IDENTITIES IN ITERATED RASCAL TRIANGLES

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ABSTRACT. In this manuscript, we explore new binomial identities within iterated Rascal triangles, revealing a connection between Vandermonde convolution and iterated Rascal numbers. We also provide new identities in terms of finite differences of iterated Rascal numbers and Binomial coefficients. Apart that, the manuscript provides a proof of the row sums conjecture in iterated Rascal triangles. Also, we establish and explore a connection between iterated Rascal triangles and $(1, q)$-Binomial coefficients, showing related OEIS sequences. All result can be validated through the supplementary Mathematica programs.

CONTENTS

1. Introduction

Rascal triangle is Pascal-like numeric triangle developed in 2010 by three middle school students, Alif Anggoro, Eddy Liu, and Angus Tulloch [1]. During math classes they were challenged to provide the next row for the following number triangle

$$
\begin{array}{ccccccc}
& & & 1 & & & \\
& & 1 & & 1 & & \\
& 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1 \\
\end{array}
$$

$$\cdots$$

The teacher anticipated that the next row would match Pascal's triangle, such as "1 4 6 4 1", by applying the binomial coefficient recurrence rule $South = East + West$. However, Anggoro, Liu, and Tulloch proposed that the next row should be "1 4 5 4 1". Instead of using Pascal's triangle rule $South = East + West$, they derived this new row using a relation they termed the diamond formula

$$South = \frac{East \cdot West + 1}{North} \tag{1.1}$$

By applying the recurrence relation from equation (1.1), the students successfully generated an entirely new triangular sequence, now referred to as the Rascal triangle.

n/k	0	1	2	3	4	5	6	7	8	9
0	1									
1	1	1								
2	1	2	1							
3	1	3	3	1						
4	1	4	5	4	1					
5	1	5	7	7	5	1				
6	1	6	9	10	9	6	1			
7	1	7	11	13	13	11	7	1		
8	1	8	13	16	17	16	13	8	1	
9	1	9	15	19	21	21	19	15	9	1

Table 1. Rascal triangle. Sequence A077028 in OEIS [2].

For example, the fourth row is "1 4 5 4 1" because $4 = \frac{1 \cdot 3 + 1}{1}$ and $5 = \frac{3 \cdot 3 + 1}{2}$. Moreover, the Rascal triangle, as presented in table (1), represents the first and foundational instance of a new family of Pascal-like triangles. This family, known as *iterated Rascal triangles*, was first introduced by J. Gregory in her master's thesis [3].

We define the k-th element in the n-th row of an iterated Rascal triangle as $\binom{n}{k}_i$, where i represents the number of iterations. The integer sequence produced by $\binom{n}{k}_i$ is referred to as an *iterated Rascal triangle Ri*, and each $\binom{n}{k}_i$ is termed an *iterated Rascal number*. Therefore, the Rascal triangle shown in table (1) corresponds to the iterated Rascal triangle $R1$, generated by the formula $\binom{n}{k}_1 = k(n-k) + 1$. While the iterated Rascal number $\binom{n}{k}_i$ is defined by the diamond rule (1.1), which differs from the standard binomial coefficient recurrence, it still maintains a significant connection with the binomial coefficients $\binom{n}{k}$, as demonstrated by

$$\binom{n}{k}_i = \sum_{m=0}^{i} \binom{n-k}{m}\binom{k}{m} \tag{1.2}$$

For example, $\binom{7}{4}_3 = 35$, $\binom{12}{7}_5 = 792$, $\binom{11}{5}_5 = 462$.

Example 1.1. Rascal triangle R2 generated by $\binom{n}{k}_2$

n/k	0	1	2	3	4	5	6	7	8	9
0	1									
1	1	1								
2	1	2	1							
3	1	3	3	1						
4	1	4	6	4	1					
5	1	5	10	10	5	1				
6	1	6	15	19	15	6	1			
7	1	7	21	31	31	21	7	1		
8	1	8	28	46	53	46	28	8	1	
9	1	9	36	64	81	81	64	36	9	1

Table 2. Rascal triangle R2. Sequence A374378 in OEIS [4].

Example 1.2. Rascal triangle R3 generated by $\binom{n}{k}_3$

n/k	0	1	2	3	4	5	6	7	8	9
0	1									
1	1	1								
2	1	2	1							
3	1	3	3	1						
4	1	4	6	4	1					
5	1	5	10	10	5	1				
6	1	6	15	20	15	6	1			
7	1	7	21	35	35	21	7	1		
8	1	8	28	56	69	56	28	8	1	
9	1	9	36	84	121	121	84	36	9	1

Table 3. Rascal triangle R3. Sequence A374452 in OEIS [5].

