**Bachelor's Thesis** 



F3

Faculty of Electrical Engineering Department of Radioelectronics

# Distributed signal processing in radio communication networks

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### **Acknowledgement** / **Declaration**

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Thank you all.		I annound In Prague, 3	te I made this . 12. 2016	thing alone.

### Abstrakt / Abstract

Pokus napsat bakalarku a take ji obhajit.

Klíčová slova: konsensus algoritmus

Fireflyes are really amazing.

**Keywords:** averaging consensus algorithm

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# Chapter 1 Introduction

#### 1.1 Motivation

To begin with an idea of an average consensus algorithm, let's make a thought experiment. We are looking for an average quantity, for example an average temperature in a room, with a group of wireless communication devices, that can exchange informations, provided they are in range to reach each other. We deploy these thermometers in the room randomly, with no special requirements on a topology. Next, let's consider, that for each pair of the thermometers we can decide, whether they can exchange information or not - meaning we know all neighbours of all devices, that are mutually in range to communicate.

Now, we can encode our experiment settings to a graph. A very natural way to represent this graph is drawing it. To do so, we simply take all thermometers as different vertices. Of course, every vertex always knows a result of its own measurement. By an edge between two vertices we mark the situation that these two nodes can exchange informations. Which means, every node knows also the value that measured it's neighbour. This ought to be only a very simple illustration how to transfer a physical experiment to the terms of graphs.

Finally, as we shall see, if we fulfill some basic convergence conditions on the properties of this graph, the average consensus algorithm acts like this: You synchronously update the value in each node by some increment, which depends only on the old value in the node and the values of its direct neighbours in graph. By doing this long enough, you obtain in each node a value going in limit to the average of all initially measured values.



#### 1.2 Outline

A Graph theory provides a very elegant way to represent informations encoded by graphs as matrices. In the first chapter we will provide some basic definitions to graph theory and properties of these important matrices. Using matrices, we will also briefly mention some very useful results from Matrix analysis, because also a serious object of our interest will be topic of eigenvalues of matrices. We will define a Laplacian of a graph and show some of it's basic properties.

In next chapter, we will in detail provide the average consensus algorithm description and show some examples with graphically illustrated solution.

In last part of this thesis, we will try to implement this algorithm to easily solve some typical problems in area of wireless digital communication - time synchronisation or carrier frequency synchronisation. In a very simple case, we can also show how do the nonidealities change the result of algorithm (additive zero-mean noise).

# Chapter 2 Graph theory

### 2.1 Brief historical review

Euler

### 2.2 Definitions

Graph

Adjacency matrix

Degree

 $Incidence\ matrix$ 

### 2.3 Laplacian matrix

Definition

Properties

Gerschgorin discs

## Chapter 3

### Linear consensus algorithms

### 3.1 Motivation

Let's think about an experiment, where few nodes aim to provide only one result of measurement based on many local measurements. For example we measure an average temperature in a room. Very acurate measure devices are expensive. We can generally try to replace small number of very good devices by some probably bigger number of less reliable devices whose benefit will be an interchange of information between near nodes.

They We want to replace a number of nodes, that exchange informatMany less accurate a reliable nodes as an alternative to very accurate and very reliable but also very expensive nodes.

- 3.2 Discrete and continuous time
- 3.3 Convergenece analysis

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