

BINOMIAL IDENTITIES

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ABSTRACT. Binomial identities

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1. BINOMIAL IDENTITIES

1.1. Part 1.

$$\begin{aligned}\binom{n}{k} &= \binom{n-1}{k} + \binom{n-1}{k-1} \\ \binom{n}{k} &= \frac{n^{\underline{k}}}{k!} \\ \sum_{r=0}^n \binom{r}{c} &= \binom{n+1}{c+1} \\ \sum_{k=0}^n \binom{r+k}{k} &= \binom{r+n+1}{n} \\ \sum_{k=0}^m \binom{n-k}{m-k} &= \binom{n+1}{m}\end{aligned}$$

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$$\begin{aligned}
\sum_{k=0}^n \binom{n-k}{k} &= f_{n+1} \\
k \binom{n}{k} &= n \binom{n-1}{k-1} \\
\binom{n}{m} \binom{m}{k} &= \binom{n}{k} \binom{n-k}{m-k} \\
\sum_{j=0}^n \binom{n}{j} \binom{m}{k-j} &= \binom{n+m}{k}
\end{aligned}$$

1.2. Part2.

$$\begin{aligned}
k \binom{n}{k} &= n \binom{n-1}{k-1} \\
\frac{k}{n} \binom{n}{k} &= \binom{n-1}{k-1} \\
\frac{k+1}{n+1} \binom{n+1}{k+1} &= \binom{n}{k} \\
\binom{n+1}{k+1} &= \frac{n+1}{k+1} \binom{n}{k}
\end{aligned}$$

1.3. Part 3.

$$\begin{aligned}
\binom{t}{r} \binom{r}{k} &= \binom{t}{k} \binom{t-k}{r-k} = \binom{t}{k} \binom{t-k}{t-r} = \binom{t}{t-k} \binom{t-k}{t-r} \\
\binom{t}{r} \binom{r}{k} &= \binom{t}{k} \binom{t-k}{r-k} = \binom{t}{t-k} \binom{t-k}{r-k} = \binom{t}{r-k} \binom{t-r+k}{t-r} = \binom{t}{r-k} \binom{t-r+k}{k}
\end{aligned}$$