BINOMIAL IDENTITIES

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ABSTRACT. Binomial identities

CONTENTS

1. BINOMIAL IDENTITIES

1.1. **Part 1.**

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \tag{1.1}$$

$$\binom{n}{k} = \frac{n^{\underline{k}}}{k!} \tag{1.2}$$

$$\sum_{r=0}^{n} \binom{r}{c} = \binom{n+1}{c+1} \tag{1.3}$$

$$\sum_{k=0}^{n} \binom{r+k}{k} = \binom{r+n+1}{n} \tag{1.4}$$

$$\sum_{k=0}^{m} \binom{n-k}{m-k} = \binom{n+1}{m} \tag{1.5}$$

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$$\sum_{k=0}^{n} \binom{n-k}{k} = f_{n+1} \tag{1.6}$$

$$k\binom{n}{k} = n\binom{n-1}{k-1} \tag{1.7}$$

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k} \tag{1.8}$$

$$\sum_{j=0}^{n} \binom{n}{j}\binom{m}{k-j} = \binom{n+m}{k} \tag{1.9}$$

1.2. **Part2.**

$$k\binom{n}{k} = n\binom{n-1}{k-1} \tag{1.10}$$

$$\frac{k}{n}\binom{n}{k} = \binom{n-1}{k-1} \tag{1.11}$$

$$\frac{k+1}{n+1}\binom{n+1}{k+1} = \binom{n}{k} \tag{1.12}$$

$$\binom{n+1}{k+1} = \frac{n+1}{k+1}\binom{n}{k} \tag{1.13}$$

1.3. **Part 3.**

$$\binom{t}{r}\binom{r}{k} = \binom{t}{k}\binom{t-k}{r-k} \tag{1.14}$$

$$\binom{t}{k}\binom{t-k}{r-k} = \binom{t}{t-k}\binom{t-k}{r-k} = \binom{t}{r-k}\binom{t-r+k}{t-r} \tag{1.15}$$

By the symmetry of binomial coefficients we get

$$\binom{t}{k}\binom{t-k}{r-k} = \binom{t}{k}\binom{t-k}{t-r} \tag{1.16}$$

$$\binom{t}{k}\binom{t-k}{t-r} = \binom{t}{t-k}\binom{t-k}{t-r} \tag{1.17}$$

1.4. **Part 4.**

$$\binom{t}{r}\binom{r}{k} = \binom{t}{k}\binom{t-k}{r-k} = \binom{t}{t-k}\binom{t-k}{r-k} = \binom{t}{r-k}\binom{t-r+k}{t-r} = \binom{t}{r-k}\binom{t-r+k}{k} \tag{1.18}$$