PLOTS OF CLOSED FORMS

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Contents

1. Introduction

$$P(m,X,N)=\sum_{r=0}^{m}\sum_{k=1}^{N}\mathbf{A}_{m,r}k^r(X-k)^r$$

$$Q(m,X,N)=\sum_{r=0}^{m}\sum_{k=0}^{N-1}\mathbf{A}_{m,r}k^r(X-k)^r$$

$$P(m,N,N) = N^{2m+1}$$

$$Q(m,N,N) = N^{2m+1}$$

$$P(m,N+1,N) = (N+1)^{2m+1} - 1 \qquad (verified)$$

$$Q(m,N-1,N) = (N-1)^{2m+1} + 1 \qquad (verified)$$

1.1. **Polynomials P(1,n,k).**

$$P(1, X, 0) = 0$$

$$P(1, X, 1) = 6X - 5$$

$$P(1, X, 2) = 18X - 28$$

$$P(1, X, 3) = 36X - 81$$

$$P(1, X, 4) = 60X - 176$$

$$P(1, X, 5) = 90X - 325$$

$$P(1, X, 6) = 126X - 540$$

$$P(1, X, 7) = 168X - 833$$

$$P(1, X, 8) = 216X - 1216$$

$$P(1, X, 9) = 270X - 1701$$

$$P(1, X, 10) = 330X - 2300$$

$$P(1, X, 11) = 396X - 3025$$

$$P(1, X, 12) = 468X - 3888$$

$$P(1, X, 13) = 546X - 4901$$

$$P(1, X, 14) = 630X - 6076$$

$$P(1, X, 15) = 720X - 7425$$

$$P(1, X, 16) = 816X - 8960$$

$$P(1, X, 17) = 918X - 10693$$

$$P(1, X, 18) = 1026X - 12636$$

$$P(1, X, 19) = 1140X - 14801$$

$$P(1, X, 20) = 1260X - 17200$$

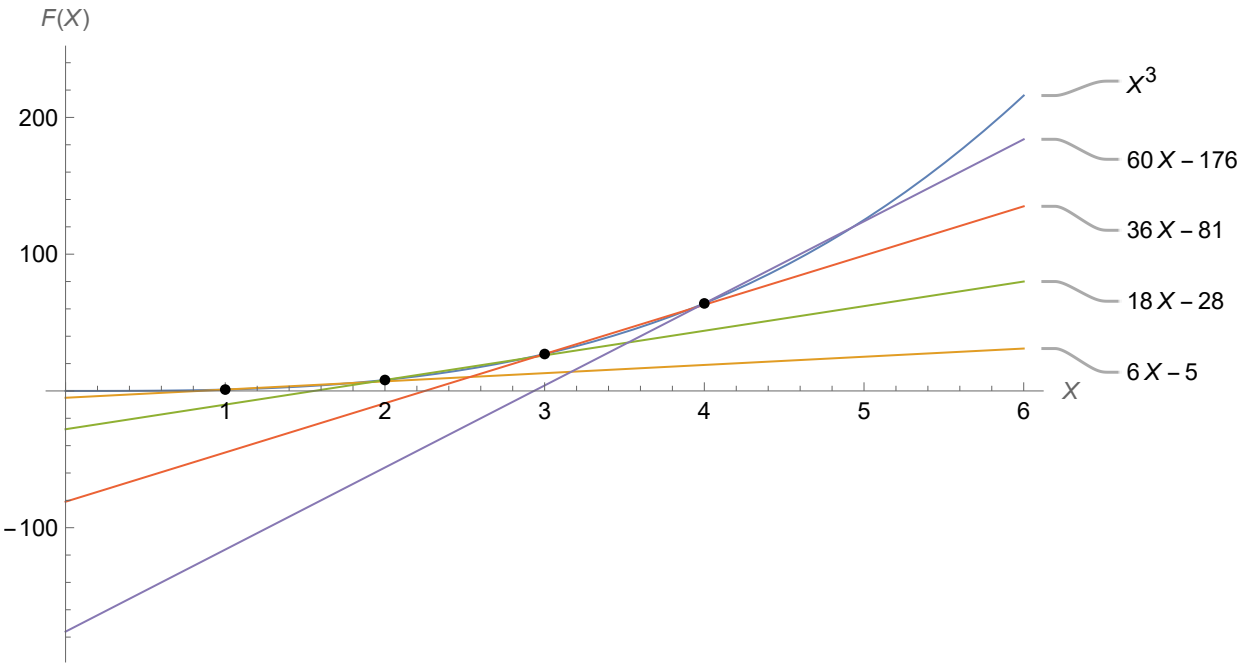

Figure 1. Polynomials P(1, n, k)

