

BINOMIAL IDENTITIES

PETRO KOLOSOV

ABSTRACT. Binomial identities

CONTENTS

1. Binomial identities	1
1.1. Part 1	1
1.2. Part2	2
1.3. Part 3	3
1.4. Part 4	3

1. BINOMIAL IDENTITIES

1.1. Part 1.

$$\begin{aligned}\binom{n}{k} &= \frac{n!}{k!(n-k)!} = \frac{1}{k!} \frac{1 \cdot 2 \cdot 3 \cdots (n-k) \cdot (n-k+1) \cdots n}{1 \cdot 2 \cdot 3 \cdots (n-k)} \\ &= \frac{1}{k!} (n-k+1)(n-k+2)(n-k+3) \cdots n \\ &= \frac{1}{k!} n(n-1)(n-2) \cdots (n-k+1)\end{aligned}\tag{1.1}$$

$$= \frac{1}{k!} \prod_{j=1}^k (n-j+1) = \frac{1}{k!} \prod_{j=0}^{k-1} (n-j)$$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}\tag{1.2}$$

Date: July 23, 2023.

2010 Mathematics Subject Classification. 26E70, 05A30.

Key words and phrases. Polynomials, Polynomial identities, Faulhaber's formula, Central Factorial Numbers .

$$\begin{aligned}
\binom{n}{k} &= \frac{n^k}{k!} = \frac{1}{k!} n(n-1)(n-2) \cdots (n-k+1) \\
&= \frac{1}{k!} \prod_{j=1}^k (n-j+1) = \frac{1}{k!} \prod_{j=0}^{k-1} (n-j)
\end{aligned} \tag{1.3}$$

$$\sum_{r=0}^n \binom{r}{c} = \binom{n+1}{c+1} \tag{1.4}$$

$$\sum_{k=0}^n \binom{r+k}{k} = \binom{r+n+1}{n} \tag{1.5}$$

$$\sum_{k=0}^m \binom{n-k}{m-k} = \binom{n+1}{m} \tag{1.6}$$

$$\sum_{k=0}^n \binom{n-k}{k} = f_{n+1} \tag{1.7}$$

$$k \binom{n}{k} = n \binom{n-1}{k-1} \tag{1.8}$$

$$\binom{n}{m} \binom{m}{k} = \binom{n}{k} \binom{n-k}{m-k} \tag{1.9}$$

$$\sum_{j=0}^n \binom{n}{j} \binom{m}{k-j} = \binom{n+m}{k} \tag{1.10}$$

1.2. Part2.

$$k \binom{n}{k} = n \binom{n-1}{k-1} \tag{1.11}$$

$$\frac{k}{n} \binom{n}{k} = \binom{n-1}{k-1} \tag{1.12}$$

$$\frac{k+1}{n+1} \binom{n+1}{k+1} = \binom{n}{k} \tag{1.13}$$

$$\binom{n+1}{k+1} = \frac{n+1}{k+1} \binom{n}{k} \tag{1.14}$$

1.3. **Part 3.**

$$\binom{t}{r} \binom{r}{k} = \binom{t}{k} \binom{t-k}{r-k} \quad (1.15)$$

$$\binom{t}{k} \binom{t-k}{r-k} = \binom{t}{t-k} \binom{t-k}{r-k} = \binom{t}{r-k} \binom{t-r+k}{t-r} \quad (1.16)$$

By the symmetry of binomial coefficients we get

$$\binom{t}{k} \binom{t-k}{r-k} = \binom{t}{k} \binom{t-k}{t-r} \quad (1.17)$$

$$\binom{t}{k} \binom{t-k}{t-r} = \binom{t}{t-k} \binom{t-k}{t-r} \quad (1.18)$$

1.4. **Part 4.**

$$\binom{t}{r} \binom{r}{k} = \binom{t}{k} \binom{t-k}{r-k} = \binom{t}{t-k} \binom{t-k}{r-k} = \binom{t}{r-k} \binom{t-r+k}{t-r} = \binom{t}{r-k} \binom{t-r+k}{k} \quad (1.19)$$