POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

PETRO KOLOSOV

ABSTRACT.

CONTENTS

1. FORMULAE

From OEIS, note that this is not Central factorial number itself, this formula is in the mathematica package as `OEISFormula`

$$T_{\text{OEIS}}(n,k) = \frac{1}{m} \sum_{j=0}^{m} (-1)^j \binom{2m}{j} (m-j)^{2n}$$

where $m = n - k + 1$. So that

$$T_{\text{OEIS}}(n,k) = \frac{1}{n-k+1} \sum_{j=0}^{n-k+1} (-1)^j \binom{2(n-k+1)}{j} ([n-k+1]-j)^{2n}$$

$$(1.1)$$

$$T_{\text{OEIS}}(n,k) = \frac{1}{n-k+1} \sum_{j=0}^{n-k+1} (-1)^j \binom{2n-2k+2}{j} (n-k+1-j)^{2n}$$

Furthermore, T_{OEIS} may be turned into changing the summation order from $n - k + 1$ to k

$$T_{\text{OEIS}}(n, n-k) = \frac{1}{k} \sum_{j=0}^{k} (-1)^j \binom{2k}{j} (k-j)^{2n}$$

1.1. **Central factorials.** Central factorials

$$x^{[n]} = x \left(x + \frac{n}{2} - 1 \right)^{\overline{n-1}} \quad (Central Factorial1)$$

$$x^{[n]} = x \left(x + \frac{n}{2} - 1 \right) \left(x + \frac{n}{2} - 1 \right) \cdots \left(x + \frac{n}{2} - 1 \right)$$

$$x^{[n]} = x \prod_{k=1}^{n-1} \left(x + \frac{n}{2} - k \right)$$

Then we have power identity given by Knuth

$$x^m = \sum_{k=1}^{m} T(m,k) x^{[k]} \quad (Power Identity1)$$

Email address: kolosovp94@gmail.com

URL: https://kolosovpetro.github.io