PLOTS OF CLOSED FORMS

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Contents

1. Introduction

$$P(m, X, N) = \sum_{r=0}^{m} \sum_{k=1}^{N} \mathbf{A}_{m,r} k^r (X - k)^r$$

$$Q(m, X, N) = \sum_{r=0}^{m} \sum_{k=0}^{N-1} \mathbf{A}_{m,r} k^r (X - k)^r$$

$$P(m, N, N) = N^{2m+1}$$

$$Q(m, N, N) = N^{2m+1}$$

$$P(m, N + 1, N) = (N + 1)^{2m+1} - 1 \qquad (verified)$$

$$Q(m, N - 1, N) = (N - 1)^{2m+1} + 1 \qquad (verified)$$

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1.1. Polynomials P(1,n,k).

$$P(1, X, 0) = 0$$

$$P(1, X, 1) = 6X - 5$$

$$P(1, X, 2) = 18X - 28$$

$$P(1, X, 3) = 36X - 81$$

$$P(1, X, 4) = 60X - 176$$

$$P(1, X, 5) = 90X - 325$$

$$P(1, X, 6) = 126X - 540$$

$$P(1, X, 7) = 168X - 833$$

$$P(1, X, 8) = 216X - 1216$$

$$P(1, X, 9) = 270X - 1701$$

$$P(1, X, 10) = 330X - 2300$$

$$P(1, X, 11) = 396X - 3025$$

$$P(1, X, 12) = 468X - 3888$$

$$P(1, X, 13) = 546X - 4901$$

$$P(1, X, 14) = 630X - 6076$$

$$P(1, X, 15) = 720X - 7425$$

$$P(1, X, 16) = 816X - 8960$$

$$P(1, X, 17) = 918X - 10693$$

$$P(1, X, 18) = 1026X - 12636$$

$$P(1, X, 19) = 1140X - 14801$$
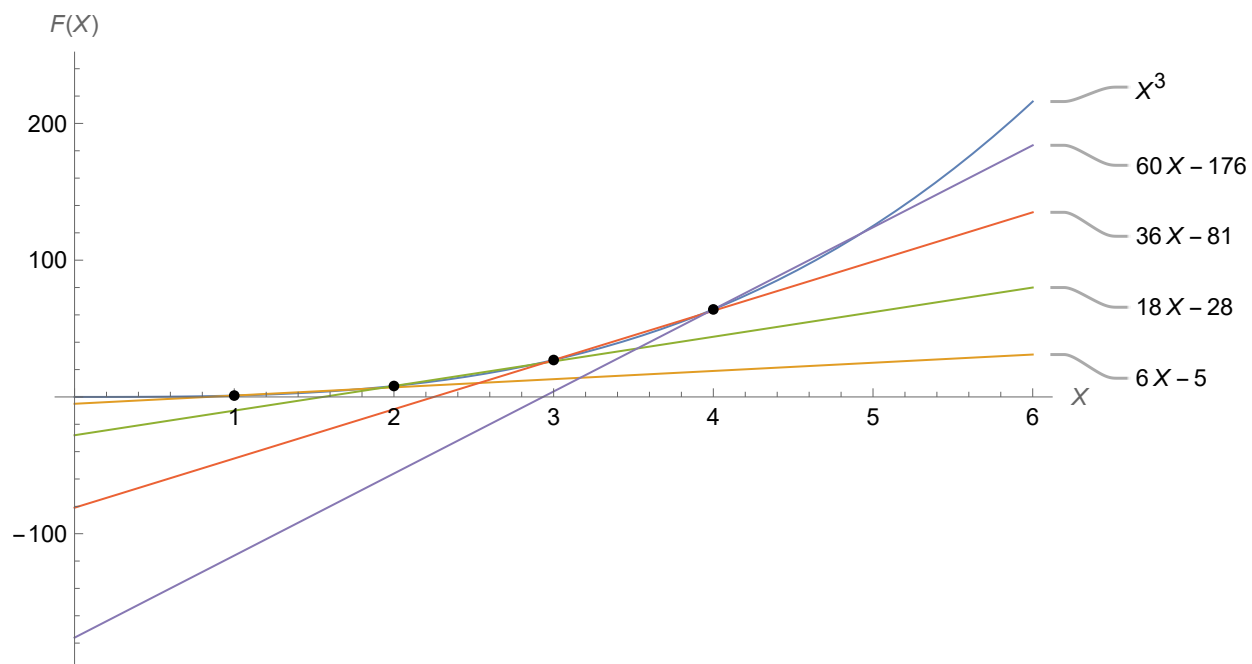
$$P(1, X, 20) = 1260X - 17200$$

