POLYNOMIAL IDENTITIES AUXILIARY

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Abstract. Polynomial identities auxiliary

Contents

1. Polynomial identities auxiliary

1.1. Central factorial numbers.

$$(2k-1)!T(2n,2k) = \frac{1}{k}\sum_{j=0}^{k}(-1)^j\binom{2k}{j}(k-j)^{2n}$$

$$(2k-1)!T(2n,2k) = \frac{1}{k}\sum_{j=0}^{k}(-1)^{k-j}\binom{2k}{k-j}j^{2n}$$

$$(2k-1)!T(2n,2k) = \frac{1}{k}2k!\left(\left\{ {2n \atop 2k} \right\} - \sum_{j=k+1}^{2k}(-1)^j\binom{2k}{j}(k-j)^{2n}\right)$$

$$T(n,k) = \frac{1}{k!}\sum_{j=0}\binom{k}{j}(-1)^j\left(\frac{1}{2}k-j\right)^n$$

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$$(2k-1)! \cdot T(2n,2k) = \frac{1}{2k} \sum_{j=0}^{2k} \binom{2k}{j} (-1)^j (k-j)^{2n}$$

1.2. Knuth's formula - approach 1 (to be verified all).

$$n^{2m-1} = \sum_{k=1}^{m} (2k-1)! T(2m,2k) \binom{n+k-1}{2k-1}$$

Substituting $(2k-1)!T(2n,2k) = \frac{1}{k} \sum_{j=0}^{k} (-1)^{k-j} \binom{2k}{k-j} j^{2n}$ we get

$$n^{2m-1} = \sum_{k=1}^{m} \sum_{j=0}^{k} \frac{(-1)^{k-j}}{k} \binom{n+k-1}{2k-1} \binom{2k}{k-j} j^{2m}$$

By means of binomial identity $\frac{k}{n}\binom{n}{k} = \binom{n-1}{k-1}$

$$n^{2m-1} = \sum_{k=1}^{m} \sum_{j=0}^{k} \frac{(-1)^{k-j}}{k} \frac{2k}{n+k} \binom{n+k}{2k} \binom{2k}{k-j} j^{2m}$$

Collapsing common terms we get

$$n^{2m-1} = 2 \sum_{k=1}^{m} \sum_{j=0}^{k} \frac{(-1)^{k-j}}{n+k} \binom{n+k}{2k} \binom{2k}{k-j} j^{2m}$$

By means of binomial identity $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$

$$n^{2m-1} = 2 \sum_{k=1}^{m} \sum_{j=0}^{k} \frac{(-1)^{k-j}}{n+k} \binom{n+k}{k-j} \binom{n+j}{k+j} j^{2m}$$