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The effect of interpolation methods in surface definition: an experimental study

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Abstract

Mappings of the earth surface and their representation in 3D (three-dimensional) models are commonly used in most recent research. Modeling research, which starts with classical surveying methods, acquires new dimensions matching the modern technologies. 3D models of any object or earth surface can be used in much visual and scientific research. A digital model of the landscape is an important part within creation of geo-information systems used in the public administration and in the commercial sphere. It is an important tool in applications such as geomorphology, hydrology, geology, cartography, ecology, mining etc.

Values of volume in terrains that do not have regular geometric structure can be obtained more accurately by using 3D models of surfaces with respect to developing technology. Basic data of 3D models must indicate 3D coordinates of the surveyed object in the reference frame. Distribution and intensity of points are important factors in modeling earth surfaces. A minimum number of points is desired in defining an object in 3D. Interpolation methods employing different mathematical models are used to obtain 3D models of terrain surfaces. In this study, the effect of interpolation methods in defining a terrain surface is investigated. For this purpose, a uniform surface, hill-shaped artificial object with a known volume is employed. The 3D surface and volume are calculated by using 12 different interpolation methods. Point distribution, point intensity and accuracy of point measurements are not considered. The same data set was used for all the interpolation methods. The interpolation methods are compared and evaluated based on the results. Copyright © 2007 John Wiley & Sons, Ltd.

Keywords: grid; volume; 3D modeling; surface; interpolation

Introduction

A digital model of the landscape is an important part within creation of geo-information systems used in the public administration and in the commercial sphere. It is an important tool in applications that model an earth surface, such as geomorphology, hydrology, geology, cartography, ecology etc. Many software products offer different interpolation methods for creation of a digital model of the landscape. Its accuracy and quality is impacted by selection of an interpolation method and precision of input data. The choice of the interpolation method and its parameters depend on many factors: for example, the type of interpolated phenomenon, the character of its surfaces (horizontal and elevation segmentation, break lines etc.) or the purpose of digital modeling (Fencik *et al.*, 2005).

Modeling the earth surface is a central issue in topographic mapping. Digital terrain models (DTMs) are a valuable data source for many applications. This is why systems for the generation and visualization of DTMs were developed early. In this context, a clear distinction has to be made between DTMs and digital elevation models (DEMs) (Kraus, 2000): DTMs describe the earth surface in the sense of the ‘bald earth’ without human artifacts such as buildings or bridges and without vegetation. The term ‘DTM’ also describes a semantic property of the object. The term DEM, however, describes a model that contains the elevations of points with respect to a reference surface, without any restriction on what the object is like. This term characterizes a modeling technique rather than the data that are described by an elevation model. In topographic mapping, DTMs can be described by DEMs. As the earth surface cannot be described by one closed analytical function in a scale appropriate for topographic mapping, it has to be divided into facets that are small enough that within them the surface can be approximated by an analytic function.

A DEM is a numerical representation of topography, usually made up of equal-sized grid cells, each with a value of elevation. Its simple data structure and widespread availability have made it a popular tool for land characterization. Because topography is a key parameter controlling the function of natural ecosystems, DEMs are used in environmental analysis (Chaplot *et al.*, 2006). DEMs are used for topographic mapping, engineering and environmental applications, spatial data visualization, orthophoto production, and integration into geographic information systems and combined analysis with other data also. The derivation of DEMs from photographs and/or digital images is one of the main tasks of photogrammetry. The photogrammetric technology may be an appropriate solution for obtaining the DEM of large areas at the present time. DEM data can easily be obtained from stereo images in photogrammetric methods.

The essential data of a DTM are the finite number of reference points, which have three-dimensional coordinates (x, y, z) in an orthogonal coordinate system or two-dimensional horizontal coordinates (x, y) and height (h). The reference points can have regular or irregular (scattered) distribution depending on the source of the data (field, contour map, arial photograph, satellite image etc.) and the data collection method (classical surveying, global positioning system, photogrammetry, digitizing, scanning etc.). Transformation of the regular data to a scattered sampling pattern or from a scattered to a regular sampling pattern is possible by using a suitable interpolation algorithm (Petrie and Kennie, 1987; Watson, 1999; Sukumar *et al.*, 2001; Yanalak, 2003).

Values of volume in terrain that have not regular geometric structure can be obtained accurately by using 3D models of terrain. If a surface is defined well, values of volume can be obtained accurately. The distribution and density of points are important factors in defining a surface. The accuracy and precision of measuring elevations are also important factors.

The aim of this study is to investigate which one is more suitable by comparing the interpolation methods used in defining terrain surfaces. For this purpose, a uniform surface hill-shaped artificial object with a known volume was employed. This object was evaluated by using the digital photogrammetric method. With the obtained 541 point, 3D model the object was obtained and its volume was calculated by using 12 different interpolation methods. The same data set was used for all the interpolation methods.

The main issue in this paper is to compare the role of interpolation methods in defining a surface, whereas volume calculation is used to carry out this aim. Interpolation methods were compared considering the principle 'the most accurate volume is obtained from the best defined surface'. The same data set was used in each method and the parameters such as point density, point distribution and measurement accuracy were kept constant; therefore, only the effect of interpolation method was investigated.

Interpolation Methods

Interpolation is the process of using known data values to estimate unknown data values. Various interpolation techniques are used in many different fields of study. One of the simplest methods, linear interpolation, requires knowledge of two points and the constant rate of change between them and beyond. The basic structure of interpolation is seen in Figure 1. With this information, it can interpolate values anywhere between these two points. More sophisticated interpolations are also available in the literature.

When a numerical derivative is needed, central difference formulae are used in the calculations. Because a central difference approach is used, values on both sides of the location for which the derivative is computed are required. This leads to blanking along the edges of the derivative grids, otherwise known as an edge effect.

