PLOTS OF CLOSED FORMS

PETRO KOLOSOV

Contents

1. Introduction

$$P(m,X,N)=\sum_{r=0}^{m}\sum_{k=1}^{N}\mathbf{A}_{m,r}k^{r}(X-k)^{r}$$

$$Q(m,X,N)=\sum_{r=0}^{m}\sum_{k=0}^{N-1}\mathbf{A}_{m,r}k^{r}(X-k)^{r}$$

$$P(m,N,N) = N^{2m+1}$$

$$Q(m,N,N) = N^{2m+1}$$

$$P(m,N+1,N) = (N+1)^{2m+1} - 1 \qquad (verified)$$

$$Q(m,N-1,N) = (N-1)^{2m+1} + 1 \qquad (verified)$$

1.1. **Polynomials P(1,n,k).**

$$P(1, X, 0) = 0$$

$$P(1, X, 1) = 6X - 5$$

$$P(1, X, 2) = 18X - 28$$

$$P(1, X, 3) = 36X - 81$$

$$P(1, X, 4) = 60X - 176$$

$$P(1, X, 5) = 90X - 325$$

$$P(1, X, 6) = 126X - 540$$

$$P(1, X, 7) = 168X - 833$$

$$P(1, X, 8) = 216X - 1216$$

$$P(1, X, 9) = 270X - 1701$$

$$P(1, X, 10) = 330X - 2300$$

$$P(1, X, 11) = 396X - 3025$$

$$P(1, X, 12) = 468X - 3888$$

$$P(1, X, 13) = 546X - 4901$$

$$P(1, X, 14) = 630X - 6076$$

$$P(1, X, 15) = 720X - 7425$$

$$P(1, X, 16) = 816X - 8960$$

$$P(1, X, 17) = 918X - 10693$$

$$P(1, X, 18) = 1026X - 12636$$

$$P(1, X, 19) = 1140X - 14801$$

$$P(1, X, 20) = 1260X - 17200$$

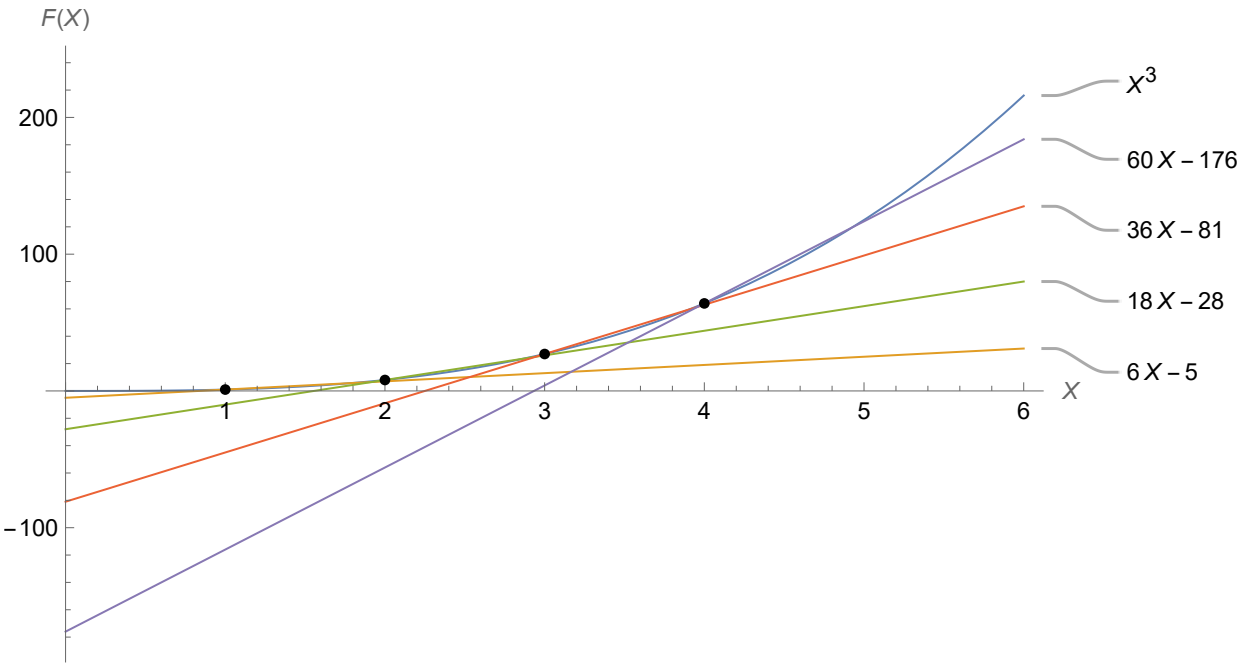

Figure 1. Polynomials P(1, n, k)

