

GOLDBACH'S CONJECTURE VIA BERTRAND'S POSTULATE

PETRO KOLOSOV

ABSTRACT. In this manuscript the relations between Bertrand's postulate and both ternary and binary Goldbach's conjectures are discussed. As a result, a three Goldbach-like conjectures are proposed and discussed. Verification programs are attached to the last section.

1. INTRODUCTION

In number theory, Bertrand's postulate is a statement that was first conjured in 1845 by Joseph Bertrand [1].

Theorem 1.1. (*Bertrand–Chebyshev theorem.*) *For every positive integer $n > 1$ exists at least one prime p such that*

$$n < p < 2n$$

Bertrand's postulate completely proved by Chebyshev in 1852 [2]. From Bertrand's postulate immediately follows

Lemma 1.1. (*Even Bertrand's partition.*) *By Bertrand's postulate, for every positive integer $n > 1$ there is at least one partition such that*

$$2n = p + \text{odd},$$

where odd is odd part of Even Bertrand's partition.

Now we have to recall ternary Goldbach's conjecture or namely Goldbach's weak conjecture

Conjecture 1.2. (*Ternary Goldbach's conjecture.*) *Every odd number greater than 7 can be expressed as the sum of three odd primes.*

$$2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k, \quad n > 2,$$

where $\mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k$ is ternary Goldbach partition.

Ternary Goldbach's conjecture was proved by H.A Helfgott in 2013 [3]. From this prospective, we are able to express the lemma 1.1 for positive odd numbers,

Lemma 1.2. (*Odd Bertrand's partition.*) *By Bertrand's postulate, for every positive integer $n > 1$ there is at least one partition such that*

$$2n + 1 = p + \text{odd} + 1 = p + \text{even},$$

where even is even part of Odd Bertrand's partition.

Date: June 20, 2021.

2010 Mathematics Subject Classification. 11P32, 11A41, 97F60.

Key words and phrases. Bertrand's Postulate, Goldbach Conjecture, Ternary Goldbach Conjecture.

Then we have the following relation between odd Bertrand's partition 1.2 and Ternary Goldbach's conjecture 1.2, for every positive integer $n > 2$

$$2n + 1 = p + \text{odd} + 1 = \mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k \quad (1)$$

From equation (1) implies the Goldbach-like conjectures

Conjecture 1.3. *For every prime $p < 2n - 1$ in positive odd $2n + 1$, $n > 2$ Bertrand's partition 1.2, exists ternary Goldbach's partition such that*

$$2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + p$$

Conjecture 1.4. *For every positive odd $2n + 1$, $n > 2$ Bertrand's partition 1.2, the prime $p < 2n - 1$ is always a member of ternary Goldbach partition $2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k$*

$$\bigvee_{t \in \{i, j, k\}} \mathfrak{p}_t = p$$

Conjecture 1.5. *For every positive odd $2n + 1$, $n > 2$ Bertrand's partition 1.2 the even part is always a sum of two primes*

$$2n + 1 = p + \text{even} \rightarrow \text{even} = \mathfrak{p}_j + \mathfrak{p}_k, \quad n > 2$$

2. DISCUSSION

3. VERIFICATION

REFERENCES

- [1] J. Bertrand. *Mémoire sur le nombre de valeurs que peut prendre une fonction: quand on y permute les lettres qu'elle renferme*. Bachelier, 1845.
- [2] Tchebichef. Mémoire sur les nombres premiers. *Journal de Mathématiques Pures et Appliquées*, pages 366–390, 1852.
- [3] H. A. Helfgott. The ternary goldbach conjecture is true, 2014.

Email address: kolosovp94@gmail.com

URL: <https://kolosovpetro.github.io>