'Compass-based' grid notation is use to indicate the neighboring grid nodes used for many of the grid calculus operations, as illustrated below (Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Using this grid notation, we can write the difference equation approximations for the necessary derivatives at location Z as follows (Surfer 8 Software Online (Surfer 8 Manual, 2006)):

$$\begin{gathered}
\frac{dz}{dx} \approx \frac{Z_E - Z_W}{2\Delta x} \quad \frac{dz}{dy} \approx \frac{Z_N - Z_S}{2\Delta y} \quad \frac{d^2 z}{dx^2} \approx \frac{Z_E - 2Z + Z_W}{\Delta x^2} \\
\frac{d^2 z}{dy^2} \approx \frac{Z_N - 2Z + Z_S}{\Delta y^2} \quad \frac{d^2 z}{dx dy} \approx \frac{Z_{NE} - Z_{NW} - Z_{SE} + Z_{SW}}{4\Delta x \Delta y} \\
\frac{d^4 z}{dx^4} \approx \frac{Z_{WW} - 4Z_W + 6Z - 4Z_E + Z_{EE}}{\Delta x^4} \quad \frac{d^4 z}{dy^4} \approx \frac{Z_{NN} - 4Z_N + 6Z - 4Z_S + 4Z_{SS}}{\Delta y^4} \\
\frac{d^4 z}{dx^2 dy^2} \approx \frac{Z_{NW} - 2Z_N + Z_{NE} - 2Z_W + 4Z - 2Z_E + Z_{SW} - 2Z_S + Z_{SE}}{4\Delta x \Delta y}
\end{gathered} \tag{1}$$

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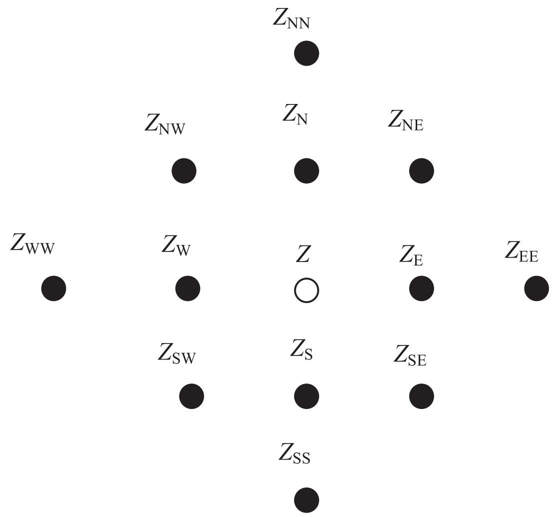

Figure 1. The basic structure of interpolation.

There are a number of different interpolation algorithms, for example inverse distance to a power, kriging, minimum curvature, modified Shepard method, natural neighbor, nearest neighbor, polynomial regression, radial basis function, triangulation with linear interpolation and so on. We can find detailed information on these interpolation methods in all given references.

Inverse Distance to a Power

The inverse distance to a power interpolation method is a weighted average interpolator, and can be either an exact or a smoothing interpolator. With inverse distance to a power, data are weighted during interpolation such that the influence of one point relative to another declines with distance from the grid node. Weighting is assigned to data through the use of a weighting power that controls how the weighting factors drop off as distance from a grid node increases. The greater the weighting power, the less effect points far from the grid node have during interpolation. As the power increases, the grid node value approaches the value of the nearest point. For a smaller power, the weights are more evenly distributed among the neighboring data points. The smoothing parameter is a mechanism for buffering this behavior. The equation used for inverse distance to a power is (Franke, 1982; Davis, 1986; Surfer 8 Software Online (Surfer 8 Manual, 2006))

$$\hat{Z}_j = \frac{\sum_{i=1}^{n} \frac{Z_i}{h_{ij}^{\beta}}}{\sum_{i=1}^{n} \frac{1}{h_{ij}^{\beta}}} \tag{2}$$

$$h_{ij} = \sqrt{d_{ij}^2 + \delta^2}$$

where

h_{ij} is the effective separation distance between grid node j and the neighboring point i,
$\hat{Z}_j$ is the interpolated value for grid node j,
Z_i are the neighboring points,

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d_{ij} is the distance between the grid node j and the neighboring point i,
β is the weighting power (the *power* parameter) and
δ is the *smoothing* parameter.

Weighted Averaging. The interpolation methods use weighted average interpolation algorithms. This means that, with all other factors being equal, the closer a point is to a grid node, the more weight it carries in determining the Z value at that grid node. The difference between interpolation methods is how the weighting factors are computed and applied to data points during grid node interpolation.

To understand how weighted averages are applied, consider the equation shown here. Given N data values (Franke, 1982; Davis, 1986; Surfer 8 Software Online (Surfer 8 Manual, 2006)),

$$\{Z_1, Z_2, \ldots, Z_n\} \tag{3}$$

the interpolated value at any grid node (for example, G_j) can be computed as the weighted average of the data values:

$$G_j = \sum_{i=1}^{N} w_{ij} Z_i \tag{4}$$

where

G_j is the interpolated grid node value at node j,
N is the number of points used to interpolate at each node,
Z_i is the Z value at the ith point and
w_{ij} is the weight associated with the ith data value when computing G_j.

The weighted method is designed to visualize very large data sets where the data density is very high relative to the output grid. This is a fast method, which uses only the data values and makes no attempt to estimate the gradients of the data. In this method, the grid node value is a weighted average of the data points surrounding it, and secondary gridding is used to fill in the unassigned node values. This method is suitable for either faulted or non-faulted data sets.

Kriging Method

Kriging is similar to inverse distance power in that it weights the surrounding measured values to derive a prediction for an unmeasured location. The general formula for both interpolators is formed as a weighted sum of the data:

$$\hat{Z}(s_0) = \sum_{i=1}^{N} \lambda_i Z(s_i) \tag{5}$$

where

$Z(s_i)$ is the measured value at the ith location,
λ_i is an unknown weight for the measured value at the ith location,
s_0 is the prediction location and
N is the number of measured values.

Kriging is a geostatistical interpolation method that has proven useful and popular in many fields. This method produces visually appealing maps from irregularly spaced data. Kriging is a very flexible interpolation method. Kriging can be either an exact or a smoothing interpolator depending on the user-specified parameters. There are two kriging methods. These are ordinary kriging and universal kriging.

Ordinary Kriging. This method assumes that the data set has a stationary variance but also a non-stationary mean value within the search radius. Ordinary kriging is highly reliable and is recommended for most data sets.