1.2. **Polynomials P(1,n,k) example n = 6.**

1.3. **Polynomials Q(1,n,k).**

$$Q(1, X, 0) = 0$$

$$Q(1, X, 1) = 1$$

$$Q(1, X, 2) = 6X - 4$$

$$Q(1, X, 3) = 18X - 27$$

$$Q(1, X, 4) = 36X - 80$$

$$Q(1, X, 5) = 60X - 175$$

$$Q(1, X, 6) = 90X - 324$$

$$Q(1, X, 7) = 126X - 539$$

$$Q(1, X, 8) = 168X - 832$$

$$Q(1, X, 9) = 216X - 1215$$

$$Q(1, X, 10) = 270X - 1700$$

$$Q(1, X, 11) = 330X - 2299$$

$$Q(1, X, 12) = 396X - 3024$$

$$Q(1, X, 13) = 468X - 3887$$

$$Q(1, X, 14) = 546X - 4900$$

$$Q(1, X, 15) = 630X - 6075$$

$$Q(1, X, 16) = 720X - 7424$$

$$Q(1, X, 17) = 816X - 8959$$

$$Q(1, X, 18) = 918X - 10692$$

$$Q(1, X, 19) = 1026X - 12635$$

$$Q(1, X, 20) = 1140X - 14800$$

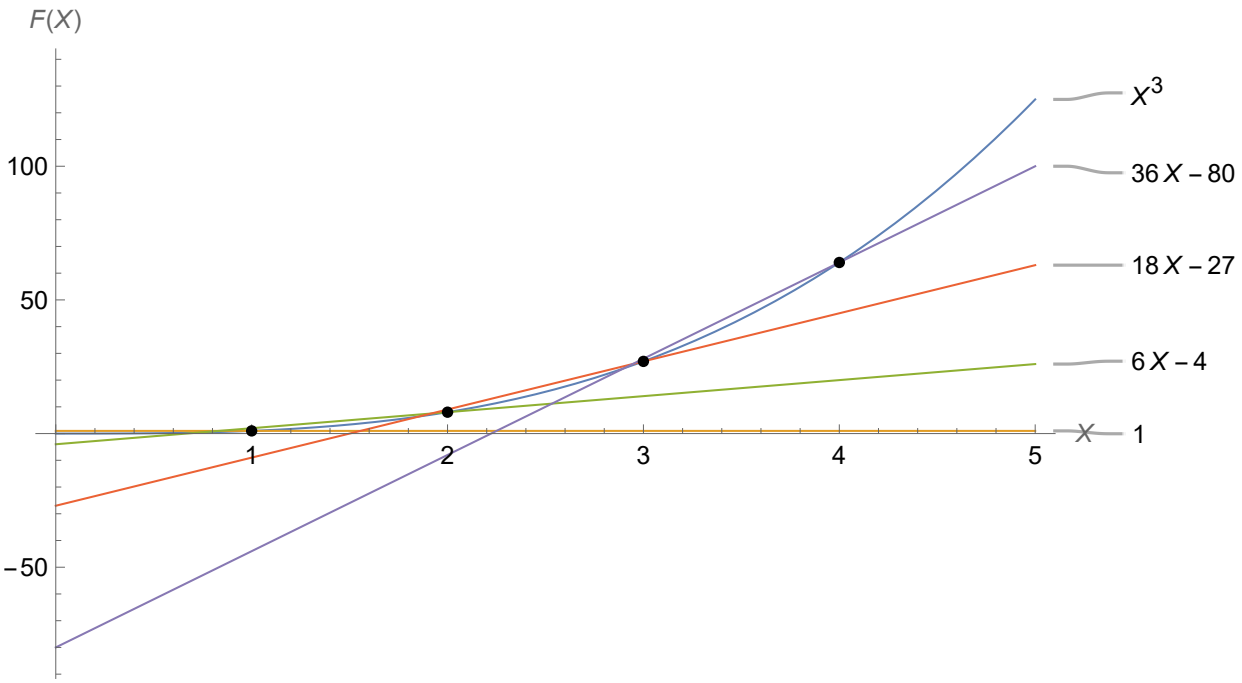

Figure 2. Polynomials Q(1, n, k)