1.2. **Polynomial P(1,n,k) Table n = 6.**

1.3. **Polynomials Q(1,n,k).**

$$Q(1, X, 0) = 0$$

$$Q(1, X, 1) = 1$$

$$Q(1, X, 2) = 6X - 4$$

$$Q(1, X, 3) = 18X - 27$$

$$Q(1, X, 4) = 36X - 80$$

$$Q(1, X, 5) = 60X - 175$$

$$Q(1, X, 6) = 90X - 324$$

$$Q(1, X, 7) = 126X - 539$$

$$Q(1, X, 8) = 168X - 832$$

$$Q(1, X, 9) = 216X - 1215$$

$$Q(1, X, 10) = 270X - 1700$$

$$Q(1, X, 11) = 330X - 2299$$

$$Q(1, X, 12) = 396X - 3024$$

$$Q(1, X, 13) = 468X - 3887$$

$$Q(1, X, 14) = 546X - 4900$$

$$Q(1, X, 15) = 630X - 6075$$

$$Q(1, X, 16) = 720X - 7424$$

$$Q(1, X, 17) = 816X - 8959$$

$$Q(1, X, 18) = 918X - 10692$$

$$Q(1, X, 19) = 1026X - 12635$$

$$Q(1, X, 20) = 1140X - 14800$$

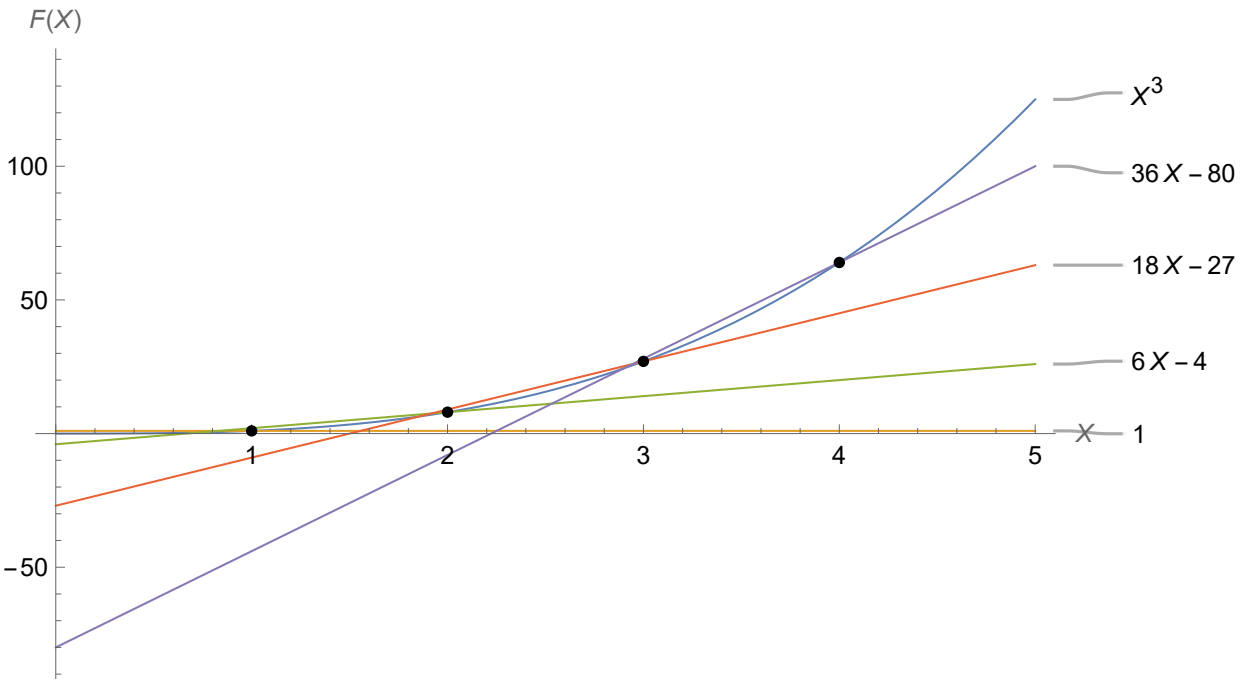

Figure 2. Polynomials Q(1, n, k)