Universal Kriging. This method represents a true geostatistical approach to interpolating a trend surface of an area. The method involves a two-stage process, where the surface representing the drift of the data is built in the first stage and the residuals for this surface are calculated in the second stage. The recommended setting is a first degree polynomial, which will avoid unpredictable behaviour at the outer margins of the data set (Cressie, 1990; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

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Minimum Curvature Method

Minimum curvature interpolation was used by Briggs (1974) to interpolate gravity values on the corners of a regular grid using scattered gravity data and finally to produce a contour map. The theory of minimum curvature interpolation, which is a gridding method, is based on minimization of the total curvature at the grid points. The total square curvature of a surface is defined as

$$C(z) = \iint \left[\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right] \mathrm{d}x\mathrm{d}y \tag{6}$$

Briggs (1974) proved that the z function, which minimizes the total squared curvature, must satisfy

$$\frac{\partial^4 z}{\partial x^4} + 2\frac{\partial^4 z}{\partial y^2 x^2} + \frac{\partial^4 z}{\partial y^4} \tag{7}$$

The iterative solution of the differential equation (7) with the use of the boundary conditions was given by Briggs (1974) and by Smith and Wessel (1990).

Minimum curvature is widely used in the earth sciences. The interpolated surface generated by minimum curvature is analogous to a thin, linearly elastic plate passing through each of the data values with a minimum amount of bending.

Minimum curvature produces a grid by repeatedly applying an equation over the grid in an attempt to smooth the grid. Each pass over the grid is counted as one iteration. The grid node values are recalculated until successive changes in the values are less than the maximum residuals value, or the maximum number of iterations is reached.

This routine first fits a simple planar model using least squares regression:

$$AX + BY + C = Z(X, Y) \tag{8}$$

The minimum curvature algorithm generates the surface that interpolates the available data and solves the modified biharmonic differential equation with tension (Briggs, 1974; Smith and Wessel, 1990; Surfer 8 Software Online (Surfer 8 Manual, 2006)):

$$(1 - T_\mathrm{i})\nabla^2(\nabla^2 Z) - (T_\mathrm{i})\nabla^2 Z = 0 \tag{9}$$

There are three sets of associated boundary conditions.

$$\text{On the edges: } (1 - T_\mathrm{b})\frac{\delta^2 Z}{\delta n^2} + (T_\mathrm{b})\frac{\delta Z}{\delta n} = 0 \tag{10}$$

$$\text{On the edges: } \frac{\delta(\nabla^2 Z)}{\delta n} = 0 \tag{11}$$

$$\text{At the corners: } \frac{\delta^2 Z}{\delta x \delta y} = 0 \tag{12}$$

where

∇^2 is the Laplacian operator
n is the boundary normal
T_i is the internal tension
T_b is the boundary tension.

Modified Shepard Method

The modified Shepard method uses an inverse distance weighted least squares method. As such, the modified Shepard method is similar to the inverse distance to a power interpolator, but the use of local least squares eliminates or

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reduces the ‘bull’s-eye’ appearance of the generated contours. The modified Shepard method can be either an exact or a smoothing interpolator (Shepard, 1968; Franke, 1982; Renka, 1988; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Quadratic Neighbors. The modified Shepard method starts by computing a local least squares fit of a quadratic surface around each observation. The quadratic neighbors parameter specifies the size of the local neighborhood by specifying the number of local neighbors. The local neighborhood is a circle of sufficient radius to include exactly this many neighbors.

Weighting Neighbors. The interpolated values are generated using a distance-weighted average of the previously computed quadratic fits associated with neighboring observations. The weighting neighbors parameter specifies the size of the local neighborhood by specifying the number of local neighbors. The local neighborhood is a circle of sufficient radius to include exactly this many neighbors.

Natural Neighbor Method

The natural neighbor interpolation method is quite popular in some fields. It cen be consider a set of Thiessen polygons (the dual of a Delaunay triangulation). If a new point (target) were added to the data set, these Thiessen polygons would be modified. In fact, some of the polygons would shrink in size, while none would increase in size. The area associated with the target’s Thiessen polygon that was taken from an existing polygon is called the ‘borrowed area’. The natural neighbor interpolation algorithm uses a weighted average of the neighboring observations, where the weights are proportional to the ‘borrowed area’ (Sibson, 1981; Owen, 1992; Watson, 1994; Sukumar *et al.*, 2001; Yanalak, 2003; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

The basic equation used in natural neighbor interpolation is identical to the one used in inverse distance power interpolation:

$$G(x, y) = \sum_{i=1}^{n} w_i f(x_i, y_i) \tag{13}$$

where

$G(x, y)$ is the natural neighbor estimation at (x, y),
n is the number of nearest neighbors used for interpolation,
$f(x_i, y_i)$ is the observed value at (x_i, y_i) and
w_i is the weight associated with $f(x_i, y_i)$.

As with inverse distance power interpolation, the nodal functions can be either constants, gradient planes, or quadratics. The difference between inverse distance power interpolation and natural neighbor interpolation is the method used to compute the weights and the method used to select the subset of scatter points used for interpolation.

Nearest Neighbor Method

The nearest neighbor interpolation method assigns the value of the nearest point to each grid node. This method is useful when data are already evenly spaced. Alternatively, in cases where the data are nearly on a grid with only a few missing values, this method is effective for filling in the holes in the data.

Sometimes with nearly complete grids of data there are areas of missing data that we want to exclude from the grid file. In this case, the search ellipse can be set to a value so the areas of no data are assigned the blanking value in the grid file. By setting the search ellipse radii to values less than the distance between data values in file, the blanking value is assigned at all grid nodes where data values do not exist (Yanalak, 2003; Yanalak and Baykal, 2003; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Polynomial Regression Method

Polynomial regression is used to define large-scale trends and patterns in data. Polynomial regression is not really an interpolator because it does not attempt to predict unknown Z values. There are several options that define the type of trend surface (Briggs, 1974; Atkinson, 1988; Brutman, 1997; Endre and Mayers, 2003; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Simple planar surface

$$Z(x, y) = A + Bx + Cy \tag{14}$$

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Bi-linear surface

$$Z(x, y) = A + Bx + Cy + Dxy \tag{15}$$

Quadratic surface

$$Z(x, y) = A + Bx + Cy + Dx^2 + Exy + Fy^2 \tag{16}$$

Cubic surface

$$Z(x, y) = A + Bx + Cy + Dx^2 + Exy + Fy^2 + Gx^3 + Hx^2y + Ixy^2 + Jy^3 \tag{17}$$

Radial Basis Function Method

Radial basis function interpolation is a diverse group of data interpolation methods. In terms of the ability to fit data and to produce a smooth surface, the multiquadric method is considered by many to be the best. All of the radial basis function methods are exact interpolators, so they attempt to honor data. It can introduce a smoothing factor to all the methods in an attempt to produce a smoother surface (Hardy, 1990; Powell, 1990; Carlson and Foley, 1991; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Function Types. The basis kernel functions are analogous to variograms in kriging. The basis kernel functions define the optimal set of weights to apply to the data points when interpolating a grid node.

$$\text{Inverse multiquadric} \quad B(h) = \frac{1}{\sqrt{h^2 + R^2}} \tag{18}$$

$$\text{Multilog} \quad B(h) = \log(h^2 + R^2) \tag{19}$$

$$\text{Multiquadratic} \quad B(h) = \sqrt{h^2 + R^2} \tag{20}$$

$$\text{Natural cubic spline} \quad B(h) = (h^2 + R^2)^{3/2} \tag{21}$$

$$\text{Thin plate spline} \quad B(h) = (h^2 + R^2)\log(h^2 + R^2) \tag{22}$$

where

h is the anisotropically rescaled, relative distance from the point to the node and
R^2 is the smoothing factor specified by the user.