1.4. **Polynomials P(2,n,k).**

$$P(2,X,0) = 0$$

$$P(2,X,1) = 30X^2 - 60X + 31$$

$$P(2,X,2) = 150X^2 - 540X + 512$$

$$P(2,X,3) = 420X^2 - 2160X + 2943$$

$$P(2,X,4) = 900X^2 - 6000X + 10624$$

$$P(2,X,5) = 1650X^2 - 13500X + 29375$$

$$P(2,X,6) = 2730X^2 - 26460X + 68256$$

$$P(2,X,7) = 4200X^2 - 47040X + 140287$$

$$P(2,X,8) = 6120X^2 - 77760X + 263168$$

$$P(2,X,9) = 8550X^2 - 121500X + 459999$$

$$P(2,X,10) = 11550X^2 - 181500X + 760000$$

$$P(2,X,11) = 15180X^2 - 261360X + 1199231$$

$$P(2,X,12) = 19500X^2 - 365040X + 1821312$$

$$P(2,X,13) = 24570X^2 - 496860X + 2678143$$

$$P(2,X,14) = 30450X^2 - 661500X + 3830624$$

$$P(2,X,15) = 37200X^2 - 864000X + 5349375$$

$$P(2,X,16) = 44880X^2 - 1109760X + 7315456$$

$$P(2,X,17) = 53550X^2 - 1404540X + 9821087$$

$$P(2,X,18) = 63270X^2 - 1754460X + 12970368$$

$$P(2,X,19) = 74100X^2 - 2166000X + 16879999$$

$$P(2,X,20) = 86100X^2 - 2646000X + 21680000$$

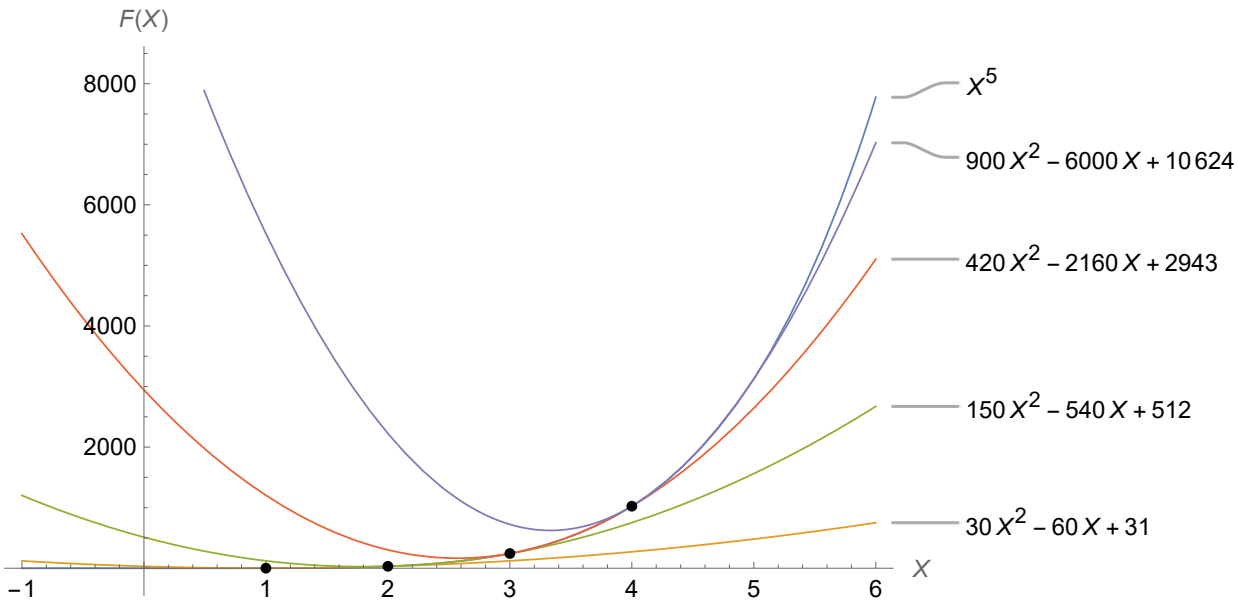

Figure 3. Polynomials P(2, n, k)

