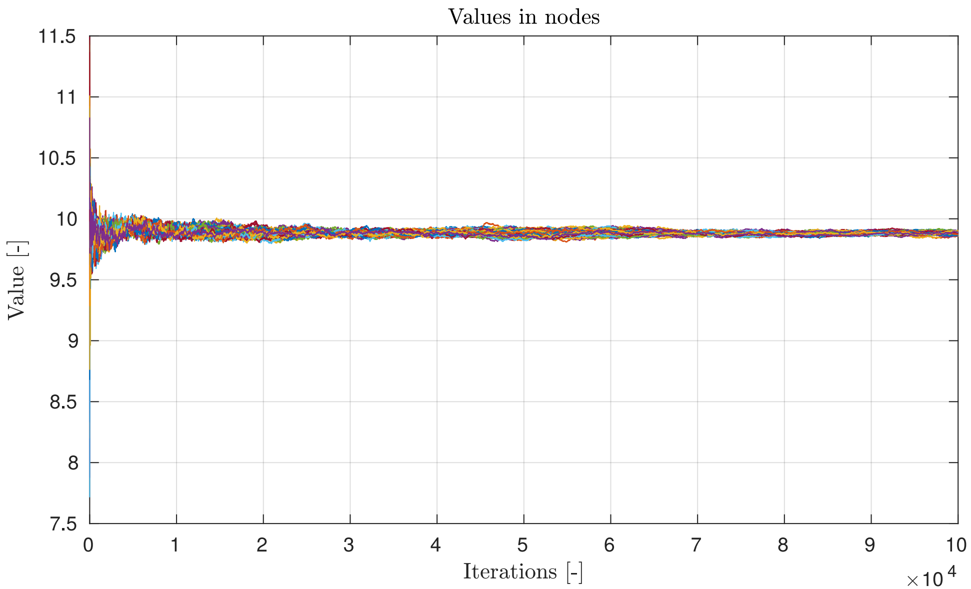

Figure 4 Run of the algorithm with decreasing step size

Noisy updates - solution (2)

- Converges in the sense of decreasing variance

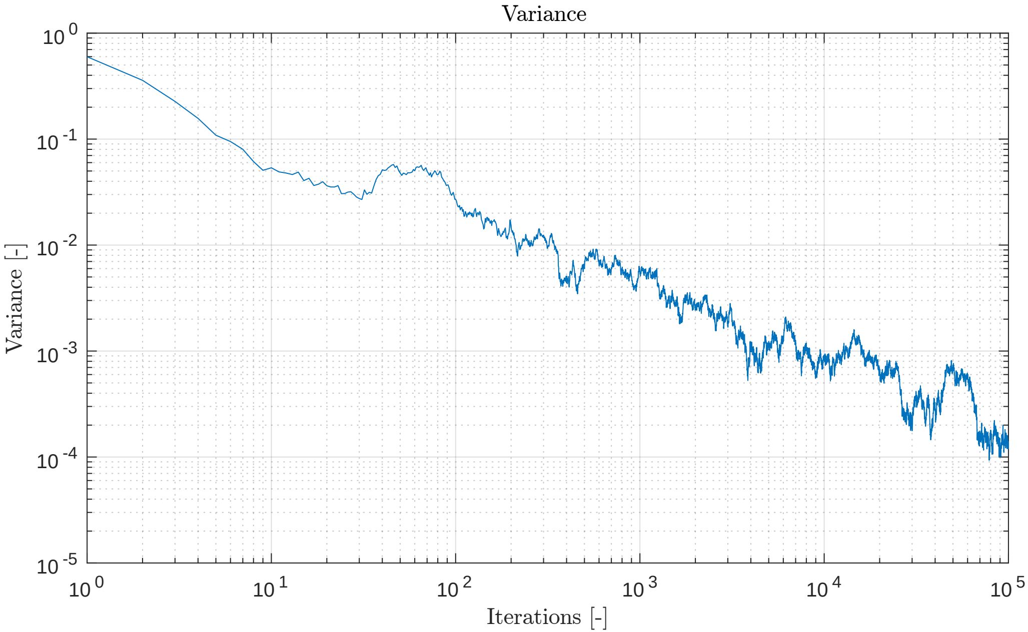

Figure 5 Decreasing variance of values in nodes from the previous example using decreasing step size

Example of application: Distributed time base synchronization (1)

- Motivation: we want to use TDMA
- Nodes need to have mutually synchronized time base