Default R^2 Value. The default value for R^2 in the radial basis function interpolation algorithm is calculated as follows:

(length of diagonal of the data extent)2/(25 × number of data points)

2·9 Triangulation with Linear Interpolation Method

The triangulation with linear interpolation method uses the optimal Delaunay triangulation. The algorithm creates triangles by drawing lines between data points. The original points are connected in such a way that no triangle edges are intersected by other triangles. The result is a patchwork of triangular faces over the extent of the grid. This method is an exact interpolator (Lawson, 1977; Lee and Schachter, 1980; Guibas and Stolfi, 1985; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Linear interpolation in a net of triangles uses the reference points as the vertices of nonoverlapping triangles that cover the interpolation area. The most common interpolation method on triangles is linear interpolation. A plane is defined in a rectangular coordinate system as

$$z = a_{00} + a_{10}x + a_{01}y \tag{23}$$

The constants a_{00}, a_{10} and a_{01} are calculated using three corner points of the triangle. The interpolated height z_0 is calculated by substituting (x_0, y_0) for (x, y) in the equation.

Triangulation with linear interpolation works best when data are evenly distributed over the grid area. Data sets that contain sparse areas result in distinct triangular facets on the map (Lawson, 1977; Lee and Schachter, 1980; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Local Polynomial Method

The local polynomial interpolation method assigns values to grid nodes by using a weighted least squares fit with data within the grid node's search ellipse.

For each grid node, the neighboring data are identified by the user-specified sector search. Using only these identified data, a local polynomial is fit using weighted least squares, and the grid node value is set equal to this value. Local polynomials can be of order one, two or three.

The form of these polynomials is (Harlan, 1982; Kidner *et al.*, 1999)

order 1

$$F(X,\ Y) = a + bX + cY \tag{24}$$

order 2

$$F(X,\ Y) = a + bX + cY + dXY + eX^2 + fY^2 \tag{25}$$

order 3

$$F(X,\ Y) = a + bX + cY + dXY + eX^2 + fY^2 + gX^2Y + hXY^2 + iX^3 + jY^3 \tag{26}$$

The polynomial coefficients are calculated by using known coordinates of the reference points. An adjustment is necessary when the number of reference points exceeds that of the polynomial coefficients. The coefficients of the polynomial are calculated by least squares estimation (Petrie and Kennie, 1987; Yanalak and Baykal, 2003).

Data Metrics Methods

The collection of data metric interpolation methods creates grids of information about the data on a node-by-node basis. The data metric interpolation methods are not, in general, weighted average interpolators of the Z values (Yang *et al.*, 2004).

Data metrics use the local data set including breaklines, for a specific grid node for the selected data metric. The local data set is defined by the search parameters. These search parameters are applied to each grid node to determine the local data set. In the following descriptions, when computing the value of a grid node (r, c), the local data set $S(r, c)$ consists of data within the specified search parameters centered at the specific grid node only. The set of selected data at the current grid node (r, c) can be represented by $S(r, c)$, where

$$S(r,\ c) = \{(x_1,\ y_1,\ z_1),\ (x_2,\ y_2,\ z_2),\ \ldots,\ (x_n,\ y_n,\ z_n)\} \tag{27}$$

n is thenumber of data points in the local data set.

The $Z(r, c)$ location refers to a specific node within the grid.

Moving Average Method

The moving average interpolation method assigns values to grid nodes by averaging the data within the grid node's search elipse (Yang *et al.*, 2004; Surfer 8 Software Online (Surfer 8 Manual, 2006)).

Search Ellipse. To use moving average, a search ellipse is defined and a minimum number of data is specified. For each grid node, the neighboring data are identified by centering the search ellipse on the node. The output grid node value is set equal to the arithmetic average of the identified neighboring data. If there are fewer than the specified minimum number of data within the neighborhood, the grid node is blanked.

The search ellipse defines the local neighborhood of points to consider when interpolating each grid node. Data points outside the search ellipse are not considered during grid node interpolation (Surfer 8 Software Online (Surfer 8 Manual, 2006)).

The search ellipse is an elliptical shape in theory, but it is generally taken as a circle in practical applications. An ellipse is chosen to use more data in one direction. A circle was taken as the search ellipse in this paper. The radius of the circle is taken so as to cover all the data in the data set (Figure 2).

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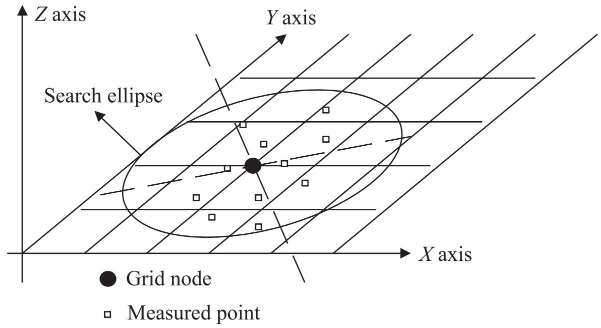

Figure 2. Grid structure and search ellipse.

Case Study

In application, an artificial object with a known volume was used. The volume of the artificial object was obtained experimentally. Then, the volume of the artificial object was calculated by using the above mentioned interpolation methods. Information on artificial object, its applications and evaluation of the results are summarized below.