1.4. **Polynomial Q(1,n,k) Table n = 6.**

1.5. **Polynomials P(2,n,k).**

$$P(2,X,0) = 0$$

$$P(2,X,1) = 30X^2 - 60X + 31$$

$$P(2,X,2) = 150X^2 - 540X + 512$$

$$P(2,X,3) = 420X^2 - 2160X + 2943$$

$$P(2,X,4) = 900X^2 - 6000X + 10624$$

$$P(2,X,5) = 1650X^2 - 13500X + 29375$$

$$P(2,X,6) = 2730X^2 - 26460X + 68256$$

$$P(2,X,7) = 4200X^2 - 47040X + 140287$$

$$P(2,X,8) = 6120X^2 - 77760X + 263168$$

$$P(2,X,9) = 8550X^2 - 121500X + 459999$$

$$P(2,X,10) = 11550X^2 - 181500X + 760000$$

$$P(2,X,11) = 15180X^2 - 261360X + 1199231$$

$$P(2,X,12) = 19500X^2 - 365040X + 1821312$$

$$P(2,X,13) = 24570X^2 - 496860X + 2678143$$

$$P(2,X,14) = 30450X^2 - 661500X + 3830624$$

$$P(2,X,15) = 37200X^2 - 864000X + 5349375$$

$$P(2,X,16) = 44880X^2 - 1109760X + 7315456$$

$$P(2,X,17) = 53550X^2 - 1404540X + 9821087$$

$$P(2,X,18) = 63270X^2 - 1754460X + 12970368$$

$$P(2,X,19) = 74100X^2 - 2166000X + 16879999$$

$$P(2,X,20) = 86100X^2 - 2646000X + 21680000$$

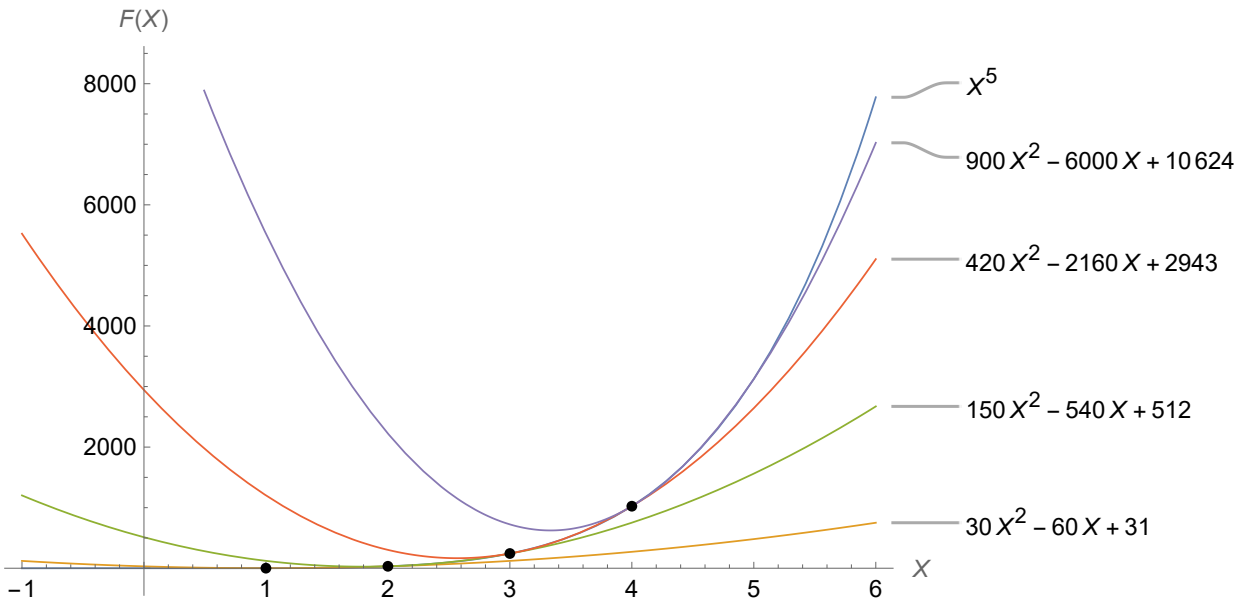

Figure 3. Polynomials P(2, n, k)

