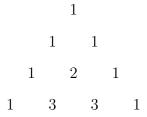# IDENTITIES IN ITERATED RASCAL TRIANGLES

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ABSTRACT. In this manuscript we show new binomial identities in iterated rascal triangles. In particular, iterated rascal numbers are closely related to $(1, q)$-binomial coefficients. Finally, we state an open conjecture about the relation between iterated rascal numbers and (p, q)-binomial coefficients.

CONTENTS

1. INTRODUCTION

In 2010, three middle school students, Alif Anggoro, Eddy Liu, and Angus Tulloch [1], were challenged to provide the next row for the number triangle shown below:

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1
\end{array}
$$

While the expected answer was "1 4 6 4 1" Anggoro, Liu, and Tulloch suggested "1 4 5 4 1" instead. They devised this new row via so-called diamond formula:

$$\textbf{South} = \frac{\textbf{East} \cdot \textbf{West} + 1}{\textbf{North}}$$

So that upcoming rows of the triangle are

n/k	0	1	2	3	4	5	6	7
0	1							
1	1	1						
2	1	2	1					
3	1	3	3	1				
4	1	4	5	4	1			
5	1	5	7	7	5	1		
6	1	6	9	10	9	6	1	
7	1	7	11	13	13	11	7	1

Table 1. Rascal triangle. See the OEIS sequence [2].

Since then, a lot of work has been done over the topic of rascal triangles. Numerous identities and relations have been revealed. For instance, few combinatorial interpretations of rascal numbers provided at [3], in particular, these interpretations establish a relation between rascal numbers and combinatorics of binary words. Few generalization approaches were proposed, namely generalized and iterated rascal triangles [4, 5].

2. Conclusions

Conclusions of your manuscript.

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