Figure 1. Polynomials P(1, n, k)

1.2. Polynomials P(1,n,k) example n = 6.

1.3. Polynomials Q(1,n,k).

$$Q(1, X, 0) = 0$$

$$Q(1, X, 1) = 1$$

$$Q(1, X, 2) = 6X - 4$$

$$Q(1, X, 3) = 18X - 27$$

$$Q(1, X, 4) = 36X - 80$$

$$Q(1, X, 5) = 60X - 175$$

$$Q(1, X, 6) = 90X - 324$$

$$Q(1, X, 7) = 126X - 539$$

$$Q(1, X, 8) = 168X - 832$$

$$Q(1, X, 9) = 216X - 1215$$

$$Q(1, X, 10) = 270X - 1700$$

$$Q(1, X, 11) = 330X - 2299$$

$$Q(1, X, 12) = 396X - 3024$$

$$Q(1, X, 13) = 468X - 3887$$

$$Q(1, X, 14) = 546X - 4900$$

$$Q(1, X, 15) = 630X - 6075$$

$$Q(1, X, 16) = 720X - 7424$$

$$Q(1, X, 17) = 816X - 8959$$

$$Q(1, X, 18) = 918X - 10692$$

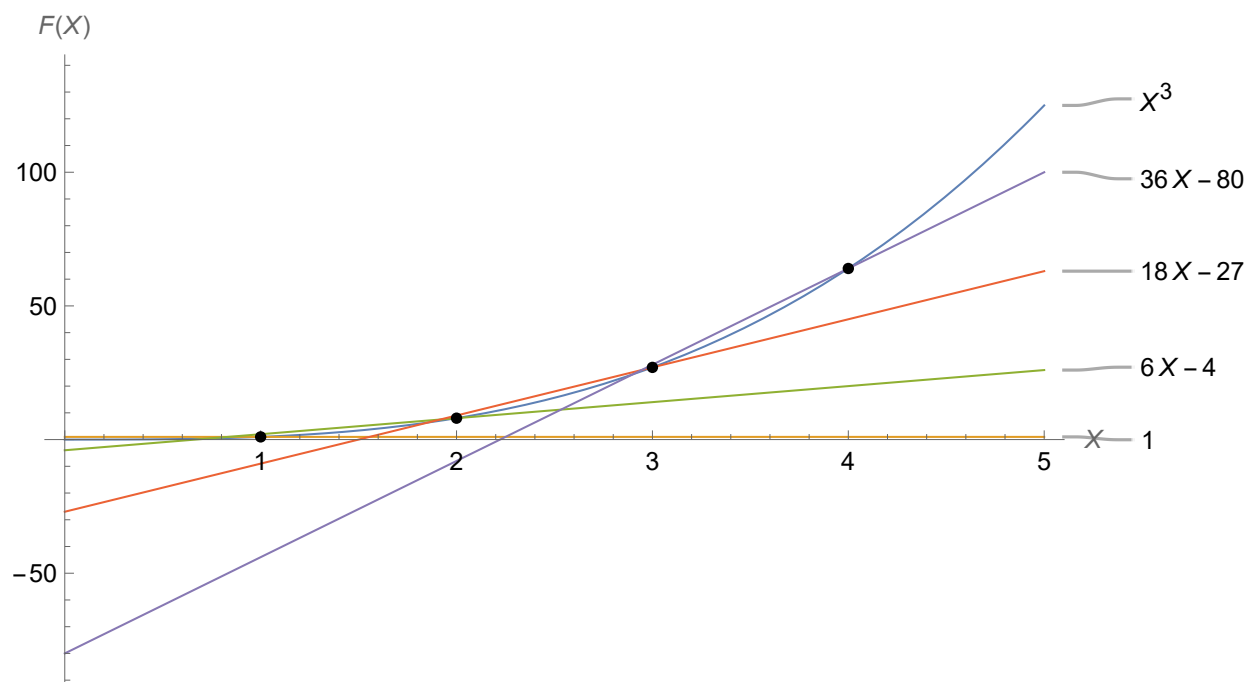$$Q(1, X, 19) = 1026X - 12635$$

$$Q(1, X, 20) = 1140X - 14800$$

Figure 2. Polynomials Q(1, n, k)