Artifical Model

The artificial object is made of gypsum with base diameter of 5 cm, height 9 cm. The artificial object has a uniform hill-shaped surface (Figure 3). In order to calculate the volume of the object, a glass of 1000 ml volume was used. First of all, the glass was filled with 100 ml water, then the artificial object put into the glass. The water level rose to 464·2 ml. By subtracting the former volume of water, the volume of the object is determined to be 364·2 ml. For surveying the control points, a test area was set (Figure 3). After surveying the coordinates (X, Y, Z) of control points, different positions of the object were photographed using an eight Megapixel DSC-F828 Sony digital camera. Calibration of the digital camera was set up. The calibration values, photos and control points were transferred to Photomodeler 5·0 software. Finally, the photogrammetric properties of the artificial object were obtained by surveying 541 points upon the surface of the object (Figure 4). Root mean square errors in control points are 0,12 cm, 0,11 cm and 0,10 cm in X, Y and Z respectively.

Application

Measured points from the object are transferred to Surfer 8 software. The volume of the object is calculated by using the previously mentioned interpolation methods. The results of volume, root mean square errors and accuracy percentage

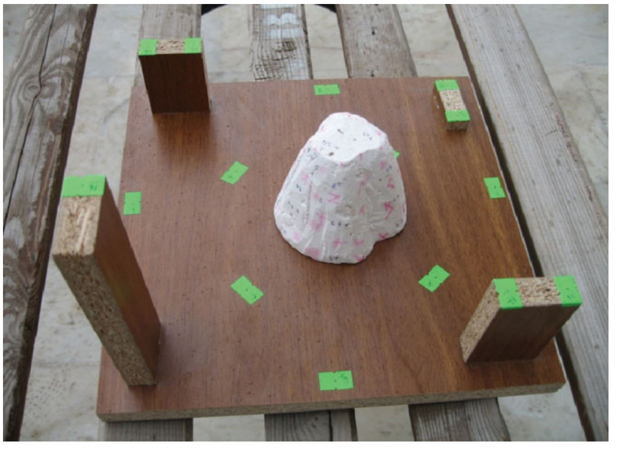

Figure 3. Artificial object and test area. This figure is available in colour online at www.interscience.wiley.com/journal/espl

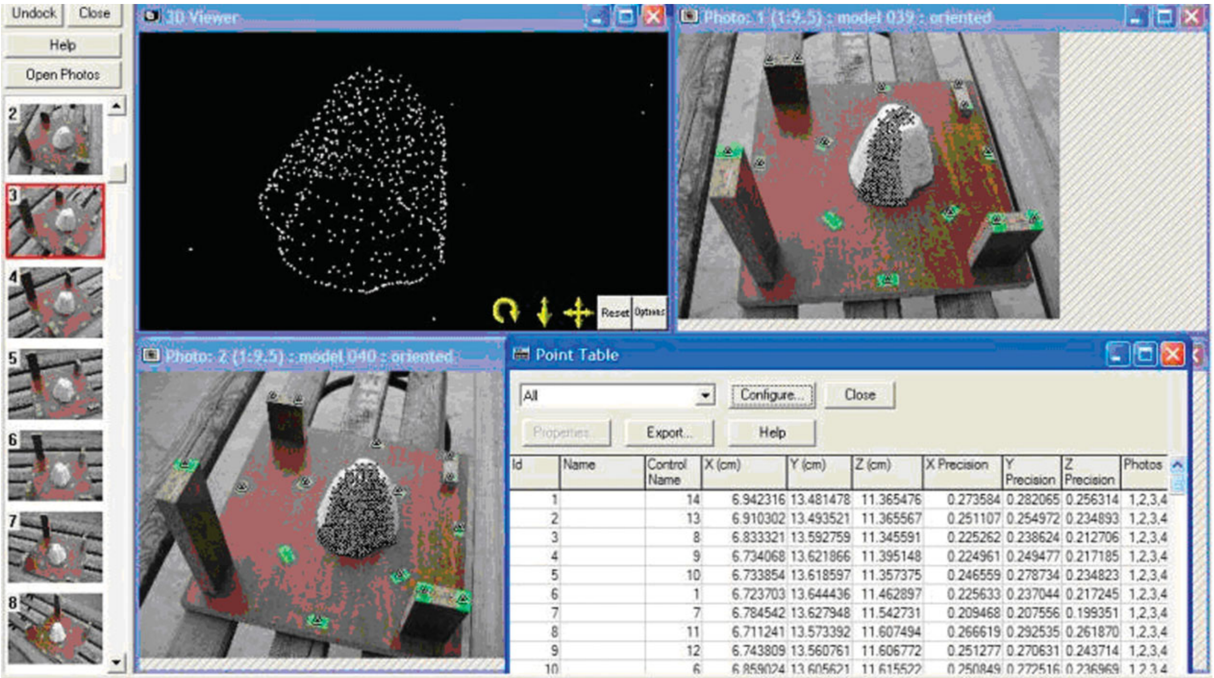

Figure 4. A view of photogrammetric evaluation in Photomodeler software. This figure is available in colour online at www.interscience.wiley.com/journal/espl

Table I. Evaluation of interpolation methods

Interpolation methods	Computing volume (cm³)	Volume difference (cm³)	Approach rate (%)	RMSE (cm)
Inverse distance to a power	368 902	−4 702	98·71	1·76
Kriging	361 303	2 897	99·20	0·94
Minimum curvature	362 671	1 529	99·58	1·32
Modified Shepard	375 163	−10 963	96·99	0·59
Natural neighbor	358 482	5 718	98·43	0·84
Nearest neighbor	367 681	−3 481	99·04	0·76
Polynomial regression	365 389	−1 189	99·67	27·73
Radial basis function	362 274	1 926	99·47	0·55
Triangulation with linear interpolation	359 527	4 673	98·72	0·92
Data metrics	556 420	−192 220	47·22	29·39
Moving average	400 179	−35 979	90·12	7·24
Local polynomial	316 995	47 205	87·04	29·31

to real value are given in Table I. Relation between RMSEs and approach rates in interpolation methods is seen in Figure 5 as graphically. Root mean square errors (RMSEs) were calculated using Equation (28).

$$\mathrm{RMSE} = \pm\sqrt{\frac{[V_i V_i]}{n-1}} \tag{28}$$

$V_i = Z_m - Z_g$
V_i = error in Z
$Z_m = Z$ (measured)
$Z_g = Z$ (calculated).

