PLOTS OF CLOSED FORMS

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CONTENTS

1. INTRODUCTION

$$P(m, b, x) = \sum_{r=0}^{m} \sum_{k=1}^{n} \mathbf{A}_{m,r} k^r (n - k)^r$$

$$Q(m, b, x) = \sum_{r=0}^{m} \sum_{k=0}^{n-1} \mathbf{A}_{m,r} k^r (n - k)^r$$

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Date: January 17, 2025.

1.1. Polynomials P(1,n,k).

$$P(1, N, 0) = 0$$

$$P(1, N, 1) = 6N - 5$$

$$P(1, N, 2) = 18N - 28$$

$$P(1, N, 3) = 36N - 81$$

$$P(1, N, 4) = 60N - 176$$

$$P(1, N, 5) = 90N - 325$$

$$P(1, N, 6) = 126N - 540$$

$$P(1, N, 7) = 168N - 833$$

$$P(1, N, 8) = 216N - 1216$$

$$P(1, N, 9) = 270N - 1701$$

$$P(1, N, 10) = 330N - 2300$$

$$P(1, N, 11) = 396N - 3025$$

$$P(1, N, 12) = 468N - 3888$$

$$P(1, N, 13) = 546N - 4901$$

$$P(1, N, 14) = 630N - 6076$$

$$P(1, N, 15) = 720N - 7425$$

$$P(1, N, 16) = 816N - 8960$$

$$P(1, N, 17) = 918N - 10693$$

$$P(1, N, 18) = 1026N - 12636$$

$$P(1, N, 19) = 1140N - 14801$$
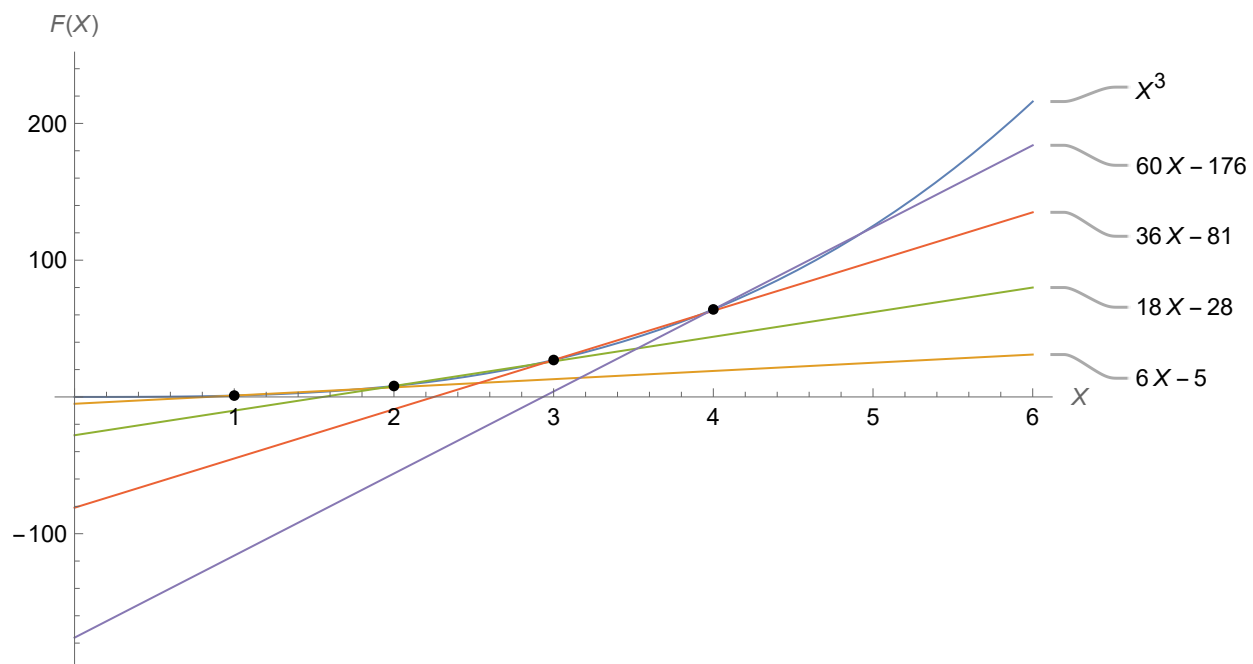
$$P(1, N, 20) = 1260N - 17200$$