1.4. **Polynomials P(2,n,k).**

$$P(2, X, 0) = 0$$

$$P(2, X, 1) = 30X^2 - 60X + 31$$

$$P(2, X, 2) = 150X^2 - 540X + 512$$

$$P(2, X, 3) = 420X^2 - 2160X + 2943$$

$$P(2, X, 4) = 900X^2 - 6000X + 10624$$

$$P(2, X, 5) = 1650X^2 - 13500X + 29375$$

$$P(2, X, 6) = 2730X^2 - 26460X + 68256$$

$$P(2, X, 7) = 4200X^2 - 47040X + 140287$$

$$P(2, X, 8) = 6120X^2 - 77760X + 263168$$

$$P(2, X, 9) = 8550X^2 - 121500X + 459999$$

$$P(2, X, 10) = 11550X^2 - 181500X + 760000$$

$$P(2, X, 11) = 15180X^2 - 261360X + 1199231$$

$$P(2, X, 12) = 19500X^2 - 365040X + 1821312$$

$$P(2, X, 13) = 24570X^2 - 496860X + 2678143$$

$$P(2, X, 14) = 30450X^2 - 661500X + 3830624$$

$$P(2, X, 15) = 37200X^2 - 864000X + 5349375$$

$$P(2, X, 16) = 44880X^2 - 1109760X + 7315456$$

$$P(2, X, 17) = 53550X^2 - 1404540X + 9821087$$

$$P(2, X, 18) = 63270X^2 - 1754460X + 12970368$$

$$P(2, X, 19) = 74100X^2 - 2166000X + 16879999$$
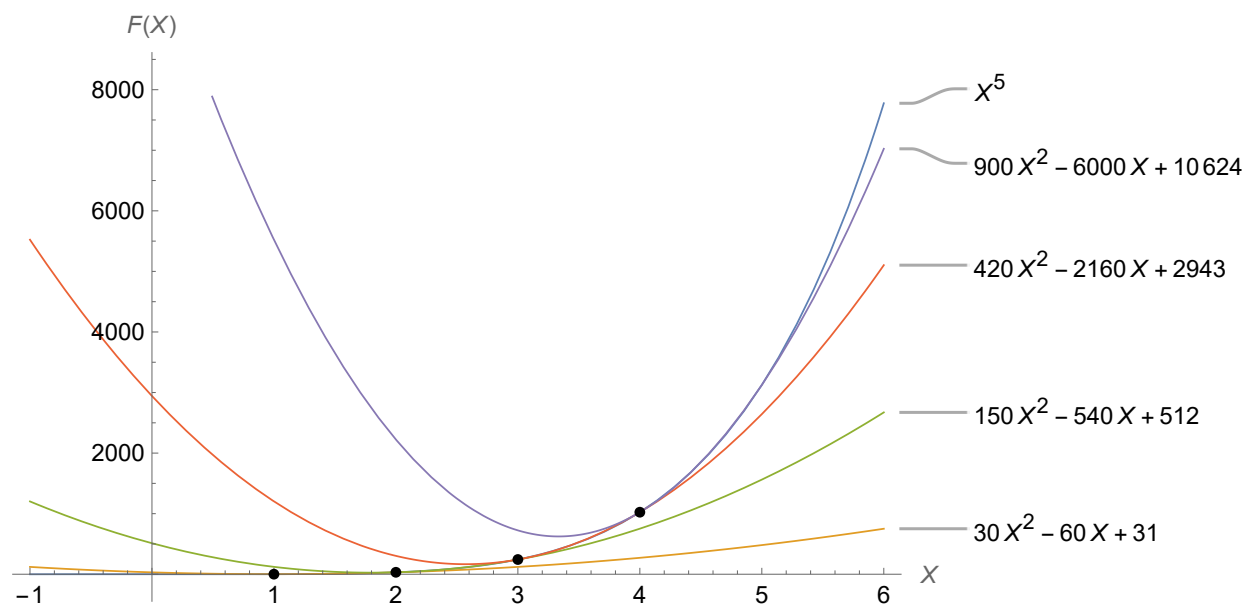
$$P(2, X, 20) = 86100X^2 - 2646000X + 21680000$$

Figure 3. Polynomials P(2, n, k)