1.6. **Polynomial P(2,n,k) Table n = 4.**

1.7. **Polynomials Q(2,n,k).**

$$Q(2,X,0) = 0$$

$$Q(2,X,1) = 1$$

$$Q(2,X,2) = 30X^2 - 60X + 32$$

$$Q(2,X,3) = 150X^2 - 540X + 513$$

$$Q(2,X,4) = 420X^2 - 2160X + 2944$$

$$Q(2,X,5) = 900X^2 - 6000X + 10625$$

$$Q(2,X,6) = 1650X^2 - 13500X + 29376$$

$$Q(2,X,7) = 2730X^2 - 26460X + 68257$$

$$Q(2,X,8) = 4200X^2 - 47040X + 140288$$

$$Q(2,X,9) = 6120X^2 - 77760X + 263169$$

$$Q(2,X,10) = 8550X^2 - 121500X + 460000$$

$$Q(2,X,11) = 11550X^2 - 181500X + 760001$$

$$Q(2,X,12) = 15180X^2 - 261360X + 1199232$$

$$Q(2,X,13) = 19500X^2 - 365040X + 1821313$$

$$Q(2,X,14) = 24570X^2 - 496860X + 2678144$$

$$Q(2,X,15) = 30450X^2 - 661500X + 3830625$$

$$Q(2,X,16) = 37200X^2 - 864000X + 5349376$$

$$Q(2,X,17) = 44880X^2 - 1109760X + 7315457$$

$$Q(2,X,18) = 53550X^2 - 1404540X + 9821088$$

$$Q(2,X,19) = 63270X^2 - 1754460X + 12970369$$

$$Q(2,X,20) = 74100X^2 - 2166000X + 16880000$$

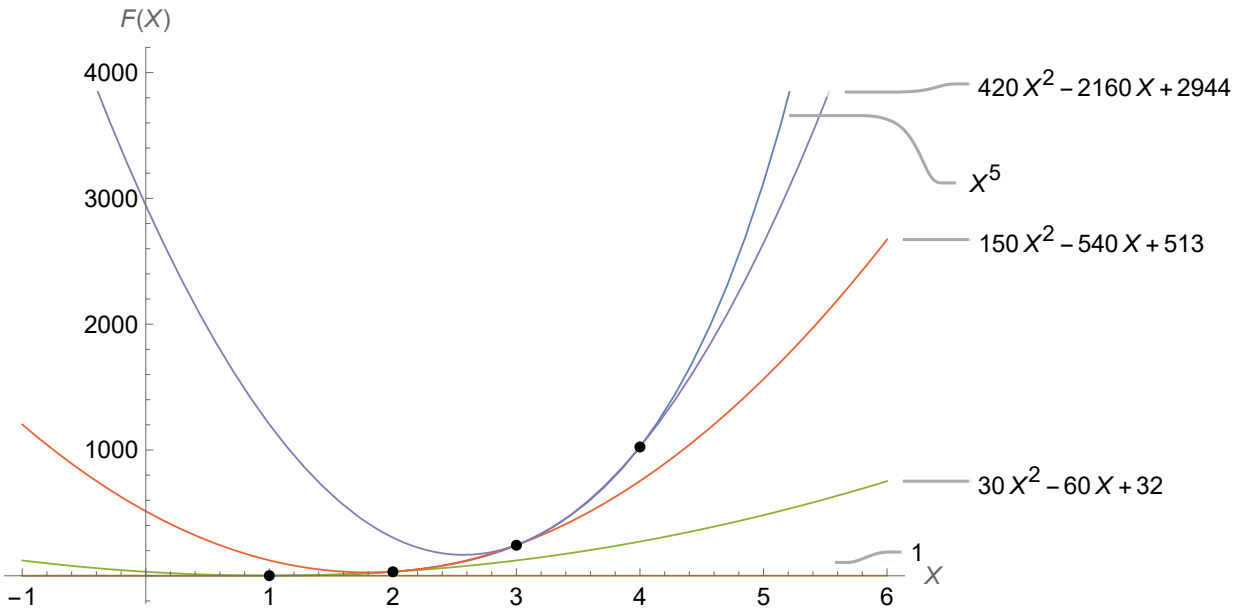

Figure 4. Polynomials Q(2, n, k)

