POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

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Abstract.

Contents

1. Formulae

From OEIS, note that this is not Central factorial number itself, this formula is in the mathematica package as `OEISFormula`

$$T_{\text{OEIS}}(n,k) = \frac{1}{m}\sum_{j=0}^{m}(-1)^j\binom{2m}{j}(m-j)^{2n}$$

where $m = n-k+1$. So that

$$T_{\text{OEIS}} = \frac{1}{n-k+1}\sum_{j=0}^{n-k+1}(-1)^j\binom{2n-2k+2}{j}(n-k+1-j)^{2n}$$

Also, OEIS sequence is defined by

$$T_{\text{OEIS}} = (2(n-k)+1)!T(2n, 2n-2k)$$

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where $T(2n, 2n-2k)$ are central factorial numbers. From stackoverflow, these are pure central factorial numbers already

$$k!T(n,k) = \sum_{j=0} \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

So that central factorial number is, this is the function `Central1(n,k)` in mathematica package

$$T(n,k) = \frac{1}{k!} \sum_{j=0} \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be $(k-1)!T(n,k)$

$$(k-1)!T(n,k) = \frac{1}{k} \sum_{j=0} \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be $(2k-1)!T(2n,2k)$

$$(2k-1)!T(2n,2k) = \frac{1}{2k} \sum_{j=0} \binom{2k}{j} (-1)^j (k-j)^{2n}$$

Since that OEIS formula defined as

$$T(2n, 2n-2k) = \frac{1}{(2n-2k)!} \sum_{j=0} \binom{2n-2k}{j} (-1)^j \left(n-k-j\right)^n$$

$$T_{\text{OEIS}} = (2(n-k)+1)!T(2n, 2n-2k) = (2n-2k+1)!T(2n, 2n-2k)$$

Then

$$T_{\text{OEIS}} = (2n-2k+1) \sum_{j=0} \binom{2n-2k}{j} (-1)^j (n-k-j)^{2n}$$

Above is `OEISFormula2` in mathematica package

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