1.5. **Polynomials P(2,n,k) example n = 4.**

1.6. **Polynomials Q(2,n,k).**

$$Q(2,X,0) = 0$$

$$Q(2,X,1) = 1$$

$$Q(2,X,2) = 30X^2 - 60X + 32$$

$$Q(2,X,3) = 150X^2 - 540X + 513$$

$$Q(2,X,4) = 420X^2 - 2160X + 2944$$

$$Q(2,X,5) = 900X^2 - 6000X + 10625$$

$$Q(2,X,6) = 1650X^2 - 13500X + 29376$$

$$Q(2,X,7) = 2730X^2 - 26460X + 68257$$

$$Q(2,X,8) = 4200X^2 - 47040X + 140288$$

$$Q(2,X,9) = 6120X^2 - 77760X + 263169$$

$$Q(2,X,10) = 8550X^2 - 121500X + 460000$$

$$Q(2,X,11) = 11550X^2 - 181500X + 760001$$

$$Q(2,X,12) = 15180X^2 - 261360X + 1199232$$

$$Q(2,X,13) = 19500X^2 - 365040X + 1821313$$

$$Q(2,X,14) = 24570X^2 - 496860X + 2678144$$

$$Q(2,X,15) = 30450X^2 - 661500X + 3830625$$

$$Q(2,X,16) = 37200X^2 - 864000X + 5349376$$

$$Q(2,X,17) = 44880X^2 - 1109760X + 7315457$$

$$Q(2,X,18) = 53550X^2 - 1404540X + 9821088$$

$$Q(2,X,19) = 63270X^2 - 1754460X + 12970369$$

$$Q(2,X,20) = 74100X^2 - 2166000X + 16880000$$

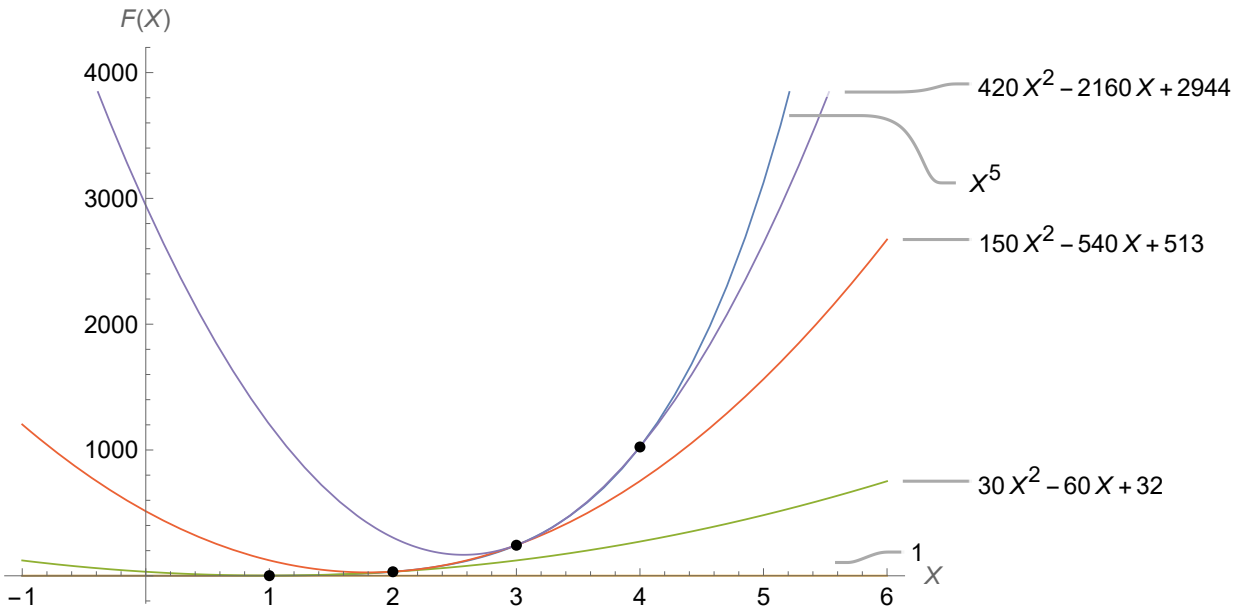

Figure 4. Polynomials Q(2, n, k)

