POLYNOMIAL IDENTITIES INVOLVING RASCAL TRIANGLE

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Abstract. Abstract

1. Definitions

Definition of generalized Rascal triangle

$$\binom{n}{k}_i = \sum_{m=0}^{i} \binom{n-k}{m} \binom{k}{m} \tag{1.1}$$

Definition of $(1, q)$-Pascal triangle

$$\begin{bmatrix} n \\ k \end{bmatrix}^q = \begin{cases} q & \text{if } k = 0, n = 0 \\ 1 & \text{if } k = 0 \\ 0 & \text{if } k > n \\ \begin{bmatrix} n-1 \\ k \end{bmatrix}^q + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}^q \end{cases}$$

2. Sides of world

$$\begin{aligned} \textbf{North} &= \binom{n-2}{k-1}_i \\ \textbf{South} &= \binom{n}{k}_i \\ \textbf{West} &= \binom{n-1}{k-1}_i \\ \textbf{East} &= \binom{n-1}{k}_i \end{aligned}$$

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Key words and phrases. Keyword1, Keyword2 .

Identity see Hotchkiss

$$\mathbf{South} = \frac{\mathbf{East} \cdot \mathbf{West} + 1}{\mathbf{North}} \tag{2.1}$$

$$\binom{n}{k}_i = \frac{\binom{n-1}{k}_i \binom{n-1}{k-1}_i + 1}{\binom{n-2}{k-1}_i} \tag{2.2}$$

Identity see Hotchkiss, for all inner $k > 0$ and $k < n$

$$\mathbf{South} = \mathbf{East} + \mathbf{West} - \mathbf{North} + 1 \tag{2.3}$$

$$\binom{n}{k}_i = \binom{n-1}{k}_i + \binom{n-1}{k-1}_i - \binom{n-2}{k-1}_i + 1 \tag{2.4}$$

3. Formulae

3.1. **Claim 1.** Generalized rascal triangle equals to Pascal's triangle up to i-th column

$$\binom{n}{k}_i = \binom{n}{k}, \quad 0 \le k \le i \tag{3.1}$$

$$\binom{n}{i-j}_i = \binom{n}{i-j}, \quad ColumnIdentity1 \tag{3.2}$$

$$\binom{n}{n-i+j}_i = \binom{n}{n-i+j}, \quad ColumnIdentity2 \tag{3.3}$$

$$\tag{3.4}$$

3.2. **Claim 2.** Generalized rascal triangle equals to Pascal's triangle up to $2i+1$-th row

$$\binom{n}{k}_i = \binom{n}{k}, \quad 0 \le n \le 2i+1 \tag{3.5}$$

$$\binom{2i+1-j}{k}_i = \binom{2i+1-j}{k} \quad RowIdentity1 \tag{3.6}$$

$$\binom{(2i+1)-j}{k}_{(2i+1)-i-1} = \binom{(2i+1)-j}{k} \tag{3.7}$$

$$\binom{t-j}{k}_{t-i-1} = \binom{t-j}{k} \quad t \ge 2i+1 \quad RowIdentity2 \tag{3.8}$$

By symmetry

$$\binom{2i+1-t}{2i+1-t-k}_i = \binom{2i+1-t}{2i+1-t-k}$$

For $k = t$

$$\binom{2i+1-t}{t}_i = \binom{2i+1-t}{t}$$
$$\binom{2i+1-t}{2i+1-2t}_i = \binom{2i+1-t}{2i+1-2t}$$

3.3. **Claim 3.**

$$\binom{j}{k} - \binom{j}{k}_i = \binom{n}{i+1}, \quad j \geq 2i+2, k = i+1 \tag{3.9}$$

$$\binom{2i+j+2}{i+1} - \binom{2i+j+2}{i+1}_i = \binom{i+j+1}{i+1} \tag{3.10}$$

$$\binom{2(i+1)+j}{i+1} - \binom{2(i+1)+j}{i+1}_i = \binom{(i+1)+j}{i+1} \tag{3.11}$$

$$\binom{2t+j}{t} - \binom{2t+j}{t}_{t-1} = \binom{t+j}{t} \tag{3.12}$$

3.4. **Claim 4.**

$$\binom{2i+3+j}{i+2} - \binom{2i+3+j}{i+2}_i = \begin{bmatrix} i+2+j \\ i+2 \end{bmatrix}^{i+2}$$
$$\binom{2(i+2)-1+j}{i+2} - \binom{2(i+2)-1+j}{i+2}_i = \begin{bmatrix} i+2+j \\ i+2 \end{bmatrix}^{i+2}$$
$$\binom{2t-1+j}{t} - \binom{2t-1+j}{t}_{t-2} = \begin{bmatrix} t+j \\ t \end{bmatrix}^{t}$$

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