ODD-POWER IDENTITY VIA MULTIPLICATION OF CERTAIN MATRICES

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Abstract. This manuscript establishes an odd-power identity expressed through matrix multiplication. Specifically, we demonstrate that a 1×1 matrix with an entry $a_{1,1} = N^{2M+1}$ can be represented as the product of three matrices: $\mathbf{J}_N$, $\mathbf{K}_{N,M}$, and $\mathbf{T}_M$, as follows

$$\begin{bmatrix} N^{2M+1} \end{bmatrix} = \mathbf{J}_N \times \mathbf{K}_{N,M} \times \mathbf{T}_M$$

Here, $\mathbf{J}_N$ denotes a unit row vector of dimension $1 \times N$, $\mathbf{K}_{N,M}$ is an $N \times M$ matrix, and $\mathbf{T}_M$ represents a column vector of dimension $M \times 1$.

Contents

Definitions

- $\mathbf{J}_N$ – unit row vector of all 1's having the dimension $1 \times N$. For example,

$$\mathbf{J}_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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Sources: https://github.com/kolosovpetro/github-latex-template

- $\mathbf{K}_{N,M}$ – matrix of dimension $N \times M$ defined by

$$\mathbf{K}_{N,M} = (k^r (N-k)^r)_{0 \leq k \leq N,\, 0 \leq r \leq M}$$

For example, given $N = 4$

$$\mathbf{K}_{4,M} = \begin{bmatrix} 0^0(4-0)^0 & 0^1(4-0)^1 & 0^2(4-0)^2 & \cdots & 0^M(4-0)^M \\ 1^0(4-1)^0 & 1^1(4-1)^1 & 1^2(4-1)^2 & \cdots & 1^M(4-1)^M \\ 2^0(4-2)^0 & 2^1(4-2)^1 & 2^2(4-2)^2 & \cdots & 2^M(4-2)^M \\ 3^0(4-3)^0 & 3^1(4-3)^1 & 3^2(4-3)^2 & \cdots & 3^M(4-3)^M \\ 4^0(4-4)^0 & 4^1(4-4)^1 & 4^2(4-4)^2 & \cdots & 4^M(4-4)^M \end{bmatrix}$$

- $\mathbf{T}_M$ – column vector of dimension $M \times 1$ defined by

$$\mathbf{T}_M = (\mathbf{A}_{M,r})_{M=\text{const},\, 0 \leq r \leq M}$$

where $\mathbf{A}_{M,r}$ is a real coefficient [1, 2, 3, 4]. For example, given $M = 3$

$$\mathbf{T}_3 = \begin{bmatrix} 1 \\ -14 \\ 0 \\ 140 \end{bmatrix}$$

1. Main theorem

2. Conclusions

Conclusions of your manuscript.

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