1.8. **Polynomial Q(2,n,k) Table n = 4.**

1.9. **Polynomials P(3,n,k).**

$$P(3,X,0) = 0$$

$$P(3,X,1) = 140X^3 - 420X^2 + 406X - 125$$

$$P(3,X,2) = 1260X^3 - 7140X^2 + 13818X - 9028$$

$$P(3,X,3) = 5040X^3 - 41160X^2 + 115836X - 110961$$

$$P(3,X,4) = 14000X^3 - 148680X^2 + 545860X - 684176$$

$$P(3,X,5) = 31500X^3 - 411180X^2 + 1858290X - 2871325$$

$$P(3,X,6) = 61740X^3 - 955500X^2 + 5124126X - 9402660$$

$$P(3,X,7) = 109760X^3 - 1963920X^2 + 12182968X - 25872833$$

$$P(3,X,8) = 181440X^3 - 3684240X^2 + 25945416X - 62572096$$

$$P(3,X,9) = 283500X^3 - 6439860X^2 + 50745870X - 136972701$$

$$P(3,X,10) = 423500X^3 - 10639860X^2 + 92745730X - 276971300$$

$$P(3,X,11) = 609840X^3 - 16789080X^2 + 160386996X - 524988145$$

$$P(3,X,12) = 851760X^3 - 25498200X^2 + 264896268X - 943023888$$

$$P(3,X,13) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774781$$

$$P(3,X,14) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907076$$

$$P(3,X,15) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591425$$

$$P(3,X,16) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398080$$

$$P(3,X,17) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653693$$

$$P(3,X,18) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360516$$

$$P(3,X,19) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778801$$

$$P(3,X,20) = 6174000X^3 - 303519720X^2 + 5179983060X - 30303773200$$

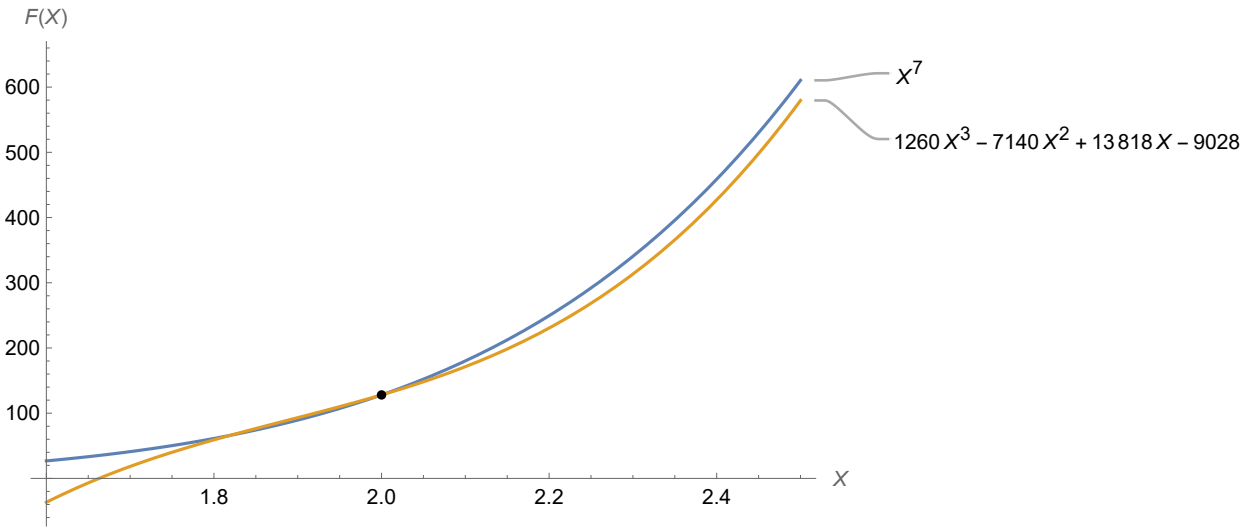

Figure 5. Polynomials P(3, n, k)