1.5. Polynomials Q(2,n,k).

$$Q(2, X, 0) = 0$$

$$Q(2, X, 1) = 1$$

$$Q(2, X, 2) = 30X^2 - 60X + 32$$

$$Q(2, X, 3) = 150X^2 - 540X + 513$$

$$Q(2, X, 4) = 420X^2 - 2160X + 2944$$

$$Q(2, X, 5) = 900X^2 - 6000X + 10625$$

$$Q(2, X, 6) = 1650X^2 - 13500X + 29376$$

$$Q(2, X, 7) = 2730X^2 - 26460X + 68257$$

$$Q(2, X, 8) = 4200X^2 - 47040X + 140288$$

$$Q(2, X, 9) = 6120X^2 - 77760X + 263169$$

$$Q(2, X, 10) = 8550X^2 - 121500X + 460000$$

$$Q(2, X, 11) = 11550X^2 - 181500X + 760001$$

$$Q(2, X, 12) = 15180X^2 - 261360X + 1199232$$

$$Q(2, X, 13) = 19500X^2 - 365040X + 1821313$$

$$Q(2, X, 14) = 24570X^2 - 496860X + 2678144$$

$$Q(2, X, 15) = 30450X^2 - 661500X + 3830625$$

$$Q(2, X, 16) = 37200X^2 - 864000X + 5349376$$

$$Q(2, X, 17) = 44880X^2 - 1109760X + 7315457$$

$$Q(2, X, 18) = 53550X^2 - 1404540X + 9821088$$

$$Q(2, X, 19) = 63270X^2 - 1754460X + 12970369$$
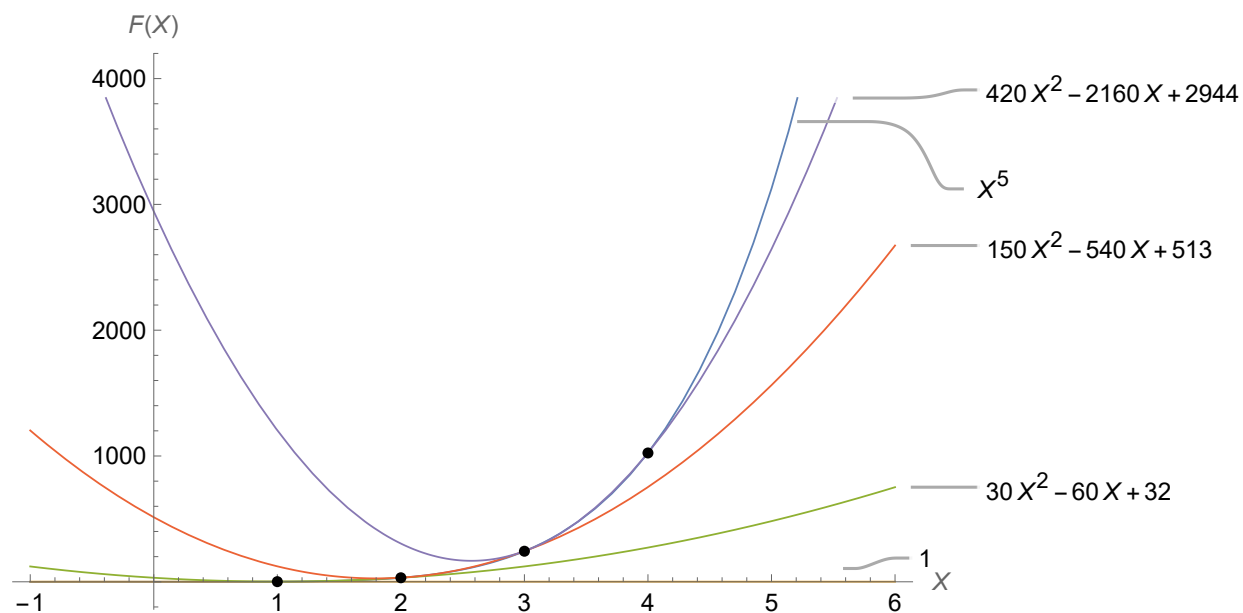
$$Q(2, X, 20) = 74100X^2 - 2166000X + 16880000$$

Figure 4. Polynomials Q(2, n, k)

