

# DEFINITIONS

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ABSTRACT. Definitions

## CONTENTS

1. Definitions	1
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### 1. DEFINITIONS

- $T(n, k)$  recursively defines central factorial numbers of the second kind (in the context of Knuth and Riordan (see references)). It is defined in mathematica package as

`CentralFactorialNumber1`

$$\left\{ \begin{array}{l} T(n, 1) = 1 \\ T(n, n) = 1 \\ T(n, k) = T(n-1, k-1) + k^2 T(n-1, k) \end{array} \right.$$

- $T(n, k)$  is central factorial number defined as `CentralFactorialNumber2` in mathematica package

$$T(n, k) = \frac{1}{k!} \sum_{j=0}^k \binom{k}{j} (-1)^j \left( \frac{1}{2}k - j \right)^n$$

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- Identity in central factorial numbers defined as `CFNIdentity1` in mathematica package

$$(2k-1)!T(2n, 2k) = \frac{1}{k} \sum_{j=0}^k (-1)^j \binom{2k}{j} (k-j)^{2n}$$

- Identity in central factorial numbers defined as `CFNIdentity2` in mathematica package

$$(2k-1)!T(2n, 2k) = \frac{1}{k} \sum_{j=0}^k (-1)^{k-j} \binom{2k}{k-j} j^{2n}$$

- Identity in central factorial numbers defined as `CFNIdentity3` in mathematica package

$$(2k-1)!T(2n, 2k) = \frac{1}{2k} \sum_{j=0}^{2k} (-1)^j \binom{2k}{j} (k-j)^{2n}$$

- Identity in odd power polynomial

$$n^{2m-1} = \sum_{k=1}^m (2k-1)!T(2m, 2k) \binom{n+k-1}{2k-1}$$

It is defined as `OddPowerIdentity1` in mathematica package

$$n^{2m-1} = \sum_{k=1}^m \text{CFNIdentity1}(m, k) \binom{n+k-1}{2k-1}$$

It is defined as `OddPowerIdentity2` in mathematica package

$$n^{2m-1} = \sum_{k=1}^m \text{CFNIdentity2}(m, k) \binom{n+k-1}{2k-1}$$

It is defined as `OddPowerIdentity3` in mathematica package

$$n^{2m-1} = \sum_{k=1}^m \text{CFNIdentity3}(m, k) \binom{n+k-1}{2k-1}$$

- Polynomial identity in odd powers defined as `OddPowerIdentity21`

$$n^{2m-1} = \sum_{k=1}^m \sum_{j=0}^k \frac{(-1)^{k-j}}{k} \binom{n+k-1}{2k-1} \binom{2k}{k-j} j^{2m}$$

- Polynomial identity in odd powers defined as `OddPowerIdentity22`

$$n^{2m-1} = \sum_{k=1}^m \sum_{j=0}^k \frac{(-1)^{k-j}}{k} \frac{2k}{n+k} \binom{n+k}{2k} \binom{2k}{k-j} j^{2m}$$