1.7. **Polynomials P(3,n,k).**

$$P(3,X,0) = 0$$

$$P(3,X,1) = 140X^3 - 420X^2 + 406X - 125$$

$$P(3,X,2) = 1260X^3 - 7140X^2 + 13818X - 9028$$

$$P(3,X,3) = 5040X^3 - 41160X^2 + 115836X - 110961$$

$$P(3,X,4) = 14000X^3 - 148680X^2 + 545860X - 684176$$

$$P(3,X,5) = 31500X^3 - 411180X^2 + 1858290X - 2871325$$

$$P(3,X,6) = 61740X^3 - 955500X^2 + 5124126X - 9402660$$

$$P(3,X,7) = 109760X^3 - 1963920X^2 + 12182968X - 25872833$$

$$P(3,X,8) = 181440X^3 - 3684240X^2 + 25945416X - 62572096$$

$$P(3,X,9) = 283500X^3 - 6439860X^2 + 50745870X - 136972701$$

$$P(3,X,10) = 423500X^3 - 10639860X^2 + 92745730X - 276971300$$

$$P(3,X,11) = 609840X^3 - 16789080X^2 + 160386996X - 524988145$$

$$P(3,X,12) = 851760X^3 - 25498200X^2 + 264896268X - 943023888$$

$$P(3,X,13) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774781$$

$$P(3,X,14) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907076$$

$$P(3,X,15) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591425$$

$$P(3,X,16) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398080$$

$$P(3,X,17) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653693$$

$$P(3,X,18) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360516$$

$$P(3,X,19) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778801$$

$$P(3,X,20) = 6174000X^3 - 303519720X^2 + 5179983060X - 30303773200$$

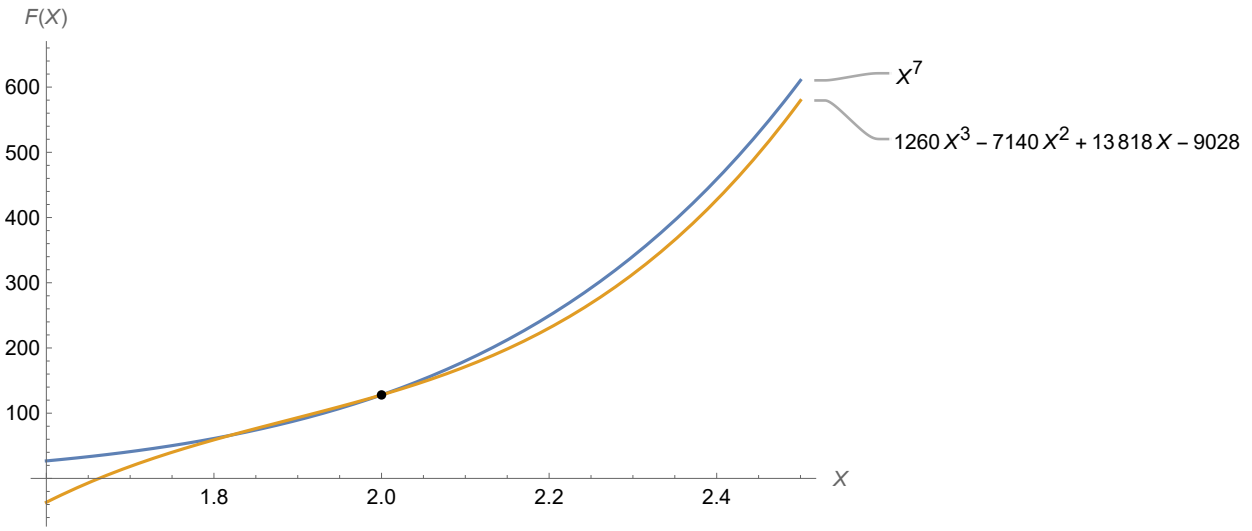

Figure 5. Polynomials P(3, n, k)