Since then, a lot of work has been done over the topic of rascal triangles. Numerous identities and relations have been revealed. For instance, a few combinatorial interpretations of rascal numbers provided at [6], in particular, these interpretations establish a relation between rascal numbers and combinatorics of binary words. Several generalization approaches were proposed, namely generalized and iterated rascal triangles [7, 8]. In particular, the

concept of iterated rascal numbers establishes a close connection between rascal numbers and binomial coefficients.

2. Binomial identities in Iterated Rascal Triangles

Prior we begin our discussion it is worth to introduce a few preliminary facts and statements. Define the iterated rascal number [3, eq. 3.2]

Definition 2.1. *Iterated rascal number*

$$\binom{n}{k}_i = \sum_{m=0}^{i} \binom{n-k}{m}\binom{k}{m} \tag{2.1}$$

The first important thing to notice is that the iterated Rascal number is a special case of the Vandermonde convolution. Consider the Vandermonde convolution [9]

$$\binom{a+b}{r} = \sum_{m=0}^{r} \binom{a}{m}\binom{b}{r-m}$$

Implies

$$\binom{n}{k} = \sum_{m=0}^{k} \binom{n-k}{m}\binom{k}{k-m}$$

Thus,

$$\binom{n}{k}_i = \sum_{m=0}^{i} \binom{n-k}{m}\binom{k}{k-m} \tag{2.2}$$

Meaning that iterated rascal number is partial case of Vandermonde convolution of $\binom{n}{k}$ with the upper summation bound equals to i. Without further hesitation consider our findings.

Proposition 2.2. *Iterated rascal triangle equals to Pascal's triangle up to i-th column. For every $k \leq i$*

$$\binom{n}{k}_i = \binom{n}{k} \tag{2.3}$$

Then binomial identity follows

$$\binom{n}{i-k}_i = \binom{n}{i-k}$$

Applying binomial coefficients symmetry principle we obtain

$$\binom{n}{n-i+k}_i = \binom{n}{n-i+k}$$

Proof. Proof of proposition (2.2) is given by [8, proposition 6.0.1]. □

Proposition 2.3. *Iterated rascal triangle equals to Pascal's triangle up to $2i+1$-th row. For every $n \leq 2i+1$*

$$\binom{n}{k}_i = \binom{n}{k}$$

For example, we notice that

- Triangle R1 generated by $\binom{n}{k}_1$ is equivalent to Pascal's triangle up to 3-rd row, see (1).
- Triangle R2 generated by $\binom{n}{k}_2$ is equivalent to Pascal's triangle up to 5-th row, see (2).
- Triangle R3 generated by $\binom{n}{k}_3$ is equivalent to Pascal's triangle up to 7-rd row, see (3).

Therefore, for every $i \geq 0$ and $n \geq 0$

$$\binom{2i+1-n}{k}_i = \binom{2i+1-n}{k} \tag{2.4}$$

Equation (2.4) is of interest because in contrast to rascal column identity (2.3) it gives relation over k for each i, so that it is true for all cases in i, k: $i < k$, $i = k$ and $k > i$. In particular, equation (2.4) implies the row sums identity in iterated rascal triangles

$$\sum_{k=0}^{\infty} \binom{2i+1-n}{k}_i = 2^{2i+1-n}$$

Given $n = 0$ we obtain

$$\sum_{k=0}^{\infty} \binom{2i+1}{k}_i = 2^{2i+1}$$

and so on. Taking $t = 2i + 1$ in (2.4) yields

$$\binom{t-n}{k}_{t-i-1} = \binom{t-n}{k}$$

Moreover, equation (2.4) gives Vandermonde-like identity

$$\binom{2i+1-n}{k} = \sum_{m=0}^{i} \binom{2i+1-n-k}{m}\binom{k}{m} \tag{2.5}$$

In particular, given $n = 0, 1$ equation (2.5) yields the following Vandermonde-like identities

$$\binom{2i+1}{k} = \sum_{m=0}^{i} \binom{2i+1-k}{m}\binom{k}{m}$$

$$\binom{2i}{k} = \sum_{m=0}^{i} \binom{2i-k}{m}\binom{k}{m}$$

Proof. Proof of proposition (2.3). We have to prove that for every i, k

$$\sum_{m=0}^{k}\binom{2i+1-n-k}{m}\binom{k}{m} - \sum_{m=0}^{i}\binom{2i+1-n-k}{m}\binom{k}{m} = 0$$

