GOLDBACH'S CONJECTURE VIA BERTRAND'S POSTULATE

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Abstract. In this manuscript the relations between Bertrand's postulate and both ternary and binary Goldbach's conjectures are discussed. As a result, a three Goldbach-like conjectures are proposed and discussed. Verification programs are attached to the last section.

1. Introduction

In number theory, Bertrand's postulate is a statement that was first conjured in 1845 by Joseph Bertrand [1].

Theorem 1.1. *(Bertrand–Chebyshev theorem.) For every positive integer $n > 1$ exists at least one prime p such that*

$$n < p < 2n$$

Bertrand's postulate completely proved by Chebyshev in 1852 [2]. From Bertrand's postulate immediately follows

Lemma 1.1. *(Even Bertrand's partition.) By Bertrand's postulate, for every positive integer $n > 1$ there is at least one partition such that*

$$2n = p + \mathtt{odd},$$

where `odd` *is odd part of Even Bertrand's partition.*

Now we have to recall ternary Goldbach's conjecture or namely Goldbach's weak conjecture

Conjecture 1.2. *(Ternary Goldbach's conjecture.) Every odd number greater than 7 can be expressed as the sum of three odd primes.*

$$2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k, \quad n > 2,$$

where $\mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k$ is ternary Goldbach partition.

Ternary Goldbach's conjecture was proved by H.A Helfgott in 2013 [3]. From this prospective, we are able to express the lemma 1.1 for positive odd numbers,

Lemma 1.2. *(Odd Bertrand's partition.) By Bertrand's postulate, for every positive integer $n > 1$ there is at least one partition such that*

$$2n + 1 = p + \mathtt{odd} + 1 = p + \mathtt{even},$$

where `even` *is even part of Odd Bertrand's partition.*

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Then we have the following relation between lemma 1.2 and Ternary Goldbach's conjecture 1.2 for every positive integer $n > 2$

$$2n + 1 = p + \texttt{odd} + 1 = \mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k \tag{1}$$

From equation (1) implies the Goldbach-like conjectures

Conjecture 1.3. *For every positive integer $n > 2$ and prime $n < p < 2n,\ p < 2n - 1$ exists ternary Goldbach's partition such that*

$$2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + p$$

Conjecture 1.4. *For every positive odd $2n + 1,\ n > 2$ Bertrand's partition 1.2, the prime $p < 2n - 1$ is always a member of ternary Goldbach partition $2n + 1 = \mathfrak{p}_i + \mathfrak{p}_j + \mathfrak{p}_k$*

$$\bigvee_{t \in \{i,j,k\}} \mathfrak{p}_t = p$$

Conjecture 1.5. *The even part of odd Bertrand's partition 1.2 is always sum of two primes*

$$2n + 1 = p + \texttt{even}, \quad \texttt{even} = \mathfrak{p}_j + \mathfrak{p}_k,\ n > 2$$

2. Discussion

3. Verification

References

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