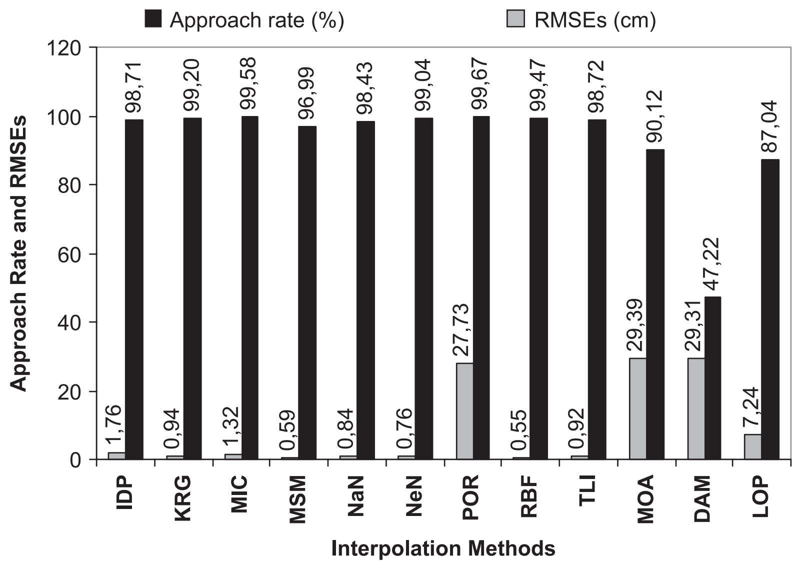

Figure 5. RMSEs and approach rates in interpolation methods.
IDP: inverse distance to a power; KRG: kriging; MIC: minimum curvature
MSM: modified Shepard method; NaN: natural neighbor; NeN: nearest neighbor
POR: polynomial regression; RBF: radial basis function; DAM: data metrics
TLI: triangulation with linear interpolation; LOP: local polynomial; MOA: moving average.

Conclusions

3D modeling of the earth surface is commonly required in most recent research. The 3D models are derived by using DTM. The DTMs are derived by using different interpolation methods. The applied interpolation methods can be changed depending on the structure of the surface and the number of control points. In this study, a comparison of different interpolation methods is interpreted to define a surface. The better the surface is described, the closer the amount of volume is to the real value.

According to Table I, the results closest to the real value of the volume are obtained from the following methods: polynomial regression (99·67%), minimum curvature (99·58%), radial basis function (99·47%), kriging (99·20%), nearest neighbor (99·04%), triangulation with linear interpolation (98·72%), inverse distance to a power (98·71%) and natural neighbor (98·43%). The poorest results were obtained from data metrics (47·22%), local polynomial (87·04%), moving average (90·12%) and modified Shepard (96·99%) methods. Methods in the first group describe the surface based on the calculated volume more accurately than the methods in the second group. The methods in the first group use mathematical modeling as a polynomial or power function. The data metric method gave the poorest result because it uses a linear function in its mathematical model. The most suitable contour map of the object is obtained from the triangulation with linear interpolation, natural neighbor methods and inverse distance to a power methods (Figure 6). The most suitable 3D model of the object is obtained from triangulation with linear interpolation, natural neighbor and inverse distance to a power methods (Figure 7). Therefore, any of the interpolation methods in the first group above may be used for surface definition and 3D modeling of uniform surface terrains. However, to be able to advise on which interpolation method is more suitable in defining what kind of a terrain surface (e.g. uniform, uneven, rough, rugged, break line etc.) with some certainity, more work with models having various surfaces is required.

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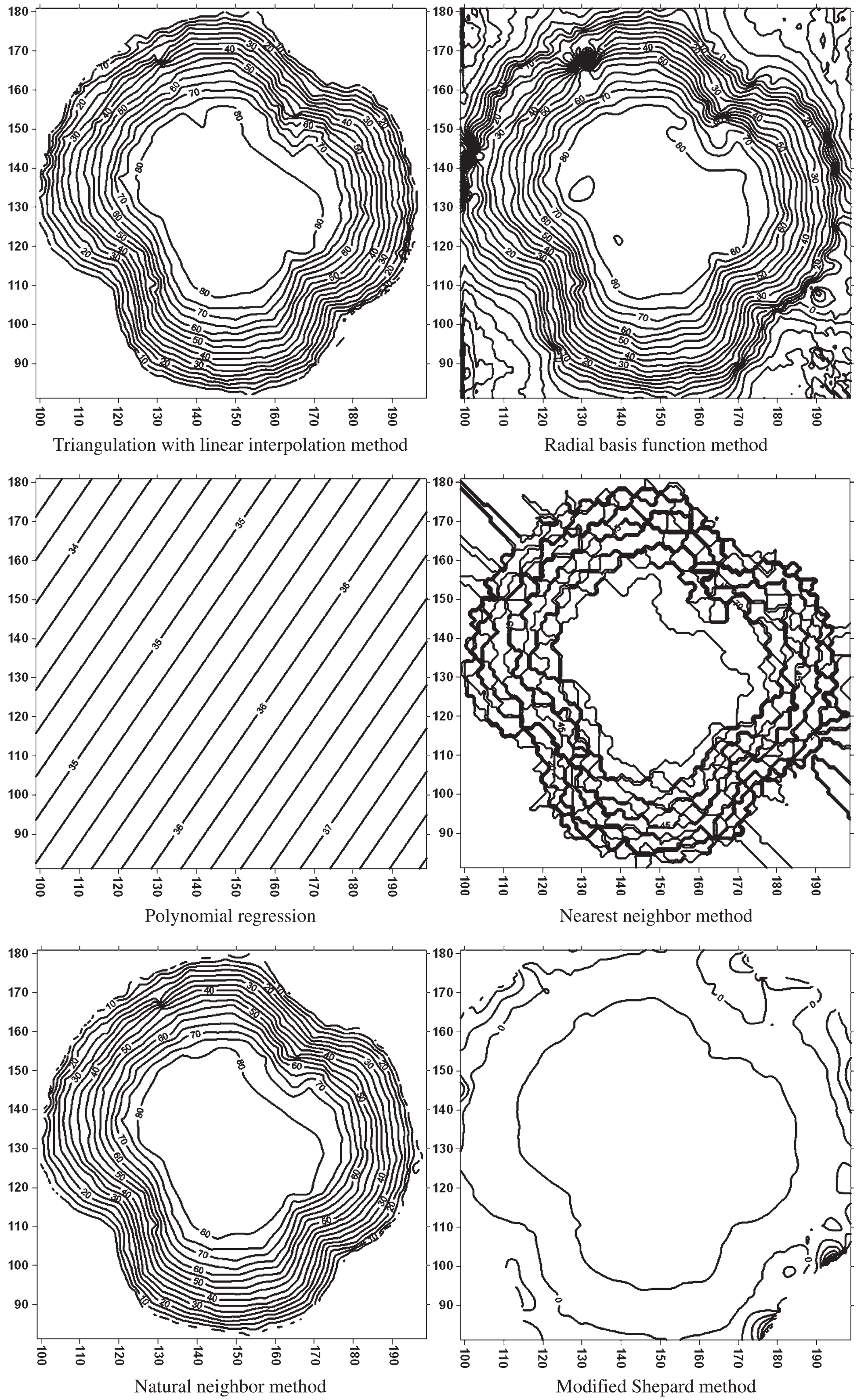

Figure 6. Contour maps of artificial object by interpolation methods.