1.10. **Polynomial P(3,n,k) Table n = 3.**

1.11. **Polynomials Q(3,n,k).**

$$Q(3,X,0) = 0$$

$$Q(3,X,1) = 1$$

$$Q(3,X,2) = 140X^3 - 420X^2 + 406X - 124$$

$$Q(3,X,3) = 1260X^3 - 7140X^2 + 13818X - 9027$$

$$Q(3,X,4) = 5040X^3 - 41160X^2 + 115836X - 110960$$

$$Q(3,X,5) = 14000X^3 - 148680X^2 + 545860X - 684175$$

$$Q(3,X,6) = 31500X^3 - 411180X^2 + 1858290X - 2871324$$

$$Q(3,X,7) = 61740X^3 - 955500X^2 + 5124126X - 9402659$$

$$Q(3,X,8) = 109760X^3 - 1963920X^2 + 12182968X - 25872832$$

$$Q(3,X,9) = 181440X^3 - 3684240X^2 + 25945416X - 62572095$$

$$Q(3,X,10) = 283500X^3 - 6439860X^2 + 50745870X - 136972700$$

$$Q(3,X,11) = 423500X^3 - 10639860X^2 + 92745730X - 276971299$$

$$Q(3,X,12) = 609840X^3 - 16789080X^2 + 160386996X - 524988144$$

$$Q(3,X,13) = 851760X^3 - 25498200X^2 + 264896268X - 943023887$$

$$Q(3,X,14) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774780$$

$$Q(3,X,15) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907075$$

$$Q(3,X,16) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591424$$

$$Q(3,X,17) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398079$$

$$Q(3,X,18) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653692$$

$$Q(3,X,19) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360515$$

$$Q(3,X,20) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778800$$

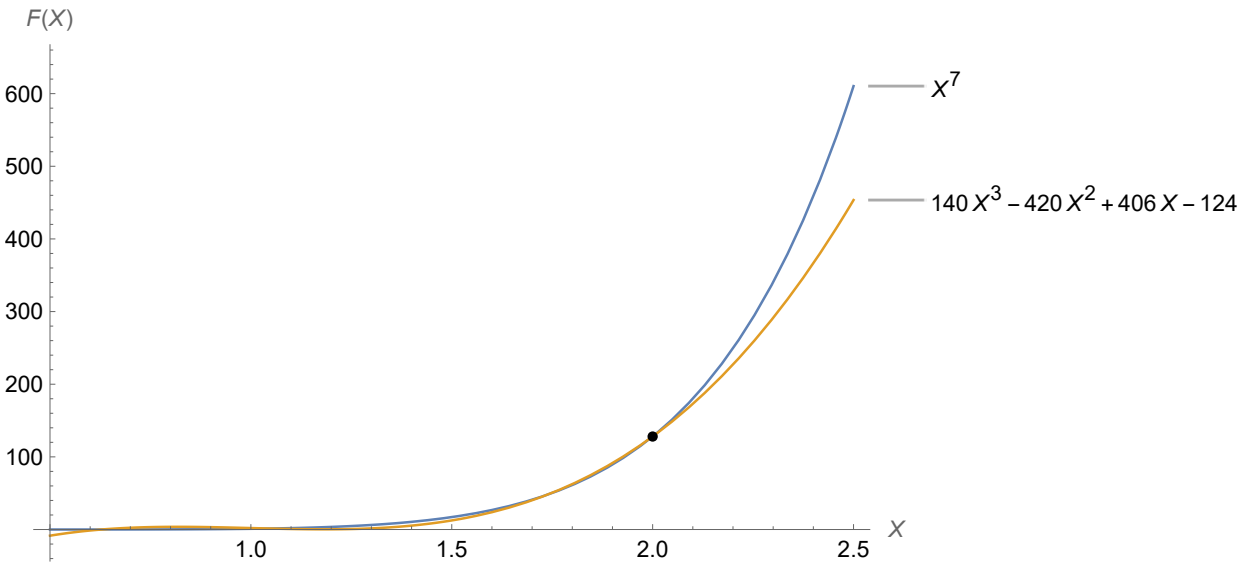

Figure 6. Polynomials Q(3, n, k)

1.12. **Polynomial Q(3,n,k) Table n = 3.**

Table 1. Comparison of X^3, $126X-540$, Difference, and Absolute Error Percentage