Figure 1. Polynomials P(1, n, k)

1.2. Polynomials Q(1,n,k).

$$Q(1, N, 0) = 0$$

$$Q(1, N, 1) = 1$$

$$Q(1, N, 2) = 6N - 4$$

$$Q(1, N, 3) = 18N - 27$$

$$Q(1, N, 4) = 36N - 80$$

$$Q(1, N, 5) = 60N - 175$$

$$Q(1, N, 6) = 90N - 324$$

$$Q(1, N, 7) = 126N - 539$$

$$Q(1, N, 8) = 168N - 832$$

$$Q(1, N, 9) = 216N - 1215$$

$$Q(1, N, 10) = 270N - 1700$$

$$Q(1, N, 11) = 330N - 2299$$

$$Q(1, N, 12) = 396N - 3024$$

$$Q(1, N, 13) = 468N - 3887$$

$$Q(1, N, 14) = 546N - 4900$$
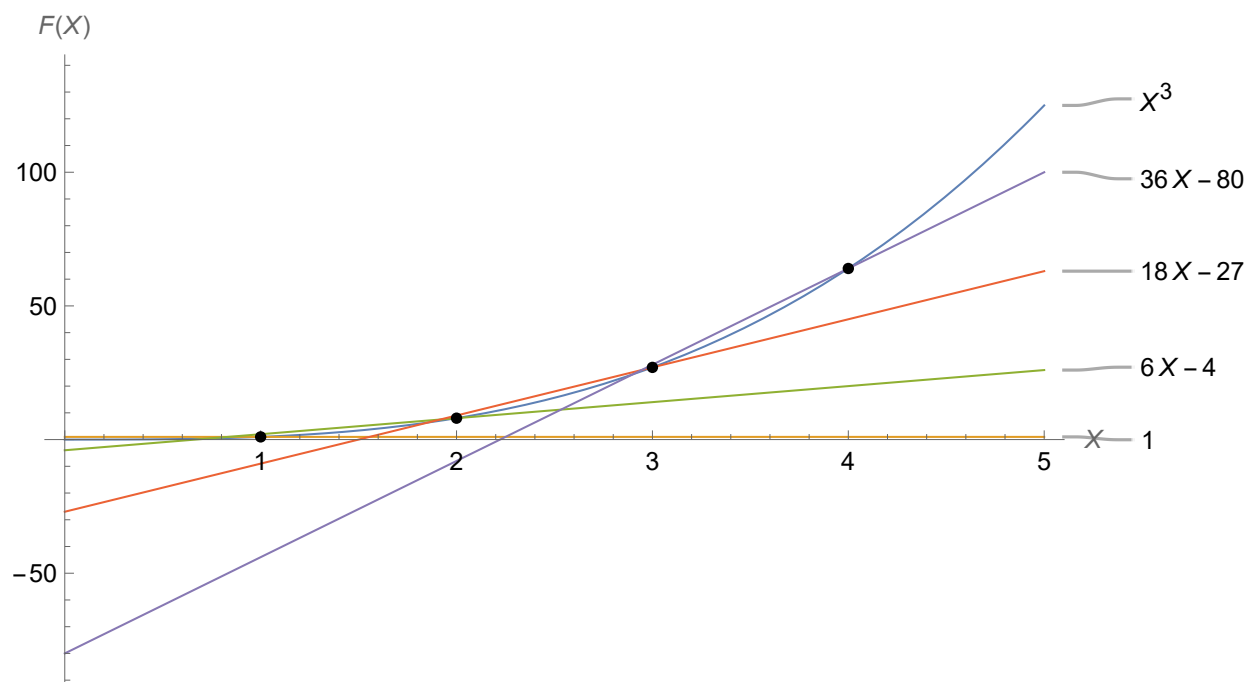
$$Q(1, N, 15) = 630N - 6075$$

$$Q(1, N, 16) = 720N - 7424$$

$$Q(1, N, 17) = 816N - 8959$$

$$Q(1, N, 18) = 918N - 10692$$

$$Q(1, N, 19) = 1026N - 12635$$

$$Q(1, N, 20) = 1140N - 14800$$

Figure 2. Polynomials Q(1, n, k)

