

# POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

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ABSTRACT.

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## 1. FORMULAE

From OEIS, note that this is not Central factorial number itself, this formula is in the mathematica package as `OEISFormula`

$$T_{\text{OEIS}}(n, k) = \frac{1}{m} \sum_{j=0}^m (-1)^j \binom{2m}{j} (m-j)^{2n}$$

where  $m = n - k + 1$ . So that

$$T_{\text{OEIS}} = \frac{1}{n-k+1} \sum_{j=0}^{n-k+1} (-1)^j \binom{2n-2k+2}{j} (n-k+1-j)^{2n}$$

Also, OEIS sequence is defined by

$$T_{\text{OEIS}} = (2(n-k)+1)!T(2n, 2n-2k)$$

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where  $T(2n, 2n - 2k)$  are central factorial numbers. From stackoverflow, these are pure central factorial numbers already

$$k!T(n, k) = \sum_{j=0}^k \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

So that central factorial number is, this is the function `Central1(n,k)` in mathematica package

$$T(n, k) = \frac{1}{k!} \sum_{j=0}^k \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be  $(k-1)!T(n, k)$

$$(k-1)!T(n, k) = \frac{1}{k} \sum_{j=0}^k \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be  $(2k-1)!T(2n, 2k)$

$$(2k-1)!T(2n, 2k) = \frac{1}{2k} \sum_{j=0}^{2k} \binom{2k}{j} (-1)^j (k-j)^{2n}$$

Since that OEIS formula defined as

$$T(2n, 2n - 2k) = \frac{1}{(2n - 2k)!} \sum_{j=0}^{2n-2k} \binom{2n-2k}{j} (-1)^j (n-k-j)^n$$

$$T_{\text{OEIS}} = (2(n-k) + 1)!T(2n, 2n - 2k) = (2n - 2k + 1)!T(2n, 2n - 2k)$$

Then

$$T_{\text{OEIS}} = (2n - 2k + 1) \sum_{j=0}^{2n-2k} \binom{2n-2k}{j} (-1)^j (n-k-j)^{2n}$$

Above is `OEISFormula2` in mathematica package—it does not work.

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