X	X^3	$126X - 540$	**Diff**	**ABS Error %**
5.3	148.877	127.800	21.077	14.1573
5.4	157.464	140.400	17.064	10.8368
5.5	166.375	153.000	13.375	8.0391
5.6	175.616	165.600	10.016	5.7034
5.7	185.193	178.200	6.993	3.7761
5.8	195.112	190.800	4.312	2.2100
5.9	205.379	203.400	1.979	0.9636
6.0	216.000	216.000	0.000	0.0000
6.1	226.981	228.600	-1.619	0.7133
6.2	238.328	241.200	-2.872	1.2051
6.3	250.047	253.800	-3.753	1.5009
6.4	262.144	266.400	-4.256	1.6235
6.5	274.625	279.000	-4.375	1.5931
6.6	287.496	291.600	-4.104	1.4275
6.7	300.763	304.200	-3.437	1.1428
6.8	314.432	316.800	-2.368	0.7531
6.9	328.509	329.400	-0.891	0.2712
7.0	343.000	342.000	1.000	0.2915
7.1	357.911	354.600	3.311	0.9251
7.2	373.248	367.200	6.048	1.6204
7.3	389.017	379.800	9.217	2.3693
7.4	405.224	392.400	12.824	3.1647
7.5	421.875	405.000	16.875	4.0000
7.6	438.976	417.600	21.376	4.8695
7.7	456.533	430.200	26.333	5.7680
7.8	474.552	442.800	31.752	6.6909
7.9	493.039	455.400	37.639	7.6341
8.0	512.000	468.000	44.000	8.5938

Table 2. Values of X^3 and $90X - 324$ for selected X

X	X^3	$90X - 324$
4.6	97.336	90.000
4.7	103.823	99.000
4.8	110.592	108.000
4.9	117.649	117.000
5.0	125.000	126.000
5.1	132.651	135.000
5.2	140.608	144.000
5.3	148.877	153.000
5.4	157.464	162.000
5.5	166.375	171.000
5.6	175.616	180.000
5.7	185.193	189.000
5.8	195.112	198.000
5.9	205.379	207.000
6.0	216.000	216.000
6.1	226.981	225.000
6.2	238.328	234.000
6.3	250.047	243.000
6.4	262.144	252.000
6.5	274.625	261.000
6.6	287.496	270.000
6.7	300.763	279.000
6.8	314.432	288.000
6.9	328.509	297.000
7.0	343.000	306.000

