POLYNOMIAL IDENTITIES INVOLVING RASCAL TRIANGLE

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Abstract. Abstract

1. Definitions

Definition of generalized Rascal triangle

$$\binom{n}{k}_i = \sum_{m=0}^{i} \binom{n-k}{m} \binom{k}{m} \tag{1.1}$$

Definition of $(1, q)$-Pascal triangle

$$\begin{bmatrix} n \\ k \end{bmatrix}^q = \begin{cases} q & \text{if } k = 0, n = 0 \\ 1 & \text{if } k = 0 \\ 0 & \text{if } k > n \\ \begin{bmatrix} n-1 \\ k \end{bmatrix}^q + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}^q & \end{cases}$$

2. Sides of world

$$\begin{aligned} \textbf{North} &= \binom{n-2}{k-1}_i \\ \textbf{South} &= \binom{n}{k}_i \\ \textbf{West} &= \binom{n-1}{k-1}_i \\ \textbf{East} &= \binom{n-1}{k}_i \end{aligned}$$

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Key words and phrases. Keyword1, Keyword2 .

Identity see Hotchkiss

$$\mathbf{South} = \frac{\mathbf{East} \cdot \mathbf{West} + 1}{\mathbf{North}}$$
$$\binom{n}{k}_i = \frac{\binom{n-1}{k}_i \binom{n-1}{k-1}_i + 1}{\binom{n-2}{k-1}_i}$$

Identity see Hotchkiss, for all inner $k > 0$ and $k < n$

$$\mathbf{South} = \mathbf{East} + \mathbf{West} - \mathbf{North} + 1$$
$$\binom{n}{k}_i = \binom{n-1}{k}_i + \binom{n-1}{k-1}_i - \binom{n-2}{k-1}_i + 1$$

3. Formulae

Claim 1

$$\binom{n}{k}_i = \binom{n}{k}, \quad 0 \leq k \leq i \tag{3.1}$$

Claim 2

$$\binom{n}{k}_i = \binom{n}{k}, \quad 0 \leq n \leq 2i + 1 \tag{3.2}$$

Claim 3

$$\binom{j}{k} - \binom{j}{k}_i = \binom{n}{i+1}, \quad j \geq 2i + 2, k = i + 1 \tag{3.3}$$

$$\binom{2i+j+2}{i+1} - \binom{2i+j+2}{i+1}_i = \binom{i+j+1}{i+1} \tag{3.4}$$

$$\binom{2(i+1)+j}{i+1} - \binom{2(i+1)+j}{i+1}_i = \binom{(i+1)+j}{i+1} \tag{3.5}$$

$$\binom{2t+j}{t} - \binom{2t+j}{t}_{t-1} = \binom{t+j}{t} \tag{3.6}$$

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