

# A CURIOSITY ABOUT POLYNOMIAL INTERPOLATION

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ABSTRACT. Interpolation of cubes expected to be

$$n^3 = 6\binom{n}{3} + 6\binom{n}{2} + \binom{n}{1} + 0\binom{n}{0}$$

but got

$$n^3 = \sum_{k=1}^n \mathbf{A}_{m,0} k^0 (n-k)^0 + \mathbf{A}_{m,1} k^1 (n-k)^1$$

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## 1. INTRODUCTION

Interpolation is a process of finding new data points based on the range of a discrete set of known data points. Interpolation has been well-developed in between 1674–1684 by Issac Newton’s fundamental works, nowadays known as foundation of classical interpolation theory [1].

At that time, in 2016, I was a first-year mechanical engineering undergraduate, so that due to lack of knowledge and perspective of view I started re-inventing interpolation formula myself, fueled by purest passion and feeling of mystery. *All the mathematical laws and*

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*Date:* January 10, 2025.

2010 *Mathematics Subject Classification.* 26E70, 05A30.

*Key words and phrases.* Keyword1, Keyword2 .

Sources: <https://github.com/kolosovpetro/ACuriosityAboutPolynomialInterpolation>

*relations exist from the very beginning, we only reveal and describe them*, I thought. That mindset truly inspired me, so my own mathematical journey has been started. Consider finite differences of cubes  $n^3$

$n$	$n^3$	$\Delta(n^3)$	$\Delta^2(n^3)$	$\Delta^3(n^3)$
0	0	1	6	6
1	1	7	12	6
2	8	19	18	6
3	27	37	24	6
4	64	61	30	6
5	125	91	36	
6	216	127		
7	343			

**Table 1.** Table of finite differences of the polynomial  $n^3$ .

The problem of interpolation of polynomials is a classical problem in mathematics and has been widely studied in literature. For instance, Concrete mathematics [2] gives interpolation of cubes by using Newton's interpolation formula

$$n^3 = 6\binom{n}{3} + 6\binom{n}{2} + \binom{n}{1} + 0\binom{n}{0}$$

because

$$f(x) = \Delta^d f(0)\binom{x}{d} + \Delta^{d-1} f(0)\binom{x}{d-1} + \cdots + f(0)\binom{x}{0} = \sum_{r=0}^d \Delta^{d-r} f(0)\binom{x}{r}$$

However, interpolation of cubes can be also done in a different way. The key point that interpolation formula above iterates over the finite difference of order  $d$ , instead it is clear that  $n^3$  can be reached as a sum of finite difference  $\Delta^1$  of first order

$$n^3 = \Delta 0^3 + \Delta 1^3 + \Delta 2^3 + \cdots + \Delta(n-1)^3 = \sum_{k=0}^{n-1} \Delta k^3$$

We know that  $\Delta^3 n^3 = 6$  is the constant for each  $n$ . The second difference of cubes  $\Delta^2 n^3$  is a linear relation in terms of third order finite difference  $\Delta^3 n^3$ .

$$\Delta^2 n^3 = (n+1)\Delta^3 n^3 = 6(n+1)$$

Finally, the first order finite difference  $\Delta n^3$  is the following relation in terms of second order finite difference

$$\Delta n^3 = \Delta 0^3 + \Delta^2(n-1)^3 = 1 + \Delta^2(n-1)^3 = 1 + 6(n-1)$$

Meaning that

$$\begin{aligned}\Delta(0^3) &= 1 + 6 \cdot 0 \\ \Delta(1^3) &= 1 + 6 \cdot 0 + 6 \cdot 1 \\ \Delta(2^3) &= 1 + 6 \cdot 0 + 6 \cdot 1 + 6 \cdot 2 \\ \Delta(3^3) &= 1 + 6 \cdot 0 + 6 \cdot 1 + 6 \cdot 2 + 6 \cdot 3\end{aligned}\tag{1.1}$$

Finally reaching its generic form

$$\Delta(n^3) = 1 + 6 \cdot 0 + 6 \cdot 1 + 6 \cdot 2 + 6 \cdot 3 + \cdots + 6 \cdot n = 1 + 6 \sum_{k=0}^n k\tag{1.2}$$

Because

$$\Delta(n^3) = \Delta(n-1)^3 + \Delta^2(n-1)^3$$

Having the relation  $n^3 = \Delta 0^3 + \Delta 1^3 + \Delta 2^3 + \cdots + \Delta(n-1)^3$ , we get

$$\begin{aligned}n^3 &= [1 + 6 \cdot 0] + [1 + 6 \cdot 0 + 6 \cdot 1] + [1 + 6 \cdot 0 + 6 \cdot 1 + 6 \cdot 2] + \cdots \\ &\quad + [1 + 6 \cdot 0 + 6 \cdot 1 + 6 \cdot 2 + \cdots + 6 \cdot (n-1)]\end{aligned}\tag{1.3}$$

By rearranging the terms of equation (1.3) it turns into summation in terms of  $k(n-k)$

$$\begin{aligned}n^3 &= n + [(n-0) \cdot 6 \cdot 0] + [(n-1) \cdot 6 \cdot 1] + [(n-2) \cdot 6 \cdot 2] + \cdots \\ &\quad \cdots + [(n-k) \cdot 6 \cdot k] + \cdots + [1 \cdot 6 \cdot (n-1)]\end{aligned}$$

Giving an identity for cubes  $n^3$

$$n^3 = \sum_{k=1}^n 6k(n-k) + 1 \quad (1.4)$$

## 2. CONCLUSIONS

Conclusions of your manuscript.

## REFERENCES

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**Version:** Local-0.1.0

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