1.3. Polynomials P(2,n,k).

$$P(2, N, 0) = 0$$

$$P(2, N, 1) = 30N^2 - 60N + 31$$

$$P(2, N, 2) = 150N^2 - 540N + 512$$

$$P(2, N, 3) = 420N^2 - 2160N + 2943$$

$$P(2, N, 4) = 900N^2 - 6000N + 10624$$

$$P(2, N, 5) = 1650N^2 - 13500N + 29375$$

$$P(2, N, 6) = 2730N^2 - 26460N + 68256$$

$$P(2, N, 7) = 4200N^2 - 47040N + 140287$$

$$P(2, N, 8) = 6120N^2 - 77760N + 263168$$

$$P(2, N, 9) = 8550N^2 - 121500N + 459999$$

$$P(2, N, 10) = 11550N^2 - 181500N + 760000$$

$$P(2, N, 11) = 15180N^2 - 261360N + 1199231$$

$$P(2, N, 12) = 19500N^2 - 365040N + 1821312$$

$$P(2, N, 13) = 24570N^2 - 496860N + 2678143$$

$$P(2, N, 14) = 30450N^2 - 661500N + 3830624$$

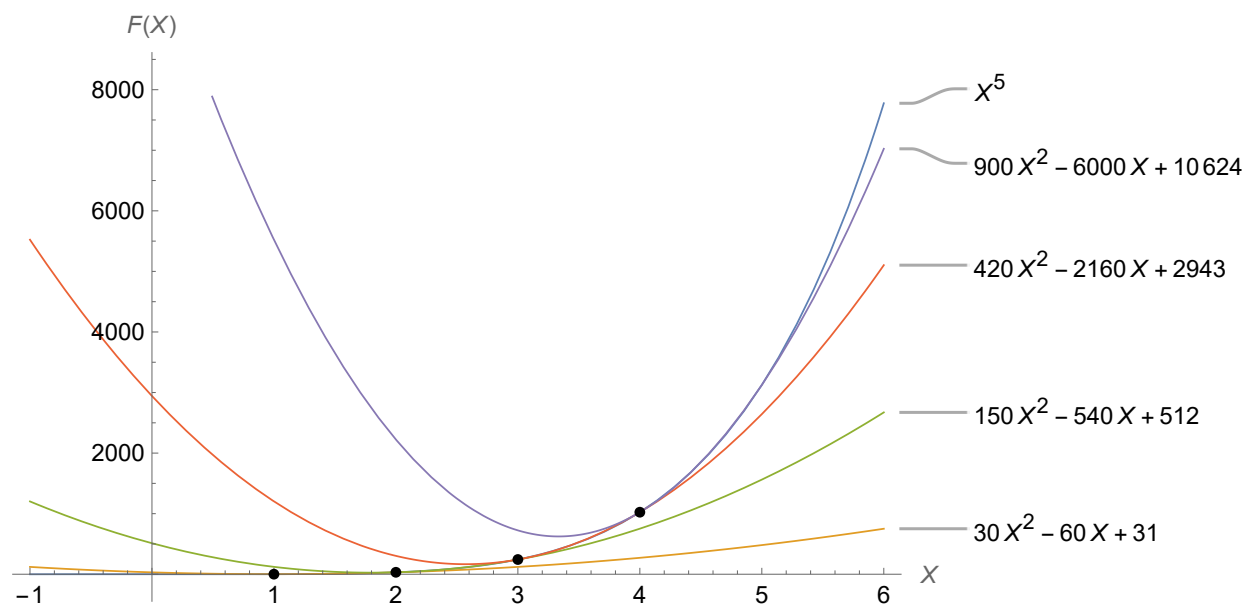$$P(2, N, 15) = 37200N^2 - 864000N + 5349375$$

$$P(2, N, 16) = 44880N^2 - 1109760N + 7315456$$

$$P(2, N, 17) = 53550N^2 - 1404540N + 9821087$$

$$P(2, N, 18) = 63270N^2 - 1754460N + 12970368$$

$$P(2, N, 19) = 74100N^2 - 2166000N + 16879999$$

$$P(2, N, 20) = 86100N^2 - 2646000N + 21680000$$

Figure 3. Polynomials P(2, n, k)

