LATEX TEMPLATE FOR GITHUB

PETRO KOLOSOV

1. Definitions

amsmathGoldbach conjecture states that every even integer N greater than 2 is a sum of two primes.

$$N = p_i + p_j$$

where (p_i, p_j) is called Goldbach pair.

Goldbach pair is not unique for some even integers, meaning that there can be multiple goldbach pairs for even integer N.

For example: $10 = 3 + 7$ and $10 = 5 + 5$ and $10 = 7 + 3$ where goldbach pairs are $(3, 7), (5, 5), (7, 3)$.

Minimal goldbach pair is the pair having minimal p_i across all goldbach pairs for even integer N.

For even integer 10 we have three pairs $(3, 7), (5, 5), (7, 3)$ while the minimal is $(3, 7)$ because 3 is the minimal value in the p_i set: $3, 5, 7$

1.1. **Function F.** $F_n(P)$ counts the number of minimal goldbach pairs (p_i, p_j) such that $p_i = P$ within the interval $6 \leq k \leq n$, where P is a prime. For example, consider the case $F_{20}(3)$. First, we get a set of minimal goldbach pairs within the range $6 \leq k \leq 20$, that is

Date: April 17, 2025.

$$6 = 3 + 3,$$

$$8 = 3 + 5,$$

$$10 = 3 + 7,$$

$$12 = 5 + 7,$$

$$14 = 3 + 11,$$

$$16 = 3 + 13,$$

$$18 = 5 + 13,$$

$$20 = 3 + 17$$

Therefore, the function $F_{20}(3)$ gives 6 because there are only six minimal goldbach pairs (p_i, p_j) such that $p_i = 3$, that are:

$$6 = 3 + 3,$$

$$8 = 3 + 5,$$

$$10 = 3 + 7,$$

$$14 = 3 + 11,$$

$$16 = 3 + 13,$$

$$20 = 3 + 17$$

What is also interesting to notice, is that p_j in the example above produces the consecutive sequence of prime numbers, $p_j = 3, 5, 7, 11, 13, 17 \ldots$ In general, for every $n \geq 2$

$$\pi(n) = F_{n+3}(3) + 1$$

References