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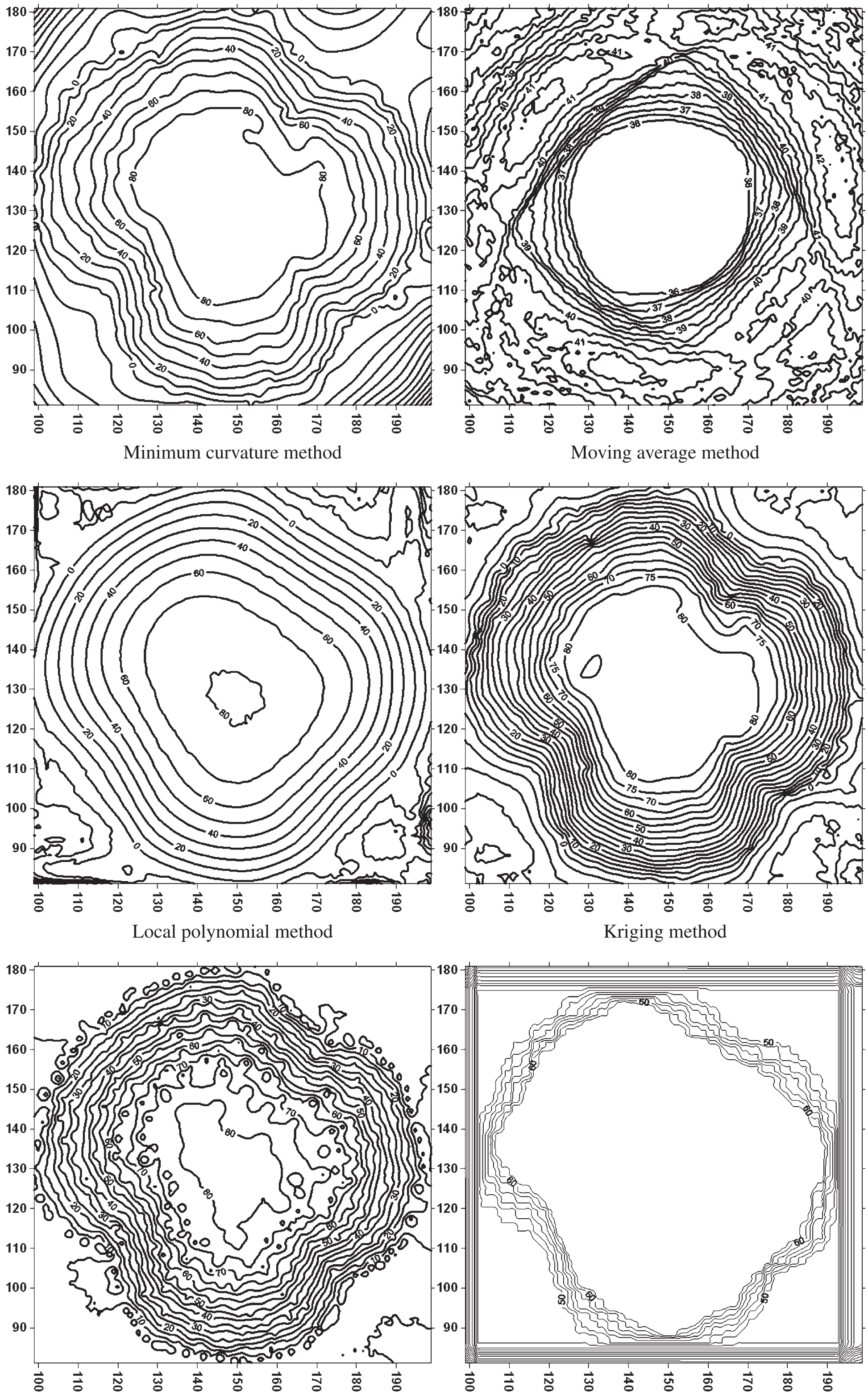

