FINITE DIFFERENCE OF CONSECUTIVE ODD POWERS MINUS 1 IS ALWAYS DIVISIBLE BY 6

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Main results

We show that for every odd power P and non-negative integer N it is always true that

$$(N+1)^P - N^P - 1 \equiv 0 \bmod 6$$

To prove the statement we have to show that for every non-negative integer N and odd power P

$$(N+1)^P - N^P - 1$$

is always divisible by both 2 and 3. For every non-negative integer N, either N is even, and $N+1$ is odd, and vise versa. It means that $(N+1)^P - N^P$ is always a positive odd number. Therefore,

$$(N+1)^P - N^P - 1 \equiv 0 \bmod 2$$

Let be $F(N) = (N+1)^P - N^P - 1$, then we want to show that for every non-negative N

$$F(N) \equiv 0 \bmod 3$$

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Since every integer N is congruent to either 0, 1, or 2 modulo 3, we check each case. Consider the case $N \equiv 0 \bmod 3$

$$N \equiv 0 \bmod 3$$

$$N + 1 \equiv 1 \bmod 3$$

$$N^p \equiv 0^p = 0 \bmod 3$$

$$(N + 1)^p \equiv 1^p = 1 \bmod 3$$

Therefore,

$$f(N) = (N + 1)^p - N^p - 1 \equiv 1 - 0 - 1 = 0 \bmod 3$$

Consider the case $N \equiv 1 \bmod 3$

$$N \equiv 1 \bmod 3$$

$$N + 1 \equiv 2 \bmod 3$$

$$N^p \equiv 1^p = 1 \bmod 3$$

$$(N + 1)^p \equiv 2^p \bmod 3$$

Since p is odd, $2^p \equiv 2 \bmod 3$, so

$$f(N) \equiv 2 - 1 - 1 = 0 \bmod 3$$

Consider the case $N \equiv 2 \bmod 3$

$$N \equiv 2 \bmod 3$$

$$N + 1 \equiv 0 \bmod 3$$

$$N^p \equiv 2^p \equiv 2 \bmod 3$$

$$(N + 1)^p \equiv 0^p = 0 \bmod 3$$

$$f(N) \equiv 0 - 2 - 1 = -3 \equiv 0 \bmod 3$$