1.6. **Polynomials P(3,n,k).**

$$P(3, X, 0) = 0$$

$$P(3, X, 1) = 140X^3 - 420X^2 + 406X - 125$$

$$P(3, X, 2) = 1260X^3 - 7140X^2 + 13818X - 9028$$

$$P(3, X, 3) = 5040X^3 - 41160X^2 + 115836X - 110961$$

$$P(3, X, 4) = 14000X^3 - 148680X^2 + 545860X - 684176$$

$$P(3, X, 5) = 31500X^3 - 411180X^2 + 1858290X - 2871325$$

$$P(3, X, 6) = 61740X^3 - 955500X^2 + 5124126X - 9402660$$

$$P(3, X, 7) = 109760X^3 - 1963920X^2 + 12182968X - 25872833$$

$$P(3, X, 8) = 181440X^3 - 3684240X^2 + 25945416X - 62572096$$

$$P(3, X, 9) = 283500X^3 - 6439860X^2 + 50745870X - 136972701$$

$$P(3, X, 10) = 423500X^3 - 10639860X^2 + 92745730X - 276971300$$

$$P(3, X, 11) = 609840X^3 - 16789080X^2 + 160386996X - 524988145$$

$$P(3, X, 12) = 851760X^3 - 25498200X^2 + 264896268X - 943023888$$

$$P(3, X, 13) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774781$$

$$P(3, X, 14) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907076$$

$$P(3, X, 15) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591425$$

$$P(3, X, 16) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398080$$

$$P(3, X, 17) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653693$$

$$P(3, X, 18) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360516$$

$$P(3, X, 19) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778801$$

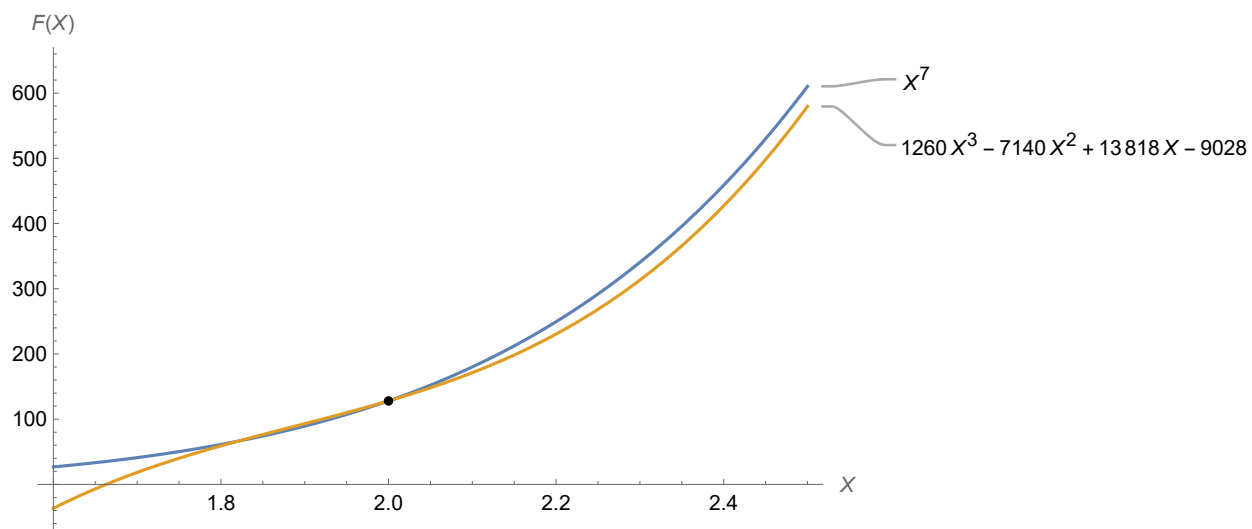$$P(3, X, 20) = 6174000X^3 - 303519720X^2 + 5179983060X - 30303773200$$

Figure 5. Polynomials P(3, n, k)