Figure 6. *Continued*

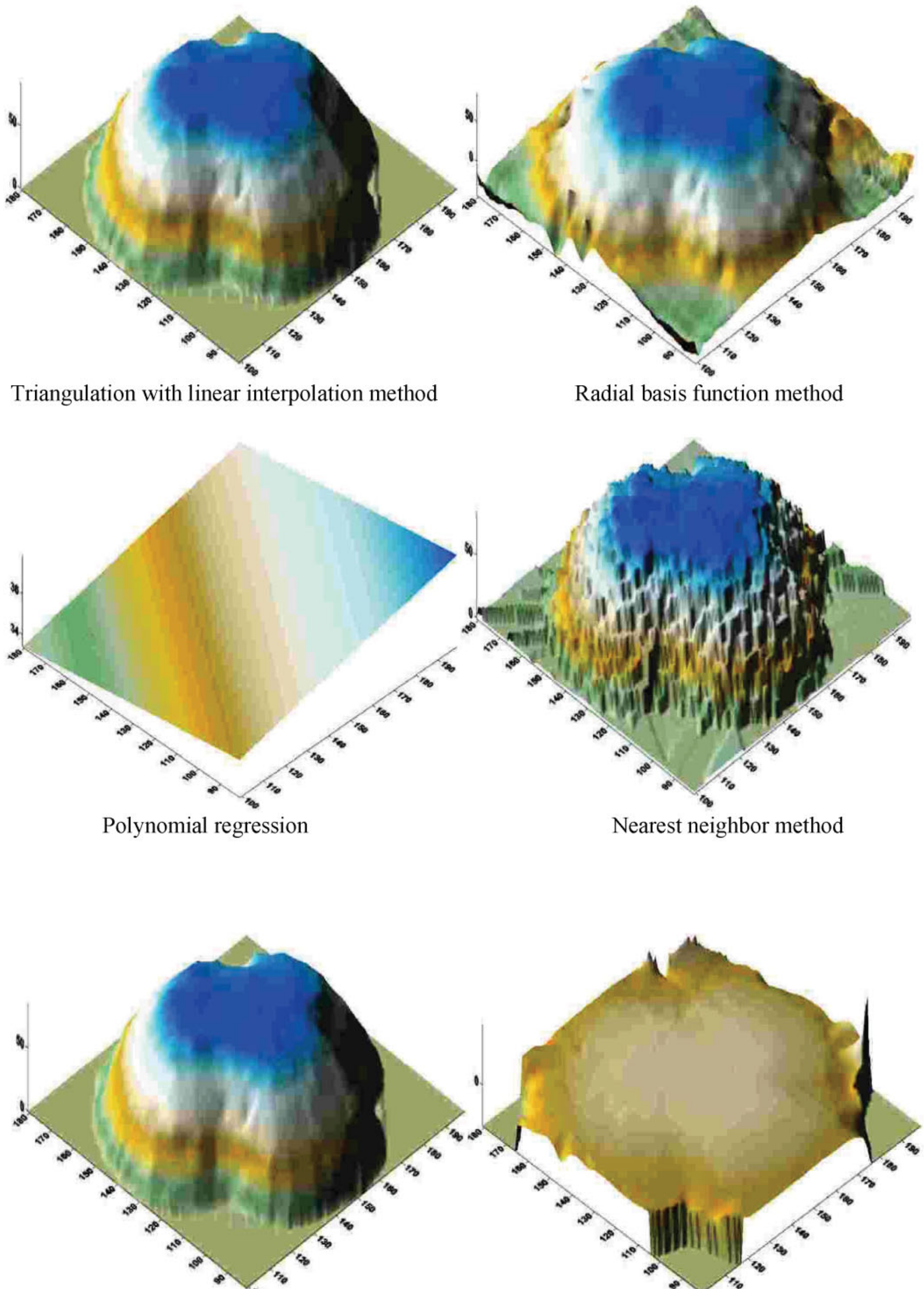

Figure 7. 3D models of artificial object by interpolation methods. This figure is available in colour online at www.interscience.wiley.com/journal/espl

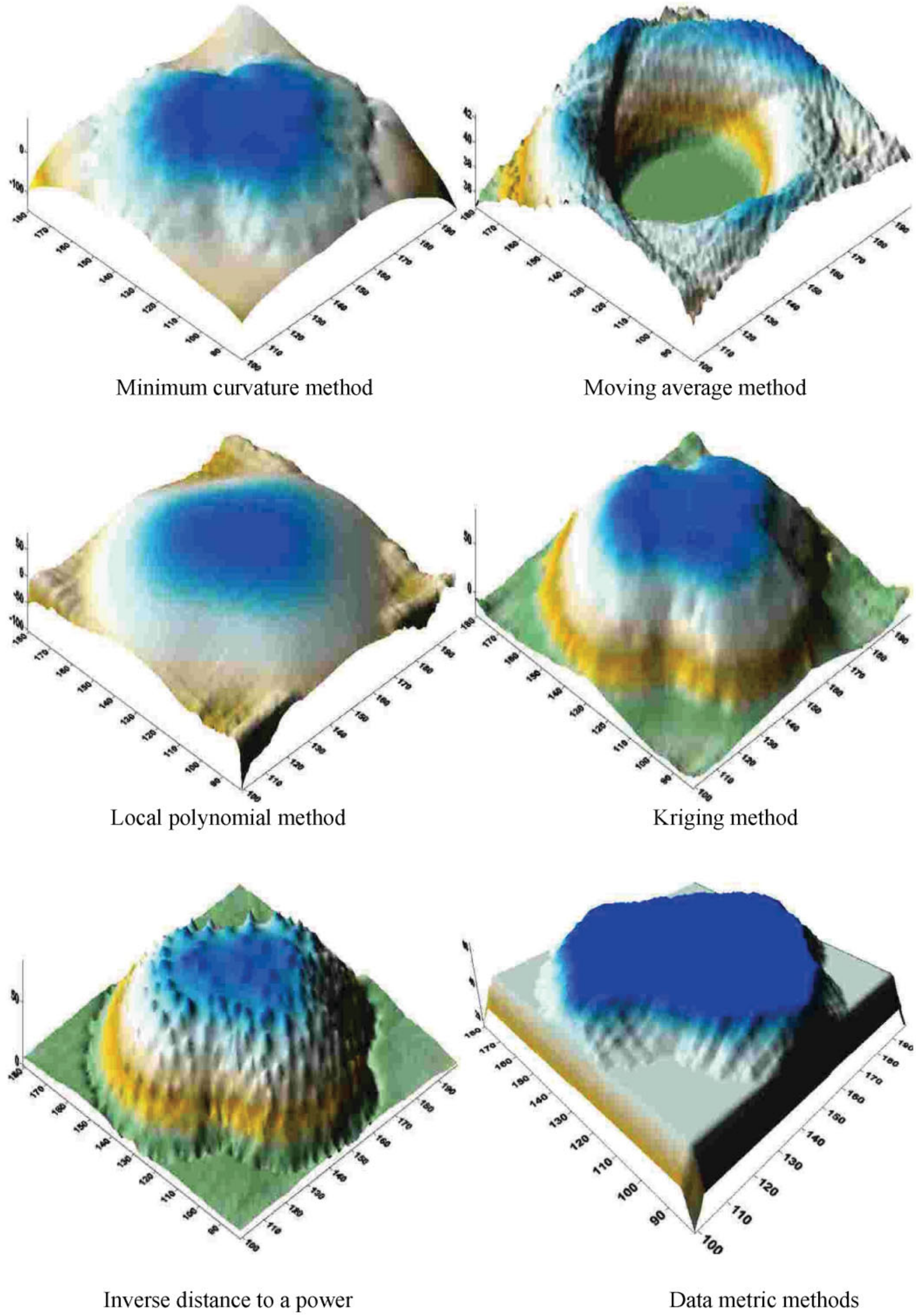

Figure 7. *Continued*

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