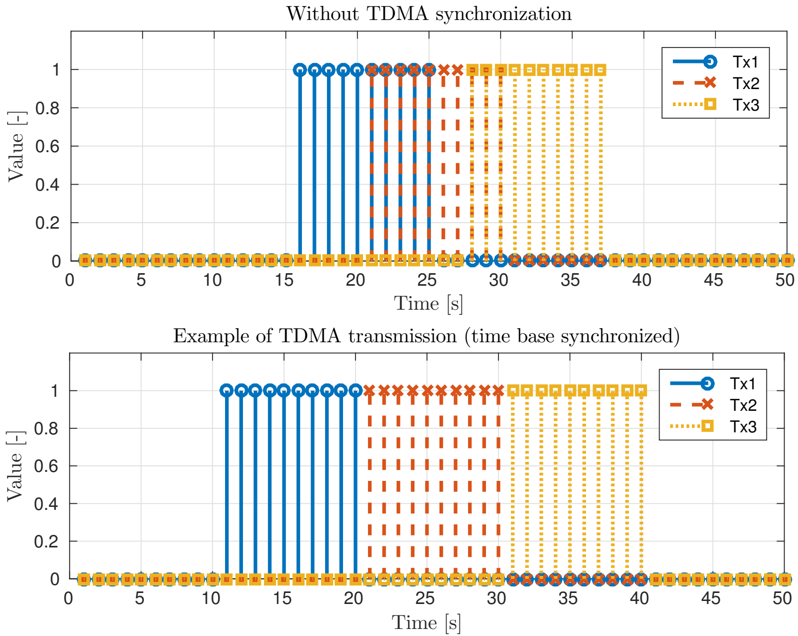

Figure 6 Pictures to explain Time Division Multiple Access

Example of application: Distributed time base synchronization (2)

- Each vertex transmits an impulse at defined moment (e.g. maximum of amplitude)
- We can apply the Average consensus algorithm on the offsets of impulses received from neighbors (see Figure bellow)

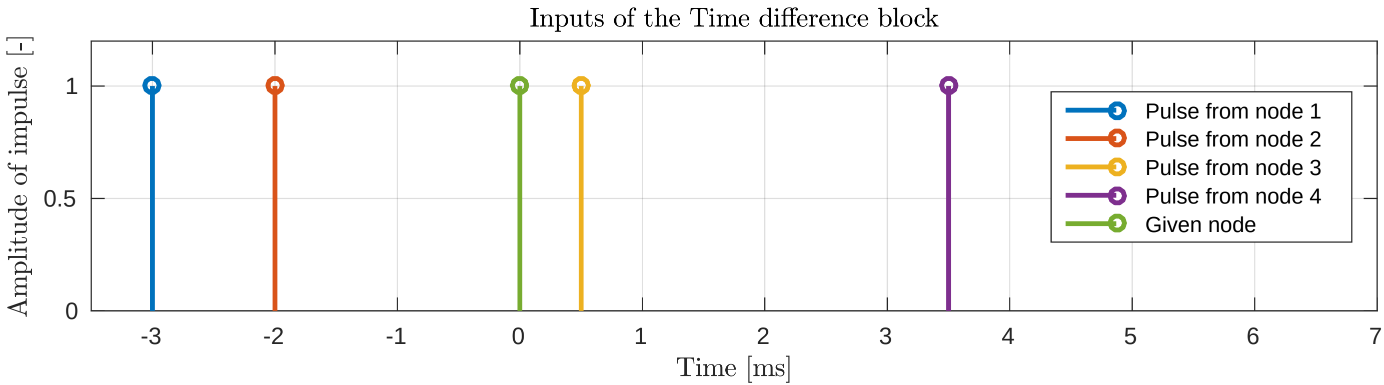

Figure 7 Figure to explain detection of time offsets

Example of application: Distributed time base synchronization (3)

- Modified update equation

$$t_i(n+1) = t_i(n) + T_i + \sum_{j \,\in\, Neighbors} p_{ij}(t_j(n) - t_i(n)), \tag{10}$$

