

GOLDBACH'S CONJECTURE VIA BERTRAND'S POSTULATE

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ABSTRACT. In this manuscript the relations between Bertrand's postulate and both ternary and binary Goldbach's conjectures are discussed. As a result, a three Goldbach-like conjectures are proposed and discussed. Verification programs are attached to the last section.

1. INTRODUCTION

In number theory, Bertrand's postulate is a statement that was first conjured in 1845 by Joseph Bertrand (ref). Bertrand's postulate may be expressed as follows

Theorem 1.1. (*Bertrand–Chebyshev theorem.*) *For every positive integer $n > 1$ exists at least one prime p such that*

$$n < p < 2n$$

Bertrand's postulate completely proved by Chebyshev in 1852 (ref). From Bertrand's postulate immediately follows

Lemma 1.1. (*Even Bertrand's partition.*) *By Bertrand's postulate, for every positive integer $n > 1$ there is at least one partition such that*

$$2n = p + \text{odd},$$

where odd is odd part of Even Bertrand's partition.

Now we have to recall ternary Goldbach's conjecture or namely Goldbach's weak conjecture

Conjecture 1.2. (*Ternary Goldbach's conjecture.*) *Every odd number greater than 7 can be expressed as the sum of three odd primes.*

$$2n + 1 = p_i + p_j + p_k, \quad n > 2$$

Ternary Goldbach's conjecture was proved by H.A Helfgott in 2013, (ref). From this prospective, we are able to express the lemma 1.1 for positive odd numbers,

Lemma 1.2. (*Odd Bertrand's partition.*) *By Bertrand's postulate, for every positive integer $n > 1$ there is at least one partition such that*

$$2n + 1 = p + \text{odd} + 1 = p + \text{even},$$

where even is even part of Odd Bertrand's partition.

Then we have the following relation between lemma 1.2 and Ternary Goldbach's conjecture 1.2 for every positive integer $n > 2$

$$2n + 1 = p + \text{odd} + 1 = p_i + p_j + p_k \tag{1}$$

From equation (1) implies the Goldbach-like conjectures

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Conjecture 1.3. *For every positive integer $n > 2$ and prime $n < p < 2n$, $p < 2n - 1$ exists ternary Goldbach's partition such that*

$$2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + p$$

Conjecture 1.4. *For every odd Bertrand's partition 1.2, the prime p such that $n < p < 2n$, $p < 2n - 1$, $n > 2$ is always a member of ternary Goldbach partition $2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k$*

$$\bigvee_{t \in \{i, j, k\}} \mathfrak{p}_t = p$$

Conjecture 1.5. *The even part of odd Bertrand's partition 1.2 is always sum of two primes*

$$2n + 1 = p + \text{even}, \quad \text{even} = \mathfrak{p}_j + \mathfrak{p}_k, \quad n > 2$$

2. DISCUSSION

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