

POLYNOMIAL IDENTITIES INVOLVING RASCAL TRIANGLE

PETRO KOLOSOV

ABSTRACT. Abstract

1. DEFINITIONS

Definition of generalized Rascal triangle

$$\binom{n}{k}_i = \sum_{m=0}^i \binom{n-k}{m} \binom{k}{m} \quad (1.1)$$

Definition of $(1, q)$ -Pascal triangle

$$\begin{bmatrix} n \\ k \end{bmatrix}^q = \begin{cases} q & \text{if } k = 0, n = 0 \\ 1 & \text{if } k = 0 \\ 0 & \text{if } k > n \\ \begin{bmatrix} n-1 \\ k \end{bmatrix}^q + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}^q & \end{cases}$$

2. SIDES OF WORLD

$$\mathbf{North} = \binom{n-2}{k-1}_i$$

$$\mathbf{South} = \binom{n}{k}_i$$

$$\mathbf{West} = \binom{n-1}{k-1}_i$$

$$\mathbf{East} = \binom{n-1}{k}_i$$

Date: June 29, 2024.

2010 Mathematics Subject Classification. 26E70, 05A30.

Key words and phrases. Keyword1, Keyword2 .

Identity see Hotchkiss

$$\mathbf{South} = \frac{\mathbf{East} \cdot \mathbf{West} + 1}{\mathbf{North}}$$

$$\binom{n}{k}_i = \frac{\binom{n-1}{k}_i \binom{n-1}{k-1}_i + 1}{\binom{n-2}{k-1}_i}$$

Identity see Hotchkiss, for all inner $k > 0$ and $k < n$

$$\mathbf{South} = \mathbf{East} + \mathbf{West} - \mathbf{North} + 1$$

$$\binom{n}{k}_i = \binom{n-1}{k}_i + \binom{n-1}{k-1}_i - \binom{n-2}{k-1}_i + 1$$

3. FORMULAE

3.1. **Claim 1.** Generalized rascal triangle equals to Pascal's triangle up to i -th column

$$\binom{n}{k}_i = \binom{n}{k}, \quad 0 \leq k \leq i \quad (3.1)$$

3.2. **Claim 2.** Generalized rascal triangle equals to Pascal's triangle up to $2i + 1$ -th row

$$\binom{n}{k}_i = \binom{n}{k}, \quad 0 \leq n \leq 2i + 1 \quad (3.2)$$

3.3. **Claim 3.**

$$\binom{j}{k} - \binom{j}{k}_i = \binom{n}{i+1}, \quad j \geq 2i + 2, k = i + 1 \quad (3.3)$$

$$\binom{2i+j+2}{i+1} - \binom{2i+j+2}{i+1}_i = \binom{i+j+1}{i+1} \quad (3.4)$$

$$\binom{2(i+1)+j}{i+1} - \binom{2(i+1)+j}{i+1}_i = \binom{(i+1)+j}{i+1} \quad (3.5)$$

$$\binom{2t+j}{t} - \binom{2t+j}{t}_{t-1} = \binom{t+j}{t} \quad (3.6)$$

3.4. **Claim 4.**

$$\begin{aligned}
\binom{2i+3+j}{i+2} - \binom{2i+3+j}{i+2}_i &= \begin{bmatrix} i+2+j \\ i+2 \end{bmatrix}^{i+2} \\
\binom{2(i+2)-1+j}{i+2} - \binom{2(i+2)-1+j}{i+2}_i &= \begin{bmatrix} i+2+j \\ i+2 \end{bmatrix}^{i+2} \\
\binom{2t-1+j}{t} - \binom{2t-1+j}{t}_{t-2} &= \begin{bmatrix} t+j \\ t \end{bmatrix}^t
\end{aligned}$$

SOFTWARE DEVELOPER, DEVOPS ENGINEER

Email address: kolosovp94@gmail.com

URL: <https://kolosovpetro.github.io>