1.4. **Polynomials Q(2,n,k).**

$$Q(2, N, 0) = 0$$

$$Q(2, N, 1) = 1$$

$$Q(2, N, 2) = 30N^2 - 60N + 32$$

$$Q(2, N, 3) = 150N^2 - 540N + 513$$

$$Q(2, N, 4) = 420N^2 - 2160N + 2944$$

$$Q(2, N, 5) = 900N^2 - 6000N + 10625$$

$$Q(2, N, 6) = 1650N^2 - 13500N + 29376$$

$$Q(2, N, 7) = 2730N^2 - 26460N + 68257$$

$$Q(2, N, 8) = 4200N^2 - 47040N + 140288$$

$$Q(2, N, 9) = 6120N^2 - 77760N + 263169$$

$$Q(2, N, 10) = 8550N^2 - 121500N + 460000$$

$$Q(2, N, 11) = 11550N^2 - 181500N + 760001$$

$$Q(2, N, 12) = 15180N^2 - 261360N + 1199232$$

$$Q(2, N, 13) = 19500N^2 - 365040N + 1821313$$

$$Q(2, N, 14) = 24570N^2 - 496860N + 2678144$$

$$Q(2, N, 15) = 30450N^2 - 661500N + 3830625$$
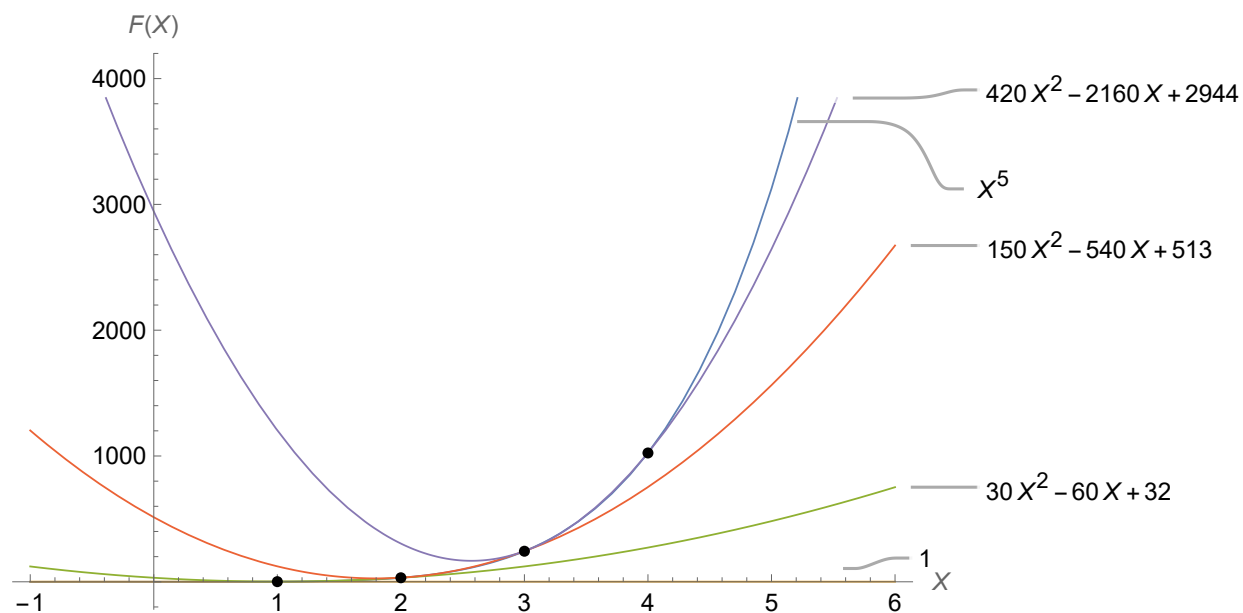
$$Q(2, N, 16) = 37200N^2 - 864000N + 5349376$$

$$Q(2, N, 17) = 44880N^2 - 1109760N + 7315457$$

$$Q(2, N, 18) = 53550N^2 - 1404540N + 9821088$$

$$Q(2, N, 19) = 63270N^2 - 1754460N + 12970369$$

$$Q(2, N, 20) = 74100N^2 - 2166000N + 16880000$$

Figure 4. Polynomials P(2, n, k)

