

# POLYNOMIAL IDENTITIES INVOLVING CENTRAL FACTORIAL NUMBERS

PETRO KOLOSOV

ABSTRACT.

## CONTENTS

1. Formulae	1
-------------	---

## 1. FORMULAE

From OEIS, note that this is not Central factorial number itself, this formula is in the mathematica package as `OEISFormula`

$$T_{\text{OEIS}}(n, k) = \frac{1}{m} \sum_{j=0}^m (-1)^j \binom{2m}{j} (m-j)^{2n}$$

where  $m = n - k + 1$ . So that

$$\begin{aligned} T_{\text{OEIS}}(n, k) &= \frac{1}{n-k+1} \sum_{j=0}^{n-k+1} (-1)^j \binom{2(n-k+1)}{j} ([n-k+1] - j)^{2n} \\ T_{\text{OEIS}}(n, k) &= \frac{1}{n-k+1} \sum_{j=0}^{n-k+1} (-1)^j \binom{2n-2k+2}{j} (n-k+1-j)^{2n} \end{aligned} \tag{1.1}$$

Furthermore,  $T_{\text{OEIS}}$  may be turned into changing the summation order from  $n - k + 1$  to  $k$

$$T_{\text{OEIS}}(n, n-k) = \frac{1}{k} \sum_{j=0}^k (-1)^j \binom{2k}{j} (k-j)^{2n}$$

---

*Date:* July 19, 2023.

2010 *Mathematics Subject Classification.* 26E70, 05A30.

*Key words and phrases.* Polynomials, Polynomial identities, Faulhaber's formula, Cental Factorial Numbers .

Also, OEIS sequence is defined by

$$T_{\text{OEIS}} = (2(n - k) + 1)!T(2n, 2n - 2k)$$

where  $T(2n, 2n - 2k)$  are central factorial numbers. From stackoverflow, these are pure central factorial numbers already

$$k!T(n, k) = \sum_{j=0}^k \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

So that central factorial number is, this is the function `Central1(n,k)` in mathematica package and it is true and holds in mathematica program

$$T(n, k) = \frac{1}{k!} \sum_{j=0}^k \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be  $(k - 1)!T(n, k)$

$$(k - 1)!T(n, k) = \frac{1}{k} \sum_{j=0}^k \binom{k}{j} (-1)^j \left(\frac{1}{2}k - j\right)^n$$

Let be  $(2k - 1)!T(2n, 2k)$  in is true and checked in mathematica as `KnuthCoefficient2`

$$\begin{aligned} (2k - 1)! \cdot T(2n, 2k) &= \frac{1}{2k} \sum_{j=0}^{2k} \binom{2k}{j} (-1)^j (k - j)^{2n} \\ &= \frac{1}{k} \sum_{j=0}^k \binom{2k}{j} (-1)^j (k - j)^{2n} \end{aligned}$$

*Email address:* kolosovp94@gmail.com

*URL:* <https://kolosovpetro.github.io>