For the case $k < i$ proof is given by [8, proposition 6.0.1]. For the case $k = i$ proof is trivial. Thus, the remaining case is $k > i$ yields

$$\sum_{m=i+1}^{k}\binom{2i+1-n-k}{m}\binom{k}{m} = 0 \qquad (2.6)$$

If (2.6) is true for each $k > i$, then its sum over k should be zero as well. Introducing sum in k to (2.6) we get

$$\sum_{k}\sum_{m=i+1}^{k}\binom{2i+1-n-k}{m}\binom{k}{m} = 0$$

The sum $\sum_{k}\binom{2i+1-n-k}{m}\binom{k}{m}$ appears to match the equation (5.26) in Concrete mathematics [10, eq. 5.26]

$$\sum_{k=0}^{\ell}\binom{\ell-k}{m}\binom{q+k}{n} = \binom{\ell+q+1}{m+n+1} \qquad (2.7)$$

Therefore,

$$\sum_{k}\binom{2i+1-n-k}{m}\binom{k}{m} = \binom{2i+2-n}{2m+1}$$

Thus, our main assumption is equivalent to

$$\sum_{m=i+1}^{k}\sum_{k}\binom{2i+1-n-k}{m}\binom{k}{m} \equiv \sum_{m=i+1}^{k}\binom{2i+2-n}{2m+1}$$

Hence, we have to prove that

$$\sum_{m=i+1}^{k}\binom{2i+2-n}{2m+1} = 0 \qquad (2.8)$$

Substituting $m = i + 1 + m$ into (2.8), we get

$$\sum_{m=0}^{k} \binom{2i+2-n}{2(i+1+m)+1} = \sum_{m=0}^{k} \binom{2i+2-n}{2i+3+2m} = 0$$

Which is indeed true because $\binom{2i+2-n}{2i+3+2m} = 0$ for every $m, n \geq 0$. Thus, the proposition (2.4) is true. □

Now, let's smoothly switch our focus to finite differences of binomial coefficients and iterated rascal numbers. Considering the table of differences $\binom{n}{k} - \binom{n}{k}_3$

In[67]:= `Grid[Table[Binomial[n, k] - RascalNumber[n, k, 3], {n, 0, 20}, {k, 0, n}], Frame → All]`

Out[67]=

k=4 ... A000332 ... n=8

k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18	k=19	k=20
0																				
0	0																			
0	0	0																		
0	0	0	0																	
0	0	0	0	0																
0	0	0	0	0	0															
0	0	0	0	0	0	0														
0	0	0	0	0	0	0	0													
0	0	0	0	1	0	0	0	0												
0	0	0	0	5	5	0	0	0	0											
0	0	0	0	15	26	15	0	0	0	0										
0	0	0	0	35	81	81	35	0	0	0	0									
0	0	0	0	70	196	262	196	70	0	0	0	0								
0	0	0	0	126	406	658	658	406	126	0	0	0	0							
0	0	0	0	210	756	1414	1716	1414	756	210	0	0	0	0						
0	0	0	0	330	1302	2730	3830	3830	2730	1302	330	0	0	0	0					
0	0	0	0	495	2112	4872	7680	8885	7680	4872	2112	495	0	0	0	0				
0	0	0	0	715	3267	8184	14232	18525	18525	14232	8184	3267	715	0	0	0	0			
0	0	0	0	1001	4862	13101	24816	35697	40186	35697	24816	13101	4862	1001	0	0	0	0		
0	0	0	0	1365	7007	20163	41217	64713	80587	80587	64713	41217	20163	7007	1365	0	0	0	0	
0	0	0	0	1820	9828	30030	65780	111705	152020	168230	152020	111705	65780	30030	9828	1820	0	0	0	0

Figure 1. Difference $\binom{n}{k} - \binom{n}{k}_3$. Highlighted column is $\binom{n}{4}$. Sequence A000332 in the OEIS [11].

We can spot that having $i = 3$ the $k = 4$-th column gives binomial coefficient $\binom{n}{4}$. Indeed, this rule is true for every i.

