POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

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Abstract. Central factorial numbers often appear in literature, for instance, in Riordan's "Combinatorial identities", D. E. Knuth's work entitled "Johann Faulhaber and Sums of Powers" and many others. In this manuscript, we start our discussion from definition of central factorial numbers (recursive and iterative), continuing with a set of identities used further in the manuscript. Then, based on odd power identities given by D. E. Knuth, we show other variations of these identities rewriting them applying derived previously identities in terms of central factorial numbers. Finally, we provide a comprehensive way to validate the results of the manuscript via supplementary Mathematica programs.

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1. Introduction

Central factorial numbers quietly often appear in literature, like for instance, in Riordan's Combinatorial identities [1]

$$T(n,k) = \frac{1}{k!}\sum_{j=0}^{k}\binom{k}{j}(-1)^j\left(\frac{1}{2}k-j\right)^n$$

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Also, the book [2] references central factorial numbers as

$$K_{rs} = \frac{1}{(2s)!} \sum_{t=0}^{2s} (-1)^t \binom{2s}{t} (s-t)^{2r+2}$$

D. E. Knuth gives the following recurrence for the central factorial numbers [3]

$$T(2m+2, 2k) = k^2 T(2m, 2k) + T(2m, 2k-2)$$

In The On-Line Encyclopedia of Integer Sequences, central factorial numbers appear to be defined via the following recurrence

$$\begin{cases} T(n,1) & = 1 \\ T(n,n) & = 1 \\ T(n,k) & = T(n-1,k-1) + k^2 T(n-1,k) \end{cases}$$

It is important to note that central factorial numbers are closely related to the central difference operator δ, Newton interpolation formula and central factorials $x^{[n]}$ and could be derived respectively. The derivation of central factorial numbers by means of central difference operator δ, Newton interpolation formula and central factorials $x^{[n]}$ is shown at [4].

2. Conclusions

Conclusions of your manuscript.

References

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