POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

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Abstract.

Contents

1. Formulae

From OEIS, note that this is not Central factorial number itself, this formula is in the mathematica package as `OEISFormula`

$$T_{\text{OEIS}}(n,k) = \frac{1}{m}\sum_{j=0}^{m}(-1)^j\binom{2m}{j}(m-j)^{2n}$$

where $m = n-k+1$. So that

$$\begin{aligned} T_{\text{OEIS}}(n,k) &= \frac{1}{n-k+1}\sum_{j=0}^{n-k+1}(-1)^j\binom{2(n-k+1)}{j}([n-k+1]-j)^{2n} \\ T_{\text{OEIS}}(n,k) &= \frac{1}{n-k+1}\sum_{j=0}^{n-k+1}(-1)^j\binom{2n-2k+2}{j}(n-k+1-j)^{2n} \end{aligned} \tag{1.1}$$

Furthermore, T_{OEIS} may be turned into changing the summation order from $n-k+1$ to k

$$T_{\text{OEIS}}(n,n-k) = \frac{1}{k}\sum_{j=0}^{k}(-1)^j\binom{2k}{j}(k-j)^{2n}$$

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Also, OEIS sequence is defined by

$$T_{\text{OEIS}} = (2(n-k)+1)!T(2n, 2n-2k)$$

where $T(2n, 2n-2k)$ are central factorial numbers. From stackoverflow, these are pure central factorial numbers already

$$k!T(n,k) = \sum_{j=0} \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

So that central factorial number is, this is the function `Central1(n,k)` in mathematica package and it is true and holds in mathematica program

$$T(n,k) = \frac{1}{k!} \sum_{j=0} \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be $(k-1)!T(n,k)$

$$(k-1)!T(n,k) = \frac{1}{k} \sum_{j=0} \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be $(2k-1)!T(2n,2k)$ in is true and checked in mathematica as `KnuthCoefficient2`

$$(2k-1)! \cdot T(2n,2k) = \frac{1}{2k} \sum_{j=0}^{2k} \binom{2k}{j} (-1)^j (k-j)^{2n}$$

Let be $(2k-1)!T(2n,2k)$ in is true and checked in mathematica as `KnuthCoefficient3`

$$(2k-1)! \cdot T(2n,2k) = \frac{1}{k} \sum_{j=0}^{k} \binom{2k}{j} (-1)^j (k-j)^{2n}$$

Let be $(2k-1)!T(2n,2k)$ in is true and checked in mathematica as `KnuthCoefficient4`

$$(2k-1)! \cdot T(2n,2k) = \frac{1}{k} \sum_{j=0}^{k} \binom{2k}{k-j} (-1)^{k-j} j^{2n}$$

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