BINOMIAL IDENTITIES

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ABSTRACT. Binomial identities

CONTENTS

1. BINOMIAL IDENTITIES

1.1. Part 1.

$$
\begin{aligned}
\binom{n}{k} = \frac{n!}{k!(n-k)!} &= \frac{1}{k!}\frac{1\cdot 2\cdot 3\cdots(n-k)\cdot(n-k+1)\cdots n}{1\cdot 2\cdot 3\cdots(n-k)} \\
&= \frac{1}{k!}(n-k+1)(n-k+2)(n-k+3)\cdots n \\
&= \frac{1}{k!}n(n-1)(n-2)\cdots(n-k+1) \\
&= \frac{1}{k!}\prod_{j=1}^{k}(n-j+1) = \frac{1}{k!}\prod_{j=0}^{k-1}(n-j)
\end{aligned}
\tag{1.1}
$$

$$
\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \tag{1.2}
$$

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$$\binom{n}{k} = \frac{n^{\underline{k}}}{k!} = \frac{1}{k!} n(n-1)(n-2)\cdots(n-k+1)$$
$$= \frac{1}{k!}\prod_{j=1}^{k}(n-j+1) = \frac{1}{k!}\prod_{j=0}^{k-1}(n-j) \tag{1.3}$$

$$\sum_{r=0}^{n}\binom{r}{c} = \binom{n+1}{c+1} \tag{1.4}$$

$$\sum_{k=0}^{n}\binom{r+k}{k} = \binom{r+n+1}{n} \tag{1.5}$$

$$\sum_{k=0}^{m}\binom{n-k}{m-k} = \binom{n+1}{m} \tag{1.6}$$

$$\sum_{k=0}^{n}\binom{n-k}{k} = f_{n+1} \tag{1.7}$$

$$k\binom{n}{k} = n\binom{n-1}{k-1} \tag{1.8}$$

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k} \tag{1.9}$$

$$\sum_{j=0}^{n}\binom{n}{j}\binom{m}{k-j} = \binom{n+m}{k} \tag{1.10}$$

1.2. **Part2.**

$$k\binom{n}{k} = n\binom{n-1}{k-1} \tag{1.11}$$

$$\frac{k}{n}\binom{n}{k} = \binom{n-1}{k-1} \tag{1.12}$$

$$\frac{k+1}{n+1}\binom{n+1}{k+1} = \binom{n}{k} \tag{1.13}$$

$$\binom{n+1}{k+1} = \frac{n+1}{k+1}\binom{n}{k} \tag{1.14}$$

1.3. Part 3.

$$\binom{t}{r}\binom{r}{k} = \binom{t}{k}\binom{t-k}{r-k} \tag{1.15}$$

$$\binom{t}{k}\binom{t-k}{r-k} = \binom{t}{t-k}\binom{t-k}{r-k} = \binom{t}{r-k}\binom{t-r+k}{t-r} \tag{1.16}$$

By the symmetry of binomial coefficients we get

$$\binom{t}{k}\binom{t-k}{r-k} = \binom{t}{k}\binom{t-k}{t-r} \tag{1.17}$$

$$\binom{t}{k}\binom{t-k}{t-r} = \binom{t}{t-k}\binom{t-k}{t-r} \tag{1.18}$$

1.4. Part 4.

$$\binom{t}{r}\binom{r}{k} = \binom{t}{k}\binom{t-k}{r-k} = \binom{t}{t-k}\binom{t-k}{r-k} = \binom{t}{r-k}\binom{t-r+k}{t-r} = \binom{t}{r-k}\binom{t-r+k}{k} \tag{1.19}$$