POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

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Abstract.

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1. Formulae

From OEIS, note that this is not Central factorial number itself, this formula is in the mathematica package as `OEISFormula`

$$T_{\text{OEIS}}(n,k) = \frac{1}{m}\sum_{j=0}^{m}(-1)^j\binom{2m}{j}(m-j)^{2n}$$

where $m = n-k+1$. So that

$$\begin{aligned}T_{\text{OEIS}}(n,k) &= \frac{1}{n-k+1}\sum_{j=0}^{n-k+1}(-1)^j\binom{2(n-k+1)}{j}([n-k+1]-j)^{2n}\\ T_{\text{OEIS}}(n,k) &= \frac{1}{n-k+1}\sum_{j=0}^{n-k+1}(-1)^j\binom{2n-2k+2}{j}(n-k+1-j)^{2n}\end{aligned} \tag{1.1}$$

Furthermore, T_{OEIS} may be turned into changing the summation order from $n-k+1$ to k

$$T_{\text{OEIS}}(n,n-k) = \frac{1}{k}\sum_{j=0}^{k}(-1)^j\binom{2k}{j}(k-j)^{2n}$$

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1.1. **Central factorials.** Central factorials

$$x^{[n]} = x\left(x + \frac{n}{2} - 1\right)^{\underline{n-1}} \quad (CentralFactorial1)$$

$$x^{[n]} = x\left(x + \frac{n}{2} - 1\right)\left(x + \frac{n}{2} - 1\right)\cdots\left(x + \frac{n}{2} - 1\right)$$

$$x^{[n]} = x\prod_{k=1}^{n-1}\left(x + \frac{n}{2} - k\right)$$

Then we have power identity given by Knuth

$$x^m = \sum_{k=1}^{m} T(m,k)x^{[k]} \quad (PowerIdentity1)$$

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