Table 3. Values of X^5 and $900X^2 - 6000X + 10624$ for selected X

X	X^5	$900X^2 - 6000X + 10624$
3.5	525.219	649.000
3.6	604.662	688.000
3.7	693.440	745.000
3.8	792.352	820.000
3.9	902.242	913.000
4.0	1024.000	1024.000
4.1	1158.560	1153.000
4.2	1306.910	1300.000
4.3	1470.080	1465.000
4.4	1649.160	1648.000
4.5	1845.280	1849.000
4.6	2059.630	2068.000
4.7	2293.450	2305.000
4.8	2548.040	2560.000
4.9	2824.750	2833.000
5.0	3125.000	3124.000
5.1	3450.250	3433.000
5.2	3802.040	3760.000
5.3	4181.950	4105.000
5.4	4591.650	4468.000
5.5	5032.840	4849.000

Table 4. Values of X^5 and $420X^2 - 2160X + 2944$ for selected X

X	X^5	$420X^2 - 2160X + 2944$
2.7	143.489	173.800
2.8	172.104	188.800
2.9	205.111	212.200
3.0	243.000	244.000
3.1	286.292	284.200
3.2	335.544	332.800
3.3	391.354	389.800
3.4	454.354	455.200
3.5	525.219	529.000
3.6	604.662	611.200
3.7	693.440	701.800
3.8	792.352	800.800
3.9	902.242	908.200
4.0	1024.000	1024.000
4.1	1158.560	1148.200
4.2	1306.910	1280.800
4.3	1470.080	1421.800
4.4	1649.160	1571.200
4.5	1845.280	1729.000
4.6	2059.630	1895.200
4.7	2293.450	2069.800
4.8	2548.040	2252.800
4.9	2824.750	2444.200
5.0	3125.000	2644.000

Table 5. Values of X^7 and $5040X^3 - 41160X^2 + 115836X - 110961$ for selected X

X	X^7	$5040X^3 - 41160X^2 + 115836X - 110961$
2.7	1046.040	942.120
2.8	1349.290	1323.480
2.9	1724.990	1728.360
3.0	2187.000	2187.000
3.1	2751.260	2729.640
3.2	3435.970	3386.520
3.3	4261.840	4187.880
3.4	5252.340	5163.960
3.5	6433.930	6345.000
3.6	7836.420	7761.240
3.7	9493.190	9442.920
3.8	11441.600	11420.300
3.9	13723.100	13723.600
4.0	16384.000	16383.000
4.1	19475.400	19428.800
4.2	23053.900	22891.300
4.3	27181.900	26800.700
4.4	31927.800	31187.200
4.5	37366.900	36081.000
4.6	43581.800	41512.400
4.7	50662.300	47511.700
4.8	58706.800	54109.100
4.9	67822.300	61334.800
5.0	78125.000	69219.000

Table 6. Values of X^7 and $5040X^3 - 41160X^2 + 115836X - 110961$ for selected X

X	X^7	$5040X^3 - 41160X^2 + 115836X - 110961$
2.7	1046.040	942.120
2.8	1349.290	1323.480
2.9	1724.990	1728.360
3.0	2187.000	2187.000
3.1	2751.260	2729.640
3.2	3435.970	3386.520
3.3	4261.840	4187.880
3.4	5252.340	5163.960
3.5	6433.930	6345.000
3.6	7836.420	7761.240
3.7	9493.190	9442.920
3.8	11441.600	11420.300
3.9	13723.100	13723.600
4.0	16384.000	16383.000
4.1	19475.400	19428.800
4.2	23053.900	22891.300
4.3	27181.900	26800.700
4.4	31927.800	31187.200
4.5	37366.900	36081.000
4.6	43581.800	41512.400
4.7	50662.300	47511.700
4.8	58706.800	54109.100
4.9	67822.300	61334.800
5.0	78125.000	69219.000