

# POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

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ABSTRACT.

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## 1. FORMULAE

From OEIS, note that this is not Central factorial number itself, this formula is in the mathematica package as `OEISFormula`

$$T_{\text{OEIS}}(n, k) = \frac{1}{m} \sum_{j=0}^m (-1)^j \binom{2m}{j} (m-j)^{2n}$$

where  $m = n - k + 1$ . So that

$$\begin{aligned} T_{\text{OEIS}}(n, k) &= \frac{1}{n-k+1} \sum_{j=0}^{n-k+1} (-1)^j \binom{2(n-k+1)}{j} ([n-k+1] - j)^{2n} \\ T_{\text{OEIS}}(n, k) &= \frac{1}{n-k+1} \sum_{j=0}^{n-k+1} (-1)^j \binom{2n-2k+2}{j} (n-k+1-j)^{2n} \end{aligned} \tag{1.1}$$

Furthermore,  $T_{\text{OEIS}}$  may be turned into changing the summation order from  $n - k + 1$  to  $k$

$$T_{\text{OEIS}}(n, n-k) = \frac{1}{k} \sum_{j=0}^k (-1)^j \binom{2k}{j} (k-j)^{2n}$$

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### 1.1. Central factorials. Central factorials

$$x^{[n]} = x \left( x + \frac{n}{2} - 1 \right)^{\overline{n-1}} \quad (CentralFactorial1)$$

$$x^{[n]} = x \left( x + \frac{n}{2} - 1 \right) \left( x + \frac{n}{2} - 1 \right) \cdots \left( x + \frac{n}{2} - 1 \right)$$

$$x^{[n]} = x \prod_{k=1}^{n-1} \left( x + \frac{n}{2} - k \right)$$

Then we have power identity given by Knuth

$$x^m = \sum_{k=1}^m T(m, k) x^{[k]} \quad (PowerIdentity1)$$

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