1.7. **Polynomials Q(3,n,k).**

$$Q(3, X, 0) = 0$$

$$Q(3, X, 1) = 1$$

$$Q(3, X, 2) = 140X^3 - 420X^2 + 406X - 124$$

$$Q(3, X, 3) = 1260X^3 - 7140X^2 + 13818X - 9027$$

$$Q(3, X, 4) = 5040X^3 - 41160X^2 + 115836X - 110960$$

$$Q(3, X, 5) = 14000X^3 - 148680X^2 + 545860X - 684175$$

$$Q(3, X, 6) = 31500X^3 - 411180X^2 + 1858290X - 2871324$$

$$Q(3, X, 7) = 61740X^3 - 955500X^2 + 5124126X - 9402659$$

$$Q(3, X, 8) = 109760X^3 - 1963920X^2 + 12182968X - 25872832$$

$$Q(3, X, 9) = 181440X^3 - 3684240X^2 + 25945416X - 62572095$$

$$Q(3, X, 10) = 283500X^3 - 6439860X^2 + 50745870X - 136972700$$

$$Q(3, X, 11) = 423500X^3 - 10639860X^2 + 92745730X - 276971299$$

$$Q(3, X, 12) = 609840X^3 - 16789080X^2 + 160386996X - 524988144$$

$$Q(3, X, 13) = 851760X^3 - 25498200X^2 + 264896268X - 943023887$$

$$Q(3, X, 14) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774780$$

$$Q(3, X, 15) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907075$$

$$Q(3, X, 16) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591424$$

$$Q(3, X, 17) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398079$$

$$Q(3, X, 18) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653692$$

$$Q(3, X, 19) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360515$$

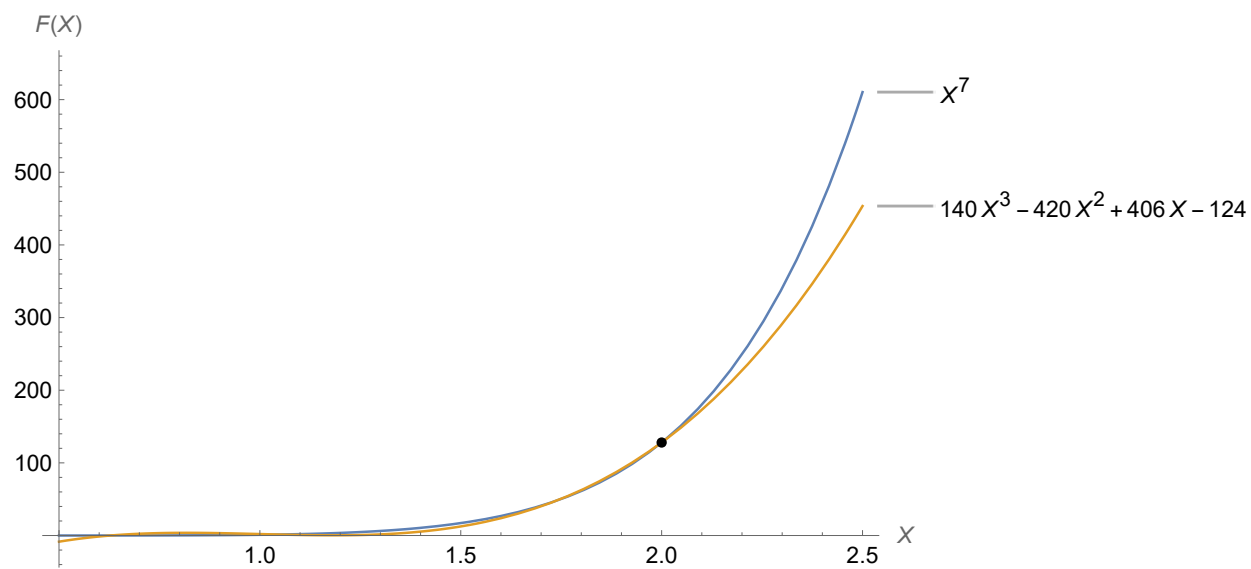$$Q(3, X, 20) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778800$$

Figure 6. Polynomials Q(3, n, k)

Table 1. Values of X^3 and $126X - 540$ for selected X

X	X^7	$126X - 540$
5.0	125.000	90.000
5.1	132.651	102.600
5.2	140.608	115.200
5.3	148.877	127.800
5.4	157.464	140.400
5.5	166.375	153.000
5.6	175.616	165.600
5.7	185.193	178.200
5.8	195.112	190.800
5.9	205.379	203.400
6.0	216.000	216.000
6.1	226.981	228.600
6.2	238.328	241.200
6.3	250.047	253.800
6.4	262.144	266.400
6.5	274.625	279.000
6.6	287.496	291.600
6.7	300.763	304.200
6.8	314.432	316.800
6.9	328.509	329.400
7.0	343.000	342.000
7.1	357.911	354.600
7.2	373.248	367.200
7.3	389.017	379.800
7.4	405.224	392.400
7.5	421.875	405.000