1.8. **Polynomials P(3,n,k) example n = 3.**

1.9. **Polynomials Q(3,n,k).**

$$Q(3,X,0) = 0$$

$$Q(3,X,1) = 1$$

$$Q(3,X,2) = 140X^3 - 420X^2 + 406X - 124$$

$$Q(3,X,3) = 1260X^3 - 7140X^2 + 13818X - 9027$$

$$Q(3,X,4) = 5040X^3 - 41160X^2 + 115836X - 110960$$

$$Q(3,X,5) = 14000X^3 - 148680X^2 + 545860X - 684175$$

$$Q(3,X,6) = 31500X^3 - 411180X^2 + 1858290X - 2871324$$

$$Q(3,X,7) = 61740X^3 - 955500X^2 + 5124126X - 9402659$$

$$Q(3,X,8) = 109760X^3 - 1963920X^2 + 12182968X - 25872832$$

$$Q(3,X,9) = 181440X^3 - 3684240X^2 + 25945416X - 62572095$$

$$Q(3,X,10) = 283500X^3 - 6439860X^2 + 50745870X - 136972700$$

$$Q(3,X,11) = 423500X^3 - 10639860X^2 + 92745730X - 276971299$$

$$Q(3,X,12) = 609840X^3 - 16789080X^2 + 160386996X - 524988144$$

$$Q(3,X,13) = 851760X^3 - 25498200X^2 + 264896268X - 943023887$$

$$Q(3,X,14) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774780$$

$$Q(3,X,15) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907075$$

$$Q(3,X,16) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591424$$

$$Q(3,X,17) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398079$$

$$Q(3,X,18) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653692$$

$$Q(3,X,19) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360515$$

$$Q(3,X,20) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778800$$

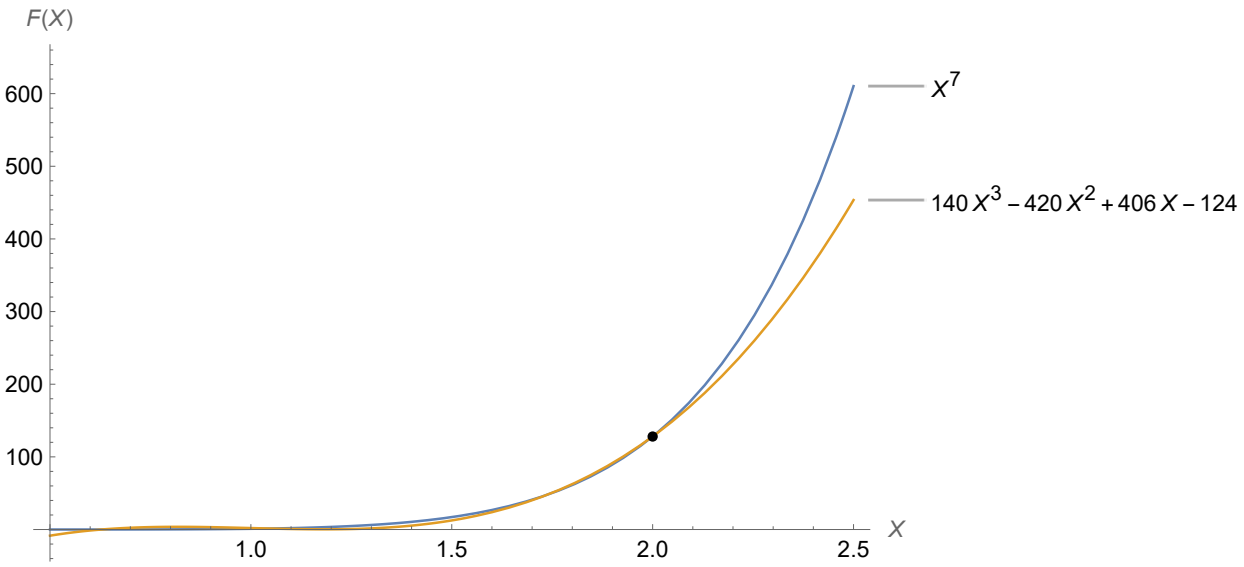

Figure 6. Polynomials Q(3, n, k)

Table 1. Values of X^3 and $126X - 540$ for selected X

X	X^7	$126X - 540$
5.0	125.000	90.000
5.1	132.651	102.600
5.2	140.608	115.200
5.3	148.877	127.800
5.4	157.464	140.400
5.5	166.375	153.000
5.6	175.616	165.600
5.7	185.193	178.200
5.8	195.112	190.800
5.9	205.379	203.400
6.0	216.000	216.000
6.1	226.981	228.600