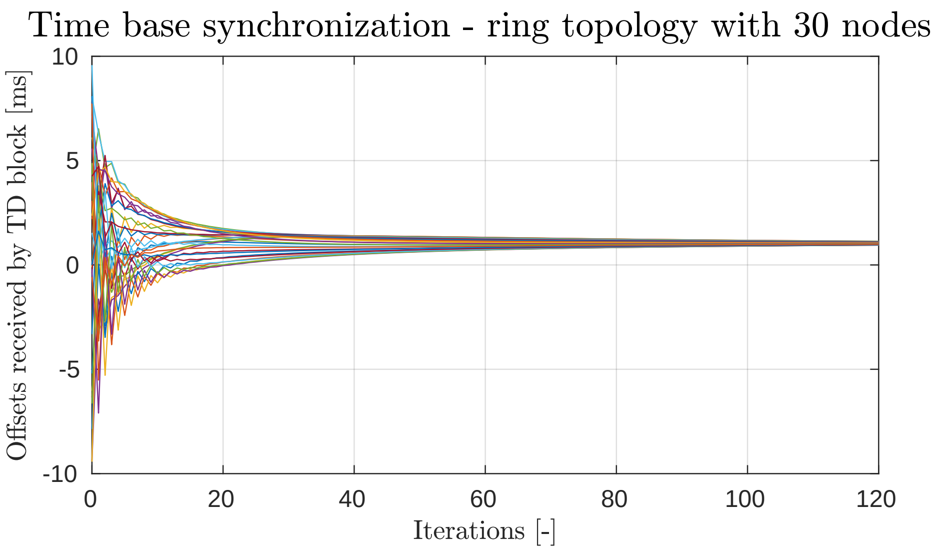

Figure 8 A run of the Time base synchronization example

Example of application: Distributed time base synchronization (4)

- Solution may be viewed as a Phase-Locked Loop implementation

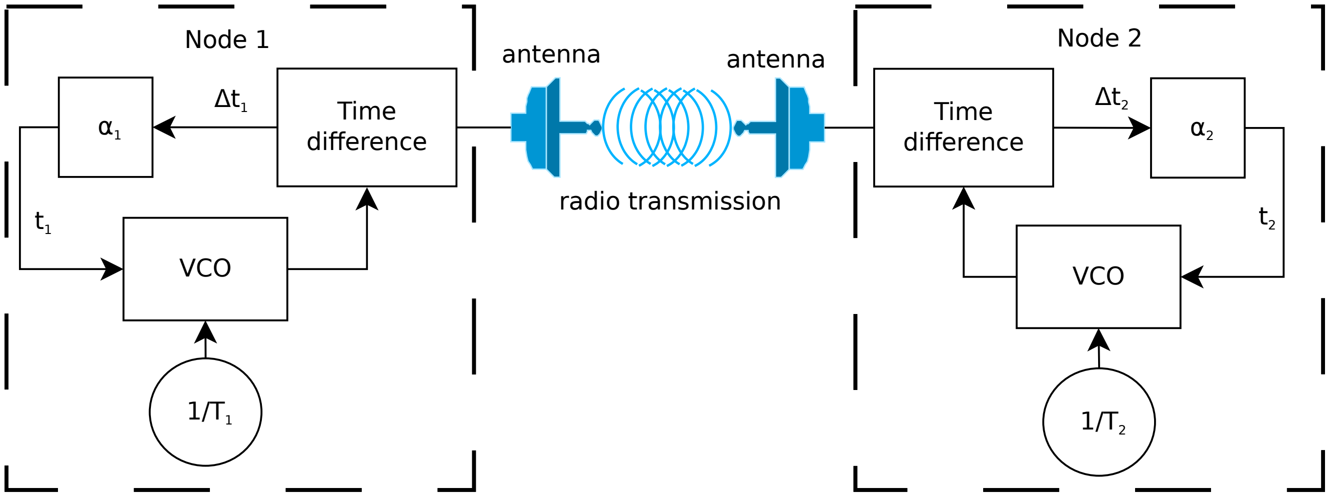

Figure 9 A sketch of an idea of Phase-Locked Loop implementation for two nodes

Conclusion & My Contribution

I have

- studied the Linear average consensus algorithm on the graph
- implemented the algorithm in several examples
- observed, that the experimentally obtained outputs are consistent with the previous theoretical part

Thank you for your attention.

Otázka

- Je možné využít analytického aparátu pro zpracování signálu, viz [1], nad grafem při analýze chování distribuovaných algoritmů?
- Ano, ale zatím spíše jen velmi omezeně [1]:
  - Operace pro zpracování signálu nad grafy (harmonická analýza) mohou být zobecněny zavedením příslušných operátorů
  - Problém s nepravidelnostmi grafů
    - Jak graf sestavit v souladu s příslušnou transformací
    - Jak využít současné znalosti zpracování signálu
    - Výpočetně efektivní řešení
  - Nejednoznačný a heuristický přístup (normalizace Laplaciánu)
  - Analýza (často) vyžaduje určení vlastních vektorů Laplaciánu
- Jedná se o výzkumně otevřený problém [1]

References

[1] David I. Shuman, Sunil K. Narang, Pascal Frossard, Antonio Ortega, and Pierre Vandergheynst. Signal Processing on Graphs: Extending High-Dimensional Data Analysis to Networks and Other Irregular Data Domains. CoRR. 2012, abs/1211.0053