a) Let the image be $x[m,n]$. Its fourier transform is

$$\Im(x[m,n]) = F[u,v] = 1/MN \ . \sum_{m=1}^{M}\sum_{n=1}^{N} x(m,n) \ .e^{-j2\frac{\pi}{M}mu} \ . \ e^{-j2\frac{\pi}{N}nv}$$

If $x[m,n]$ is scaled by a constant a

$$F[u,v] \ = \Im(ax[m,n]) = a \ \sum_{m=1}^{M}\sum_{n=1}^{N} x(m,n) \ .e^{-j2\frac{\pi}{M}mu} \ . \ e^{-j2\frac{\pi}{N}nv} \ = aF[u,v]$$

$$F[u,v] = a \ \sum_{m=1}^{M}\sum_{n=1}^{N} x(m,n) \ .e^{-j2\frac{\pi}{M}mu} \ . \ e^{-j2\frac{\pi}{N}nv}$$

b) $\Im(x[m,n]) = F[u,v] = \iint x(m,n) \ .e^{-j2\frac{\pi}{M}mu} \ . \ e^{-j2\frac{\pi}{N}nv}$

$$\Im(x[am,an]) = \int\int x(am,nn) \ .e^{-j2\frac{\pi}{M}mu} \ . \ e^{-j2\frac{\pi}{N}nv} \ dm \ dn$$

let $am = k => a. \ dm = dk$

$$bn = l => b. \ dn = dl$$

$$\Im(x[am,an]) = \iint x(k,l) \ .e^{-j2\frac{\pi}{M}k\frac{u}{a}} \ . \ e^{-j2\frac{\pi}{N}l\frac{v}{b}} \ \frac{dm}{a} \ \frac{dn}{b}$$

$$\Im(x[am,an]) = \frac{F(\frac{u}{a}, \frac{v}{b})}{ab}$$

c)

$$\Im(x[m,n]) = F[u,v] = 1/MN \ . \ \sum_{m=1}^{M} \sum_{n=1}^{N} x(m,n) \ .e^{-j2\frac{\pi}{M}(m+a)u} \ . \ e^{-j2\frac{\pi}{N}(n+b)v}$$

$$\Im(ax[m + a, n + b])$$

$$= 1/MN \ . \ \sum_{m=1}^{M} \sum_{n=1}^{N} x(m,n) \ .e^{-j2\frac{\pi}{M}mu} \ . \ e^{-j2\frac{\pi}{N}nv} .e^{-j2\pi(\frac{1}{M}au+\frac{1}{N}bv)}$$

$$= F[u,v] \ * \ e^{-j2\pi(\frac{1}{M}au+\frac{1}{N}bv)}$$

This shows that there is change only in the phase of the image spectrum leaving the magnitude intact.

d)

$$\Im(x[m,N - n) = F[u,N - v] = 1/MN \ . \ \sum_{m=1}^{M} \sum_{n=1}^{N} x(m,n) \ .e^{-j2\frac{\pi}{M}(m+a)u} \ . \ e^{-j2\frac{\pi}{N}(n}$$

$$\Im(ax[m + a, n + b])$$

$$= 1/MN \ . \ \sum_{m=1}^{M} \sum_{n=1}^{N} x(m,n) \ .e^{-j2\frac{\pi}{M}mu} \ . \ e^{-j2\frac{\pi}{N}nv} .e^{-j2\pi(\frac{1}{M}au+\frac{1}{N}bv)}$$

$$= F[u,v] \ * \ e^{-j2\pi(\frac{1}{M}au+\frac{1}{N}bv)}$$

This shows that there is change only in the phase of the image spectrum leaving the magnitude intact.

e) The pattern on the magnitude spectrum gets inversed. A line at the centre implies that the magnitude there is the same.