6.2	238.328	241.200
6.3	250.047	253.800
6.4	262.144	266.400
6.5	274.625	279.000
6.6	287.496	291.600
6.7	300.763	304.200
6.8	314.432	316.800
6.9	328.509	329.400
7.0	343.000	342.000
7.1	357.911	354.600
7.2	373.248	367.200
7.3	389.017	379.800
7.4	405.224	392.400
7.5	421.875	405.000

Table 2. Values of X^5 and $900X^2 - 6000X + 10624$ for selected X

X	X^5	$900X^2 - 6000X + 10624$
3.0	243.000	724.000
3.1	286.292	673.000
3.2	335.544	640.000
3.3	391.354	625.000
3.4	454.354	628.000
3.5	525.219	649.000
3.6	604.662	688.000
3.7	693.440	745.000
3.8	792.352	820.000
3.9	902.242	913.000
4.0	1024.000	1024.000
4.1	1158.560	1153.000
4.2	1306.910	1300.000
4.3	1470.080	1465.000
4.4	1649.160	1648.000
4.5	1845.280	1849.000
4.6	2059.630	2068.000
4.7	2293.450	2305.000
4.8	2548.040	2560.000
4.9	2824.750	2833.000
5.0	3125.000	3124.000
5.1	3450.250	3433.000
5.2	3802.040	3760.000
5.3	4181.950	4105.000
5.4	4591.650	4468.000
5.5	5032.840	4849.000
5.6	5507.320	5248.000
5.7	6016.920	5665.000

Table 3. Values of X^7 and $5040X^3 - 41160X^2 + 115836X - 110961$ for selected X

X	X^7	$5040X^3 - 41160X^2 + 115836X - 110961$
2.0	128.000	-3609.000
2.1	180.109	-2545.560
2.2	249.436	-1670.280
2.3	340.483	-952.920
2.4	458.647	-363.240
2.5	610.352	129.000
2.6	803.181	554.040
2.7	1046.040	942.120
2.8	1349.290	1323.480
2.9	1724.990	1728.360
3.0	2187.000	2187.000
3.1	2751.260	2729.640
3.2	3435.970	3386.520
3.3	4261.840	4187.880
3.4	5252.340	5163.960
3.5	6433.930	6345.000
3.6	7836.420	7761.240
3.7	9493.190	9442.920
3.8	11441.600	11420.300
3.9	13723.100	13723.600
4.0	16384.000	16383.000
4.1	19475.400	19428.800
4.2	23053.900	22891.300
4.3	27181.900	26800.700
4.4	31927.800	31187.200
4.5	37366.900	36081.000
4.6	43581.800	41512.400
4.7	50662.300	47511.700