Proposition 2.4. *(Row-column difference.) For every $i \geq 0$*

$$\binom{n+2i}{i} - \binom{n+2i}{i}_{i-1} = \binom{n+i}{i}$$

Proof. We have previously stated that iterated rascal numbers are closely related to Vandermonde convolution (2.2). Thus, proposition (2.4) can be rewritten as

$$\sum_{m=0}^{i} \binom{n+i}{m}\binom{i}{i-m} - \sum_{m=0}^{i-1} \binom{n+i}{m}\binom{i}{i-m} = \binom{n+i}{i}\binom{i}{0}$$

Therefore, $\binom{n+2i}{i} - \binom{n+2i}{i}_{i-1} = \binom{n+i}{i}$ is indeed true. $\square$

Proposition (2.4) yields to few more identities. Applying binomial coefficients symmetry

$$\binom{n+2i}{n+i} - \binom{n+2i}{n+i}_{i-1} = \binom{n+i}{n}$$

Taking $j = n + i$ gives

$$\binom{j+i}{j} - \binom{j+i}{j}_{i-1} = \binom{j}{j-i}$$

By symmetry

$$\binom{j+i}{i} - \binom{j+i}{i}_{i-1} = \binom{j}{i}$$

Proposition (2.4) can be generalized even further, for every $i < k$ and $i > k$.

Proposition 2.5. *(Finite difference of binomial coefficients and iterated rascal numbers for $i < k$.) For every $i < k$*

$$\binom{n}{k} - \binom{n}{k}_i = \sum_{m=i+1}^{k} \binom{n-k}{m}\binom{k}{k-m}$$

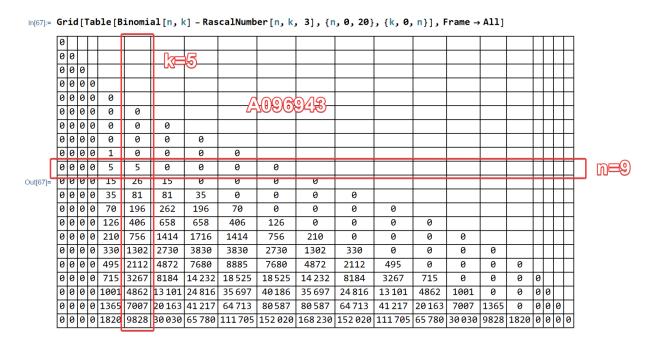*Proof.* It is true by means of Vandermonde convolution. $\square$

Proposition 2.6. *(Finite difference of binomial coefficients and iterated rascal numbers for $i > k$.) For every $i > k$*

$$\binom{n}{k} - \binom{n}{k}_i = \sum_{m=k+1}^{i} \binom{n-k}{m}\binom{k}{k-m}$$

Proof. It is true by means of Vandermonde convolution. $\square$

3. Q-Binomial identities in Iterated Rascal Triangles

Consider the table of differences of binomial coefficients and iterated rascal numbers one more time as there is another pattern we can spot.

```
In[67]:= Grid[Table[Binomial[n, k] - RascalNumber[n, k, 3], {n, 0, 20}, {k, 0, n}], Frame → All]
```

(k=5 column highlighted; A096943; row n=9 highlighted)

```
Out[67]=
0
0 0
0 0 0
0 0 0 0
0 0 0 0    0
0 0 0 0    0     0
0 0 0 0    0     0     0
0 0 0 0    0     0     0     0
0 0 0 0    1     0     0     0     0
0 0 0 0    5     5     0     0     0     0
0 0 0 0   15    26    15     0     0     0     0
0 0 0 0   35    81    81    35     0     0     0     0
0 0 0 0   70   196   262   196    70     0     0     0     0
0 0 0 0  126   406   658   658   406   126     0     0     0     0
0 0 0 0  210   756  1414  1716  1414   756   210     0      0      0     0
0 0 0 0  330  1302  2730  3830  3830  2730  1302   330      0      0     0     0
0 0 0 0  495  2112  4872  7680  8885  7680  4872  2112    495      0     0     0    0
0 0 0 0  715  3267  8184 14232 18525 18525 14232  8184   3267    715     0     0    0   0
0 0 0 0 1001  4862 13101 24816 35697 40186 35697 24816  13101   4862  1001     0    0   0 0
0 0 0 0 1365  7007 20163 41217 64713 80587 80587 64713  41217  20163  7007  1365    0   0 0 0
0 0 0 0 1820  9828 30030 65780 111705 152020 168230 152020 111705 65780 30030 9828 1820 0 0 0 0
```

Figure 2. Difference $\binom{n}{k} - \binom{n}{k}_3$. Highlighted column is $(1,5)$-binomial coefficient $\begin{bmatrix} n \\ k \end{bmatrix}^5$. Sequence A096943 in the OEIS [12].

The $(1,q)$-binomial coefficients $\begin{bmatrix} n \\ k \end{bmatrix}^q$ are special kind of binomial coefficients defined by

Definition 3.1. $(1,q)$-*Binomial coefficient*

$$
\begin{bmatrix} n \\ k \end{bmatrix}^q = \begin{cases} q & \text{if } k = 0, n = 0 \\ 1 & \text{if } k = 0 \\ 0 & \text{if } k > n \\ \begin{bmatrix} n-1 \\ k \end{bmatrix}^q + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}^q \end{cases}
\tag{3.1}
$$

Indeed, the relation shown in Figure (2) is true for every i, so that it establishes a relation between $(1,q)$-binomial coefficients and iterated rascal numbers.

