# DEFINITIONS

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ABSTRACT. Definitions

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## 1. DEFINITIONS

- $T(n,k)$ recursively defines central factorial numbers of the second kind (in the context of Knuth and Riordan (see references)). It is defined in mathematica package as `CentralFactorialNumber1`

$$\begin{cases} T(n,1) &= 1 \\ T(n,n) &= 1 \\ T(n,k) &= T(n-1,k-1) + k^2 T(n-1,k) \end{cases}$$

- $T(n,k)$ is central factorial number defined as `CentralFactorialNumber2` in mathematica package

$$T(n,k) = \frac{1}{k!} \sum_{j=0}^{k} \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

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- Identity in central factorial numbers defined as `CFNIdentity1` in mathematica package

$$(2k-1)!T(2n,2k) = \frac{1}{k}\sum_{j=0}^{k}(-1)^j\binom{2k}{j}(k-j)^{2n}$$

- Identity in central factorial numbers defined as `CFNIdentity2` in mathematica package

$$(2k-1)!T(2n,2k) = \frac{1}{k}\sum_{j=0}^{k}(-1)^{k-j}\binom{2k}{k-j}j^{2n}$$

- Identity in central factorial numbers defined as `CFNIdentity3` in mathematica package

$$(2k-1)!T(2n,2k) = \frac{1}{2k}\sum_{j=0}^{2k}(-1)^j\binom{2k}{j}(k-j)^{2n}$$

- Identity in odd power polynomial

$$n^{2m-1} = \sum_{k=1}^{m}(2k-1)!T(2m,2k)\binom{n+k-1}{2k-1}$$

It is defined as `OddPowerIdentity1` in mathematica package

$$n^{2m-1} = \sum_{k=1}^{m}\texttt{CFNIdentity1}(m,k)\binom{n+k-1}{2k-1}$$

It is defined as `OddPowerIdentity2` in mathematica package

$$n^{2m-1} = \sum_{k=1}^{m}\texttt{CFNIdentity2}(m,k)\binom{n+k-1}{2k-1}$$

It is defined as `OddPowerIdentity3` in mathematica package

$$n^{2m-1} = \sum_{k=1}^{m}\texttt{CFNIdentity3}(m,k)\binom{n+k-1}{2k-1}$$

- Polynomial identity in odd powers defined as `OddPowerIdentity21`

$$n^{2m-1} = \sum_{k=1}^{m}\sum_{j=0}^{k}\frac{(-1)^{k-j}}{k}\binom{n+k-1}{2k-1}\binom{2k}{k-j}j^{2m}$$

- Polynomial identity in odd powers defined as `OddPowerIdentity22`

$$n^{2m-1} = \sum_{k=1}^{m}\sum_{j=0}^{k}\frac{(-1)^{k-j}}{k}\frac{2k}{n+k}\binom{n+k}{2k}\binom{2k}{k-j}j^{2m}$$