ODD-POWER IDENTITY VIA MULTIPLICATION OF CERTAIN MATRICES

PETRO KOLOSOV

Abstract. In this manuscript, we show an odd-power identity in terms of certain matrix multiplication. More precisely, the matrix of size 1×1 such that $a_{1,1} = N^{2M+1}$ is a result of multiplication of three matrices $\mathbf{J}_N \times \mathbf{K}_{N,M} \times \mathbf{T}_M$, where $\mathbf{J}_N$ is unit row vector of size $1 \times N$, $\mathbf{K}_{N,M}$ is a matrix of size $N \times M$, and $\mathbf{T}_M$ is a column vector of size $M \times 1$, so that

$$\begin{bmatrix} N^{2M+1} \end{bmatrix} = \mathbf{J}_N \times \mathbf{K}_{N,M} \times \mathbf{T}_M$$

Contents

Definitions

- $\mathbf{J}_N$ – unit row vector of all 1's having the size $1 \times N$. For example,

$$\mathbf{J}_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- $\mathbf{K}_{N,M}$ – matrix of size $N \times M$ defined by

$$\mathbf{K}_{N,M} = (k^r (N-k)^r)_{0 \leq k \leq N,\, 0 \leq r \leq M}$$

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Key words and phrases. Keyword1, Keyword2 .

- $\mathbf{T}_M$ – column vector of size $M \times 1$ defined by

$$\mathbf{T}_M = (\mathbf{A}_{M,r})_{M=\text{const},\ 0\le r\le M}$$

  where $\mathbf{A}_{M,r}$ is rational value (literature).
- Row vector
- Column vector
- $\mathbf{A} = (a_{ij})$

1. Introduction

Your introduction here. Include some references [1, 2, 3, 4, 5, 6]. Lorem Ipsum is simply dummy text of the printing and typesetting industry. Lorem Ipsum has been the industry's standard dummy text ever since the 1500s, when an unknown printer took a galley of type and scrambled it to make a type specimen book. It has survived not only five centuries, but also the leap into electronic typesetting, remaining essentially unchanged. It was popularised in the 1960s with the release of Letraset sheets containing Lorem Ipsum passages, and more recently with desktop publishing software like Aldus PageMaker including versions of Lorem Ipsum.

Figure example

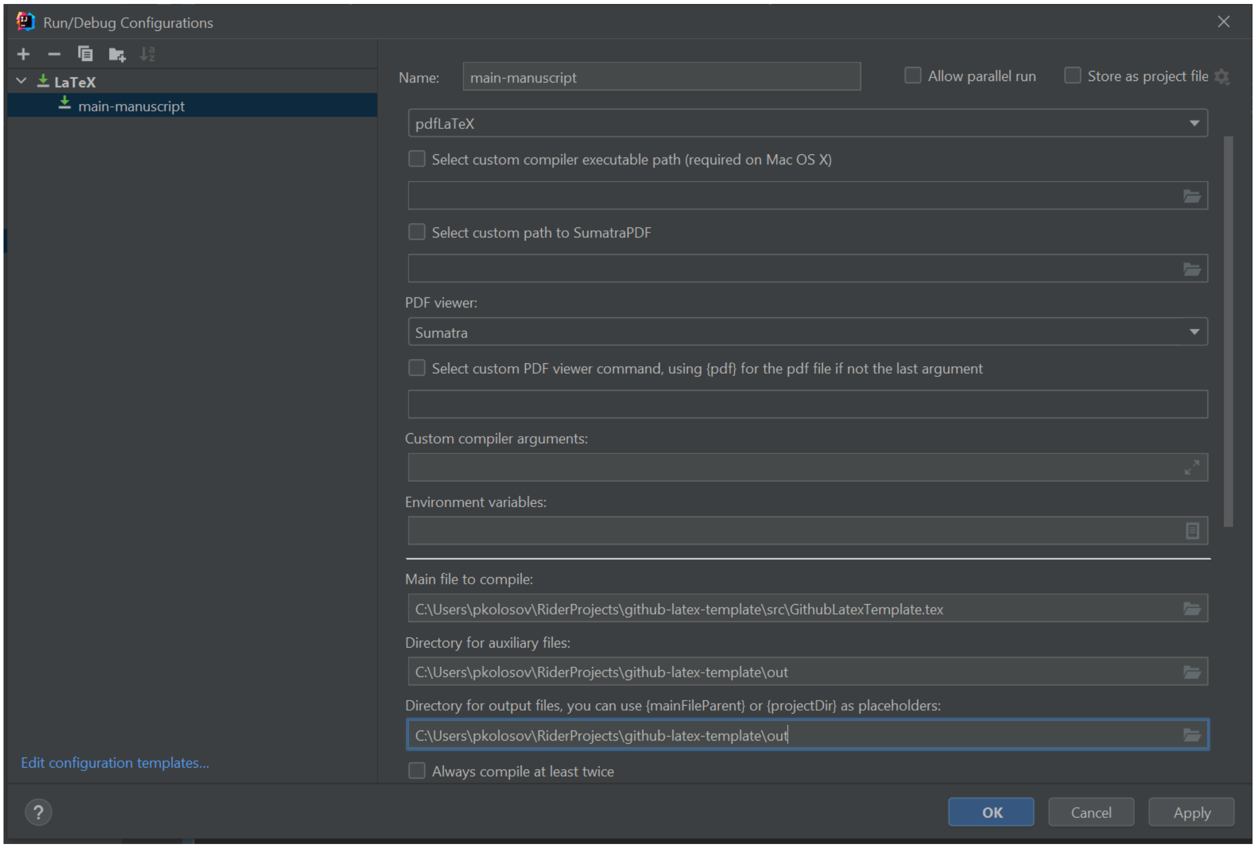

Figure 1. Figure example.

$$\left[\!\left[\begin{matrix} a \\ b \end{matrix}\right]\!\right]_m$$
$$\left[\!\left[\begin{matrix} a \\ b \end{matrix}\right]\!\right]_m$$

And for any natural m we have polynomial identity

$$x^m = \sum_{k=1}^{m} T(m,k) x^{[k]} \tag{1.1}$$

where $x^{[k]}$ denotes central factorial defined by

$$x^{[n]} = x \left(x + \frac{n}{2} - 1 \right)^{\underline{n-1}}$$

where $(n)^{\underline{k}} = n(n-1)(n-2)\cdots(n-k+1)$ denotes falling factorial in Knuth's notation. In particular,

$$x^{[n]} = x\left(x+\frac{n}{2}-1\right)\left(x+\frac{n}{2}-1\right)\cdots\left(x+\frac{n}{2}-n-1\right) = x\prod_{k=1}^{n-1}\left(x+\frac{n}{2}-k\right)$$

2. Conclusions

Conclusions of your manuscript.

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Software Developer, DevOps Engineer

Email address: kolosovp94@gmail.com

URL: https://kolosovpetro.github.io