Proposition 3.2. *(Relation between iterated rascal numbers and $(1, q)$-binomial coefficients.)*
For every $i \geq 0$

$$\binom{2i+3+j}{i+2} - \binom{2i+3+j}{i+2}_i = \left[\begin{matrix} i+2+j \\ i+2 \end{matrix}\right]^{i+2}$$

Taking $t = i + 2$ in (3.2) yields

$$\binom{2t-1+j}{t} - \binom{2t-1+j}{t}_{t-2} = \left[\begin{matrix} t+j \\ t \end{matrix}\right]^{t}$$

In particular,

- Having $i = 1$ proposition (3.2) gives the OEIS sequence A006503 [13] such that third column of $(1, 3)$-Pascal triangle A095660 [14].

- Having $i = 3$ proposition (3.2) gives the OEIS sequence A096943 [12] such that third column of $(1, 5)$-Pascal triangle A096940 [15].

- Having $i = 5$, the proposition (3.2) yields the OEIS sequence A097297 [16] such that seventh column of $(1, 6)$-Pascal triangle A096956 [17].

4. Row sums conjecture

In [8] the authors propose the following conjecture for row sums of iterated rascal triangles.

Conjecture 4.1. *(Conjecture 7.5 in [8].) For every i*

$$\sum_{k=0}^{4i+3} \binom{4i+3}{k}_i = 2^{4i+2}$$

Proof. Rewrite conjecture statement explicitly as

$$\sum_{k=0}^{4i+3} \sum_{m=0}^{i} \binom{4i+3-k}{m}\binom{k}{m} = 2^{4i+2}$$

Rearranging sums and omitting summation bounds yields

$$\sum_{m=0}^{i} \sum_{k} \binom{4i+3-k}{m}\binom{k}{m} = 2^{4i+2} \qquad (4.1)$$

We can observe the pattern (2.7) in equation (4.1). Thus, the sum $\sum_k \binom{4i+3-k}{m}\binom{k}{m}$ equals to

$$\sum_k \binom{4i+3-k}{m}\binom{k}{m} = \binom{4i+4}{2m+1}$$

Therefore, conjecture (4.1) is equivalent to

$$\sum_{m=0}^{i} \binom{4i+4}{2m+1} = 2^{4i+2}$$

Note that

$$\sum_{m=0}^{2i+1} \binom{4i+4}{2m+1} = \sum_{m=0}^{2i+1} \left[\binom{4i+3}{2m+1} + \binom{4i+3}{2m} \right] = 2^{4i+3}$$

So that

$$\frac{1}{2} \sum_{m=0}^{2i+1} \binom{4i+4}{2m+1} = \sum_{m=0}^{i} \binom{4i+4}{2m+1} = 2^{4i+2}$$

This completes the proof. □

Proposition 4.2. *For every* i

$$\sum_{k=0}^{4i+3} \binom{4i+3}{k}_i = 2^{4i+2}$$

In particular, equation (2.7) assumes the following identity in row sums of iterated rascal triangles

$$\sum_{k=0}^{n} \binom{n}{k}_i = \sum_{m=0}^{i} \binom{n+1}{2m+1}$$

Decomposing $\binom{n+1}{2m+1}$ in above equation yields

Proposition 4.3. *(Iterated rascal triangles row sums.) For every* i

$$\sum_{k=0}^{n} \binom{n}{k}_i = \sum_{m=0}^{2i+1} \binom{n}{m}$$

Proof.

$$\sum_{k=0}^{n} \binom{n}{k}_i = \sum_{m=0}^{i} \binom{n+1}{2m+1} = \sum_{m=0}^{i} \left[\binom{n}{2m} + \binom{n}{2m+1} \right] = \sum_{m=0}^{2i+1} \binom{n}{m}$$

□

5. Conclusions

In this manuscript we have discussed new binomial identities in iterated rascal triangles (2.4), (2.4), (2.5), revealing a connection between the Vandermonde convolution formula and iterated rascal numbers. We also present Vandermonde-like binomial identities (2.5). Furthermore, we establish a relation between iterated rascal triangles and $(1, q)$-binomial coefficients (3.2). All the results can be validated using supplementary Mathematica scripts at [18].

6. Acknowledgements

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Version: Local-0.1.0

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