1.5. **Polynomials Q(3,n,k).**

$$P(3, N, 0) = 0$$

$$P(3, N, 1) = 140N^3 - 420N^2 + 406N - 125$$

$$P(3, N, 2) = 1260N^3 - 7140N^2 + 13818N - 9028$$

$$P(3, N, 3) = 5040N^3 - 41160N^2 + 115836N - 110961$$

$$P(3, N, 4) = 14000N^3 - 148680N^2 + 545860N - 684176$$

$$P(3, N, 5) = 31500N^3 - 411180N^2 + 1858290N - 2871325$$

$$P(3, N, 6) = 61740N^3 - 955500N^2 + 5124126N - 9402660$$

$$P(3, N, 7) = 109760N^3 - 1963920N^2 + 12182968N - 25872833$$

$$P(3, N, 8) = 181440N^3 - 3684240N^2 + 25945416N - 62572096$$

$$P(3, N, 9) = 283500N^3 - 6439860N^2 + 50745870N - 136972701$$

$$P(3, N, 10) = 423500N^3 - 10639860N^2 + 92745730N - 276971300$$

$$P(3, N, 11) = 609840N^3 - 16789080N^2 + 160386996N - 524988145$$

$$P(3, N, 12) = 851760N^3 - 25498200N^2 + 264896268N - 943023888$$

$$P(3, N, 13) = 1159340N^3 - 37493820N^2 + 420839146N - 1618774781$$

$$P(3, N, 14) = 1543500N^3 - 53628540N^2 + 646725030N - 2672907076$$

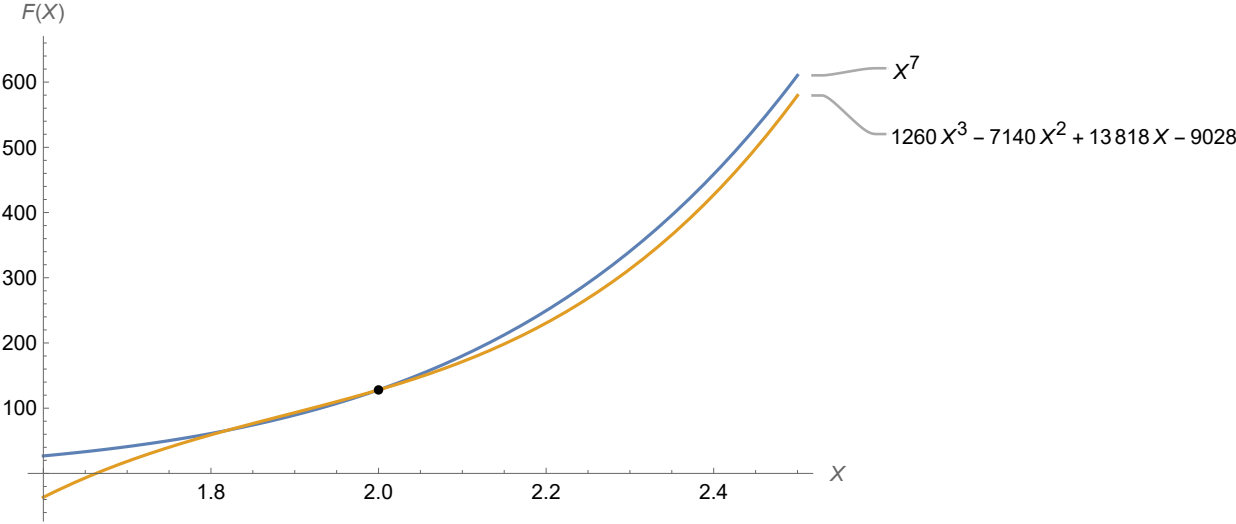$$P(3, N, 15) = 2016000N^3 - 74891040N^2 + 965662320N - 4267591425$$

$$P(3, N, 16) = 2589440N^3 - 102416160N^2 + 1406064016N - 6616398080$$

$$P(3, N, 17) = 3277260N^3 - 137494980N^2 + 2002403718N - 9995653693$$

$$P(3, N, 18) = 4093740N^3 - 181584900N^2 + 2796022026N - 14757360516$$

$$P(3, N, 19) = 5054000N^3 - 236319720N^2 + 3835983340N - 21343778801$$

$$P(3, N, 20) = 6174000N^3 - 303519720N^2 + 5179983060N - 30303773200$$

Figure 5. Polynomials P(1, n, k)

REFERENCES